$$
\begin{aligned}
P_5(x) &= \frac{63x^5 - 70x^3 + 15x}{8} \\
P_5'(x) &= \frac{(63\times 5)x^4 - (70\times 3)x^2 + 15}{8} \\
&= \frac{(15\times 7)\times(3x^4 - 2x^2 + \frac{1}{7})}{8} \\
P_5''(x) &= \frac{(15\times 7)\times(12x^3 - 4x)}{8} \\
&= \frac{(15\times 7\times 4)\times(3x^3 - x)}{8}
\end{aligned}
$$

$P_5''(x) = 0$的解为$-\frac{\sqrt{3}}{3}, 0, \frac{\sqrt{3}}{3}$，且可得出表

	-1	**(-1,$-\frac{\sqrt{3}}{3}$)**	$-\frac{\sqrt{3}}{3}$	**($-\frac{\sqrt{3}}{3}$,0)**	**0**	**(0, $\frac{\sqrt{3}}{3}$)**	$\frac{\sqrt{3}}{3}$	**($\frac{\sqrt{3}}{3}$, 1)**	**1**
$P_5''(x)$	-	-	0	+	0	-	0	+	+
$P_5'(x)$	1.14286	+,-	-0.190476	-,+	0.142857	+,-	-0.190476	-,+	1.14286

求解$P_5'(x)$的代码。

```
double x;
while(cin >> x)
cout << 3*pow(x, 4)-2*pow(x, 2)+1.0/7 << endl;
return 0;
```

二分法求得 $P_5'(x) = 0$的解$x = -0.765055,\ -0.285232,\ 0.285232,\ 0.765055$

二分求$P_5'(x) = 0$代码：

```cpp
double P(double x) {
    return 3*pow(x, 4)-2*pow(x, 2)+1.0/7;
}
int main() {
    double l, r, mid;
    while(cin >> l >> r) {
        while(r - l > eps) {
            mid = (l + r)/2;
            double tmp = P(mid);
            if(tmp >= 0) l = mid;
            else r = mid-eps;
        }
        cout << l << endl;
    }
    return 0;
}
```

$\therefore P_5(x)$在区间$[-1,1]$单调性为：

	-1	(-1,-0.765055)	-0.765055	(-0.765055,-0.285232)	-0.285232	(-0.285232,0.285232)	0.285232	(0.285232,-0.765055)	-0.765055	(0.765055,1)	1
$P_5(x)$	-8	↗	3.357575	↘	-2.7730218	↗	2.7730218	↘	-3.357575	↗	

```cpp
const double eps = 1e-13;
double P(double a) {
    return 63 * pow(a, 5) - 70 * pow(a, 3) + 15 * a;
}
double up(double x, double y) {
    return x <= y;
}
double down(double x, double y) {
    return x >= y;
}

double binary(double l, double r, int k) {
    double mid;
    while(r - l >= eps) {
        mid = (l + r)/2;
        double tmp = P(mid);
        if(k && down(tmp, 0)) l = mid;
        else if(!k && up(tmp, 0)) l = mid;
        else r = mid-eps;
    }
    return l;
}
int main() {
#ifndef ONLINE_JUDGE
    freopen("in.txt", "r", stdin);
    freopen("out.txt", "w", stdout);
#endif
    double l, r, mid;
    int k;
    while(cin >> l >> r >> k) {
       printf("%.8lf\n", binary(l, r, k));
    }
    return 0;
}
```

所以：$P_5(x)$的解为：

$$
\begin{aligned}
x_0 &= -0.90617985 \\
x_1 &= -0.53846931 \\
x_2 &= 0.00000000 \\
x_3 &= 0.53846931 \\
x_4 &= 0.90617985
\end{aligned}
$$