Optimization of Dynamic Ridesharing Systems

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Internet-enabled technologies are becoming more widespread; users are constantly connected to the network in every place and daily activity. Access to transportation-related features—mobile payment systems, Global Positioning System connections, real-time public transit timetables or traffic congestion information, and so on—is easy. This access results in new ways to plan mobility. In the innovative mobility systems implemented and developed with these technologies, the new real-time capabilities of dynamic ridesharing, an extended version of traditional ridesharing, can play a key role if the relevant performance is improved. In other words, although ridesharing is not a new idea, recent technological advances should increase its popularity. In this paper, a proposed ridesharing system considers the interactions between drivers or riders and the system manager and the interactions between drivers and riders. The positions and speeds of the shared vehicles and the traffic flows in which such vehicles travel are omitted. To optimize the performance of the ridesharing system, a discrete event, dynamic pickup and delivery model that represents the considered dynamics and an optimal matching problem that optimally allocates an empty seat in a vehicle to a rider are proposed. The dynamic model represents the behavior of the ridesharing system and computes the relevant performance; the optimization problem finds the best match and path in the considered transportation network to minimize the difference between the desired departure and arrival times. In this paper, after the introduction of the ridesharing model, the discussion of the solution to the optimal matching problem, a simulation model is described. A real world case study is then presented and discussed.

Internet-enabled technologies are becoming increasingly widespread. As an estimate of how telecommunications have become increasingly central in daily life, consider that in the United States, over 63 million people access the internet every day through mobile technologies (*1*).

Users are constantly connected to the network in every place and in all their daily activities; users can access transportation features and tools—toll and parking payment systems, Global Positioning System connections, real-time public transit timetables or traffic congestion information, and so on (*2, 3*)—and can then optimally plan their mobility.

Many innovative mobility systems have been implemented and developed for the mentioned technologies; these systems include dynamic ridesharing, which is an extended version of traditional ridesharing. In traditional ridesharing, drivers offer rides to users (or "riders") who are going to destinations located along the drivers' paths. The new technologies offer users the possibility of exchanging requests for rides almost in real time, instead of having to plan in advance (*4*). The possibility of finding good ride matches is therefore increased.

The dynamic ridesharing projects implemented in the United States in the past were inefficient (*5*), but such services could not rely on today's technologies, which have particularly advanced in the past 10 to 15 years. Therefore, while ridesharing is not a new idea, recent technological advances might increase its popularity and better exploit its potential (*6, 7*).

All the new applications have quite similar structures and require both a preregistration in a personal page and the download of an application (app) for smartphones. Nevertheless, the offered services show significant differences (*8*):

- Green Monkeys (www.greenmonkeys.com), active in France and Switzerland, offers a refund by proposing a taxi to the user in case of last-minute cancellations; other companies offer this service only if public transit is not available.
- VilleFluide (www.villefluide.fr), another French service, matches a ride only when at least one backup ride is possible; this process guarantees more reliability and flexibility in case of last-minute cancellations.
- Most systems are based on the exchange of money; a few of them, such as GoLoco (www.goloco.org), are equipped with automated payment systems.
- Carriva (www.carriva.org) allows users to enter the conditions under which they want to share a ride, such as willingness to travel with other riders.
- Flinc (www.flinc.org) and Avego (www.avego.it) provide an assessment of the quality of the ride through a feedback rating system.
- Avego, once the match is established, makes the real-time position of the partner available on a map.

Despite the different services and interfaces offered to users, these systems do not seem to undertake trip planning that globally maximizes the number of shared rides.

Moreover, despite the high potential of ridesharing to reduce private vehicle usage, in Europe the average occupancy rate of a private car is between 1.1 and 1.2 people (*9*). Therefore, this topic is still an object of research because a fully successful formula has not been found; all the applications implemented in the past have gained small success, mainly because of psychological barriers and the low flexibility of the service (*10, 11*). The main difficulty is to reach the critical mass (i.e., to attract a sufficient number of users to the system such that the demand of drivers and riders is always satisfied) (*12, 13*).

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The implementation of algorithms for the dynamic matching problem between drivers and riders has not attracted much interest (*14*), although this topic is one of the main components of the system. It is well known that satisfaction and the perceived utility of ridesharing are key elements in the attractiveness of the mode. In particular, satisfaction is strongly influenced by the quality of the match; that is, reaching the desired destination on time, sharing a ride with someone headed near it, and, in general, the participant having his or her preferences well considered.

In Agatz et al., the matching problem is described as the optimization of three possible objective functions (*6, 15*):

1. The minimization of the total kilometers traveled,
2. The minimization of the total travel time, and
3. The maximization of the number of users.

The multiple ride problem (i.e., more than one driver or more than one rider within a single ride) is also analyzed to consider the shared-ride chains.

The design of a disaggregated performance function allows the cost and time components to be taken into account separately; the function also makes it possible to evaluate the performance of the system attributable to the variation of a single value.

In addition, to consider the extra time caused by the pickup and the time saved through the use of dedicated lanes, travel time has been divided into the in-vehicle time and the additional time that results from other operations (*16, 17*). Finally, the cost of fuel and tools has been separated from the costs related to parking in the workplace.

In this paper, a dynamic ridesharing system model is proposed that considers the interactions between drivers or riders and the system manager (e.g., reservations and information exchanges) and the interactions between drivers and riders (e.g., pickup at origin and delivery at destination).

The dynamics of the positions and the speeds of all the shared vehicles and the dynamics of the traffic flows in which such vehicles travel are omitted. Therefore, because the considered dynamics involve the synchronicity and parallelism of different activities, logic states, and conditions (as is usual in manufactured systems), the modeling framework for discrete event systems (DES) appears to be the most suitable tool to represent the essential phenomena of the considered systems (*18–20*).

With the aim of optimizing the performance of the dynamic ridesharing system, a discrete event, dynamic pickup and delivery model and an optimal matching problem are proposed (*21*). The pickup and delivery model represents the dynamics of the considered ridesharing system and, by means of its integration with the optimization problem, provides the relevant performance evaluation.

The optimal matching problem optimally allocates an empty seat in a vehicle to a rider, according to the user-requested arrival times at the origin and the destination. The proposed optimization problem finds the best match and path in the considered transportation network to minimize the difference between the desired departure and arrival times; the problem also provides, by means of suitably defined auxiliary variables, the generalized costs of each trip.

The paper is organized as follows. First, DES and the main characteristics of dynamic ridesharing systems are introduced. After the statement of the optimal matching problem and the discussion on its solution and relationship with modal choice models, a simulation model is presented. A real world case study, from Genoa, Italy, is then presented.

BASICS OF DES

DES can be defined as those systems in which the events (e_h, $h = 1, \ldots, H$) that can be informally considered to be something instantaneous cause a transition from one state (x_k) to another state (x_{k+1}) of the system. The states can assume nonnumerical or logic values. The set of all the admissible events (the "space of events") is indicated by E; the set of all the possible system states (the "space of states") is indicated by X. The system dynamics are given by the state equation

$$x_{k+1} = \delta(x_k, e_k) \qquad k = 0, 1, 2, 3, \ldots \tag{1}$$

This equation provides the $(k + 1)$th state (x_{k+1}) as a function of the kth event ($e_k \in E$) and the kth state (x_k). In such an equation, the generic state function [$\delta(\cdot)$] may be expressed as a flow chart, a computer program, a mathematical equation, and so on.

In DES, the time variable t depends on the event occurrence (i.e., t is updated when, and only when, an event occurs). The sequences of the couples (x_k, t_k), in which t_k is the time instant at which the event e_k occurs, indicate the state trajectories.

DES allow the easy representation of the macrochanges that occur in a system, such as the beginning and the end of an operation, and neglect the microchanges that occur continuously. Such a characteristic makes the modeling of very large systems possible. In addition, DES can explicitly represent synchronization (two or more conditions that have to be met so that an event can occur) and parallelism (two or more separate system subdynamics that can evolve without interactions for a certain period of time).

With reference to ridesharing systems, such dynamics may be the realization of all the conditions that allow a ride proposal or reservation to take place or the independent behavior of the different vehicles.

DES are usually too complex to be studied analytically; they are often simulated by computer simulation programs (*22*).

DYNAMIC RIDESHARING SYSTEM MODEL

In this section, the considered dynamic ridesharing system model is presented. The model's architecture and dynamics, as well as the optimization variables, are described.

Dynamic Ridesharing System Architecture and Dynamics

The ridesharing manager mainly consists of a computer server that receives the ride proposal and requests, finds the optimal match based on the model described in the following section, and proposes the match to the users, together with the pickup and delivery times and the economic contributions the riders have to provide to the drivers.

If the users accept, then the ridesharing manager communicates the paths to the drivers and confirms the rides. The ridesharing manager can also collect a posteriori information on the rides and update the users' reliability scores to keep finding matches that suit users the most.

Finally, the ridesharing manager manages the payment system, which in the proposed architecture consists of a fare provided by the riders to the system, and the economic contribution computed on the basis of the mileage of the trip, tolls, and so on.

The details of the events that characterize the ridesharing dynamics and the optimization problem will be described later; however, it is useful to say the following:

- Some events, such as ride bookings and cancellations, modify the number of ride requests, and a new run of the optimization procedure is then required.
- The optimization problem is solved only after an event that modifies the number of ride requests.
- In some conditions, no solution for the optimization problem exists, for instance, when there are too many riders' requests and not enough drivers, or when drivers or riders (one or more) do not accept the only possible match. In these cases, the ridesharing manager tries to solve the optimization problem without the last request of the riders or drivers, if requests are sorted by requested departure time.
- The optimal solution, accepted by all the users, is only partially applied. As is typical in a rolling horizon framework, when an event occurs, the system is optimized for the whole time horizon; that is, requests for nonimmediate trips (up to the last one, which can be booked some hours in advance) are also taken into account. Nevertheless, only the matches determined for the immediate future (i.e., for those trips that occur within a fixed time period) are communicated to users; therefore, the system can take into account new pieces of information (traffic state updates, new bookings, etc.) when they become available.

This behavior is summarized in Figure 1, in which the general algorithm for DES simulation is reported, and its connection with the optimization procedure is highlighted.

Dynamic Ridesharing System Network

The transportation network structure that underlies the ridesharing system consists of virtually the whole real road network of the considered zone; users can meet everywhere. Nevertheless, it is possible to assume, for the sake of simplicity, that users can meet only at a priori fixed places (the pickup and delivery stations), such as near bus stops, intersections, the corners of squares, and so on. Under this assumption, it is possible to define a reduced graph $G = \{N, L, W(t)\}$, where N is the set of the pickup and delivery stations, L is the set of arcs that link the stations, and $W(t)$: $L \rightarrow \mathbb{R}$ is a function that assigns a generalized time-dependent cost (mainly consisting of a monetary cost and a travel time, depending on the flows) to each arc in L. A suitable choice for L consists of only the shortest paths, computed for the real network, among all the couples of nodes in N. With this choice, the function $W(t)$ assigns the minimum path costs, computed at time t, to each arc in L. As a result of private traffic flows, the travel times on the real network links change as time passes, and the shortest path and the relevant costs therefore change. Then, dynamically, the graph G evolves, and for each statement of the optimization problem the graph can be different. From the point of view of drivers and riders, such system behavior is negligible, because they simply follow the path the ridesharing manager assigns them.

An example of such a graph is reported in Figure 2. To consider the departure time to be an optimization variable, for each pickup and delivery station that represents an origin for a user u (either driver or rider), a supplementary fictitious node is also considered. The arc that links such a node with a pickup and delivery station has weight σ_u, which represents the optimal departure time of user u, computed with respect to the initial time ($t = 0$).

OPTIMAL MATCHING PROBLEM

The problem of finding the optimal match of drivers and riders can be thought of as a static pickup and delivery problem, in which vehicles are represented by drivers and transportation requests from given origins to given destinations are represented by riders (*19*). Because drivers and riders are transportation users, it is of paramount importance to express the performance the ridesharing system can provide them separately.

With this aim, the problem of finding the optimal match between drivers and users can be formulated as described below. Assume that

D = set of drivers;
R = set of riders;
$U = R \cup D$ = general set of dynamic ridesharing system users;
σ_u = departure time of user u, $\forall u \in U$, from origin o to be optimized;
S_u = desired departure time of user u, $\forall u \in U$, from origin o;
τ_u = optimal arrival time of user u, $\forall u \in U$, at destination d to be optimized;
T_u = desired arrival time of user u, $\forall u \in U$, at destination d;
c_j = capacity (i.e., the maximum number of riders allowed at one time) of the vehicle of driver j, $\forall j \in D$;
$x^u_{h,l}$ = binary variable set to one if the arc $(h, l) \in L$ belongs to the optimal path of user u, $\forall u \in U$, and set to zero otherwise;
FS(h) = forward star of the node h; that is, the set that gathers all the nodes l, such that the arc (h, l) belongs to the set L;
BS(h) = backward star of the node h; that is, the set that gathers all the nodes l, such that the arc (l, h) belongs to the set L;
$t^u_{h,l}$ = travel time on arc $(h, l) \in L$, continuously updated on the basis of real traffic data (such a term can be generalized to a more representative cost that considers fuel consumption, fare, tolls, and so on); and
$\omega_{j,k}$ = binary variable that is set to one if the rider k is matched with the driver j, $\forall k \in R$, $\forall j \in D$, and set to zero otherwise.

To define the cost function, consider Figure 3, which shows a comparison function between the optimal departure time (σ_u) and the user desired departure time (S_u). A possible analytic formulation for such a function is

$$f_u(\sigma_u, S_u) = \max\{0, a_u(S_u - h_u) - a_u\sigma_u\} + \max\{0, a_u\sigma_u - a_u(S_u + h_u)\} \tag{2}$$

This formulation makes all the values of σ_u in the tolerance interval $(S_u - h_u, S_u + h_u)$ without cost; the costs grow linearly (with slope a_u) outside. The subscript u indicates that it is possible to consider different parameters for each user. Analogously, it is possible to define a similar function, namely $g_u(\tau_u, T_u)$, that weights the difference between the optimal arrival time (τ_u) at destination d and the user desired arrival time (T_u).

With these assumptions, the problem of satisfying as many requests as possible can be written as

$$\min \sum_{u \in U} f_u(\sigma_u, S_u) + g_u(\tau_u, T_u) \tag{3}$$

subject to

$$\tau_u = \sigma_u + \sum_{\forall (h,l) \in L} t_{h,l} x^u_{h,l} \qquad \forall u \in U \tag{4}$$

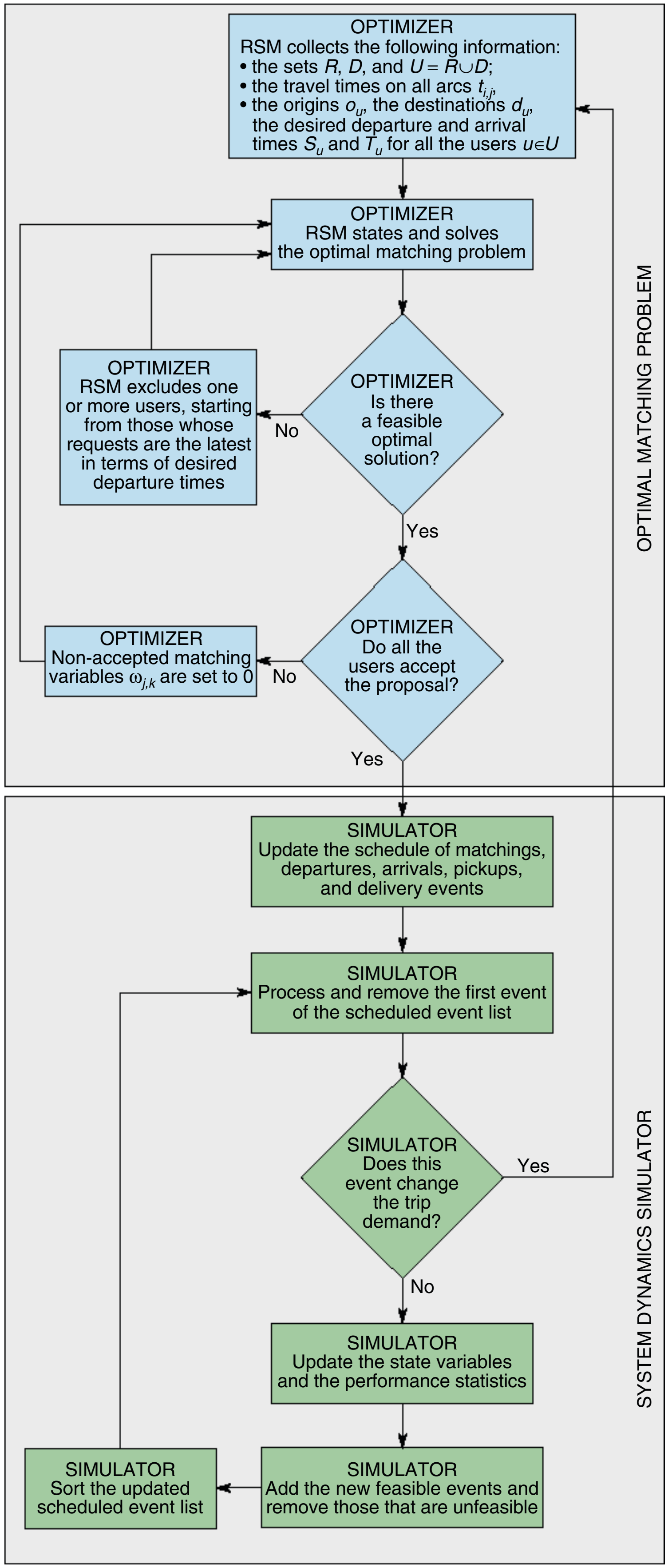

FIGURE 1 Ridesharing simulation process and its interaction with the optimization procedure [RMS = ridesharing manager; D = set of drivers; R = set of riders; ($U = R \cup D$) = general set of system users].

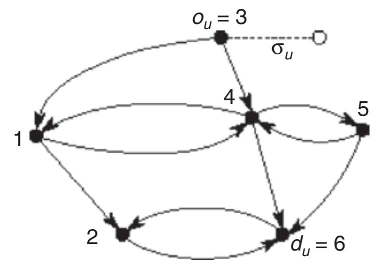

FIGURE 2 Example of graph with origin of user u (o_u) and destination of user u (d_u).

$$\sum_{j \in N} \omega_{j,k} = 1 \qquad \forall k \in R, \forall j \in D \tag{5}$$

$$\sum_{k \in R} \omega_{j,k} \leq c_j \qquad \forall j \in D \tag{6}$$

$$\sum_{l \in \mathrm{FS}(h)} x_{h,l}^u - \sum_{l \in \mathrm{BS}(h)} x_{l,h}^u = \begin{cases} 1 & h \equiv o_u \\ -1 & h \equiv d_u \\ 0 & \text{otherwise} \end{cases} \qquad \forall h \in N; \forall u \in U \tag{7}$$

$$\begin{cases} \omega_{j,k} + x_{h,l}^k - 1 \leq x_{h,l}^j \\ \omega_{j,k} + x_{h,l}^j \neq 2 + 2x_{h,l}^k \end{cases} \qquad \forall j \in D; \forall k \in R; \forall (h,l) \in L \tag{8}$$

$$x_{h,l}^u = \{0,1\} \qquad \forall u \in U; \forall (h,l) \in L \tag{9}$$

$$\omega_{j,k} = \{0,1\} \qquad \forall j \in D; \forall k \in R \tag{10}$$

$$\sigma_u \in \mathbb{R}_+ \qquad \forall u \in U \tag{11}$$

In these equations,

- The cost function in Equation 3 aims to minimize the difference in the desired and real departure and arrival times of all the users by means of the tolerance function (f_u) defined in Equation 2 for the requested departure time, and a similar function (g_u) for the requested arrival time;
- The constraints in Equation 4 define the arrival times at the destinations for all the users $u \in U$;
- The constraints in Equation 5 state that riders can ride in only one vehicle at a time;
- The constraints in Equation 6 limit the capacities of the vehicles;
- The constraints in Equation 7 define separate graph structures for each user $u \in U$. The variables that refer to different graphs are constrained to assume the same values for the matched users by means of the constraints in Equation 8 that guarantee that

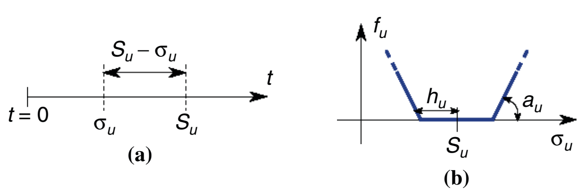

FIGURE 3 Cost function: (a) definitions of variables and (b) shape of the function f_u.

–The variables $x_{h,l}^k$ and $x_{h,l}^j$ assume the same value for all the links (h, l) that belong to the path on which rider k and driver j travel together and

–The variables $x_{h,l}^k$ and $x_{h,l}^j$ can assume different values for all the links (h, l) that belong to the paths on which rider k and driver j do not travel together; and

- The constraints in Equations 9 to 11 define the variables.

The resulting problem is, then, a mixed continuous-integer linear programming problem that can be easily solved by commercial optimization software because of the problem's relatively limited number of variables and constraints. As an example, the network considered in the following case study—characterized by about two dozen arcs and about 10 users in the network at a time—has approximately 150 variables and 200 to 300 constraints and can be solved with the software CPLEX in less than 1 s.

Although the problem provides a solution for all the drivers and riders in the system at a time, only the variables (the optimal matching and the paths) that refer to near-future trips are taken into account and communicated to users; the result is therefore fixed in the future problem statement. The remaining (far-future) trips will be reconsidered in the future solutions, as typical in a rolling horizon framework.

Therefore, this approach maximizes the utility of the whole system rather than the utility of individual drivers and riders. The maximization of the system utility is defined as the minimization of the total waiting time—defined as the time that elapses between the users' desired departure times and the actual departure times—for all users. The system is intended to satisfy the maximum number of requests and distribute the costs between the maximum number of users. Under this approach, some users could be penalized and wait for a longer time.

Conversely, single-user utility minimizes the user's own travel time. For instance, according to the proposed approach, it might be more convenient that four riders start their trips with a delay of 1 min each, rather than for three riders to depart on time and one rider have a delay of 5 min. Therefore, system utility is almost never equal to the sum of the utilities of individual system components.

Such an approach does not guarantee that users will accept the proposed matches, but the performance computed for each user through Equations 3 to 11 allows, dynamically at each match proposal, the determination of incentives to increase the probability of user acceptance (*23*). However, the focus of this paper is the optimization model; further development and optimization of such incentives is left to future work.

SIMULATION MODEL

As mentioned, the dynamic ridesharing system considered here can be represented by DES and characterized by the following events:

- Driver booking. The proposal of a driver traveling in a given zone to share his or her vehicle; the driver also communicates his or her origin, destination, and desired departure and arrival times;
- Rider booking. The request of a rider for a trip from a given origin to a given destination, with the desired departure and arrival times;
- Driver departure. The departure of the driver along the optimal path, communicated to him or her by the ridesharing system manager;
- Driver arrival. The arrival of the driver at the destination;
- Rider pickup. The pickup of the rider by the matched driver; and
- Rider delivery. The delivery of the rider at the destination.

Such simple events occur at random instants that are characterized by different stochastic distributions normally used to model such phenomena. In particular,

- The interval between two proposals or reservations of drivers or riders are exponentially distributed, with a rate represented by the number of requests per hour, and
- The departure, arrival, and pickup and delivery times can be represented by normal distributions, in which the mean is represented by the time computed by the ridesharing manager, based on information about traffic and the optimal paths, and the variance represents the stochastic phenomena that characterize traffic.

The last event to be considered is the driver or rider denial of a trip. When one user (or more than one user) does not accept the proposed optimal route, the scheduled trip cannot begin regularly; then, the system performs a new optimization and sets the relevant matching variables ($\omega_{j,k}$) to zero and proposes a new alternative to the users. Because this process reduces the number of possible matching alternatives, the process can continue until feasible solutions are available, as shown in Figure 1.

Although the above events change the state of the whole system (for instance, the number of requests, the number of users waiting for the vehicle or traveling), between two subsequent events the state remains the same, and the information necessary for the optimization problem remains constant. Therefore, the considered system can be thought of as a dynamic ridesharing system made up of a sequence of static ridesharing systems in a rolling horizon framework, each system optimized by the information available at the time.

CASE STUDY

To evaluate the performance of the proposed optimization model, a real world case study has been tested. For this purpose, a DES simulation of a portion of the city of Genoa (depicted in Figure 4) has been developed. The considered case study consists of a network of 15 pickup and delivery stations and 19 arcs that were selected by taking into account the topology of the city. Nodes consist of the intersections and squares that have sufficient space for the pickup and delivery of users; arcs represent the minimum shortest paths between each pair of nodes.

The considered area covers about 11 km^2 along a 7.5-km coastline; the area hosts about 100,000 citizens. The main traffic flows have longitudinal directions and mostly consist of commuting trips that originate during the morning peak hours from the residential areas located in the east of the city and end in the business districts located in the center; during the evening peak hours, the direction is of course reversed. Only morning and afternoon mobility demand has been taken into account in the simulation. The demand during the rest of the day is of little interest compared with the demand at the peak times.

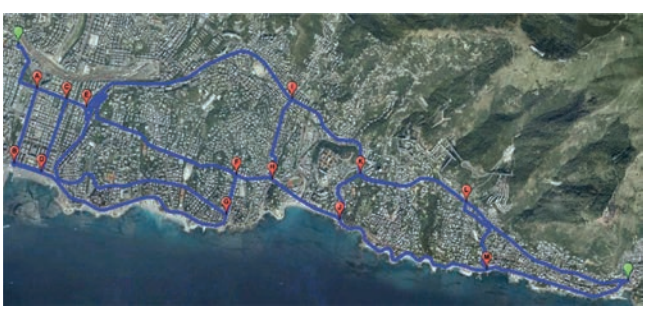

FIGURE 4 Case study network.

TABLE 1 Simulation Parameters

Parameter	Value
Number of drivers per hour	2
Number of riders per hour	5
Tolerance h_u (min)	7
User refusal probability (%)	10
Link travel time variance (min)	3

An average of two and five trip requests per hour from drivers and riders, respectively, has been assumed. Drivers can start their trips anywhere; riders can be picked up by drivers only at the pickup and delivery stations represented by the nodes, which have been marked in Figure 4. The green signs have no special meaning in the dynamic ridesharing network; they represent the railway stations of Genoa Brignole and Genoa Nervi, located on the left and the right edges of Figure 4, respectively.

Table 1 shows the relevant parameters of the simulation. The origins and destinations randomly generated for each request have higher probabilities for eastern nodes as origins and western nodes as destinations in the morning, and vice versa in the afternoon. Real travel times, measured in the network under the considered simulation conditions, have been taken into account.

Finally, even if the match between a rider and a user is good in terms of performance (i.e., the match has optimized departure and arrival times within the accepted tolerance levels with respect to the desired times), a matching refusal probability has been considered to represent the uncertainty of user behavior.

The simulation results are reported in Table 2, in which the following performance indexes have been considered:

- Mean travel time (optimized): the mean of the travel times of all the users in the considered time period, as computed by the optimization problem.
- Mean travel time (simulated): the mean of the travel times of all the users in the considered time period, as provided by the simulator and taking into account the uncertainty of the arc travel times.
- Number of unfeasible solutions: the number of times the problem in Equations 3 to 11 does not satisfy all the demands. Such a phenomenon is caused by the randomly generated demand that sometimes cannot be satisfied (too many riders or too few drivers).

TABLE 2 Simulation Results

Performance Index	Morning Peak (7:30–9:30 a.m.)	Afternoon Peak (5:00–7:00 p.m.)
Mean travel time (optimized) (min)	278	295
Mean travel time (simulated) (min)	290	313
Number of unfeasible solutions	1	2
Mean delay per user (beyond tolerance) (min)	0.9	1.2
Percentage of match refusals (with optimization)	~15	~13
Percentage of match refusals (without optimization)	~75	~76

- Mean delay per user (beyond tolerance): the mean delay between the desired departure and arrival times, with respect to the optimized times, taking into account the tolerance in Table 1. Despite this parameter being computed on the basis of the optimization results, it is an interesting indicator of user satisfaction and represents the amount of time users must wait in excess of the tolerance level.
- Percentage of match refusals (with optimization): the number of matches refused by users, given the (chosen) refusal probability when the optimization procedure is considered, when the refusal is attributable to the nonsatisfaction of their requirements (i.e., their requested departure and arrival times or their desired origins and destinations).
- Percentage of match refusals (without optimization): analogous to the previous parameter but computed through simulations performed without the optimization procedure. In these simulations, the paths and departures times of the drivers are not optimized to satisfy the requests of the riders; matches are proposed only to riders along the driver's fixed path.

From the results reported in Table 2, it is possible to state that the rolling horizon optimization guaranteed good results in the considered periods and provided relatively small delays in the user departure and arrival times. The number of refusals suggests that the relative dimensions of the sets D and R be reconsidered because if these sets are not compatible, unfeasibility (demand dissatisfaction) can occur.

The average delay indicates the necessity of better investigating the acceptance–refusal model of users. Although in the present paper such probabilities have been fixed a priori, a discrete choice model should be considered that takes into account such delays as well as the economic cost of the trips so as to better estimate the performance of the dynamic ridesharing system.

An interesting result is the significant reduction of the match refusals attributable to the better satisfaction of user requirements. Such a result was expected because, without the optimization procedure, the probability that the drivers' and users' requirements would naturally match is low and would depend only on the random generation of requests.

CONCLUSIONS

In this paper, a model based on mixed continuous-integer linear programming has been proposed to maximize the performance of dynamic ridesharing systems. The system architecture and dynamics have been described, and the optimization model has been stated. The performance of the proposed model has been analyzed through a DES–based simulation.

The proposed optimization procedure provides good results in the mean delay per user (about 1 min in both the considered scenarios); the most significant result is the significant reduction of the match refusals: to 15% or 13%, compared with 75% or 76%, respectively.

Therefore, it is possible to state that, although the results are promising, further investigations on more extended case studies should be performed; different traffic conditions should also be investigated. A discrete user choice model has to be implemented to better understand the performance of the ridesharing system; suitable incentives should also be designed to make the refusal probability as small as possible.

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