Generating Function and Others.

- Introduction & Validation.

- OGF, EGF, PGF & DGF
  - OGF & Root of Unity Filter

  - EGF
    - cf 891E
  - PGF

  - DGF & Möbius
    - Prove for Möbius Inversion.
    - PE 639.

- Multi Generating Function.
  - IMO '95/6
  - USAMO '86/5
  - * Finitization.

- My reverie

Problem 1 [Zeitz]

$$D(x)^2 = x^2(x+1)^2(x^2-x+1)^2(x^2+x+1)$$

$$A(x) = \sum_{i\geq 0} a_i \cdot x^i \qquad B(x) = \sum_{i\geq 0} b_i \cdot x^i$$

① $F(x,y) = \sum_{n,k\geq 0} f_{n,k}\, x^n y^k$

$$= (1+xy+x^2y^2+x^3y^3+\cdots)(1+x^2+x^4+\cdots)(1+x^3+x^6+\cdots)\cdots$$

$$= \frac{1}{(1-xy)(1-x^2)(1-x^3)\cdots}$$

$a_n = f_{n,1} + 2f_{n,2} + 3f_{n,3} + \cdots$

$$A(x) = \left.\frac{\partial F(x,y)}{\partial y}\right|_{y=1} = \left.\sum_{n,k\geq 0} f_{n,k}\cdot k \cdot x^n y^{k-1}\right|_{y=1} = \sum_{n,k\geq 0} k\cdot f_{n,k}\cdot x^n$$

$$\to \frac{x}{(1-x)^2(1-x^2)(1-x^3)(1-x^4)\cdots}$$

② $G(x,y) = \sum_{n,k\geq 0} g_{n,k}\, x^n \cdot y^k$

$$= (1+xy+x^2y+x^3y+\cdots)(1+x^2y+x^4y+\cdots)\cdots$$

$$= \left(1+\frac{xy}{1-x}\right)\left(1+\frac{x^2y}{1-x^2}\right)\left(1+\frac{x^3y}{1-x^3}\right)\cdots$$

$b_n = g_{n,1} + 2\cdot g_{n,2} + 3\cdot g_{n,3} + 4\cdot g_{n,4} + \cdots$

$$B(x) = \left.\frac{\partial G(x,y)}{\partial y}\right|_{y=1} = \underline{f(x)} \quad f(x^2) \quad f(x^3)$$

$$H = h_1 \cdot h_2 \cdot h_3$$

$$\frac{dH}{dx} = h_1 h_2 \cdot \frac{dh_3}{dx} + h_1 \cdot h_3 \cdot \frac{dh_2}{dx} + h_2 h_3 \cdot \frac{dh_1}{dx}$$

$$= H\left(\frac{dh_1}{dx} \cdot \frac{1}{h_1} + \frac{dh_2}{dx} \cdot \frac{1}{h_2} + \frac{dh_3}{dx} \cdot \frac{1}{h_3}\right)$$

$$G(x,y) \cdot \left[\ \_\_\_ \cdot \_\_\_ \ \_\_\_ \ \cdots \right.$$

$$B(x) = \frac{x}{(1-x)^2 (1-x^2)(1-x^3) \cdots} = A(x)$$