Pricing Carpool Rides Based on Schedule Displacement

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Pricing carpool rides based on schedule displacement

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Abstract

This paper considers a carpool matching (CaMa) problem in which participants price shared rides based on both operating cost and schedule displacement (i.e, the absolute difference between the desired and actual arrival times). By reporting their valuation of this displacement, each participant in effect bids for every possible shared ride that generates a unique value to her. The CaMa problem can be formulated as a mixed integer program (MIP) that maximizes the social welfare by choosing matching pairs and a departure time for each pair. We show the optimal departure time can be determined for each pair *a priori*, independent of the matching problem. This result reduces the CaMa problem to a standard bipartite matching problem. We prove that the classical Vickrey-Clarke-Groves (VCG) pricing policy ensures no participant is worse off or has the incentive to misreport their valuation of schedule displacement. To control the large deficit created by the VCG policy, we develop a single-side reward (SSR) pricing policy, which only compensates participants who are forced by the system to endure a schedule displacement. Under the assumption of overpricing tendency (i.e., no participant would want to underreport their value), we show the SSR policy not only generates substantial profits, but also retains the other desired properties of the VCG policy, notably truthful reporting. Even though it cannot rule out underreporting, our simulation experiments confirm that the SSR policy is a robust and deficit-free alternative to the VCG policy. Specifically, we find that (1) underreporting is not a practical concern for a carpool platform as it never reduces the number of matched pairs and its impact on profits is largely negligible; and (2) participants have very little to gain by underreporting their value.

Keywords: carpool matching; schedule displacement; single-side reward pricing; truthful reporting

1 Introduction

Traffic congestion has become a daunting challenge in many mega-cities around the world. The productivity lost to congestion, in the form of wasted time and fuel, has a staggering price tag of about $150 billion a year in the United States [Schrank et al., 2015]. Mitigating traffic congestion

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requires a concerted effort, ranging from improving operational efficiency to regulating car ownership and usage. A seemingly low-hanging fruit, however, is to increase the occupancy rate of private automobiles by encouraging ride sharing. Since 1970s the United States government has been promoting carpool through policies that favor the construction of high occupancy vehicle (HOV) lanes [e.g. Intermodal Surface Transportation Efficiency Act in 1991, see Li et al., 2007]. Judged by the outcome, however, these efforts were largely ineffective: the average car occupancy rate had declined from 1.9 in 1970 to just below 1.4 in 2010 [Sivak, 2013]. Meanwhile, the share of carpool as a transportation mode was only 10.7% in 2008 [Chan and Shaheen, 2012], a 50% drop from its peak in the 1970s that is often attributed to the declining gas price during that period of time [Ferguson, 1997].

While ridesharing offers clear economic incentives to its participants (e.g., reduced operating cost, access to HOV lanes), it also negatively impacts travel experience, of which the loss of flexibility, convenience and privacy is frequently cited [Chan and Shaheen, 2012]. Other issues include the limited pool of peers with which one can share a ride, and the challenge to split the cost fairly [Furuhata et al., 2013]. A new generation of ride sharing platforms are tearing down some of these barriers [e.g. Chan and Shaheen, 2012]. These platforms offer two types of services: *carpool* such as Bla Bla Car[1], waze carpool[2] and Didi Hitch[3], and *ridepool* such as Uber Pool and Lyft Line. The difference between the two is twofold. First, drivers take part in carpool to fulfill their own travel needs, whereas drivers for ride pool do it as a mobility service provider. Second, carpool is arranged ahead of time whereas ridepool typically occurs in real-time. The focus of this study is carpool.

By creating a virtual pool of connected users, modern carpool services promise to reduce the cost of search and coordination. However, bringing together a large number of riders and drivers also creates its own challenge: *how to match them in a fair and efficient way that promotes participation in carpool?* At the heart of this challenge lie two problems: the pricing problem and the matching/routing/scheduling problem. The latter is closely related to the multi-vehicle pick-up and delivery problem, which has been extensively studied in the literature [see Agatz et al., 2012, for a review]. The pricing problem, having received relatively less attention, is the subject of this paper.

Unlike ridepool drivers, a carpool driver does not earn a wage for her labor. Instead, she is compensated for the extra effort incurred when sharing a ride with others. Hence, a desired compensation depends on how the driver values this extra effort, and with whom she is matched. Similarly, the price a rider is willing to pay also varies with the matching partner and her private valuation of the shared ride. Because carpool relies on voluntary participation, it is essential that the service is such priced that both riders and drivers are satisfied. To determine satisfaction, one needs to know how participants value their shared rides, and the most common mechanism to elicit this information is double auction. If a carpool platform prices rides by auction, participants will be asked to report, among other things, how much they value the trip. Upon receiving these ”bids”, the platform decides (1) matched pairs (the winners of the auction); and (2) the price for each rider and the payment to each driver. Ideally, these decisions should ensure that (1) all participants are satisfied (referred to as Individual Rationality, or IR), (2) no participant has incentive to misreport her trip value (referred to as Individual Compatibility, or IC), (3) the social

[1] https://www.blablacar.com/
[2] https://www.waze.com/en-GB/carpool
[3] http://www.didachuxing.com/static/h5/didahome/index.html

welfare is maximized (referred to as Allocative Efficiency, or AE), and (4) the platform is at least revenue neutral (referred to as budget balance, BB). It is well known that [see e.g. Myerson and Satterthwaite, 1983] no pricing policy can simultaneously satisfy IR, IC, AE and BB. Hence, the key is the tradeoff between these conflicting properties.

Existing auction schemes designed for carpool typically assume that a driver should be compensated according to her *detour* cost, proportional to the distance travelled and time spent for picking up and dropping off the rider [e.g. Kleiner et al., 2011, Zhao et al., 2014]. The detour-based valuation is simple and easy to understand, but it misses some important aspects in carpool, notably the impact of schedule disruption to both drivers and riders. Existing schemes impose schedule compatibility as a hard constraint on matching. That is, only participants who have similar schedules would be considered in the same auction pool. This treatment has two shortcomings. First, it completely ignores the tradeoff between the operating cost and the inconvenience caused by a schedule displacement. Empirical evidence suggests that not only does such a tradeoff exist, it is a driving force behind morning commute [Vickrey, 1969, Small, 1982]. Second, only matching users with similar schedules do not always eliminate schedule displacement cost: that a person is willing to tolerate a 5-minute delay does not mean it is not an annoying inconvenience affecting his/her utility.

In this study, we propose that the value of a shared ride to a carpool participant should include both the operating cost and the schedule cost. The former is assumed to be public information determined based on trip distance, vehicle type and so on. The schedule cost, on the other hand, depends on an individual's schedule displacement-—which is defined as the absolute difference between the scheduled and actual arrival times, hence depends on the match—and his/her private valuation of the displacement. Consequently, this setting leads to a *double auction with transaction-dependent bids*, i.e., the bids submitted by both buyers (riders) and sellers (drivers) on the product (shared ride) depend on the actual transaction (the matching result). In other words, the private value of the product to either side of the trade depends on with whom the deal is closed. This type of auction has not been studied in the context of ride sharing, to the best of our knowledge, nor have we come across it in our reading of the economic literature on the auction theory.

We shall show the Vickrey-Clarke-Groves (VCG) pricing policy [Vickrey, 1961, Clarke, 1971, Groves, 1973] ensures AE, IC and IR in our setting. It is worth emphasizing that the proof is not straightforward, although the result is not unexpected. The challenge comes from two sources. First, both the winner's payment and the participant's utility depend on the actual departure time that is endogenous in the matching problem. Second, while a participant submits only one nominal bid (identified by their valuation of the schedule displacement), the platform effectively translates that bid to a different offer for each feasible shared ride. These unique properties considerably complicate the proof of IC.

A notable drawback of the VCG policy is that it often runs a large deficit. This may be justified when a carpool platform is expanding its user base and hence willing to offer subsidies. Eventually, however, the platform needs a different pricing policy in order to turn a profit or at least break even. To this end, we propose a novel pricing scheme built on the idea of rewarding only the participant who suffers a schedule displacement, referred to as single-side-reward (SSR) pricing. The SSR policy is proven to be IR, BB, AE and partial IC. By partial IC, we mean that it is IC only if no participant has the incentive to underreport, i.e., reporting *below* the true value of the schedule displacement. Our simulation results will show that the SSR policy is a robust and deficit-free alternative to the VCG policy. In particular, under this policy, the platform has almost nothing to lose and participants have little to gain from misreporting.

The remainder of the paper is structured as follows. Section 2 briefly reviews the related studies. In Sections 3 and 4, we present the problem setting and define the carpool matching problem, respectively. Section 5 proposes a VCG pricing policy and proves that it has all desired properties except for budget balance. Section 6 addresses the issue of deficit by designing and analyzing the SSR pricing policy. Results from both illustrative numerical examples and large-scale simulation experiments are reported and discussed in Section 7 to validate the analytical results. A summary of findings and further research directions are given in Section 8.

2 Related studies

No attempt is made here to provide a comprehensive review on ridesharing, whose literature has grown rapidly in recent years. The reader is referred to Agatz et al. [2012], Furuhata et al. [2013]. A noteworthy stream of research is concerned with the impact of ridesharing on the surface transportation system, by exploring the interactions between ridesharing price, congestion, users' mode choice and planning decisions (e.g. high occupancy lanes, congestion pricing and parking). These studies may be classified into static [e.g. Reijman et al., 2015, Xu et al., 2015a,b, Di et al., 2018, 2017] and dynamic [e.g. Qian and Zhang, 2011, Xiao et al., 2016, Liu and Li, 2017] problems, depending on the type of models adopted to describe the underlying transportation system. Another line of inquiry, which is the focus of the present study, examine the management and operational problems facing ridesharing platforms. These studies may be classified as addressing "matching" and "pricing" problem, as discussed in what follows.

Agatz et al. [2011] approaches the matching problem in dynamic ridesharing using a rolling horizon strategy. They formulate it as a maximum-weight bipartite matching problem for fixed driver/rider roles, and a general graph matching problem when a participant's role is not fixed. Both problems aim to minimize detour distance and can be solved efficiently. Lee and Savelsbergh [2015] propose to guarantee a minimum matching rate by optimally deploying dedicated drivers. Their problem, formulated as a mixed integer program, is similar to the Team Orienteering Problem with Time Windows. Building on Agatz et al. [2011], Stiglic et al. [2015] allow (1) riders to be picked up at multiple meeting points and (2) multiple riders to be matched with a single driver. The problem can still be formulated as a bipartite matching problem, by representing possible rider combinations as individual nodes in the matching problem. Their simulation results show that introducing meeting points could significantly improve matching efficiency. Cao et al. [2015] treat the matching problem as a driver recommendation problem. Passengers specify their maximum price and waiting time and the system recommends to them a set of so-called skyline drivers, who are essentially no-dominated drivers in terms of price and waiting time. Pelzer et al. [2015] consider a bipartite matching problem that confines match making within optimally configured spatial units called partitions. The optimal match maximizes the distance savings with acceptable inconvenience for all participants (measured by the extra time related to a shared ride). Stiglic et al. [2016] show that in general a greater matching efficiency can be achieved when participants are willing to tolerate greater inconvenience, such as departure time or detour time. Biswas et al. [2017] develop a greedy matching heuristic to maximize profit while giving detour-based discounts. Recognizing that both the matching rate and the profitability per shared ride depend on discounts, they also propose an algorithm that adaptively learns the optimal profit discount factor. Wang et al. [2017] add a weak stability constraint into the standard bipartite matching model to ensure matching stability, i.e., no pair would prefer each other to their current partners [see, e.g. Shapley and Shubik, 1971, Sotomayor, 1999, Gale and Shapley, 2013]. Masoud and Jayakrishnan [2017] consider a general ride-matching problem in which a passenger can be transferred between multiple drivers, and a driver can carry multiple riders. The problem is formulated as a special case of the general pick-up and delivery problem and solved using a specialized decomposition scheme. Alonso-Mora et al. [2017] propose to minimize the total delay of all passengers who request ridesharing service, plus penalty associated with unserved demand. Their algorithm represents rider combinations as individual nodes in a request-trip-vehicle (RTV) shareability graph [similar to Stiglic et al., 2015], identifies feasible trips on the graph, and then solves the matching problem.

For the pricing problem, the classical mechanism is auction. In one of the first applications of auction in ridesharing, Kamar and Horvitz [2009] propose an agent-based carpool (ABC) system in which agents can choose to be either driver or rider. Agents report their cognitive cost associated with driving as a private preference and ABC computes a payment using a variant of the VCG scheme [Vickrey, 1961, Clarke, 1971, Groves, 1973]. To balance budget, a threshold-based mechanism proposed by Parkes et al. [2001] is adopted to impose BB as a hard constraint when calculating payment. This is achieved, however, at the expense of truthful reporting (IC). In Kleiner et al. [2011], a quite different auction setting is adopted: agents must announce their intended role (passenger or driver) when they enter the system, and the auction is one side in that its primary function is to rank passengers based on their bids to a driver. To ensure IC, a second-price payment (i.e., the winner pays the price of the next bidder in the ranking) is employed. Zhao et al. [2014] consider a ridesharing system in which agents are matched based on their reported preferences for (1) departure time window, (2) number of available seats, and (3) private trip cost per unit distance. They show that the VCG pricing scheme is AE, IC and IR but leads to large deficit. Two revised pricing schemes are considered for deficit control. The first is a fixed payment scheme that ensures IC and IR but has no control over efficiency loss. Their second scheme, called two-sided reserve pricing, aims to offer a measured tradeoff between efficiency loss and deficit. It first excludes from matching the agents whose reported trip cost is higher (lower) than the reserve price as a driver (rider), and then cap the resulting VCG payment using the reserved prices. While both schemes can achieve weakly BB if detour is prohibited, their impact on the matching rate is unclear. Zhao et al. [2015] examine the pricing issue in ridesharing with uncertainty, which assumes that an agent has a privately known probability of completing a trip, in addition to its valuation. They found no mechanism that can maximize efficiency and ensure truthful reporting except in extremely restrictive cases. Instead, an ex-post payment scheme, which pays agents based on their realized actions rather than what they report, is designed. In Zhang et al. [2016], participant are assumed to bid based on their valuation of the trip's unit price. The focus is to design a discounted trade reduction (DTR) mechanism to ensure high trading volume (matching rate). Zhang et al. [2018] consider a ridesharing system in which the driver imposes a different reserve price based on original-destination information. To ensure budget balance, they design a one-sided ridesharing market with variable reserve price constraints and showed it is an individually rational, truthful, and computationally efficient mechanism. Balseiro et al. [2019] design a ridesharing pricing scheme applied over a finite horizon and impose a set of constraints to ensure periodic individual rationality, dynamic incentive compatibility, no positive transfers, and promise keeping. Their design objective, however, is to maximize the profit of the platform. Bian and Liu [2019] apply ridesharing in a first-mile problem that simultaneously makes matching, vehicle routing and pricing decisions. Their matching problem is one-sided since all drivers are controlled by a service provider. Under this setting, a VCG mechanism can ensure IC, IR, AE and BB.

A few studies consider non-auction-based pricing policies for ridesharing. Using the game-theoretic concept of the kernel, Bistaffa et al. [2015] propose a fair pricing scheme that pays an agent so as to eliminate the incentive to switch partners. Such a pricing scheme effectively enables a stable matching as in Wang et al. [2017] and Rasulkhani and Chow [2019]. Peng et al. [2018] propose a stable matching ridesharing model to minimize the total travel cost. They incorporate participants' preference for gender as a constrain into the matching model and consider equity and incentives in payment design. Furuhata et al. [2015] and Gopalakrishnan et al. [2016] consider cost-sharing schemes in cases where riders may be dynamically added to any existing group that is already executing a share-ride. Furuhata et al. [2015] suggest that such an online pricing scheme should have five desirable properties: online fairness (agents should be better off by submitting request earlier), immediate response (travelers must be provided with an upper bound on their shared cost immediately), IR, BB, and ex-post incentive compatibility (the dominating strategy is to submit the request truthfully). They design a scheme that can satisfy all five properties, as long as the matching satisfy certain conditions (e.g. minimizing the operating cost). Gopalakrishnan et al. [2016] introduce the concepts of sequential individual rationality (SIR)—similar to the online fairness criterion in Furuhata et al. [2015]—and the sequential fairness (SF), which requires that the benefit of sharing obtained from accommodating new riders is equitably distributed to all users. They show that imposing SIR-feasible constraints renders the real-time matching problem NP-hard.

To summarize, no existing pricing policy for ridesharing had allowed carpoolers to monetize the inconvenience associated with schedule displacement, as widely assumed in the literature of transportation economics. Another issue overlooked in the literature is the impact of deficit control measures on matching efficiency and truthful reporting. We set out to fill these gaps.

3 Problem setting

Consider a carpool platform that receives matching requests from a set of participants in a decision period. For simplicity, we assume that participants must declare their intended role $\iota \in \{d, r\}$ (d and r stand for driver and rider respectively), when they submit the request. Each participant intends to complete a trip in a network that consists of a set of nodes N. A trip is described by a *schedule*, defined as follows.

Definition 1 (Schedule). *The schedule of a participant of role ι is represented as a 4-tuple $(s^\iota, t^\iota, \tau^\iota, b^\iota)$, where $s^\iota \in N$, $t^\iota \in N$ and τ^ι denote the origin, the destination, and the desired arrival time of the trip, respectively. $b^\iota \geq 0$ is the participant's self-reported sensitivity to schedule displacement (£ per unit time).*

In this study, we assume that a participant always truthfully reports $s^\iota \in N$, $t^\iota \in N$ and τ^ι, but has the incentive to misreport b^ι. It is difficult to envision a scenario in which a person can benefit from lying to the platform about her origin, destination or desired arrival time, even if it is feasible. If they misreport those elements, they may well end up in a wrong place at a wrong time. Treating b^ι as the participant's bid is appropriate for two reasons. First, this information is less transparent and more heterogeneous compared to the detour distance and operating cost.

Second, b^i directly affects how much detour a driver is willing to accept and how much a rider is willing to pay for a shared ride. Hence, it can drive the matching and pricing decisions of the platform.

We make the following assumption about participants' bidding behavior, following the classic literature on auction [e.g. Myerson and Satterthwaite, 1983].

Assumption 1. *Participants would report false valuation if doing so benefits them, but would not collude with others.*

The schedule of driver $i \in I = \{1, \cdots, n\}$ is denoted as $\left(s_i^d, t_i^d, \tau_i^d, b_i^d\right)$ and the schedule of rider $j \in J = \{1, \cdots, m\}$ is denoted as $\left(s_j^r, t_j^r, \tau_j^r, b_j^r\right)$. We further define

- h_j^r as the shortest travel time from s_j^r (origin of rider j) to t_j^r (destination of rider j);
- $h_{ij}^{d\to r}$ as the shortest travel time from s_i^d (origin of driver i) to s_j^r (origin of rider j);
- $h_{ji}^{r\to d}$ be the shortest travel time from t_j^r (destination of rider j) to t_i^d (destination of driver j);
- $\theta_{ij} = h_{ij}^{d\to r} + h_j^r$; and
- $\eta_{ij} = \theta_{ij} + h_{ji}^{r\to d}$.

θ_{ij} and η_{ij} are the time required to reach the destination of rider j and driver i, respectively, when they are matched by the platform. For a matched pair ij, the reported schedule displacement is defined for driver i and rider j as

$$\xi_{ij}^d = \left|\tau_i^d - \delta_{ij} - \eta_{ij}\right| \quad \text{and} \quad \xi_{ij}^r = \left|\tau_j^r - \delta_{ij} - \theta_{ij}\right|, \tag{1}$$

respectively, where δ_{ij} is driver i 's departure time when matched with rider j. Note that δ_{ij} is determined in the matching process and hence affected by how participants report their schedules. We are now ready to introduce the participants' valuation policy based on their schedule.

Assumption 2 (Schedule-based valuation policy)**.** *Each participant values late and early arrival equally*[4]*. That is, if driver $i \in I$ is matched with rider $j \in J$ and that driver i departs at δ_{ij}, driver i and rider j value the shared trip respectively as*

$$p_{ij}^d\left(b_i^d, \delta_{ij}\right) = \alpha\eta_{ij} + b_i^d \xi_{ij}^d \quad \text{and} \quad p_{ij}^r\left(b_j^r, \delta_{ij}\right) = \beta h_j^r - b_j^r \xi_{ij}^r, \tag{2}$$

where α is a constant price per unit travel time paid to the driver and β is a constant price per unit travel time charged on the rider, both are pubic information determined by the platform.

The above valuation policy considers both the operating cost and the schedule cost. It implies that (1) a driver should be compensated more for a longer travel time; (2) participants should be compensated more for a greater schedule displacement; (3) different participants may value the same schedule displacement differently, based on their private information; (4) the platform could affect the valuations of any given pair by changing the departure time. Importantly, while

[4] In reality most people would likely value being late more than being early. However, the theoretical results presented herein can be easily extended to accommodate the case of asymmetric schedule cost.

each participant only submits a schedule that contains their private valuation of schedule disruption, the platform can convert it to as many bids as needed by applying the pricing formula defined in Assumption 2. This is the essential feature that distinguishes our study from others'.

We emphasize that the schedule displacements and valuations given in Eqs. (1) and (2) correspond to *reported* schedules by each participant. When they report truthfully, these variables will be rewritten with a $\bar{\cdot}$ or a $\tilde{\cdot}$ to highlight the difference, depending on whether the driver or the rider is the truth-telling participant. Specifically, $\bar{\xi}^{\iota}_{ij}$ ($\tilde{\xi}^{\iota}_{ij}$) and $\bar{p}^{\iota}_{ij}$ ($\tilde{p}^{\iota}_{ij}$) denote, respectively, the schedule displacement and valuation for participant ι in a matched pair ij when driver i (rider j) truthfully reports her valuation. Let $\bar{b}^d_i$ and $\tilde{b}^r_j$ be the truthful value of schedule displacement for driver i and rider j, respectively. The truthful valuations corresponding to truthful reporting by driver or rider are defined as:

$$\bar{p}^d_{ij}\left(\bar{b}^d_i, \bar{\delta}_{ij}\right) = \alpha\eta_{ij} + \bar{b}^d_i \bar{\xi}^d_{ij} \quad ; \quad \bar{p}^r_{ij}\left(b^r_j, \bar{\delta}_{ij}\right) = \beta h^r_j - b^r_j \bar{\xi}^r_{ij}. \tag{3}$$

$$\tilde{p}^d_{ij}\left(b^d_i, \tilde{\delta}_{ij}\right) = \alpha\eta_{ij} + b^d_i \tilde{\xi}^d_{ij} \quad ; \quad \tilde{p}^r_{ij}\left(\tilde{b}^r_j, \tilde{\delta}_{ij}\right) = \beta h^r_j - \tilde{b}^r_j \tilde{\xi}^r_{ij}. \tag{4}$$

4 Carpool matching problem

Upon receiving all reported schedules, the platform solves the carpool matching (CaMa) problem—also known as the winner determination problem in an auction—in order to determine (1) the matched pairs (winning bidders), (2) the departure time of each matched driver, and (3) the pricing policy: specifically, for each matched pair ij, the payment to driver i, denoted as q^d_i, and the price charged on rider j, denoted as q^r_j. Again, q^d_i and q^r_j are obtained based on reported schedules. Assuming truthful reporting from driver i (rider j), the payment (price) to driver i is denoted as $\bar{q}^d_i$ ($\tilde{q}^d_i$) and the charge (price) on rider j is denoted as $\bar{q}^d_j$ ($\tilde{q}^r_j$).

To formulate the CaMa problem, let x_{ij} represent the allocation incidence, i.e., $x_{ij} = 1$ if driver i is matched with rider j, and 0 otherwise. The goal of the platform is to maximize the allocative efficiency, or social welfare, which can also be interpreted as the maximal profit that the platform could earn by pricing all matches directly based on bids. In a double side market, this loosely means matching buyers (riders) with highest valuation of an item (a shared ride) and sellers (drivers) with lowest valuation of that item. Hence, for each matched pair ij, the social welfare is defined as

$$a_{ij} = p^r_{ij} - p^d_{ij}, \tag{5}$$

which is the reported valuation of driver i less that of rider j. The CaMa problem can be formulated as the following mixed integer program (MIP)

CaMa Problem
$$\max \sum_{i\in I}\sum_{j\in J} a_{ij}x_{ij} \tag{6a}$$

subject to:

$$\sum_{j\in J} x_{ij} \leq 1, \forall i \in I, \tag{6b}$$

$$\sum_{i\in I} x_{ij} \leq 1, \forall j \in J, \tag{6c}$$

$$\delta_{ij} \geq 0, \forall i \in I, \forall j \in J, \tag{6d}$$

$$x_{ij} \in \{0,1\}, \forall i \in I, \forall j \in J. \tag{6e}$$

The above formulation defines a set of feasible CaMa points, denoted as $\mathcal{Y}$. It also yields an optimal CaMa point $\boldsymbol{y}^* = [\boldsymbol{x}^*, \boldsymbol{\delta}^*, \boldsymbol{q}^*] \in \mathcal{Y}$, where $\boldsymbol{x}^*$, $\boldsymbol{\delta}^*$, $\boldsymbol{q}^*$ are vectors of optimal allocation incidence, departure time for matched drivers and pricing policy for matched participants, respectively. By default, the pricing policy $\boldsymbol{q}^*$ is set according to the reported valuation vector $\boldsymbol{p}$. This policy, however, may not ensure truthful reporting. To clarify this issue, let us first explain what it takes to be a desired CaMa point. We first define the utility of the carpool participants based on the pricing policy.

Definition 2 (Utility of carpool participants)**.** *For a matched pair ij, the utility of driver i and rider j is defined, respectively, as*

$$u_{ij}^d = q_i^d - \bar{b}_i^d \left(\zeta_{ij}^d - \bar{\zeta}_{ij}^d\right) - \bar{p}_{ij}^d \quad \textit{and} \quad u_{ij}^r = \tilde{p}_{ij}^r - \tilde{b}_j^r \left(\zeta_{ij}^r - \tilde{\zeta}_{ij}^r\right) - q_j^r. \tag{7}$$

Any participant who is not matched has a utility of zero.

It is worth emphasizing that the typical definition of utility in auction does not have the second term in the above. Take the driver's utility for example. The first term is the payment from the platform and the third is the true valuation. In a typical auction setting, the difference between the two would define the utility of the driver. In our case, this is not sufficient because the driver's bid could affect the schedule displacement. If misreporting helps reduce the displacement, i.e. $\zeta_{ij}^d - \bar{\zeta}_{ij}^d < 0$, then the driver's utility would increase; hence the need to include the second term in the utility. We shall see that having this term in the utility does not violate the well-known properties of the VCG-type pricing, but it does complicate the proof of these properties considerably.

Eq. (7) defines the utility according to reported schedules. Assuming that driver i reports truthfully, the utilities are redefined as

$$\bar{u}_{ij}^d = \bar{q}_i^d - \bar{p}_{ij}^d \quad \text{and} \quad \bar{u}_{ij}^r = \bar{p}_{ij}^r - \bar{q}_j^r. \tag{8}$$

Also when rider j truthfully reports, the utilities are

$$\tilde{u}_{ij}^d = \tilde{q}_i^d - \tilde{p}_{ij}^d \quad \text{and} \quad \tilde{u}_{ij}^r = \tilde{p}_{ij}^r - \tilde{q}_j^r. \tag{9}$$

With the definition of the utility we can now describe the desired properties of any feasible CaMa point.

Definition 3 (Desired properties of a feasible CaMa point)**.** *Let $\boldsymbol{y} \in \mathcal{Y}$ be a feasible CaMa point, and $\tilde{I}$ and $\tilde{J}$ be the set of matched drivers and riders corresponding to $\boldsymbol{y}$. $\boldsymbol{y}$ is said to be*

budget balancing (BB) *if $\sum_{i\in\tilde{I}} q_i^d = \sum_{j\in\tilde{J}} q_j^r$ or weak budget balancing (WBB) if $\sum_{i\in\tilde{I}} q_i^d \le \sum_{j\in\tilde{J}} q_j^r$;*

individual rational (IR) *if $q_i^d \ge p_{ij}^d$ for each driver $i \in \tilde{I}$ and $q_j^r \le p_{ij}^r$ for each rider $j \in \tilde{J}$;*

individual compatible (IC) *if no participant can improve her utility by misreporting her value of schedule displacement; and*

allocative efficient (AE) *if the total social welfare is maximized, i.e., $\boldsymbol{y}$ solves Problem 6.*

5 Pricing policy ensuring truthful reporting

In this section, we first show that the optimal departure time $\boldsymbol{\delta}^*$ can be determined a priori, independent of the CaMa Problem (6). This means that (6) can be reduced from MIP to an integer program. We then prove that replacing the default optimal pricing policy $\boldsymbol{q}^*$ in a solution to (6), i.e. $\boldsymbol{y}^* = [\boldsymbol{x}^*, \boldsymbol{\delta}^*, \boldsymbol{q}^*]$, with a VCG-type policy $\boldsymbol{q}^+$ [Vickrey, 1961, Clarke, 1971, Groves, 1973] would lead to a CaMa point $\boldsymbol{y}^+ = [\boldsymbol{x}^*, \boldsymbol{\delta}^*, \boldsymbol{q}^+]$ that satisfies three of the four desired properties listed in Definition 3: IC, IR and AE, despite that bid valuations vary significantly with the matching result.

5.1 Property of optimal departure time

While $\boldsymbol{\delta}$ is a decision vector in Problem (6), it has an important property that allows us to determine it independently.

Lemma 1 (Property of optimal departure time)**.** *Let* $[\boldsymbol{x}^*, \boldsymbol{\delta}^*]$ *be an optimal solution to Problem (6),* δ^*_{ij} *must satisfy the following condition for any matched pair ij:*

$$\begin{cases} \delta^*_{ij} = \tau^r_j - \theta_{ij}, & b^r_j > b^d_i; \\ \delta^*_{ij} = \tau^d_i - \eta_{ij}, & b^r_j < b^d_i; \\ \delta^*_{ij} \in [\kappa_1, \kappa_2], & b^r_j = b^d_i, \end{cases} \tag{10}$$

where $\kappa_1 = \min\left(\tau^r_j - \theta_{ij}, \tau^d_i - \eta_{ij}\right)$ *and* $\kappa_2 = \max\left(\tau^r_j - \theta_{ij}, \tau^d_i - \eta_{ij}\right)$.

Proof: Suppose that for the optimal solution $[\boldsymbol{x}^*, \boldsymbol{\delta}^*]$ there exists a matched pair ij such that Condition (10) is not satisfied. Recalling Eqs. (1, 2, 5), we have

$$a_{ij} = \beta h^r_j - b^r_j \left|\tau^r_j - \delta_{ij} - \theta_{ij}\right| - \alpha\eta_{ij} - b^d_i \left|\tau^d_i - \delta_{ij} - \eta_{ij}\right|.$$

Maximizing a_{ij} equals minimizing $b^r_j \left|c_1 - \delta_{ij}\right| + b^d_i \left|c_2 - \delta_{ij}\right|$, where $c_1 = \tau^r_j - \theta_{ij}$ and $c_2 = \tau^d_i - \eta_{ij}$.

When $b^r_j > b^d_i$, the function is minimized when δ_{ij} is set such that the displacement of rider j is zero (i.e., $\delta_{ij} = c_1$, point B in Figure 1); otherwise, it should set the displacement of driver i as zero (i.e., $\delta_{ij} = c_2$, point A in Figures 1). For visualization, Figure 1 plots $b^r_j \left|c_1 - \delta_{ij}\right| + b^d_i \left|c_2 - \delta_{ij}\right|$ as a function of δ_{ij} when $b^r_j > b^d_i$. When $b^r_j = b^d_i$, the line AB in Figure 1 would be horizontal, hence a_{ij} will be maximized on any point on AB, i.e., when $\delta^*_{ij} \in [\kappa_1, \kappa_2]$. Therefore, if Condition (10) is not satisfied, one can always increase a_{ij} by changing the departure time δ_{ij}, hence increase the objective function value in Problem (6). This suggests that $[\boldsymbol{x}^*, \boldsymbol{\delta}^*]$ is not an optimal solution, a contradiction. □

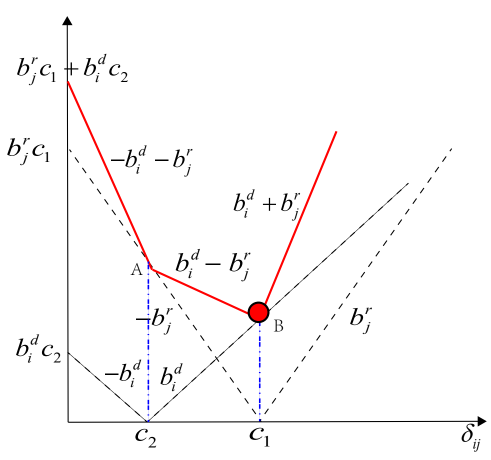


Figure 1: Illustration of the function $b^r_j \left|c_1 - \delta_{ij}\right| + b^d_i \left|c_2 - \delta_{ij}\right|$ (represented by the thick red line).

Lemma 1 implies that the optimal departure time may not be unique. To avoid this non-uniqueness, we redefine the optimal departure time as follows:

$$\begin{cases} \delta_{ij}^* = \tau_j^r - \theta_{ij}, & b_j^r > b_i^d; \\ \delta_{ij}^* = \tau_i^d - \eta_{ij}, & b_j^r \leq b_i^d. \end{cases} \tag{11}$$

With this result, Problem (6) can be reduced from a mixed integer program to a standard bipartite matching problem, as asserted below.

Proposition 1. *Problem (6) is equivalent to the 0-1 integer program (12), with a_{ij}^* in (12a) being defined by the departure time condition of (11).*

$$\textbf{\textit{Reduced CaMa Problem}} \qquad \max \sum_{i \in I} \sum_{j \in J} a_{ij}^* x_{ij} \tag{12a}$$

$$\textit{subject to: (6b, 6c, 6e)} \tag{12b}$$

Proof: Lemma 1 implies that the determination of the departure time (by the optimal condition (11)) is not affected by the allocation. Hence, we can set a_{ij} based on (11) a priori, and then solve a reduced version of (6) as defined by (12). □

We note that the feasible region is not empty since the platform can simply choose to do nothing. Therefore, the solution existence of the CaMa problem is obvious. The reduced CaMa problem is a standard bipartite matching problem that can be reduced to a linear program (LP), since its coefficient matrix satisfies total unimodularity. Thus, the problem should always produces a unique maximum social welfare, even though there may be multiple ways to pair the participants to achieve it.

In what follows, we will use $V = \sum_{i \in I} \sum_{j \in J} a_{ij}^* x_{ij}^*$ to denote the optimal objective function value of Problem (12), or the maximum social welfare generated by the system. Accordingly, V_{-i}, V_{-j} and V_{-ij} are introduced to represent, respectively, the maximum social welfare of the system when driver i, rider j, and both driver i and rider j withdraw their requests from the platform.

5.2 VCG pricing policy

Once Problem (12) is solved, the platform obtains $\boldsymbol{x}^*$, i.e., the winning participants in matched pairs. Yet, the platform cannot charge riders and pay drivers simply based on their reported valuations, owing to the possibility of misreporting. A conventional remedy is the VCG pricing policy that is known to ensure truthful reporting in classical auctions. However, the CaMa problem differs from the classical settings in two aspects. First, because the auction is double sided, misreporting means not only that a winner can become a loser (or vice versa), but also that a winner can be matched with a different partner. Second and more important, whenever a mismatch occurs, the participants involved end up buying (or selling) a completely different product, since the valuation of a shared ride depends on the matching partners. We show in this section that the VCG policy is still valid in this non-orthodox setting.

We first describe how VCG pricing policy may be implemented. For each optimally matched pair ij, the VCG charge on rider j is defined as q_j^{r+}, and the VCG payment to driver i as q_i^{d+}. The implementation of the VCG policy is summarized in **Algorithm** 1. We next show that the CaMa point $\boldsymbol{y}^+ = [\boldsymbol{x}^*, \boldsymbol{\delta}^*, \boldsymbol{q}^+]$ is IR, IC and AE.

Algorithm 1 VCG pricing policy

Input: schedules submitted from driver $i \in I$, $\left(s_i^d, t_i^d, \tau_i^d, b_i^d\right)$, and rider $j \in J$, $\left(s_j^r, t_j^r, \tau_j^r, b_j^r\right)$.
Output: the optimal matching pair $\boldsymbol{x}^*$, the optimal departure time $\boldsymbol{\delta}^*$ and VCG pricing $\boldsymbol{q}^+$.
Winner determination problem:
For any pair ij, set a_{ij}^* by determining $\boldsymbol{\delta}^*$ using Eqs. (1, 2, 5, 11).
Solve Problem (12) to obtain V and $\boldsymbol{x}^*$. Let $\tilde{I}$ and $\tilde{J}$ denote the set of matched drivers and riders in $\boldsymbol{x}^*$.
Set VCG price:
for every driver $i \in \tilde{I}$ **do**
    Calculate V_{-i} by re-solving Problem (12) without driver i.
    Set the bonus for driver i as $\rho_i^{d+} = V - V_{-i}$.
    Set the VCG payment to driver i as $q_i^{d+} = p_{ij}^d + \rho_i^{d+}$.
end for
for every rider $j \in \tilde{J}$ **do**
    Calculate V_{-j} by re-solving Problem (12) without rider j.
    Set the discount for rider j as $\rho_j^{r+} = V - V_{-j}$.
    Set the VCG charge on rider j as $q_j^{r+} = p_{ij}^r - \rho_j^{r+}$.
end for
return $\boldsymbol{x}^*$, $\boldsymbol{\delta}^*$ and $\boldsymbol{q}^+$.

That $\boldsymbol{y}^+$ is AE is self-evident, since it corresponds to $\boldsymbol{x}^*$ which is obtained by solving Problem (12). The proof of IR is also straightforward.

Proposition 2. *The CaMa point* $\boldsymbol{y}^+ = [\boldsymbol{x}^*, \boldsymbol{\delta}^*, \boldsymbol{q}^+]$ *is IR.*

Proof: It is easy to see that $V \geq V_{-i}$ and $V \geq V_{-j}$, since V_{-i} and V_{-j} are both the social welfare of a subset of the users included in the system that yields V. Hence, ρ_i^{d+} and ρ_j^{r+} must be non-negative as per **Algorithm** 1. According to Definition 3, $\boldsymbol{y}^+$ is IR. □

In order to prove that the VCG pricing policy ensures truthful reporting (i.e. IC), we first give the following lemmas.

Lemma 2. *With the VCG pricing policy, if driver i is not matched when she truthfully reports her valuation of schedule displacement $\bar{b}_i^d$, she is also not matched when she reports the valuation as $b_i^d = \bar{b}_i^d + \Delta$, where $\Delta > 0$.*

Proof: See Appendix B. □

Lemma 3. *With the VCG pricing policy, if driver i is not matched when she truthfully reports her valuation of schedule displacement $\bar{b}_i^d$, she cannot increase her utility by reporting the valuation as $b_i^d = \bar{b}_i^d - \Delta$ where $\Delta > 0$.*

Proof: See Appendix C. □

Lemma 4. *With the VCG pricing policy, suppose that driver i is matched with rider j when she truthfully reports her bid valuation $\bar{b}_i^d$ and she is matched with rider k when she misreports her valuation $b_i^d = \bar{b}_i^d + \Delta$ where $\Delta \neq 0$. Then, $\bar{u}_{ij}^d \geq u_{ik}^d$, i.e. she cannot improve her utility by misreporting.*

Proof. See Appendix D. □

We are now ready to present the main result.

Proposition 3. *The CaMa point* $\boldsymbol{y}^+ = [\boldsymbol{x}^*, \boldsymbol{\delta}^*, \boldsymbol{q}^+]$ *is IC.*

Proof: Let's first consider the case when driver i is not matched with truthful reporting. If she reports higher than the true value, she will not be matched, as shown in Lemma 2. In this case, her utility would not change. If she reports a valuation lower than her true value, Lemma 3 asserts that she cannot improver her utility either.

Then suppose that driver i is matched with rider j with truthful reporting. If misreporting results in a failure to match, her utility would become zero and definitely not increase. If instead she is matched with misreporting, Lemma 4 shows that under this condition she also cannot improve her utility. Therefore, we conclude that, with the VCG pricing policy, under no circumstance could the driver improve the utility by lying about her private valuation. In other words, bid truthfully is a dominant strategy for any driver. It is easy to see that the same argument can be repeated for each rider. The proof is completed. □

The VCG pricing policy achieves truthful reporting by promising drivers and riders rewards on top of their stated value of a shared ride. The problem is that this success could come with a high cost in the form of external subsidies. The amount of subsidy required to run the VCG pricing policy can be computed as follows:

$$\Phi = V - \sum_{i \in \tilde{I}} \rho_i^{b+} - \sum_{j \in \tilde{J}} \rho_j^{r+}, \tag{13}$$

where V is the optimal objective function value in Problem (12), and ρ_j^{r+} and ρ_i^{d+} are rewards given to riders and drivers defined in **Algorithm** 1. However, soon or later the platform will have to control or eliminate the budget deficit, i.e. achieving BB, by striking a balance between the desired properties. We now turn to addressing to this important issue.

6 Single-side-reward (SSR) pricing

In order to control deficit, we need to reduce either the bonus for drivers or the discount for riders. Since only one participant of any matched pair would be subject to schedule displacement when Problem (12) is solved (as per Lemma 1), we postulate that it may be unnecessary to reward the participant whose punctual arrival is guaranteed. In what follows, we shall design such a single-side-reward (SSR) pricing policy to balance the budget, at the expense of potentially weakening the protection against misreporting.

To completely rule out misreporting under the SSR pricing, a stronger behavioral assumption is introduced as follows.

Assumption 3 (Overpricing tendency)**.** *When a participant attempts to game the carpool system by misreporting her valuation of schedule displacement, she always reports a value higher than her true valuation.*

Algorithm 2 presents the SSR pricing policy. In what follows, we denote $\boldsymbol{y}^- = \{\boldsymbol{x}^*, \boldsymbol{\delta}^*, \boldsymbol{q}^-\}$ as an optimal CaMa point associated with the SSR pricing $\boldsymbol{q}^-$.

Since $\boldsymbol{y}^-$ corresponds to a solution to Problem (12), it is clearly allocative efficient. We next show that it is also IR, IC and weak BB.

Proposition 4. *The CaMa point* $\boldsymbol{y}^- = \{\boldsymbol{x}^*, \boldsymbol{\delta}^*, \boldsymbol{q}^-\}$ *obtained from **Algorithm** 2 is IR.*

Algorithm 2 Single-side-reward (SSR) pricing policy

```
Input: schedules submitted from driver i ∈ I, (s_i^d, t_i^d, τ_i^d, b_i^d), and rider j ∈ J, (s_j^r, t_j^r, τ_j^r, b_j^r).
Output: the optimal matching pair x*, the optimal departure time δ* and SSR pricing q^-.
Winner determination problem:
For any pair ij, set a*_ij by determining δ* using Eqs. (1, 2, 5, 11).
Solve Problem (12) to obtain V and x*. Let Ĩ and J̃ denote the set of matched drivers and riders in x*.
Set SSR price:
for every winning pair (i, j) do
    if b_i^d < b_j^r then
        Calculate V_{-i} by re-solving Problem (12) without driver i.
        Set the bonus for driver i as ρ_i^{d-} = V − V_{-i}.
        Set the SSR payment to driver i as q_i^{d-} = p_ij^d + ρ_i^{d-}.
        Set the SSR charge on rider j as q_j^{r-} = p_ij^r.
    else
        Calculate V_{-j} by re-solving Problem (12) without rider j.
        Set the SSR payment to driver i as q_i^{d-} = p_ij^d.
        Set the discount for rider j as ρ_j^{r-} = V − V_{-j}.
        Set the SSR charge on rider j as q_j^{r-} = p_ij^r − ρ_j^{r-}.
    end if
end for
return x*, δ* and q^-.
```

Proof: If $b_i^d < b_j^r$, we have $\delta_{ij}^* = \tau_j^r - \theta_{ij}$ according to Lemma 1. In this case, driver i essentially receives a VCG payment, which satisfies IR as per Proposition 2. Rider j's charge is set to her valuation, hence also satsifies IR. The case of $b_i^d \geq b_j^r$ can be proven similarly. □

Proposition 5. *The CaMa point $\boldsymbol{y}^- = \{\boldsymbol{x}^*, \boldsymbol{\delta}^*, \boldsymbol{q}^-\}$ obtained from **Algorithm** 2 is IC, if Assumption 3 holds.*

Proof: We shall prove bidding truthfully is a dominant strategy for any driver i. The proof for rider j can be carried out similarly. Assumption 3 implies that $b_i^d = \bar{b}_i^d + \Delta$ where $\Delta > 0$. In this case, if driver i is not matched with truthful bid, she will not be matched when she misreports her valuation, according to Lemma 2.

Assuming that driver i is matched with rider j when she truthfully bids, the following two cases must be considered separately.

Case 1 $\bar{b}_i^d < b_j^r$. When driver i truthfully bids, her utility equals the bonus $\bar{\rho}_i^{d-} = \bar{V} - \bar{V}_{-i} \geq 0$. When she misreports the valuation, there are two possibilities as discussed below:

- she is not matched. In this case, her utility is 0, meaning she is not better off.
- she is matched with rider k.
  - If her bid $b_i^d < b_k^r$, we have

$$\bar{u}_{ij}^{d-} = \bar{u}_{ij}^{d+} \geq u_{ik}^{d+} = u_{ik}^{d-}. \tag{14}$$

Here the first equality and second equality hold because the SSR policy rewards her with a VCG payment as per **Algorithm** 2 in this case; the inequality holds because of Proposition 3.

– If her bid $b_i^d \geq b_k^r$, we have

$$\bar{u}_{ij}^{d-} = \bar{u}_{ij}^{d+} \geq u_{ik}^{d+} \geq u_{ik}^{d-}. \tag{15}$$

The last inequality holds because the SSR policy gives no bonus to driver i in this case whereas the VCG policy gives a non-negative bonus.

Case 2 $\bar{b}_i^d \geq b_j^r$. With truthful reporting driver i's utility would be zero. If driver i misreports her valuation as $b_i^d = \bar{b}_i^d + \Delta$ where $\Delta > 0$, we claim that driver i will always be matched with rider j, hence her utility remains the same. To see this, note that $b_i^d > \bar{b}_i^d \geq b_j^r$, we have $\bar{\delta}_{ij}^* = \delta_{ij}^* = \tau_i^d - \eta_{ij}$ and then we have

$$\begin{aligned}
&\bar{a}_{ij} - a_{ij} \\
=& - b_j^r \left|\tau_j^r - \bar{\delta}_{ij}^* - \theta_{ij}\right| - \bar{b}_i^d \left|\tau_i^d - \bar{\delta}_{ij}^* - \eta_{ij}\right| + b_j^r \left|\tau_j^r - \delta_{ij}^* - \theta_{ij}\right| + b_i^d \left|\tau_i^d - \delta_{ij}^* - \eta_{ij}\right| \\
=& \Delta \left|\tau_i^d - \delta_{ij}^* - \eta_{ij}\right| = 0.
\end{aligned} \tag{16}$$

For any $g \neq j$ where $g \in J$, here are two possibilities as discussed:

- $\bar{\delta}_{ig}^* = \delta_{ig}^*$. In this case, we have

$$\bar{a}_{ig} - a_{ig} = \Delta \left|\tau_i^d - \delta_{ig}^* - \eta_{ig}\right| \geq 0. \tag{17}$$

- $\bar{\delta}_{ig}^* \neq \delta_{ig}^*$. Using the similar argument in the proof for Lemma 2, we claim $\bar{b}_i^d < b_g^r \leq b_i^d$ and then $\bar{\delta}_{ig}^* = \tau_g^r - \theta_{ig}$ and $\delta_{ig}^* = \tau_i^d - \eta_{ij}$. Thus we have

$$\begin{aligned}
\bar{a}_{ig} - a_{ig} &= -\bar{b}_i^d \left|\tau_i^d - \bar{\delta}_{ig}^* - \eta_{ig}\right| + b_g^r \left|\tau_g^r - \delta_{ig}^* - \theta_{ig}\right| \\
&= \left(b_g^r - \bar{b}_i^d\right) \left|\tau_i^d - \tau_g^r + \theta_{ig} - \eta_{ig}\right| \geq 0.
\end{aligned} \tag{18}$$

Combing Eq. (16), (17) and (18), we obtain

$$\bar{a}_{ij} - a_{ij} \leq \bar{a}_{ig} - a_{ig}, \forall g \in J. \tag{19}$$

Therefore, matching i with any $g \neq j$ cannot increase the objective function value of Problem (12).

Combining the two cases above proves that bidding truthfully is a dominant strategy for any driver $i \in I$. We can show the same for rider $j \in J$. This completes the proof. □

We are finally ready to present the main result, i.e., the SSR policy could at least break even.

Proposition 6. *The CaMa point $\boldsymbol{y}^- = \{\boldsymbol{x}^*, \boldsymbol{\delta}^*, \boldsymbol{q}^-\}$ obtained from **Algorithm** 2 is weak BB.*

Proof: Suppose pair ij is matched. It is clear that

$$a_{ij} = p_{ij}^r - p_{ij}^d \geq 0, \tag{20}$$

for otherwise the platform is better off not matching them at all. We also have

$$V = a_{ij} + V_{-ij} \leq a_{ij} + V_{-i}, \tag{21}$$

because V_{-ij} is the social welfare of a subset of the users included in the system that yields V_{-i}. If $b_i^d < b_j^r$, rider $j's$ charge is $q_j^{r-} = p_{ij}^r$ and driver $i's$ payment is $q_i^{d-} = p_{ij}^d + V - V_{-i}$, based on the SSR pricing policy. The difference between the two

$$q_j^{r-} - q_i^{d-} = p_{ij}^r - p_{ij}^d - V + V_{-i} \geq p_{ij}^r - p_{ij}^d - a_{ij} = 0, \quad (22)$$

where the first inequality is due to Eq. (21) and the second is due to Eq. (20). The proof is similar when $b_i^d \geq b_j^r$. Since for each matched pair ij, the platform will never pay the driver i more than it charges rider j, the profit is always non-negative. This completes the proof. □

What happens to the observation of IC when Assumption 3 (overpricing tendency) is violated? First, if with a truthful bid driver i is not matched, then bidding below her true value would not increase her utility. To see why this is the case, note that driver i's utility under the SSR policy is always equal to or lower than that under the VCG policy. Per Lemma 3, the VCG policy cannot improve driver i's utility under this circumstance; hence neither could the SSR policy.

Now suppose that driver i is matched with rider j by truthful reporting. We can show that in this case bidding below her true valuation would not help her either, if her true value $\bar{b}_i^d < b_j^r$ (see Case 1 in the proof of Proposition 5). Only when $\bar{b}_i^d \geq b_j^r$ may driver i benefit from underreporting her evaluation. Such a benefit would only materialize if she is matched with another rider k such that $b_i^d < b_k^r$ (since under the SSR policy, the driver will receive a bonus in this case).

From the above analysis we can see that individuals do have an incentive to violate Assumption 3 under the SSR policy. The incentive is that they could extract a bonus (or a discount) by switching to a different partner. However, from the platform's point view, this practice neither reduces the number of matched pairs nor necessarily undercuts the profits. In fact, our simulation results show that, as more participants underreport their valuation, the overall profits of the platform tends to increase. We now turn to these and other numerical results.

7 Numerical experiments

In this section, numerical experiments are conducted to examine the properties of the proposed pricing policies. We first confirm the analytical results using an illustrative example. Then simulation experiments are performed to demonstrate the effectiveness of the SSR policy in reducing the platform's deficit. Since the SSR policy does not guarantee truthful reporting, another focus of the simulation study is to examine the actual impact of under-reporting under this policy.

7.1 Illustrative example

In this example, we consider a CaMa problem with one driver and two riders. The participant's schedule and other parameters are shown in Figure 2. We assume that driver 1's true valuation of her schedule displacement is 1.8, but she might also report it as 1 or 4. The schedules reported by the two riders are taken as given in this analysis. Suppose that driver 1 reports truthfully and is matched with rider 1. In this case, to ensure driver 1 arrives on time, she must depart at $\bar{\delta}_{11} = \tau_1^d - \eta_{11} = 1$ and for rider 1 to arrive on time, the departure time is $\bar{\delta}_{11} = \tau_1^r - \theta_{11} = 5$. Since driver 1 reports a schedule displacement value (1.8) higher than that of rider 1 (1.5), Lemma 1 dictates that $\bar{\delta}_{11}^* = 1$. The valuation of driver 1 and rider 1 is $\bar{p}_{11}^d = \alpha\eta_{11} + \bar{b}_1^d \bar{\xi}_{11}^d = 1 \times 9 + 1.8 \times 0 = 9$ and $\bar{p}_{11}^r = \beta h_1^r - b_1^r \bar{\xi}_{11}^r = 3 \times 6 - 1.5 \times 4 = 12$. Hence, the social welfare obtained from matching driver 1 and rider 1 is $\bar{a}_{11} = \bar{p}^r_{11} - \bar{p}^d_{11} = 12 - 9 = 3$. The calculation can be carried for the other matched pair (driver 1-rider 2), and for different values of b^d_1. The results are summarized in Table 1.

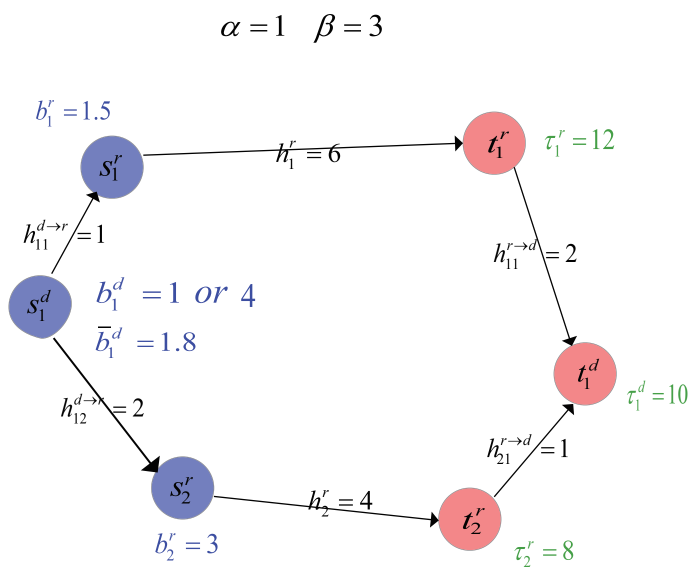


Figure 2: Configuration and parameters in the illustrative example (see Appendix A for notations used in the plot)

Consider the VCG pricing policy (see **Algorithm 1**). We first assume that driver 1 bids at her true value of 1.8. In this case, driver 1 is matched with rider 2. This is because $\bar{a}_{12} = 3.2$ is larger than $\bar{a}_{11} = 3$ (see Table 1). In other words, matching driver 1 and rider 2 maximizes the social welfare. According to the VCG policy, the bonus for driver 1 would be 3.2 (since there will be no matching when the driver is removed) and the discount for rider 2 is 0.2 (since the social welfare will be reduced from 3.2 to 3 when rider 2 is removed), hence the solution is IR. Then, under this policy, the total payment to driver 1 would be 8.8 + 3.2 = 12 and the charge on rider 2 would be 12 - 0.2 = 11.8. We can verify that the utility for driver 1 is $\bar{u}^{d+}_{12} = q^{d+}_1 - b^d_1(\xi^d_{12} - \bar{\xi}^d_{12}) - \bar{p}^d_{ij} = 12 - 8.8 = 3.2$. The profits of the platform, however, is $q^{r+}_2 - q^{d+}_1 = 11.8 - 12 = -0.2$.

Table 1: Optimal departure time, ride valuation and social welfare corresponding to different matching results and value of schedule displacement reported by driver 1.

	1				1.8				4			
	δ^*_{ij}	p^d_{ij}	p^r_{ij}	a_{ij}	$\bar{\delta}^*_{ij}$	$\bar{p}^d_{ij}$	$\bar{p}^r_{ij}$	$\bar{a}_{ij}$	δ^*_{ij}	p^d_{ij}	p^r_{ij}	a_{ij}
Driver 1-Rider 1	5	13	18	5	1	9	12	3	1	9	12	3
Driver 1-Rider 2	2	8	12	4	2	8.8	12	3.2	3	7	9	2

We now turn to the SSR pricing policy (see **Algorithm 2**). Since the policy would not change the optimal matching result, drive 1 is still matched with rider 2. Because $\bar{b}^d_1 < b^r_2$, rider 2 will be set to arrive on time and the bonus (equal to 3.2) is given to driver 1 to offset the schedule cost. Consequently, the payment to driver 1 and the charge on rider 2 are both 12, meaning that the platform now breaks even. Since driver 1 receives the same bonus, her utility remains positive.

Table 2: Driver 1's utility associated with different reported values of schedule displacement under the VCG or SSR policies.

b_i^d	Winning pair	$\bar{p}_{ij}^d$	$\bar{\xi}_{ij}^d$	ξ_{ij}^d	p_{ij}^d	VCG			SSR		
						ρ_i^{d+}	q_i^{d+}	u_{ij}^{d+}	ρ_i^{d-}	q_i^{d-}	u_{ij}^{d-}
1	Driver 1-Rider 1	9	0	4	13	5	18	1.8	5	18	1.8
1.8	Driver 1-Rider 2	8.8	2	2	8.8	3.2	12	3.2	3.2	12	3.2
4	Driver 1-Rider 1	9	0	0	9	3	12	3	0	9	0

Table 2 presents the results of the above analysis when driver 1 bids at the two other values. If driver 1 reports her value as $1 < 1.8$, she will be matched with rider 1. By underreporting her value, driver 1 is subject to a four-minute delay and hence a greater valuation of the ride (13 instead of 9). Under the VCG policy, she will receive a bonus of 5 (equal to a_{11} in Table 1). Her utility becomes $u_{11}^{d+} = q_1^{d+} - \bar{b}_1^d \left(\xi_{11}^d - \bar{\xi}_{11}^d\right) - \bar{p}_{11}^d = 18 - 1.8 \times (4 - 0) - 9 = 1.8 < 3.2$. Under the SSR policy, since she is perceived as valuing the schedule displacement lower than rider 1, driver 1 still receives the same bonus and her utility remains at 1.8. Clearly, underreporting in this case is not desirable because it significantly reduces driver 1's utility.

For over-reporting (at 4), the driver again will be matched with rider 1. In this case, she will receive a smaller bonus (3) under the VCG policy and no bonus at all under the SSR policy. Her utility would be 3 and 0 under the VCG and SSR policy, respectively. She is always worse off by over-reporting, consistent with our main theoretical result.

7.2 Simulation results

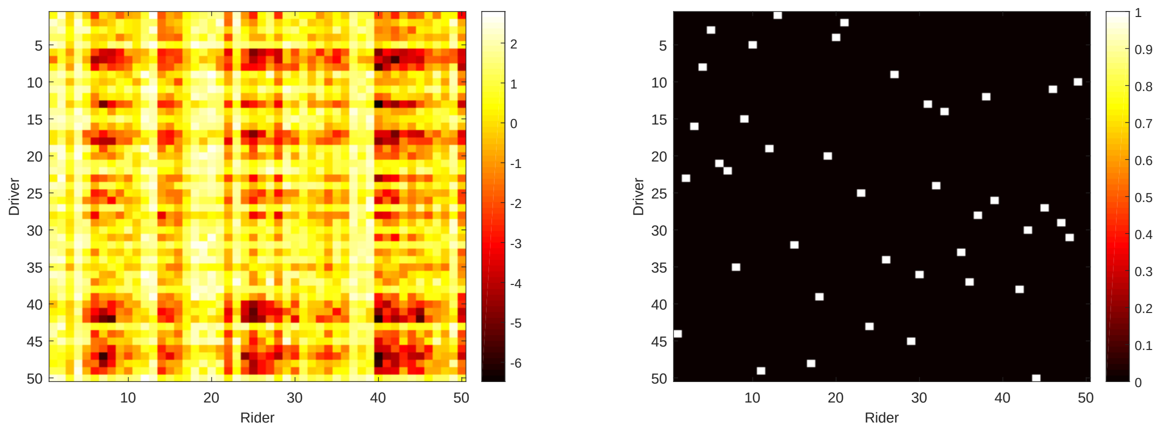
In this section we create CaMa problems by drawing random numbers from independent distributions to form the schedules of participants in each problem. We consider two simulation settings, as shown in Table 3. Unless otherwise specified, the platform's fixed price parameter is $\alpha = 0.5$ (for driver) and $\beta = 1.5$ (for rider), and 100 simulation runs (i.e. 100 random CaMa problems are solved) are performed in each experiment.

Table 3: Scenarios of simulation settings. U$[c, d]$ represents a uniform distribution with c and d being the lower and upper bounds. Log-N(μ, σ^2) represents a log-normal distribution with μ and σ being the location and scale parameters.

Scenario	α	β	b_i^d / b_j^r	h_j^r	$h_{ij}^{d\to r} / h_{ji}^{r\to d}$	τ_i^d / τ_j^r
1	0.5	1.5	U[0,3]	U[3,4]	U[1,2]	U[10,12]
2			Log-N(1,σ^2)			

7.2.1 Benchmark

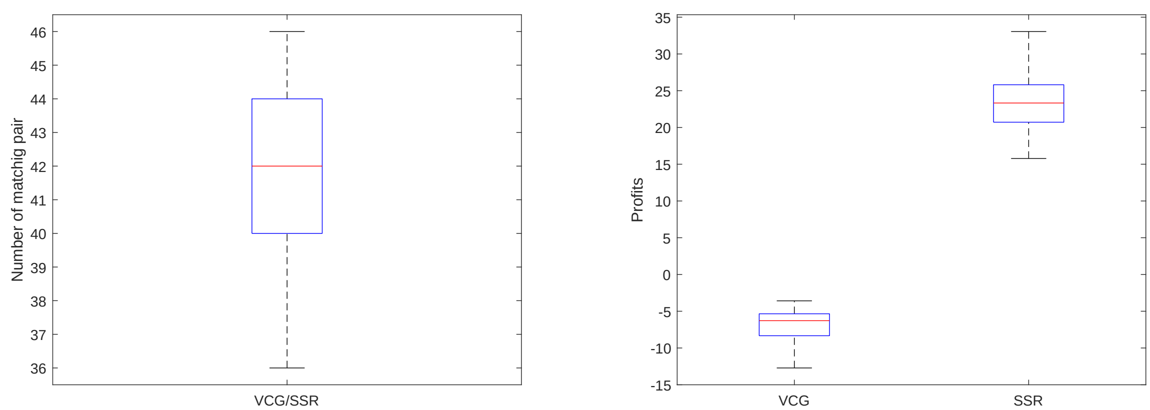
As a benchmark case, we consider 50 drivers and 50 riders ($n = m = 50$). In each simulation run we draw the parameters for each of the one hundred participants, independently from the distributions specified in Scenario 1 of Table 3. We then find the optimal CaMa Point corresponding to both VCG and SSR pricing policies, i.e., $\boldsymbol{y}^+$ and $\boldsymbol{y}^-$.



(a) Social welfare of any matched pairs (a_{ij}). Color represents the value of a_{ij}.

(b) Matched pairs at optimum (x^*_{ij}). Color represents the value of x_{ij}.

Figure 3: Visualization of a CaMa solution corresponding to a random sample.

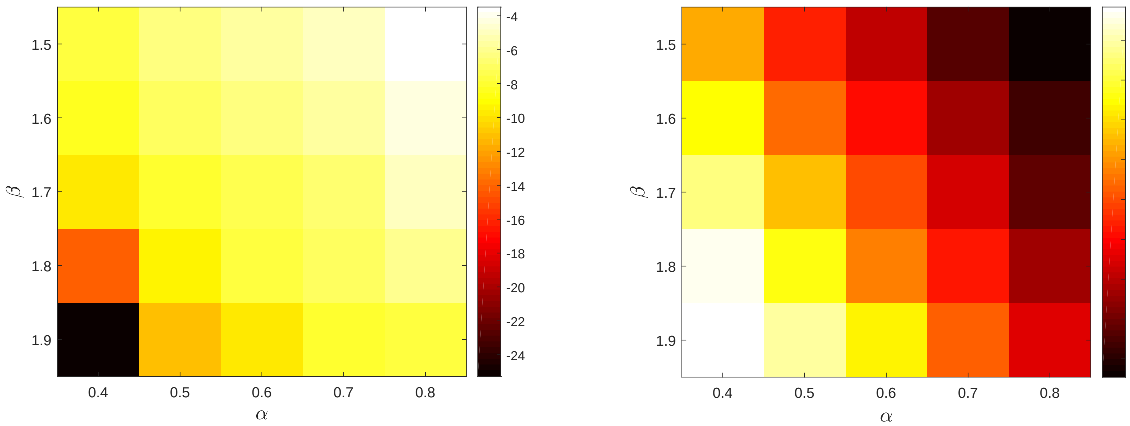


(a) Number of matching pairs

(b) Profits

Figure 4: Boxplot of matching efficiencies and profits (produced based on 100 simulation runs drawn from distributions defined in Scenario 1 in Table 3).

Figures 3 (a) and (b) visualize, respectively the social welfare of all matched pairs, and the optimal matching result from one run. It is easy to see from the plots that the optimally matched pairs are those that generate highest social welfare. Figure 4 (a) shows the number of matched pairs ranges between 36 and 46, with an average about 41.75. Figure 4 (b) shows VCG produces deficits in all runs (ranging between -12.72 and -3.58), whereas SSR always yields a profits (range between 15.78 and 33.05). The profit by SSR averages at 23.33 whereas the deficit by VCG averages at -6.90, a gap close to 30 (or about 0.72 per matched pair). Also, the profits produced by SSR is subject to a greater variability than the deficit by VCG.



(a) Profits with VCG policy.

(b) Profits with SSR policy.

Figure 6: Effect of α and β on system profits.

7.2.2 Sensitivity analysis

We next perform two sensitivity analysis using the benchmark case. The first is concerned with how α and β (i.e. the platform's pricing on the fixed cost of shared rides) affect the matching efficiency and profits, and the second addresses the effect of schedule flexibility. In both analyses, we set $m = n = 50$, and use Scenario 1 in Table 1 to set the simulation.

In the first analysis, we set $\alpha = [0.4; 0.5; 0.6; 0.7; 0.8]$ and $\beta = [1.5; 1.6; 1.7; 1.8; 1.9]$. This leads to 36 cases, each corresponding to a different combination of α and β values. Figure 5 shows the number of matching pairs increase when β (the price per unit time charged on the riders) increase while holding α constant; or when α (the price per unit time paid to drivers) decreases at a constant β. This is intuitive since giving drivers a less compensation or riders a higher price would make it easier to find mutually beneficial matches. Figure 6 (a) shows higher matching efficiency creates larger deficit under the VCG policy. Yet, the opposite is true for the SSR policy: the more are participants matched, the greater profit the platform enjoys (Figure 6 (b)).

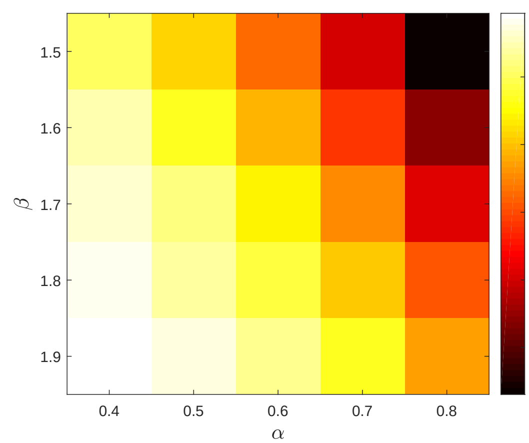
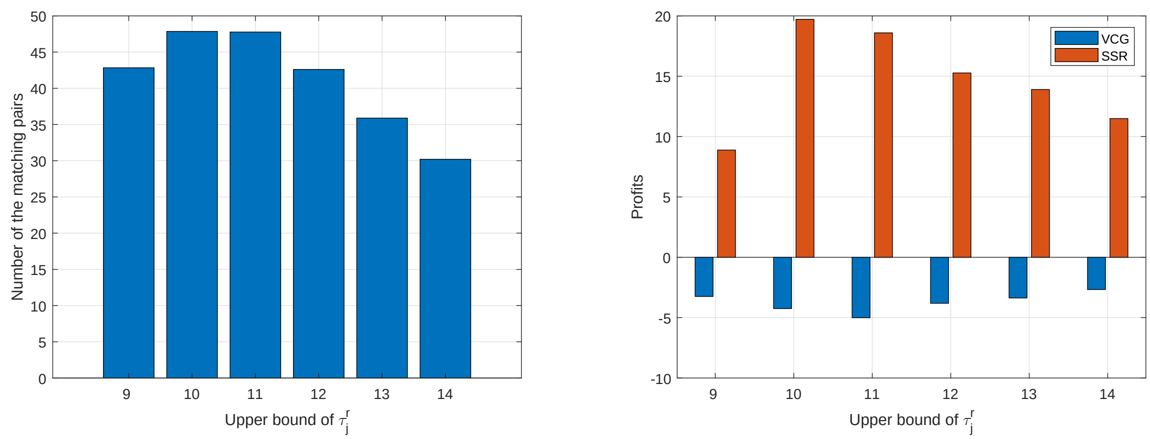


Figure 5: Effects of α and β on the matching efficiency. Color represents the number of matched pairs.

In the second analysis, we study how the differences between the arrival times of drivers and riders affect the matching efficiency and profits. We fix each driver's arrival time at 11, and the lower bound of each rider's arrival time at 8. Then we select the upper bound of the rider' arrival time from the set $UB = [9; 10; 11; 12; 13; 14]$. For each upper bound $ub \in UB$, a sample of 100 riders are drawn from a uniform distribution U[8,ub]. Effectively, a small upper bound means larger variances in the desired arrival time. As the upper bound increases, the matching efficiency first rises, peaks when the upper bound is about 11, and then begins to drop (Figure 7 (a)). The profit under the SSR policy demonstrates a somewhat similar pattern (Figure 7 (b)). However, the peak profits under SSR policy is achieved at 10. As to the VCG policy, the deficit peaks when the upper bound is around 11 and then begins to drop.



(a) Matching efficiency

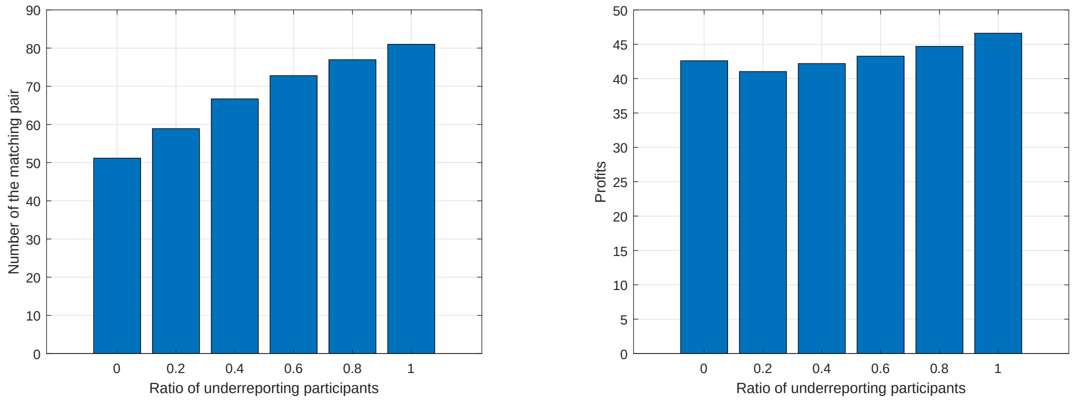
(b) Profits

Figure 7: Effects of the variance in the participants' arrival time on system matching efficiency and profits.

Very narrowly distributed arrival times hurt matching efficiency because drivers must detour to pick up and drop off riders. If they tend to have the same arrival time, at least one would have to suffer a large displacement. There is a "sweet spot", in terms of the arrival time variance, which maximizes the opportunity that the participants' schedule, relative to the detour distance, would "click" to minimize the total displacement. Beyond that spot, the difference becomes too great to reconcile, hence the matching efficiency begins to drop. In our example, the shortest travel time $h_{ji}^{r \to d}$ from rider's destination t_j^r to driver's destination t_i^d obeys uniform distribution U[1, 2]. When the upper bound of rider's arrival time is around 11, intuitively, it should hit the sweet spot since driver's arrival time 11 equals to the mean of rider's arrival time 9.5 plus 1.5 (the mean of $h_{ji}^{r \to d}$).

7.2.3 Impact of underreporting

This experiment considers cases with $m = n = 100$. To configure the simulations we still use Scenario 1 in Table 3. We focus on the SSR policy since it cannot rule out underreporting. All participants' true value of schedule disruption is set as 3, and the number of participants who underreport the value is gradually increased from 0. For each misreporting participant, her value of schedule displacement is drawn from five uniform distributions U [0.5, 1.0], U [1.0, 1.5], U [1.5, 2.0], and U [2.0, 2.5], corresponding to a probability of 0.1, 0.2, 0.3, and 0.4, respectively. In this way, we capture the fact that those who misreport this value are more likely to report in a range closer to the true value than a ranger further away. Figure 8 (a) shows that the number of matched pairs increases when more participants underreport their value. The profits under the SSR policy also tend to rise with the ratio of underreporting participants, although the lowest profit appears

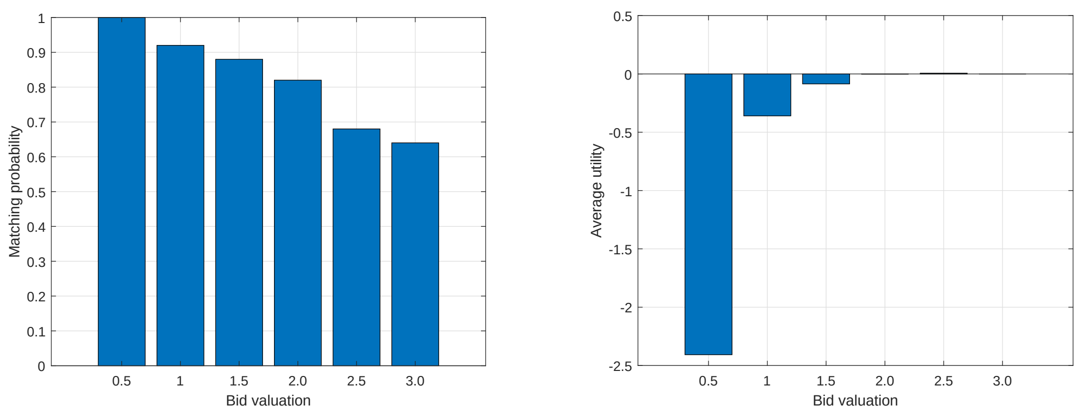


(a) Matching efficiency

(b) Profits

Figure 8: Effect of underreporting on system matching efficiency and profits.

to occur when that ratio is around 0.2. Overall, the trend suggests that a platform implementing the SSR policy need not worry too much about underreporting, both in terms matching efficiency and revenue.

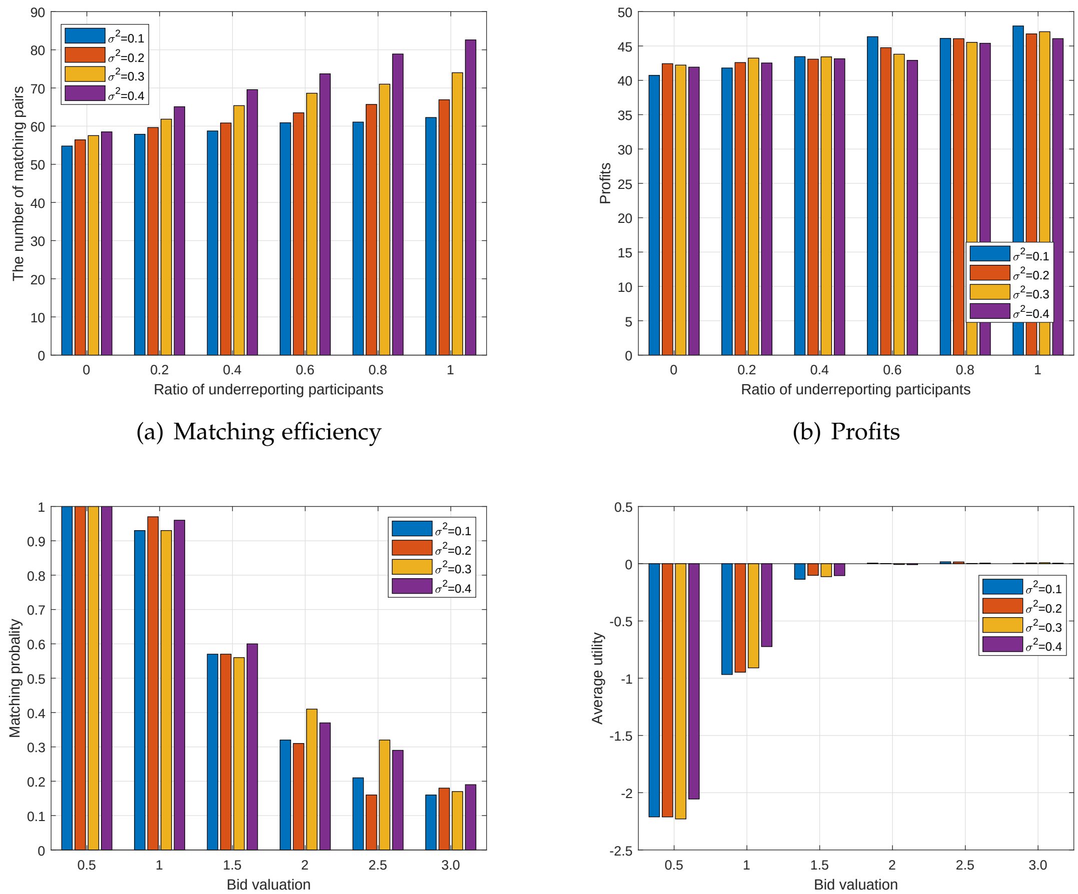


(a) Likelihood of success matching

(b) Utility

Figure 9: Effect of underreporting on individual matching efficiency and utility.

We now examine how likely an individual will benefit from underreporting, assuming that it is common knowledge among participants that this practice could potentially improve individual utility. Without loss of generality, we pick a driver (called driver 1) and set her true value of schedule displacement as 3 and her desired arrival time as $\tau_1^d = 11$. We then change driver 1's bid value from 0.5 to 3, with 0.5 increment. For each value, we perform 100 simulation runs, count the number of times when driver 1 is successfully matched and compute the average utility. These results are reported in Figure 9. Figure 9 (a) shows that lowering the bid value always makes matching easier for driver 1. If driver 1 reports her value as 0.5 (1/6 of her true value), she is almost guaranteed to be matched. Yet, Figure 9 (b) indicates that such a high matching probability is not necessarily a blessing, since severe underreporting leads to significant utility loss. There is indeed a range (around 2.5) within which underreporting is helpful. However, the average gains in the utility is negligible.



(a) Matching efficiency

(b) Profits

(c) Likelihood of success matching

(d) Utility

Figure 10: Effects of underreporting on system matching efficiency and profits (Log-Normal distribution).

Finally, we check whether or not the above results are sensitive the type of distributions used to generate the random samples. To this end, we use the setting of Scenario 2 in Table 3. For the Log-Normal distribution from which we draw random bid values, four different variances (of its natural logarithm) are considered: 0.1, 0.2, 0.3 and 0.4. Figures 10 (a) and (b) produce the same plot as in Figure 8, whereas Figure 10 (c) and (d) produce the same plots as in Figure 9. Comparing Figure 10 with Figures 8 and 9, these different distributions of the schedule displacement value do slightly affect the performance of platforms and individuals. However, the general findings remain largely intact. They all show that underreporting is neither a practical concern for the platform, nor an effective tool for cheating. In other words, the SSR policy is a robust deficit-free alternative to the VCG policy.

8 Conclusions

We studied a carpool matching (CaMa) problem in which carpool participants price the shared rides based on both operating cost and schedule displacement. The first part is considered to be public information and the second is private. By reporting their private valuation of the schedule displacement, each participant in effect bids for every possible potential carpool ride that generates a unique value to her. Importantly, the value of each bid depends on with whom the participant is matched, which distinguishes our setting from most existing double auction problems.

The CaMa problem can be formulated as a mixed integer program (MIP) that maximizes the social welfare by choosing matching pairs and a departure time for each pair. We showed that the optimal departure time can be determined for each pair a priori, independent of the matching problem. This reduces the CaMa problem to a standard bipartite matching problem that is much easier to solve. We proved that the classical VCG pricing policy ensures that no participant would have a lower utility or the incentive to misreport their valuation of schedule displacement. While this may not sound surprising, we note that the existence of transaction-dependent bids complicates the proof considerably. To control the large deficit typically accompanying the VCG policy, we developed a single-side reward (SSR) pricing policy, which only compensates participants who are forced by the system to endure schedule displacement. Under the assumption of overpricing tendency (i.e., no participant would want to underreport their value), we proved that the SSR policy not only generates substantial profits, but also retain the other desired properties of the VCG policy, notably truthful reporting. Even though it cannot rule out underreporting, our simulation experiments confirm that the SSR policy is a robust and deficit-free alternative to the VCG policy. Specifically, we found

- as more participant underreport their value of schedule displacement, both the number of matched pairs and the total profits tend to increase (hence, underreporting is not a major practical concern for a carpool platform); and
- participants have very little to gain by underreporting their value in all tested cases.

So far we have assumed that all travel times are deterministic parameters. In reality, travel times may be disrupted by random incidents such as major crashes and inclement weather conditions. If participants are to hedge against such uncertainty, the platform may need very different strategies to achieve the desired matching outcome. Hence, designing pricing policies under uncertainty is an interesting problem for further research. A future study may also extend the modeling framework to accommodate alternative design objectives (e.g., profit maximization) and other ridesharing settings.

Acknowledgments

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A Notations of main variables

Table 4: Notations of main variables.

Symbol	Descriptions
$\iota \in \{d, r\}$	d and r stand for driver and rider, respectively.
$(s^\iota, t^\iota, \tau^\iota, b^\iota)$	a participant's origin, destination, desired arrival time and bid valuation.
$I = \{1, \cdots, n\}$ and $J = \{1, \cdots, m\}$	the driver set and the rider set.
$\bar{b}_i^d$ $(\tilde{b}_j^r)$	driver $i's$ truthful bid valuation (rider $j's$ truthful bid valuation).
$h_{ij}^{d\to r}$	shortest travel time from s_i^d (origin of the driver i) to s_j^r (origin of rider j).
h_j^r	shortest travel time from s_j^r (origin of the rider j) to t_j^r (destination of rider j)
$h_{ji}^{r\to d}$	shortest travel time from t_j^r (destination of rider j) to t_i^d (destination of driver i).
θ_{ij}	travel time from s_i^d (origin of the driver i) to t_j^r (destination of rider j).
η_{ij}	travel time from s_i^d (origin of the driver i) to t_i^d (destination of rider j).
β and α	unit price charged on the rider and paid to the driver by the platform, respectively.
δ_{ij}	driver i 's departure time when she is matched with rider j.
$\bar{\delta}_{ij}$ $(\tilde{\delta}_{ij})$	If driver i (rider j) truthful reporting, driver i 's departure time when matched with rider j.
ξ_{ij}^d	schedule displacement for driver i when matched with rider j.
ξ_{ij}^r	schedule displacement for rider j when matched with driver i.
$\bar{\xi}_{ij}^d$ $(\tilde{\xi}_{ij}^d)$ and $\bar{\xi}_{ij}^r$ $(\tilde{\xi}_{ij}^r)$	schedule displacement for driver i and rider j when driver i (rider j) truthful reports, respectively.
p_{ij}^d (p_{ij}^r)	bid price of driver i (rider j) for rider j (driver i).
$\bar{p}_{ij}^d$ $(\tilde{p}_{ij}^d)$ and $\bar{p}_{ij}^r$ $(\tilde{p}_{ij}^r)$	driver i' bid price and rider $j's$ bid price if driver i (rider j) truthfully reports.
a_{ij}	social welfare of a matched pair ij
$\bar{a}_{ij}$	social welfare when driver i truthfully reports
x_{ij}	allocation incidence. $x_{ij} = 1$ if driver i is matched with rider j, and 0 otherwise.
q_i^d and q_j^r	payment to driver i and the price charged on rider j, respectively.
u_{ij}^d and u_{ij}^r	utility of driver i and rider j when driver i is matched with rider j, respectively.
ρ_i^{d+} (ρ_i^{d-}) and ρ_j^{r+} (ρ_j^{r-})	the bonus for driver i and rider j under VCG (SSR) policy.
V	maximum payoff of the system corresponding to the optimal value of Problem (12).
$V_{-i}(V_{-j})$	maximum payoff of the system when driver i (rider j) is removed from the platform.
V_{-ij}	maximum payoff of the system when driver i and rider j are both removed from the platform.

B Proof of Lemma 2

Proof: We prove the result using reduction to absurdity. Let $\bar{a}_{ik} = \bar{p}^r_{ik} - \bar{p}^d_{ik}$. $\bar{V}_{-ik}$ and $\bar{V}_{-i}$ denote the maximum payoff of the system corresponding to driver i truthful reporting. When driver i is not matched with truthful bid valuation $\bar{b}^d_i$, the following inequality holds for any $k \in J$,

$$\bar{a}_{ik} + \bar{V}_{-ik} < \bar{V}_{-i}, \forall k \in J, \tag{23}$$

since otherwise there exists at least one k such that matching i with k would give social welfare equal to or greater than that of leaving i out. When driver i misreports her bid valuation as $b^d_i = \bar{b}^d_i + \Delta$ where $\Delta > 0$, we have

$$V_{-ik} = \bar{V}_{-ik}, \forall k \in J; \tag{24}$$

$$V_{-i} = \bar{V}_{-i}. \tag{25}$$

If in this case, driver i is matched with rider j, then

$$a_{ij} + V_{-ij} \geq V_{-i}. \tag{26}$$

We next show that this is impossible. To this end we first prove that $a_{ij} \leq \bar{a}_{ij}$, whether or not the truthful reporting affects the optimal departure time. Let $\bar{\delta}^*_{ij}$ and δ^*_{ij} be the optimal departure time corresponding to truthful reporting and misreporting from driver i, respectively.

Case 1 If $\delta^*_{ij} = \bar{\delta}^*_{ij}$, we have

$$\begin{aligned} a_{ij} &= \beta h^r_j - b^r_j \left|\tau^r_j - \delta^*_{ij} - \theta_{ij}\right| - \alpha\eta_{ij} - (\bar{b}^d_i + \Delta)\left|\tau^d_i - \delta^*_{ij} - \eta_{ij}\right| \\ &= \bar{a}_{ij} - \Delta\left|\tau^d_i - \delta^*_{ij} - \eta_{ij}\right| \leq \bar{a}_{ij}. \end{aligned} \tag{27}$$

Case 2 If $\delta^*_{ij} \neq \bar{\delta}^*_{ij}$, we claim that $\bar{b}^d_i < b^r_j \leq b^d_i$. To see why this is the case, note that if $b^r_j \leq \bar{b}^d_i < b^d_i$, $\delta^*_{ij} = \bar{\delta}^*_{ij} = \tau^d_i - \eta_{ij}$ per Lemma 1 and Eq. (11); if $\bar{b}^d_i < b^d_i < b^r_j$, then $\delta^*_{ij} = \bar{\delta}^*_{ij} = \tau^r_j - \theta_{ij}$. Accordingly, we have $\bar{\delta}^*_{ij} = \tau^r_j - \theta_{ij}$ and $\delta^*_{ij} = \tau^d_i - \eta_{ij}$. Thus,

$$\begin{aligned} a_{ij} &= \beta h^r_j - b^r_j \left|\tau^r_j - \delta^*_{ij} - \theta_{ij}\right| - \alpha\eta_{ij} - b^d_i\left|\tau^d_i - \delta^*_{ij} - \eta_{ij}\right| \\ &= \beta h^r_j - b^r_j \left|\tau^r_j - \tau^d_i + \eta_{ij} - \theta_{ij}\right| - \alpha\eta_{ij} \\ &\leq \beta h^r_j - \bar{b}^d_i \left|\tau^r_j - \tau^d_i + \eta_{ij} - \theta_{ij}\right| - \alpha\eta_{ij} = \bar{a}_{ij}. \end{aligned} \tag{28}$$

Therefore, using Eq (24) yields

$$a_{ij} + V_{-ij} \leq \bar{a}_{ij} + V_{-ij} = \bar{a}_{ij} + \bar{V}_{-ij}, \tag{29}$$

and from Eqs. (23) and (25), we obtain

$$\bar{a}_{ij} + \bar{V}_{-ij} < \bar{V}_{-i} = V_{-i}. \tag{30}$$

Adding (29) and (30) together yields

$$a_{ij} + V_{-ij} < V_{-i}, \tag{31}$$

a contradiction with Eq. (26). This completes the proof. □

C Proof of Lemma 3

Proof: Since driver i is not matched when she truthful bids her valuation $\bar{b}_i^d$, we have

$$\bar{a}_{ik} + \bar{V}_{-ik} < \bar{V}_{-i} = \bar{V}, \forall k \in J. \tag{32}$$

Now suppose that the driver misreports the valuation as $b_i^d = \bar{b}_i^d - \Delta$ with $\Delta > 0$. If she is still not matched, her utility is always zero, hence the statement is correct. Below we focus on the case when she is matched with a rider j by misreporting. Using the similar argument in the proof for Lemma 2, we can show either $\delta_{ij}^* = \bar{\delta}_{ij}^*$, or $b_i^d < b_j^r \leq \bar{b}_i^d$, and accordingly

$$\bar{\delta}_{ij}^* = \tau_i^d - \eta_{ij}, \delta_{ij}^* = \tau_j^r - \theta_{ij}, \bar{\xi}_{ij}^d = 0, \xi_{ij}^d = \left|\tau_i^d - \tau_j^r + \theta_{ij} - \eta_{ij}\right|. \tag{33}$$

Case 1 If $\delta_{ij}^* = \bar{\delta}_{ij}^*$, we have

$$p_{ij}^d = \bar{p}_{ij}^d - \Delta\left|\tau_i^d - \delta_{ij}^* - \eta_{ij}\right|, \tag{34}$$

$$a_{ij} = \bar{a}_{ij} + \Delta\left|\tau_i^d - \delta_{ij}^* - \eta_{ij}\right|, \tag{35}$$

$$V_{-ij} = \bar{V}_{-ij}, \tag{36}$$

$$\xi_{ij}^d = \bar{\xi}_{ij}^d. \tag{37}$$

Combining Eq. (32), (35) and (36) yields

$$V - \Delta\left|\tau_i^d - \delta_{ij}^* - \eta_{ij}\right| = a_{ij} + V_{-ij} - \Delta\left|\tau_i^d - \delta_{ij}^* - \eta_{ij}\right| = \bar{a}_{ij} + \bar{V}_{-ij} < \bar{V}. \tag{38}$$

The utility of the driver i is

$$\begin{aligned} u_{ij}^{d+} &= q_i^{d+} - \bar{b}_i^d\left(\xi_{ij}^d - \bar{\xi}_{ij}^d\right) - \bar{p}_{ij}^d = p_{ij}^d + V - V_{-i} - \bar{p}_{ij}^d = -\Delta\left|\tau_i^d - \delta_{ij}^* - \eta_{ij}\right| + V - V_{-i} \\ &< \bar{V} - V_{-i} = \bar{V} - \bar{V}_{-i} = 0. \end{aligned} \tag{39}$$

Here the second equality is due to the VCG payment and Eq. (37), the third equality is due to Eq. (34), the inequality is due to Eq. (38) and the last two equalities are due to Eqs. (32) and (25). Hence, in this case driver i's utility decreases from zero to negative when she is matched by misreporting.

Case 2 If $\delta_{ij}^* \neq \bar{\delta}_{ij}^*$, we have

$$\begin{aligned} a_{ij} &= \beta h_j^r - b_j^r\left|\tau_j^r - \delta_{ij}^* - \theta_{ij}\right| - \alpha\eta_{ij} - b_i^d\left|\tau_i^d - \delta_{ij}^* - \eta_{ij}\right| \\ &= \beta h_j^r - \alpha\eta_{ij} - b_i^d\left|\tau_i^d - \tau_j^r + \theta_{ij} - \eta_{ij}\right| \\ &= \beta h_j^r - \alpha\eta_{ij} - \left(\bar{b}_i^d - \Delta\right)\left|\tau_i^d - \tau_j^r + \theta_{ij} - \eta_{ij}\right| \\ &\leq \beta h_j^r - \alpha\eta_{ij} - b_j^r\left|\tau_i^d - \tau_j^r + \theta_{ij} - \eta_{ij}\right| + \Delta\left|\tau_i^d - \tau_j^r + \theta_{ij} - \eta_{ij}\right| \\ &= \bar{a}_{ij} + \Delta\left|\tau_i^d - \tau_j^r + \theta_{ij} - \eta_{ij}\right|. \end{aligned} \tag{40}$$

Here the second equality is due to Eq. (33), the inequality is due to $b_j^r \leq \bar{b}_i^d$. Hence

$$V - \Delta \left| \tau_i^d - \delta_{ij}^* - \eta_{ij} \right| = a_{ij} + V_{-ij} - \Delta \left| \tau_i^d - \delta_{ij}^* - \eta_{ij} \right| \leq \bar{a}_{ij} + \bar{V}_{-ij} < \bar{V}.$$

The first inequality above is due to Eq. (40) and the second is due to Eq. (32). In this case, the utility of driver i is also negative, i.e.,

$$\begin{aligned}
u_{ij}^{d+} &= q_i^{d+} - \bar{b}_i^d \left(\xi_{ij}^d - \bar{\xi}_{ij}^d \right) - \bar{p}_{ij}^d = p_{ij}^d + V - V_{-i} - \bar{b}_i^d \left| \tau_i^d - \tau_j^r + \theta_{ij} - \eta_{ij} \right| - \bar{p}_{ij}^d \\
&= b_i^d \left| \tau_i^d - \tau_j^r + \theta_{ij} - \eta_{ij} \right| + V - V_{-i} - \bar{b}_i^d \left| \tau_i^d - \tau_j^r + \theta_{ij} - \eta_{ij} \right| - \bar{b}_i^d \left| \tau_i^d - \tau_i^d + \eta_{ij} - \eta_{ij} \right| \\
&= -\Delta \left| \tau_i^d - \tau_j^r + \theta_{ij} - \eta_{ij} \right| + V - V_{-i} \\
&< \bar{V} - V_{-i} = \bar{V} - \bar{V}_{-i} = 0.
\end{aligned} \tag{41}$$

Combing Eqs. (39) and (41) completes the proof. □

D Proof of Lemma 4

Proof: When driver i misreports her bid valuation and she is matched with rider k, from **Algorithm 1**, her bonus and payment are respectively

$$\rho_i^{d+} = V - V_{-i} = a_{ik} + V_{-ik} - V_{-i}; q_i^{d+} = p_{ik}^d + \rho_i^{d+}. \tag{42}$$

The utility of driver i is

$$u_{ik}^{d+} = q_i^{d+} - \bar{b}_i^d \left(\xi_{ik}^d - \bar{\xi}_{ik}^d \right) - \bar{p}_{ik}^d = p_{ik}^d + a_{ik} + V_{-ik} - V_{-i} - \bar{b}_i^d \left(\xi_{ik}^d - \bar{\xi}_{ik}^d \right) - \bar{p}_{ik}^d. \tag{43}$$

Case 1 If $\delta_{ik}^* = \bar{\delta}_{ik}^*$, we have $\xi_{ik}^d = \bar{\xi}_{ik}^d, p_{ik}^r = \bar{p}_{ik}^r$, hence

$$p_{ik}^d + a_{ik} - \bar{b}_i^d \left(\xi_{ik}^d - \bar{\xi}_{ik}^d \right) - \bar{p}_{ik}^d = p_{ik}^d + \bar{p}_{ik}^r - p_{ik}^d - \bar{p}_{ik}^d = \bar{a}_{ik}. \tag{44}$$

Case 2 If $\delta_{ik}^* \neq \bar{\delta}_{ik}^*$, here we need to consider two subcases: $\Delta < 0$ or $\Delta > 0$. Suppose $\Delta < 0$ and using the similar argument in the proof for Lemma 3, we have $b_i^d < b_k^r \leq \bar{b}_i^d$. Similar to Eq. (33), we have

$$\bar{\delta}_{ik}^* = \tau_i^d - \eta_{ik}, \delta_{ik}^* = \tau_k^r - \theta_{ik}, \bar{\xi}_{ik}^d = 0, \xi_{ik}^d = \left| \tau_i^d - \tau_k^r + \theta_{ik} - \eta_{ik} \right|,$$

and then

$$\begin{aligned}
&p_{ik}^d + a_{ik} - \bar{b}_i^d \left(\xi_{ik}^d - \bar{\xi}_{ik}^d \right) - \bar{p}_{ik}^d \\
=&p_{ik}^d + p_{ik}^r - p_{ik}^d - \bar{b}_i^d \left| \tau_i^d - \tau_k^r + \theta_{ik} - \eta_{ik} \right| - \bar{p}_{ik}^d \\
=&\beta h_k^r - b_k^r \left| \tau_k^r - \delta_{ik}^* - \theta_{ik} \right| - \bar{b}_i^d \left| \tau_i^d - \tau_j^r + \theta_{ik} - \eta_{ik} \right| - \bar{p}_{ik}^d \\
\leq&\beta h_k^r - b_k^r \left| \tau_k^r - \tau_k^r + \theta_{ik} - \theta_{ik} \right| - b_k^r \left| \tau_i^d - \tau_k^r + \theta_{ik} - \eta_{ik} \right| - \bar{p}_{ik}^d = \bar{p}_{ik}^r - \bar{p}_{ik}^d = \bar{a}_{ik}.
\end{aligned} \tag{45}$$

Similarly, we can show that $p_{ik}^d + a_{ik} - \bar{b}_i^d \left(\xi_{ik}^d - \bar{\xi}_{ik}^d \right) - \bar{p}_{ik}^d \leq \bar{a}_{ik}$ holds when $\Delta > 0$.

Combining Eq. (44) and (45), Eq. (43) becomes

$$u_{ik}^{d+} \leq \bar{a}_{ik} + V_{-ik} - V_{-i}. \tag{46}$$

When driver i truthfully reports the bid valuation, her utility is

$$\begin{aligned} \bar{u}_{ij}^{d+} &= \bar{q}_i^{d+} - \bar{b}_i^d \left(\bar{\xi}_{ij}^d - \bar{\xi}_{ij}^d \right) - \bar{p}_{ij}^d \\ &= \bar{a}_{ij} + \bar{V}_{-ij} - \bar{V}_{-i}. \end{aligned} \tag{47}$$

Adding Eqs. (46) and (47), we have

$$\bar{u}_{ij}^{d+} - u_{ik}^{d+} \geq \bar{a}_{ij} + \bar{V}_{-ij} - \bar{V}_{-i} - \bar{a}_{ik} - V_{-ik} + V_{-i} = \bar{a}_{ij} + \bar{V}_{-ij} - \bar{a}_{ik} - \bar{V}_{-ik}.$$

Because driver i is matched with rider j when she truthfully reports her valuation,

$$\bar{a}_{ij} + \bar{V}_{-ij} \geq \bar{a}_{ig} + \bar{V}_{-ig}, \forall g \in J,$$

This leads to

$$\bar{u}_{ij}^{d+} - u_{ik}^{d+} \geq 0,$$

and completes this proof. $\square$