Composing music with quantum computers

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I will be summarizing and detailing a paper by Volkmar Putz and Karl Svozil called *Quantum music* [1]. They illustrate some ideas of how to map a quantum system to music. I extend this idea to using quantum computers to map a quantum algorithm to a song. I plan to pursue this topic for my final project (if you approve), and am quite entertained by this work.

I. ABOUT THE AUTHORS

There is not much work in the area of quantum music or using quantum computers to compose music. In fact, this is the only significant paper I could find on the subject. Similarly, these two authors, to the best of my knowledge, have published no other papers relating to this subject. Nonetheless, I find it to be a very interesting application. I will focus on Karl Svozil, because I can't find much information on Volkmar Putz in English!

Karl Svozil is currently a professor of theoretical physics at the Vienna University of Technology in Vienna, Austria. His most cited work is related to the study of randomness in quantum physics.

Svozil did his PhD at the University of Vienna and Heidelberg in theoretical physics, and then was a visiting scholar at the Lawrence Berkeley National Laboratory (associated with Berkeley University) for two years from 1982-1983, before becoming staff at the University of Vienna where he is now a professor. The lab is currently very involved in quantum computing, and has always had a strong presence in quantum information.

Svozil is an editor for various journals in physics and computer science. He is also religious and interested in philosophy, and recently published a paper on how science can guide theology to a more complete understanding of our existence (currently a preprint [2]).

II. QUANTUM MUSIC

Putz and Svozil mention a simple mapping between quantum states and musical notes, but I find it unnecessary and uninteresting. In what follows, I will summarize what I find to be the most interesting part of their paper.

A. Fermi model of tones

In physics, a fermion is a class of particles that obey the Pauli Exclusion Principle; that is, no two fermions can occupy the same quantum state. In many-body quantum systems composed of fermions, we can represent the state with the occupation number formalism.

Consider a tensor product wavefunction over n subspaces. In the occupation number formalism, we choose the basis wavevectors to be of the form

$$|\psi\rangle = |\alpha_0\rangle |\alpha_1\rangle ... |\alpha_{n-1}\rangle \,, \tag{1}$$

where each α_i represents the occupation number of the i^{th} state, ie how many particle are in the i^{th} state. But because the particles are fermions and no two fermions can occupy the same state, each α_i can only be either 0 or 1.

Putz and Svozil suggest mapping fermionic occupation numbers to musical note booleans in a chord. For example, we can create a mapping from the fermionic basis to notes in one octave of the C major scale with

$$|\alpha\rangle = |\alpha_c\rangle_c |\alpha_d\rangle_d |\alpha_e\rangle_e |\alpha_f\rangle_f |\alpha_g\rangle_g |\alpha_a\rangle_a |\alpha_b\rangle_b \,. \tag{2}$$

With this mapping, each basis wavevector maps to a chord in C major (see Table I). To encode more notes with this mapping, we just need higher dimensional wavevectors.

chord	notes	wavevector
I	c, e, g	$\|1\rangle_c \|0\rangle_d \|1\rangle_e \|0\rangle_f \|1\rangle_g \|0\rangle_a \|0\rangle_b$
I^7	c, e, g, b	$\|1\rangle_c \|0\rangle_d \|1\rangle_e \|0\rangle_f \|1\rangle_g \|0\rangle_a \|1\rangle_b$
V	g, b, d	$\|0\rangle_c \|1\rangle_d \|0\rangle_e \|0\rangle_f \|1\rangle_g \|0\rangle_a \|1\rangle_b$
N/A	c, f	$\|1\rangle_c \|0\rangle_d \|0\rangle_e \|1\rangle_f \|0\rangle_g \|0\rangle_a \|0\rangle_b$
...	...	...

TABLE I. Some example mappings from tonal chords or sets of notes to wavevectors in the Fermi model of tones

So far we have mapped a basis set of wavevectors to musical notes. But in quantum mechanics, a wavevector can be a superposition of basis wavevectors. For example, we can have a wavevector that is

$$|\psi\rangle = \cos\theta \, |\alpha_1\rangle + e^{i\phi} \sin\theta \, |\alpha_2\rangle \,. \tag{3}$$

Upon measurement of $|\psi\rangle$, we will measure either $|\alpha_1\rangle$ or $|\alpha_2\rangle$, the former with probability $\cos^2\theta$ and the latter with probability $\sin^2\theta$.

More generally, we can have a wavevector that is

$$|\psi\rangle = \sum_i c_i \, |\alpha_i\rangle \tag{4}$$

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as long as $\sum_i |c_i|^2 = 1$ (to conserve probability). If we were to take measurements of this wavevector, we would measure each $|\alpha_i\rangle$ with probability $|c_i|^2$. In other words, a single wavevector $|\psi\rangle$ can encode many different chords and notes. Every time we measure the wavevector, we will find one particular chord or note given by some α_j. The measurement itself is intrinsically random, but we can set up $|\psi\rangle$ such that different chords or notes are more probable to be measured, and will therefore show up in our song more often.

The main point here is that $|\psi\rangle$ encodes an entire song, and by making measurements of $|\psi\rangle$ we determine which chord or note comes next. Thus, the song will be played back differently every time it is played because the measurement follows some probability distribution.

B. The sound of quantum algorithms

I will give an extremely brief explanation of quantum computers just for the purposes of quantum music, but it is by no means meant to be comprehensive.

Quantum computers manipulate a tensor product state of qubits. A qubit is a two state system, often represented in terms of spin, which has basis states spin up $|\uparrow\rangle$ or spin down $|\downarrow\rangle$. Quantum algorithms evolve a quantum state of qubits and then measure them in this spin basis. A system of n qubits has basis states

$$|\psi_n\rangle = |\alpha_0\rangle |\alpha_1\rangle \dots |\alpha_{n-1}\rangle \tag{5}$$

where each α_i can be either $\uparrow$ or $\downarrow$. But notice that a simple relabeling of $\uparrow$ to 1 and $\downarrow$ to 0 gives us the exact basis states as before in Equation 1.

In other words, after running a quantum algorithm we are left with a wavevector $|\psi\rangle$ exactly of the form of Equation 4. We can measure the output of a quantum computer after running a quantum algorithm, and the output will be exactly some $|\alpha\rangle$ as in Equation 2; a note or chord!

C. Method

A quantum algorithm is a sequence of quantum logic gates. A quantum computer runs this sequence, and then samples from the output, which is a wavevector in superposition of many basis wavevectors, where we understand now that each basis wavevector can be mapped to a note or chord. Thus, by running a given quantum algorithm multiple times and measuring each time, we get a sequence of notes and chords that are sampled from a probability distribution that is created by the algorithm. Before we said that the quantum state $|\psi\rangle$ encodes a song, and by sampling from the state we can create music. Now, $|\psi\rangle$ is a deterministic output from a quantum algorithm. Therefore, a quantum algorithm encodes a song, and by running the algorithm and sampling from the output we can create music.

Some more research needs to be done on how to create algorithms that encode decent sounding songs. But one particular technique that Putz and Svozil mention is to use entanglement to enforce certain conditions. For example, consider sampling from the wavevector (here we ignore notes e - b and only use c and d)

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|1\rangle_c |0\rangle_d + |0\rangle_c |1\rangle_d \right). \tag{6}$$

The qubit representing the c note is entangled with the qubit representing the d note. The reason is as follows. If we sample from $|\psi\rangle$, we will measure $|1\rangle_c |0\rangle_d$ about 50% of the time and $|0\rangle_c |1\rangle_d$ the other 50% of the time. Thus, this song will only ever contain single notes c and d, never c and d played at the same time (represented by $|1\rangle_c |1\rangle_d$) or no notes played (represented by $|0\rangle_c |0\rangle_d$).

It is straightforward to see how this idea of entanglement could be extended to ensuring that, for example, only triads are played.

III. EXAMPLE

Let us do some relabeling for convenience. Define the C-, F-, and G-major chords as

$$|C\rangle = |1\rangle_c |0\rangle_d |1\rangle_e |0\rangle_f |1\rangle_g |0\rangle_a |0\rangle_b , \tag{7}$$

$$|F\rangle = |1\rangle_c |0\rangle_d |0\rangle_e |1\rangle_f |0\rangle_g |1\rangle_a |0\rangle_b , \tag{8}$$

$$|G\rangle = |0\rangle_c |1\rangle_d |0\rangle_e |0\rangle_f |1\rangle_g |0\rangle_a |1\rangle_b . \tag{9}$$

Note that these chords are represented with seven qubits. Consider a quantum algorithm that creates the quantum wavevector

$$|S\rangle = \frac{1}{\sqrt{29}} \left(4 |C\rangle + 3 |G\rangle + 2 |F\rangle \right). \tag{10}$$

This is the output of the quantum computer after running the algorithm. But of course we cannot actually access this full mathematical form of $|S\rangle$. All we can do is sample from $|S\rangle$, and in doing this we will measure the C-major chord with probability 16/29, the G-major chord with probability 9/29, and the F-major chord with probability 4/29. Thus, by running this algorithm ten times, a possible song is the chord progression (C G C G F G C F G C).

IV. REFLECTION

I plan to pursue quantum computing in graduate school and hopefully for my career. This paper was a cool application of quantum computers, and for my final project, I will code something to have IBM's quantum computer improvise music based on the work in this paper (if you think that is a good final project idea).

This work is interesting in my mind for two main reasons. Firstly, I like the idea of an abstract song represented by a wavevector, where all the data for the song is in there, but the actual musical progression varies due to the randomness of measurement. Quantum computers create songs that are constantly changing! Secondly, people are constantly trying to come up with new ways to introduce the general public or investors to quantum computing. There are many companies and universities studying and creating quantum algorithms and quantum hardware. Quantum music is an interesting and easy way to introduce people to quantum computing, hopefully stimulating more involvement in the field.

[1] V. Putz & K. Svozil. *Quantum Music*. Soft Computing, March 2017, Volume 21, Issue 6, pp 1467 - 1471.

[2] Karl Svozil. *Theology and Metaphysics in Sombre, Scientific Times* (2018). Preprint arXiv:1809.05339.