

## Final Project: Infrastructure interdependency analysis and sustainable design of electricity utilities

Due: Friday 12/14/18 11:59 pm

Version 1.0

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### Version history

- **Version 1.** Initial release. Disclaimer: We have done our best to make a realistic integrated design problem that will help you understand how mathematical programming can be used to solve practical and complex infrastructure systems design and analysis problems. There will inevitably be corrections to the problem statement based on questions you raise while working through the project. When revisions to the project description are required, we will make the appropriate changes and summarize them here.

## 1 Project overview

In this group (three people max) project, you will design an efficient operating strategy for an electric utility with several plants available to generate power, and you will also explore the dependencies between electricity generation infrastructure and other critical infrastructure assets.

In the first part of the project you will analyze the interdependencies between the electric utility and other critical infrastructure. You will use an *input-output* model to evaluate potential cascade effects that a failure in one infrastructure causes on the system (i.e., on the other infrastructure and on itself due to the presence of feedback loops).

In the second part of the project, you will design the most efficient way to operate the electric utility. To meet the energy demand, you also have the possibility to design several new plants, or add a demand side management plan, which may help you meet your efficiency objectives. When designing the operating plan for the utility, we are interested in achieving both a low cost supply of electricity with a minimal global warming potential. The final mix of the energy from the various plants is known as the *energy portfolio*.

Most questions in Part 1 and part 2 can be explored in parallel, so don't spend too much time working on one aspect without saving time for the other. You should have all the tools needed to answer part 1 already, while part 2 will require some additional background on optimization, which will be introduced soon.

Note this project is inspired by several case studies in the literature [1, 2, 3, 4], which may prove to be useful references for further work on the project.

A completed project will consist of several elements:

- A **final report** which contains responses to the questions in this project description. The report should be self contained, and typeset in word or latex. Any necessary calculations should be explained in the report and detailed in an appendix. All assumptions you make in your calculations should be highlighted and justified with the appropriate references as needed.
- A copy of any **Python code** required for the problems, and a detailed explanation on how to run the codes. As a rule of thumb, your code should be sufficiently well documented so that another group could easily run and modify your code if asked (but do not share it outside your group!). You may attach your description on how to run the code, and the code as an appendix to the report.
- A ten-minute progress report **presentation** in class on **Wednesday, November 14**. You can use this time to summarize your progress to date, as well as to explain a problem you are having difficulty solving, or a problem you believe you have solved correctly. This is time to share ideas and get input from the instructors and rest of the class to help you complete the project.
- A twenty-five minute (maximum) Powerpoint **presentation** which you will use to present your results to the class on **Thursday, December 6**. Your final written report will be due on **Friday, December 14 11:59PM CDT**, in order to give you time to make any necessary changes following the presentation. This is a hard deadline, and no extensions will be granted.
- A single, signed **statement of contributions** of each member of the group, as well as an estimate of the percentage work contribution to the project. Each member of the group must sign off on the statement. The first statement of contributions is due on **Wednesday, November 14** in class, and the second statement is due with the final project on the **Friday, December 14** deadline. Projects without signatures by all team members will not be graded.

*Please note that copying code and/or solutions without proper attribution to the original authors is considered plagiarism. All infractions of the academic code will be filed with the School of Engineering for subsequent disciplinary action, including but not limited to a failing grade in this course.* Code and document plagiarism checkers may be used to verify the authenticity of your submitted source code and your report. All group members are fully responsible for the authenticity of the entire project and all deliverables.

## 2 Infrastructure interdependency analysis

The following *input-output interdependency analysis* is based on the works of Setola et al. [3] and Haimen and Jiang [4], which may prove useful references as you answer questions related to this section.

A electric utility can be defined as *critical infrastructure* in the sense that it provides an essential service for the functioning of a city or a country. Other critical infrastructure includes wired and wireless telecommunications, rail transportation, the petroleum network, natural gas, and satellite communications, to name a few. The failure of a critical infrastructure asset can have an important impact not only in its own operation, but also in the functioning of other infrastructure, since different infrastructure assets interact with one another at various levels in order to provide goods and services efficiently and cost-effectively.

These interactions increase infrastructure dependencies. In principle, *direct dependency* mechanisms are relatively easy to identify and analyze. Practically, even direct dependencies may go unnoticed until after a major event. An example of an infrastructure dependency is the failure of the Galaxy IV satellite system in 1998, which degraded US telecommunication services, leading to cascading effects in other infrastructure assets. As a result of the satellite failure, about 40 million pagers(!) stopped working, more than 20 United Airlines flights were delayed due to the lack of high altitude weather, and interestingly, the road

transportation infrastructure was also affected because highway refueling stations were unable to process credit cards as their satellite links were down. This illustrates the significance of dependencies between critical infrastructure and the complexity of analyzing the impacts of infrastructure outages.

In the case of multiple, large-scale critical infrastructure assets, direct dependencies between elements form loops and give rise to mutual dependencies known as *interdependencies*. An example of an infrastructure interdependency is as follows. The electrical power system depends on the delivery of fuels to power generating stations through transportation services, the production of those fuels depends in turn on the use of electrical power, and those fuels are needed by the transportation services<sup>1</sup>.

The *inoperability input-output model* (IIM) provides a framework for dependency analysis and allows one to model how a given amount of inoperability in one entity (inability of the entity to perform its intended function) affects the operability of other entities, i.e., the spread of inoperability in a networked system.

Mathematically the IMM is identical to the EIO-LCA framework with which you are already familiar, but the story is slightly different. Let  $x \in R^n$  denote the vector of degradation of  $n$  elements of infrastructure, and let  $x_i$  denote the total level of degradation of the  $i^{th}$  infrastructure. We adopt the notation that  $x_i = 0$  is perfectly operating infrastructure, and  $x_i = 1$  represents completely failed infrastructure. We let  $f_i$  denote an *external shock* to the system (e.g., direct damage as a result of an earthquake) which causes an immediate degradation of the  $i^{th}$  infrastructure. However, because infrastructure  $i$  depends also on other infrastructure, which itself may suffer a loss of performance due to the damage of  $i$ , we have multiple effects that contribute to the total level of degradation. We denote the degradation of infrastructure  $i$  caused by degradation in infrastructure  $j$  as  $x_{ij}$ , and let  $o_i = \sum_j x_{ij}$  be the cumulative damage to  $i$  caused by damage in infrastructure  $j$ .

This gives rise to the following equation:

$$x_i = o_i + f_i = \sum_j x_{ij} + f_i, \quad (1)$$

Equation (1) says the total level of degradation to infrastructure  $i$  is caused by two parts. The first part is the initial degradation caused by an external disturbance or event  $f_i$  (e.g., an earthquake). The second part that contributes to degradation is the cumulative effects  $\sum_j x_{ij}$  of degradation to  $i$  caused by all other infrastructure degradation.

Next, we make an assumption that the degradation of infrastructure  $i$  caused by degradation to infrastructure  $j$  is proportional to the total level of degradation of infrastructure  $j$ :

$$x_{ij} = a_{ij}x_j, \quad (2)$$

where  $a_{ij}$  is the proportionality constant that tells how failures in  $j$  propagate to failures in  $i$ . In other words,  $a_{ij}$  represents the fraction of inoperability transmitted by the  $j^{th}$  infrastructure to the  $i^{th}$  infrastructure. Note that  $a_{ij}$  express the direct influence exerted by infrastructure  $j$  on infrastructure  $i$ . For instance, if infrastructure  $j$  is completely inoperable (i.e.,  $x_j = 1$ ), and  $a_{ij} = 0.25$ , then infrastructure  $i$  will be degraded to  $a_{ij}x_j = 0.25 \times 1 = 0.25$ . As a result of (1) and (2) (you will show this in the questions below), we obtain the matrix model:

$$x = Ax + f, \quad (3)$$

where  $x \in R^n$ , and  $f \in R^n$  are column vectors, and  $A \in R^{n \times n}$  is the matrix of influence coefficients  $a_{ij}$ .

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<sup>1</sup>This example comes directly from the US National Science Foundation's *Critical Resilient Interdependent Infrastructure Systems* (CRISP) program.

Id	Sector
1	air transportation
2	electricity
3	wireless telecommunications (TLC wireless)
4	wired telecommunications (TLC wired)
5	water management
6	rail transportation
7	finance
8	fuel & petroleum grid
9	natural gas
10	naval ports
11	satellite communication & navigation

Table 1: Critical infrastructure sectors. Source: Setola et al (2009).

Sector Id	1	2	3	4	5	6	7	8	9	10	11
1	0	0.134	0.308	0.456	0.033	0.024	0.012	0.024	0.007	0.001	0.310
2	0	0	0.010	0.023	0.001	0.003	0.000	0.002	0.178	0.008	0.004
3	0.002	0.109	0	0.120	0.002	0.005	0.002	0.002	0.004	0.002	0.007
4	0.006	0.083	0.013	0	0.004	0.003	0.004	0.001	0.004	0.002	0.005
5	0.005	0.050	0.009	0.020	0	0.005	0.008	0.008	0.008	0.005	0.020
6	0.001	0.233	0.104	0.109	0.005	0	0.007	0.006	0.001	0.002	0.004
7	0.003	0.100	0.030	0.100	0.007	0.003	0	0.003	0.007	0.003	0.008
8	0.008	0.500	0.100	0.050	0.050	0.020	0.020	0	0	0.020	0.008
9	0.002	0.030	0.009	0.005	0.005	0.000	0.002	0.005	0	0.005	0.005
10	0.005	0.030	0.020	0.020	0.030	0.030	0.020	0.010	0.020	0	0.005
11	0	0	0	0	0	0	0	0	0	0	0

Table 2: Matrix  $A$  for a 6-12 hr outage. Modified from Setola et al (2009).

Following Haimes and Jiang [4], we assume that the diagonal elements of  $A$  are equal to zero, i.e.,  $a_{ii} = 0$  for all  $i$ . In doing so we exclude from the system the possibility that degradation of infrastructure  $i$  causes additional degradation in infrastructure  $i$ .

For each infrastructure we can define the *dependency index*  $\kappa_i$  and the *influence index*  $\lambda_j$  as follows [3]:

$$\kappa_i = \frac{1}{n-1} \sum_{j \neq i} a_{ij} \text{ (row summation)} \quad (4)$$

$$\lambda_j = \frac{1}{n-1} \sum_{i \neq j} a_{ij} \text{ (column summation)} \quad (5)$$

Consider the system defined by the infrastructure assets presented in Table 1. Based on data provided by domain experts of each infrastructure sector, matrix  $A$  was determined for the system for an outage period of 6-12 hours, which is given in Table 2. Note that the experts rated the influence of infrastructure  $j$  on their own infrastructure occurring as a result of the lack of resources provided by the  $j^{th}$  infrastructure during a period of 6 to 12 hours. The coefficients could vary for a different outage period, due to technology substitutions (e.g., local diesel generators can provide power temporarily, replenishment resources can be shipped in for longer disruptions, etc.)

**Question 2.1** Given (1) and (2), show (analytically) that (3) is an equivalent matrix model.

**Question 2.2** Explain how to interpret the last row of table 2. Does this seem like a reasonable set of assumptions?

**Question 2.3** Explain how we can interpret the indices defined in equations (4) and (5) for a given infrastructure  $i$  or  $j$ .

**Question 2.4** Write a solution to equation (3) for  $x$ . Call  $S$  the matrix that multiplies vector  $f$  in the resulting equation (i.e., your solution should look something like  $x = Sf$ ). You need to define  $S$  and show how to compute it in terms of other parameters given in the problem. Can you establish any relationship between  $s_{ij}$  (the  $ij$ th element of  $S$ ) and  $a_{ij}$ ? Are there any requirements on any matrices you manipulate in order calculate your solution?

**Question 2.5** Give a written explanation of what each element in the matrix  $S$  means (i.e., define  $s_{ij}$ ).

**Question 2.6** Explain from a modeling and data collection standpoint why it is easier to obtain the values for the elements in matrix  $A$  compared to matrix  $S$ .

**Question 2.7** Define the *overall dependency index*, denoted  $\bar{\kappa}_i$ , and the *overall influence index*, denoted  $\bar{\lambda}_j$ , which are analogous to the indices defined in (4) and (5) but they operate on the elements of matrix  $S$  instead of matrix  $A$ . How can you interpret the overall indices  $\bar{\kappa}_i$  and  $\bar{\lambda}_j$ ? How do they differ from  $\kappa_i$  and  $\lambda_j$ ?

**Question 2.8** Analyze the coefficients of matrix  $A$  given in Table 2 for the *fuel & petroleum grid* and *naval port* sectors. Which infrastructure assets have the greatest influence over each of these sectors? Which infrastructure assets do *fuel & petroleum grid* and *naval port* influence the most?

**Question 2.9** Calculate the indices  $\kappa_i$  and  $\lambda_j$ , compare and analyze them for the different sectors. You may wish to plot them to illustrate the comparison. Summarize what you find.

**Question 2.10** Compute matrix  $S$  and calculate the overall indices  $\bar{\kappa}_i$  and  $\bar{\lambda}_j$ . Compare them with  $\kappa_i$  and  $\lambda_j$ . Do you observe any interesting features? Explain.

**Question 2.11** With the incorporation of *smart grid* technologies into the electricity generation infrastructure, the electricity utilities can be more efficient. At the same time, the additional technology introduces new dependencies in various infrastructure assets. For example, we expect that the dependency between the electricity infrastructure and the communications infrastructure might change. Which coefficient(s) of matrix  $A$  will change, and in which direction (increase/decrease)? You must explain any changes you believe would be necessary to model the incorporation of new technology into the electricity grid.

**Question 2.12** Assume that any coefficient(s) that you indicated in the previous question increase/decrease (as you specified) by 10%. Compute the new matrix  $S$ , and recalculate the overall indices  $\bar{\kappa}_i$  and  $\bar{\lambda}_j$ . Compare them to the previous ones and comment any interesting features that you observe. Which infrastructure assets are affected most by this change?

**Question 2.13** Consider a sudden strike of a strong snow storm. Assume it creates an external reduction of 50% of the rail transportation (cancelled and delayed trains due to the heavy snow), 10% of the electricity sector (electricity outages), 35% of air transportation (cancelled flights) and 20% of the naval port operations (shipping delays and decreased efficiency in handling inventory). What is the effect on other infrastructure assets? How much do critical infrastructure dependencies further degrade the already damaged infrastructure? Are other infrastructure assets degraded even without being directly affected by the storm?

**Question 2.14** In your opinion, is matrix  $A$  presented in Table 2 an appropriate influence matrix to analyze the effects of a heavy snow storm? What limitations does it have?

**Question 2.15** Simulate the propagation of the initial damage through the  $k^{th}$  tier of impacts using the recursive relation defined in lecture 12:

$$x(k) = Ax(k-1) + f.$$

Plot  $x(k)$  as a function of  $k$  and explain what you observe. How does  $x(k)$  compare to  $Sf$  (occasionally, always, only for some  $f$ , for all  $f$ )? Is  $x(k)$  larger or smaller than  $Sf$ ? Is it a coincidence, or is there something more fundamental about what you observe?

**Question 2.16** Assume that the values in Table 2 are the expected values of *uniform* distributions with ranges of 10% of the mean value. Perform a Monte Carlo analysis to quantify the uncertainty in  $x$  given the uncertainty in  $A$ . Plot the results and compare them to the deterministic analysis and comment any interesting features that you observe. Which infrastructure is the most robust to the uncertainty? Which one is the most sensitive to the uncertainty? Is the uncertainty on  $x$  uniform? Do you expect it to be uniform? You may also wish to look at the effect on  $S$  of uncertainty on  $A$ .

**Question 2.17** Instead of uncertainty on  $A$ , assume the magnitude of the initial disruption caused by the snowstorm is uncertain. Assume the uncertainty on  $f$  follows a uniform distribution with a range of 10% of the deterministic value used previously. What do you observe? What is worse, the 10% uncertainty on  $A$ , or the 10% uncertainty on  $f$ ?

**Question 2.18** Following the analysis of the previous two questions, rerun the models using the lower bound of each uncertain parameter or input (i.e., one run with all parameters in  $A$  at the lower bound, and one run with  $f$  at the lower bound), and again with the upper bounds of each parameter or input. Compare the outputs with what you observed from the previous questions. Explain what you find and comment generality of any properties you observe.

**Question 2.19** If the total level of degradation to air fuel & petroleum grid, TLC wireless, and finance cannot be greater than 20%, 30%, and 35%, respectively, what is the maximum initial impact  $f_{max}$  that the system can absorb? Your solution might be an analytical solution or a solution to an optimization problem, you will need to decide.

### 3 Description of the electric utility

An electric utility is composed of several existing power plants, which use various technologies to generate electricity in its portfolio. The properties of each power plant are listed in Table 3 and 4. For example, plant  $i = 1$  is a conventional coal fired power plant. It has a maximum generating capacity  $x_1^{max}$  of 250 MW, and has an expected variable cost  $\bar{c}_1^v$  of \$28.6/MWh due to fuel and various operations and maintenance expenses. The standard deviation of the variable cost  $\sigma_1^v = \$3.0/\text{MWh}$ , is due in part to uncertainty in fuel prices. Thus, we can model the variable cost as a random variable  $c_1^v \sim \mathcal{N}(\bar{c}_1^v, (\sigma_1^v)^2)$ .

Because of network constraints, it has a minimum capacity constraint  $x_1^{min}$  of 70 MW, which means that at least 70 MW of electricity must be generated at all times from plant 1. Table 3 also shows that the unplanned outage rate (or outage probability)  $o_1^u$  of the plant is 5%. This gives the plant a derated capacity of  $(1 - o_1^u)x_1^{max} = (1 - 0.05)250 = 237.5$  MW. Similarly,  $o_1^p$  provides the planned outage rate for the plant, which is used to derate the annual capacity of the plant (measured in MWh). The planned outage rate is often specified in terms of a *capacity factor*, where the capacity factor is one minus the planned outage rate. Finally, Table 3 also provides information on the operating *global warming potential* (GWP) for each plant  $g_i$  defined in terms of kg of  $CO_2$  equivalent emissions per MWh.

plant	type	$x_i^{max}$ (MW)	$x_i^{min}$ (MW)	$o_i^u$	$o_i^p$	$g_i$ kg CO <sub>2</sub> e/MWh
1	(e) Conventional coal	250	70	0.05	0.15	1001
2	(e) Advanced coal	210	0	0.04	0.15	766
3	(e) conventional combustion nat gas	180	0	0.035	0.70	443
4	(e) nuclear	400	100	0.05	0.10	16
5	(n) Advanced coal with CCS <sup>1</sup>	500 (max)	0	0.05	0.15	396
6	(n) Advanced CC <sup>2</sup> natural gas	500 (max)	0	0.05	0.15	305
7	(n) wind	300 (max)	0	0.03	0.66	12
8	(n) solar	400 (max)	0	0.03	0.8	45
9	(e) Hydroelectric dam	200	0	0.08	0.12	78

Table 3: generating plant production data (e) denotes existing plant, (n) denotes new plant. <sup>1</sup>Carbon Capture and Storage. <sup>2</sup>Combined Cycle.

plant	type	$\bar{c}_i^v$ (\$/MWh)	$\sigma_i^v$ (\$/MWh)	$\bar{c}_i^c$ (\$/kW)	$\sigma_i^c$ (\$/kW)
1	(e) Conventional coal	28.6	3.0	-	-
2	(e) Advanced coal	29.1	3.0	-	-
3	(e) conventional combustion nat gas	79.9	3.0	-	-
4	(e) nuclear	11.5	4.0	-	-
5	(n) Advanced coal with CCS	36.8	2.0	93.3	9
6	(n) Advanced CC natural gas	44.3	2.0	17.8	6
7	(n) wind	0.90	0.15	83.3	12
8	(n) solar	1.0	0.5	145.0	20
9	(e) Hydroelectric dam	1.5	0.20	-	-

Table 4: generating plant cost data (e) denotes existing plant, (n) denotes new plant.

In addition to the existing plants, you also have the option to build up to 4 proposed new plant designs (5-8). The (fixed) capital cost  $c_i^c \sim \mathcal{N}(\bar{c}_i^c, (\sigma_i^c)^2)$  (with expected cost  $\bar{c}_i^c$  and standard deviation  $\sigma_i^c$ ) of each design is assumed to be a linear function of the maximum capacity of the plant you construct, which you

will have to determine.

With the properties of the power plants available to the utility defined, we now describe the energy load the utility must serve. The power plant load blocks are defined in Table 5. A power plant load block describes the amount of energy demanded (load) in MW, and the number of hours the load is demanded. For example, load block 1 contains the 100 hours (out of the 8766 hours in a year) with the highest demand. In these peak hours, the utility must supply sufficient capacity to meet the load of 1390 MW. Note the width of the load block (number of hours) need not be consecutive. In our utility, the load is described in 6 load blocks, although the number of load blocks could be increased for a more detailed analysis.

load block $t$	1	2	3	4	5	6
number of hours, $n_t$	100	315	671	1245	2780	3655
load (MW), $l_t$	1390	1305	830	545	355	200

Table 5: Load blocks describing the annual power demand which must be served by the utility.

In addition to determining the most efficient way to produce electricity, the utility may also reduce the load in each load block through a *demand side management program* (DSM program). For example, the utility may incentivize consumers to invest in energy efficient lighting, heating, etc, or real-time load control (smart grid) technologies. These DSM programs have a maximum energy savings per load block, which is described in Table 6. Note that the table lists the maximum energy savings if the plan is fully implemented. A partial implementation of a program is also possible. If program 1 is only partially implemented (say 50%), then the energy savings in each block would be 40, 15, 12.5, 6, 2.5 and 0 in blocks 1–6 respectively. Like the other costs in this problem, the cost per MWh of the demand side management programs is a random variable  $c_k^d \sim \mathcal{N}(\bar{c}_k^d, (\sigma_k^d)^2)$ , and the data is given in Table 6.

program $k$	description	max energy savings $s_{kt}^{max}$ (MW)	cost $\bar{c}_k^d$ (\$/MWh)	cost std dev $\sigma_k^d$ (\$/MWh)
1	Energy efficiency	[80, 30, 25, 12, 5, 0]	55	15
2	Energy efficiency	[70, 30, 15, 10.5, 5, 0]	65	7
3	Load control	[100, 0, 0, 0, 0, 0]	100	20

Table 6: Demand side management programs available to the utility.



### 3.1 Electric utility management objectives

Three objectives will be considered for the management of electricity. The first objective you will consider is to minimize the expected annual cost of operating the utility. The expected cost function is made of *i*) a fixed (capital) cost of operating the plant, which is annualized; *ii*) a variable cost associated with purchasing fuel, and operations and maintenance costs, and *iii*) a variable cost associated with implementing any demand side management plans.

The second objective function you will consider is to minimize the annual global warming potential of the electric utility. The global warming potential is expressed in terms of tons of  $CO_2$  equivalent emissions associated with operating the plants to produce required electricity.

The third objective function to be considered is to minimize the variance of the cost of supplying electricity. The variance of the price of a good (for example 1 MWh of electricity) can be used as a measure of the risk of that good. Portfolio theory assumes that for a given level of risk, planners prefer lower costs to higher ones. Similarly, for a given expected cost, planners prefer less risk to more risk. By combining various goods in a portfolio, it is possible to create a portfolio with lower risk than any of the goods individually. This is known as *diversification*. Unlike the first two objectives, this objective will result in a quadratic optimization problem.

### 3.2 Electric utility problem constraints

The problem of minimizing cost or minimizing emissions is subject to the following constraints:

- **Load constraint.** The supply of power must meet the load demanded during each sub period  $t$ . Consider that  $x_{it}$  is the power supplied by plant  $i$  in load block  $t$  (in MW). Then, for each block  $t$ , we need the sum of  $x_{it}$  (over all  $i$ ) to be greater than or equal to the net power demanded during time period  $t$  ( $l_t$ ). The load reduction in block  $t$  can be computed as the sum (over all  $k$ ) of the energy savings from full implementation of DSM program  $k$  ( $s_{kt}^{max}$ ) times the DSM implementation rate  $z_k$ . This constraint applies to all load blocks ( $t = 1, \dots, T$ ).
- **Instantaneous capacity constraint.** The power generated from plant  $i$  in load block  $t$  ( $x_{it}$ ) must be less than or equal the derated capacity for each plant caused by unplanned outages: of plant  $i$ . This constraint applies to all plants in each load ( $i = 1, \dots, I, \quad t = 1, \dots, T$ ).
- **Annual energy constraint (existing plants).** The existing plants need regularly scheduled down time to repair and maintain equipment. The total annual capacity is derated for planned outages by the planned outage rate  $o_i^p$ . This is expressed as:

$$\sum_{t=1}^T n_t x_{it} - (1 - o_i^p) 8766 x_i^{max} \leq 0 \quad i = 1, 2, 3, 4, 9$$

where  $n_t$  is the width of load block  $t$ .

- **Annual energy constraint (new plants)** The new plants also have an annual energy constraint, which depends on the constructed capacity  $y_i$ . It can be expressed as:

$$\sum_{t=1}^T n_t x_{it} - (1 - o_i^p) 8766 y_i \leq 0 \quad i = 5, \dots, 8.$$

- **Minimum generation constraint.** The energy generated from each source ( $x_{it}$ ) must be larger than the *must run* (minimum) capacity  $x_i^{min}$  of each plant  $i$ . This constraint applies to all plants in each load block ( $i = 1, \dots, I, \quad t = 1, \dots, T$ ).

- **Bounds on DSM programs.** The DSM implementation rate  $z_k$  for program  $k$  is bounded between 0 (no implementation) and 1 (full implementation). This constraint is valid for all DSM programs ( $k = 1, \dots, K$ ), and is expressed as:

$$0 \leq z_k \leq 1, \quad k = 1, \dots, K.$$

- **New generation bounds.** We let  $y_i$  denote the size (MW) of any of new plant  $i$ , and write the new generation constraints as:

$$x_{it} \leq (1 - o_i^u)y_i \quad i = 5, \dots, 8, \quad t = 1, \dots, T$$

$$y_i \leq x_i^{max} \quad i = 5, \dots, 8.$$

### 3.3 Summary of notation

A summary of the problem indices, variables, and constants you will need are defined below.

- $i$  = index over the power plants,  $i = 1, \dots, I$
- $k$  = index over the demand side management programs,  $k = 1, \dots, K$
- $t$  = index over the load blocks,  $t \in 1, \dots, T$
- $n_t$  = number of hours in load block  $t$  (hours)
- $l_t$  = power load in load block  $t$  (MW)
- $c_i^c \sim \mathcal{N}(\bar{c}_i^c, (\sigma_i^c)^2)$  = annualized capital cost of plant  $i$  (\$/kW)
- $c_k^d \sim \mathcal{N}(\bar{c}_k^d, (\sigma_k^d)^2)$  = cost of implementing demand side management program  $k$  (\$/MWh)
- $s_{kt}^{max}$  = maximum energy savings available in load block  $t$  from demand side management program  $k$  (MW)
- $c_i^v \sim \mathcal{N}(\bar{c}_i^v, (\sigma_i^v)^2)$  = variable cost associated with operating plant  $i$  (\$/MWh)
- $g_i$  = global warming potential ( $CO_2$  eq/MWh) from plant  $i$
- $x_i^{max}$  = maximum capacity of power plant  $i$  (MW)
- $z_k$  = implementation rate of demand side management program  $k$ . If program  $k$  is not implemented,  $z_k = 0$ , if program  $k$  is implemented at 30%,  $z_k = .3$ .  $z_k \in [0, 1]$
- $x_{it}$  = the power (MW) produced from power plant  $i$  during load block  $t$
- $y_i$  = the design capacity of new power plant  $i$  (MW)
- $o_i^u$  = unplanned outage rate  $o_i^u \in [0, 1]$
- $o_i^p$  = planned annual outage rate  $o_i^p \in [0, 1]$

## 4 Electric utility design considerations

### 4.1 Cost and emissions optimization modeling

**Question 4.20** Identify the decision variables for managing the utility. How many decision variables do you have?

**Question 4.21** Write an expression for the objective function which minimizes the total global warming potential (tons of  $CO_2$  eq) in terms of the problem decision variables and constants identified above.

**Question 4.22** Write an expression for the objective function which minimizes the expected annual cost of operating the utility (expected capital costs, expected variable operating costs, and expected demand management costs), using the decision variables and constants defined above.

**Question 4.23** Write an expression for the load constraints described above. How many load constraints do you have?

**Question 4.24** Write an expression for the instantaneous capacity constraints described above. How many instantaneous capacity constraints do you have?

**Question 4.25** Write an expression for the minimum generation constraints described above. How many minimum generation constraints do you have?

**Question 4.26** Explain in words the meaning of the new generation bounds.

**Question 4.27** Given the constraints described above, write the complete optimization problem for minimizing expected cost subject to the production constraints. How many constraints do you have?

**Question 4.28** Solve the minimum expected cost optimization problem in Python, and describe your solution in detail. What is the cost, and which power plants and DSM programs are used to generate power and save energy? What are the  $CO_2$  eq emissions associated with your minimum cost design? What is the standard deviation of the cost? Which constraints are satisfied at equality? Perform a sensitivity analysis for this problem and determine which constraint has the highest sensitivity. Do your results make sense? Explain. *Caution to the optimization experts: If you use the Lagrange multipliers as your shadow cost, you need to verify the problem is not degenerate. If this makes no sense to you, you can safely proceed with the general sensitivity approaches you have seen in class.*

**Question 4.29** Solve the minimum environmental emissions optimization problem in Python. What is the expected cost, and which power plants and DSM programs are used to generate power and save energy? What are the  $CO_2$  eq emissions associated with your minimum emissions design? What is the standard deviation of the cost of operating the plant? Which constraints are satisfied at equality, and which constraints have slack? Perform a sensitivity analysis for this problem and determine which constraint has the highest sensitivity. Again, justify why this solution makes sense.

**Question 4.30** Perform a multi-objective (emissions and expected cost) optimization, and compute the Pareto frontier of efficient solutions. Describe any interesting features about the frontier.

**Question 4.31** An emissions cap is being considered. How should the cap be set in order to achieve the same solution as the minimum environmental emissions objective? Under what values would the emissions cap have an influence in the minimum expected cost optimization problem? Explain how the emissions cap can be embedded into your minimum expected cost optimization problem. Write an equation which describes the emissions cap, implement it in your optimization problem in Python, and demonstrate how it works.

**Question 4.32** Instead of an emissions cap, now tax on  $CO_2$  eq emissions is being considered (\$/kg  $CO_2$  eq) in the minimum cost optimization problem. Under what values will the emissions tax influence the solution?

Can you choose a tax which will result in the same solution as the minimum emissions optimization problem? Explain why or why not. If it is possible, explain how to set the price, and demonstrate in Python that your method works.

**Question 4.33** Derive an expression for the objective function which minimizes the variance of the annual cost of operating the utility.

**Question 4.34** Perform a multi-objective (cost variance and expected cost) optimization, and compute the Pareto frontier of efficient solutions. Describe any interesting features about the frontier.

**Question 4.35** Given the results of your optimization problems above, propose a single design for the electric utility which you will recommend to be implemented. This might require solving yet another optimization problem. Explain how you arrived at your decision. What is the expected cost, the  $CO_2$  eq emissions, and the standard deviation of the cost associated with your design? Which power plants and DSM programs are used?

**Question 4.36** What is the final expected cost in \$/kW to generate electricity by this utility?

**Question 4.37** Due to seasonal and inter-annual variability of the input flow to the hydroelectric dam, there is a higher uncertainty in the maximum power that this plant can generate in comparison with other power plants. For long-term optimization problems, which consider the expected cost and/or emissions during long periods of time (e.g., annually), the uncertainty could be embedded in the unplanned outage  $o^u$ . Consider that due to climate change, more intense and frequent droughts are expected. We estimate that this can be represented as an increase in  $o^u$  of the hydroelectric dam from 8% to 62%. How sensitive is your design of the electric utility to this change?

**Question 4.38** Assume carbon capture and storage has made a technological break through, reducing emissions close to zero (i.e.  $g_5 = 16$  kg  $CO_2e/MWh$ ) and reducing the cost of constructing a plant to  $\bar{c}_5^c = 43.3$  \$/kW. Other technologies plateaued and did not make any progress; assume everything else stays as described in Table 3 and Table 4. Perform a multi-objective (emissions and expected cost) optimization weighting emissions at 60% and cost at 40%. Compare these results to the answer of Question 4.30 with the same weighting scheme. Create two plant-load block profiles to compare your results and make sure to also mention what DSM programs are in use. Table 7 shows what is meant by a plant-load block profile.

plant	type	LB 1	LB 2	LB 3	LB 4	LB 5	LB 6
1	(e) Conventional coal						
2	(e) Advanced coal						
3	(e) conventional combustion nat gas						
4	(e) nuclear						
5	(n) Advanced coal with CCS						
6	(n) Advanced CC natural gas						
7	(n) wind						
8	(n) solar						
9	(e) Hydroelectric dam						

Table 7: Production mix-load block profile.

## References

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