# Taquin - Analyse - Manhattan

| 4 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 1 | 2 | 3 | 4 | 2 | 3 | 4 | 5 | 3 | 4 | 5 | 6 |
| 1 | 1 | 0 | 1 | 2 | 2 | 1 | 2 | 3 | 3 | 2 | 3 | 4 | 4 | 3 | 4 | 5 |
| 2 | 2 | 1 | 0 | 1 | 3 | 2 | 1 | 2 | 4 | 3 | 2 | 3 | 5 | 4 | 3 | 4 |
| 3 | 3 | 2 | 1 | 0 | 4 | 3 | 2 | 1 | 5 | 4 | 3 | 2 | 6 | 5 | 4 | 3 |
| 4 | 1 | 2 | 3 | 4 | 0 | 1 | 2 | 3 | 1 | 2 | 3 | 4 | 2 | 3 | 4 | 5 |
| 5 | 2 | 1 | 2 | 3 | 1 | 0 | 1 | 2 | 2 | 1 | 2 | 3 | 3 | 2 | 3 | 4 |
| 6 | 3 | 2 | 1 | 2 | 2 | 1 | 0 | 1 | 3 | 2 | 1 | 2 | 4 | 3 | 2 | 3 |
| 7 | 4 | 3 | 2 | 1 | 3 | 2 | 1 | 0 | 4 | 3 | 2 | 1 | 5 | 4 | 3 | 2 |
| 8 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 0 | 1 | 2 | 3 | 1 | 2 | 3 | 4 |
| 9 | 3 | 2 | 3 | 4 | 2 | 1 | 2 | 3 | 1 | 0 | 1 | 2 | 2 | 1 | 2 | 3 |
| 10 | 4 | 3 | 2 | 3 | 3 | 2 | 1 | 2 | 2 | 1 | 0 | 1 | 3 | 2 | 1 | 2 |
| 11 | 5 | 4 | 3 | 2 | 4 | 3 | 2 | 1 | 3 | 2 | 1 | 0 | 4 | 3 | 2 | 1 |
| 12 | 3 | 4 | 5 | 6 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 0 | 1 | 2 | 3 |
| 13 | 4 | 3 | 4 | 5 | 3 | 2 | 3 | 4 | 2 | 1 | 2 | 3 | 1 | 0 | 1 | 2 |
| 14 | 5 | 4 | 3 | 4 | 4 | 3 | 2 | 3 | 3 | 2 | 1 | 2 | 2 | 1 | 0 | 1 |
| 15 | 6 | 5 | 4 | 3 | 5 | 4 | 3 | 2 | 4 | 3 | 2 | 1 | 3 | 2 | 1 | 0 |

| | | |
|---|---|---|
| 0 | 1 | 2 |
| 3 | 4 | 5 |
| 6 | 7 | 8 |

| | | | |
|---|---|---|---|
| 0 | 1 | 2 | 3 |
| 4 | 5 | 6 | 7 |
| 8 | 9 | 10 | 11 |
| 12 | 13 | 14 | 15 |

B2 = ABS(QUOTIENT($A2;$A$1)-QUOTIENT(B$1;$A$1))+ABS(MOD($A2;$A$1)-MOD(B$1;$A$1))

Manhattan( p , m ,n ) = | m/p - n/p | + | ( m mod p ) - ( n mod p )|

# Taquin - Analyse - coups possibles

| 2 | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 1 | | | | | | | | | | | | | |
| 1 | 1 | | | 1 | | | | | | | | | | | | |
| 2 | 1 | | | 1 | | | | | | | | | | | | |
| 3 | | 1 | 1 | | | | | | | | | | | | | |
| 4 | | | 1 | | | 1 | | | | | | | | | | |
| 5 | | | | 1 | 1 | | | | | | | | | | | |
| 6 | | | | | 1 | | | 1 | | | | | | | | |
| 7 | | | | | | 1 | 1 | | | | | | | | | |
| 8 | | | | | | | 1 | | | 1 | | | | | | |
| 9 | | | | | | | | 1 | 1 | | | | | | | |
| 10 | | | | | | | | | 1 | | | 1 | | | | |
| 11 | | | | | | | | | | 1 | 1 | | | | | |
| 12 | | | | | | | | | | | 1 | | | 1 | | |
| 13 | | | | | | | | | | | | 1 | 1 | | | |
| 14 | | | | | | | | | | | | | 1 | | | 1 |
| 15 | | | | | | | | | | | | | | 1 | 1 | |

| | | |
|---|---|---|
| | 1 | 2 |
| 3 | 4 | 5 |
| 6 | 7 | 8 |

| | | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| 4 | 5 | 6 | 7 |
| 8 | 9 | 10 | 11 |
| 12 | 13 | 14 | 15 |

```
SI(OU(ET(B$1=$A2-$A$1; $A2>=$A$1);
ET( B$1=$A2-1;MOD($A2;$A$1)<>0);
ET(B$1=$A2+1;MOD($A2;$A$1)<>$A$1-1);
ET(B$1=$A2+$A$1;$A2<$A$1*$A$1-$A$1));1;0)
```

# Taquin - Analyse - coups possibles

| 3 | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | | 1 | | | | | | | | | | | | |
| 1 | 1 | | 1 | | 1 | | | | | | | | | | | |
| 2 | | 1 | | | | 1 | | | | | | | | | | |
| 3 | 1 | | | | 1 | | 1 | | | | | | | | | |
| 4 | | 1 | | 1 | | 1 | | 1 | | | | | | | | |
| 5 | | | 1 | | 1 | | | | 1 | | | | | | | |
| 6 | | | | 1 | | | | 1 | | | | | | | | |
| 7 | | | | | 1 | | 1 | | 1 | | | | | | | |
| 8 | | | | | | 1 | | 1 | | | | | | | | |
| 9 | | | | | | | 1 | | | | 1 | | | | | |
| 10 | | | | | | | | 1 | | 1 | | 1 | | | | |
| 11 | | | | | | | | | 1 | | 1 | | | | | |
| 12 | | | | | | | | | | 1 | | | | 1 | | |
| 13 | | | | | | | | | | | 1 | | 1 | | 1 | |
| 14 | | | | | | | | | | | | 1 | | 1 | | |
| 15 | | | | | | | | | | | | | 1 | | | |

| | | |
|---|---|---|
| | 1 | 2 |
| 3 | 4 | 5 |
| 6 | 7 | 8 |

| | | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| 4 | 5 | 6 | 7 |
| 8 | 9 | 10 | 11 |
| 12 | 13 | 14 | 15 |

```
SI(OU(ET(B$1=$A2-$A$1; $A2>=$A$1);
ET( B$1=$A2-1;MOD($A2;$A$1)<>0);
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ET(B$1=$A2+$A$1;$A2<$A$1*$A$1-$A$1));1;0)
```

# Taquin - Analyse - coups possibles

| 4 | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | | | 1 | | | | | | | | | | | |
| 1 | 1 | | 1 | | | 1 | | | | | | | | | | |
| 2 | | 1 | | 1 | | | 1 | | | | | | | | | |
| 3 | | | 1 | | | | | 1 | | | | | | | | |
| 4 | 1 | | | | | 1 | | | 1 | | | | | | | |
| 5 | | 1 | | | 1 | | 1 | | | 1 | | | | | | |
| 6 | | | 1 | | | 1 | | 1 | | | 1 | | | | | |
| 7 | | | | 1 | | | 1 | | | | | 1 | | | | |
| 8 | | | | | 1 | | | | | 1 | | | 1 | | | |
| 9 | | | | | | 1 | | | 1 | | 1 | | | 1 | | |
| 10 | | | | | | | 1 | | | 1 | | 1 | | | 1 | |
| 11 | | | | | | | | 1 | | | 1 | | | | | 1 |
| 12 | | | | | | | | | 1 | | | | | 1 | | |
| 13 | | | | | | | | | | 1 | | | 1 | | 1 | |
| 14 | | | | | | | | | | | 1 | | | 1 | | 1 |
| 15 | | | | | | | | | | | | 1 | | | 1 | |

| | | |
|---|---|---|
| | 1 | 2 |
| 3 | 4 | 5 |
| 6 | 7 | 8 |

| | | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| 4 | 5 | 6 | 7 |
| 8 | 9 | 10 | 11 |
| 12 | 13 | 14 | 15 |

```
SI(OU(ET(B$1=$A2-$A$1; $A2>=$A$1);
ET( B$1=$A2-1;MOD($A2;$A$1)<>0);
ET(B$1=$A2+1;MOD($A2;$A$1)<>$A$1-1);
ET(B$1=$A2+$A$1;$A2<$A$1*$A$1-$A$1));1;0)
```

$(n+1 \% 3 \neq 0) = \neg(n+1 \% 3 = 0) = \neg(n \% 3 = 2) = n\%3 \neq 2$

$\downarrow$

$\exists k \,/\, n+1 = kp$

$n+1 = (k-1)p + p$

$= k$

$n = kp - 1 = kp - 1 + p - p$

$n = (k-1)p + p - 1$

$n \overset{3}{\equiv} p-1$

$4 - \frac{4}{3} = 4 \cdot \frac{8}{3} = 2 + \frac{2}{3}$

$3$

$4 -$

$5 \cdot 4 - \frac{4}{5} =$

$\neg(n+1 \equiv 0) \Leftrightarrow \neg(n \equiv p-1)$

$\neg(n+a \equiv 0) \Leftrightarrow \neg(n \equiv p-a)$

$\neg(n-1 \equiv 2) - \neg(n \equiv 0)$ $\quad \neg(n-1$

$\exists k \,/\, n-1 = \cancel{3}pk + 2$

$n = pk + 3$

$n = 3(k+1)$

$n = 3k'$

$n \equiv 0 \ (3)$

$\begin{cases} n-1 \overset{p}{\equiv} p-1 \\ n \overset{p}{\equiv} 0 \end{cases}$ $\quad n-1 = kp + p - 1$

$n = kp + p$

$n \equiv 0$

Sur tableau $p \times p$
nb transitions possibles
$= 4p(p-1)$
pour $p^2$ cases
Soit pour une case une
moyenne de
$4(1 - \frac{1}{p}) = 4 - \frac{4}{p}$

$n+1 \equiv kp$

$n = kp - 1$ $\quad n \equiv p-1$

$n+1 = kp$.

$n < 15$

$16 \cdot 4 = 12$

$\frac{4p(p-1)}{p^2} = 4(1 - \frac{1}{p})$

transitions =
Un taux d'arborescence
de $4(1 - \frac{1}{3}) = 4 - \frac{4}{3} =$

$f(p) = 4p(p-1)$

$f(p+1) = 4(p+1)p$

$=$

$(p+1)f(p) = (p-1)f(p+1)$

$\text{Card } 4p^2 - 4p$

$9 -$

$4p^2 - 4p =$

total $= 4p(p-1)$

$3 \times 3 \to 12(2) = 24$ OK

$4 \times 4 \to 16 \cdot 3 = 48$

moyenne

| | |
|---|---|
| min = 0 | $p=1$ |
| $2$ | $p=2$ |
| $\frac{8}{3}$ | $p=3$ |
| $3$ | $p=4$ |

| | 3x3 | | | |
|---|---|---|---|---|
| | -3 | -2 | -1 | |
| -1 | 0 | 1 | 2 | 3 |
| 2 | 3 | 4 | 5 | 6 |
| 5 | 6 | 7 | 8 | 9 |
| | 9 | 10 | 11 | |

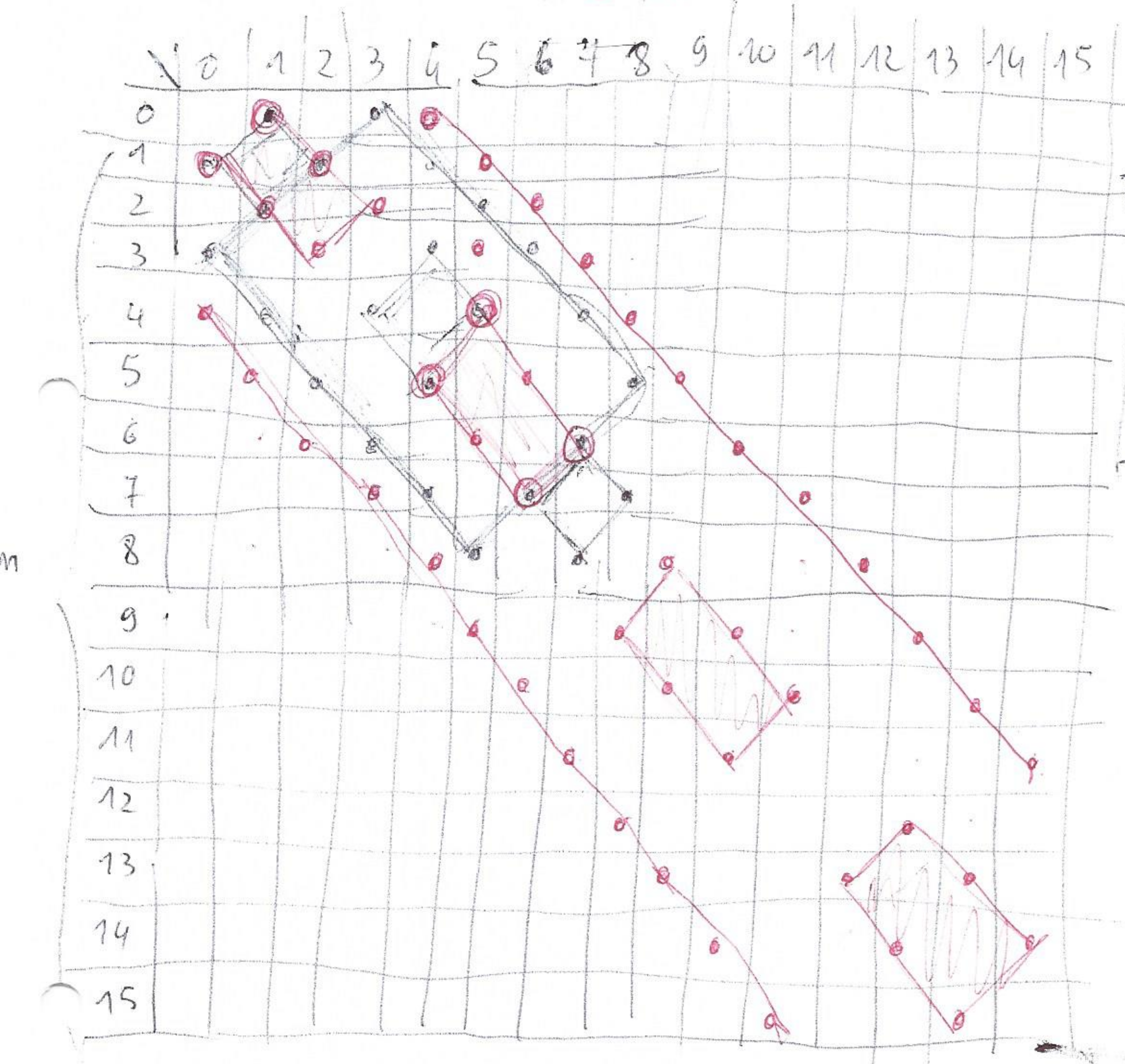
| | 4x4 | | | | |
|---|---|---|---|---|---|
| | -4 | -3 | -2 | -1 | |
| -1 | 0 | 1 | 2 | 3 | 4 |
| 3 | 4 | 5 | 6 | 7 | 5 |
| 7 | 8 | 9 | 10 | 11 | 6 |
| 11 | 12 | 13 | 14 | 15 | 7 |
| | 16 | 17 | 18 | 19 | |

$f_1$ $n-4 > 0$

$f_2$ $n-1 \bmod p \neq 3$

$f_3$ $n+1 \bmod p \neq 0$

$f_4$ $n+4 < p^2$



Formule général

$$\boxed{\begin{array}{l} n-p > 0 \\ n-1 \bmod p \neq p-1 \\ n+1 \bmod p \neq 0 \\ n+p < p^2 \end{array}}$$

$$\boxed{\begin{array}{l} n > p \\ n \bmod p \neq 0 \\ n \bmod p \neq p-1 \\ n < p^2-p \end{array}}$$

4x4 pour 5 : $5-4$, $5-1$, $5+1$, $5+4$ — $10-4$, $10-1$, $10+1$, $10+4$ — $7-3$, $7-1$, $7+1$, $7+3$

3x3 pour 5 : $5-3$, $5-1$, $(5+1)$, $5+3$

$n \rightarrow$ $n-p$, $n-1$, $n+1$, $n+p$

condition : $f > 0$, $f < p^2$

$$f(n) = \begin{cases} f3(n) = n+1 \,\%\, 3 \neq 0 \\ f2(n) = n-1 \,\%\, 3 \neq 2 \\ f1(n) = n-p > 0 \\ f4(n) = n+p < p^2 \end{cases}$$

$n \% p \neq p-1$

$n \% p \neq$

A FAIRE = ☐ Simulation EXCEL (le 24/12/2012)

| | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | 1 | | 1 | |

$f(0$

$f(5) = \quad 6$

$3$

$0$

si$\left(B = f(A)\right]$

$= 1$

$B \; B$ si$( A > \$A\$1$

et $A-$

$A$

$A$

$$\begin{cases} f(1,n) = n - p & \text{si } n-p > 0 \\ f(2,n) = n-1 & n \quad n \bmod p \neq 0 \\ f(3,n) = n+1 & \text{si } n \bmod p \neq p-1 \\ f(4,n) = n+p & \text{si } n < p^2 - p \end{cases}$$

H

G

D

B

Valeur $= 1$ si

$$\text{ou} \begin{cases} B = A-p & \text{et } A \geq p \\ B = A-1 & \text{et } A \bmod p \neq 0 \\ B = A+1 & \text{et } A \bmod p \neq p-1 \\ B = A+p & \text{et } A < p^2 - p \end{cases}$$

```
si (  ou(  ET( B$1 = $A2-$A$1 ; $A2>=$A$1 );
           ET( B$1 = $A2-1    ; MOD( $A2, $A$1)<>0 );
           ET( B$1 = $A2+1    ; MOD($A2 ; $A$1) <> $A$1-1);
           ET( B$1 = $A2+$A$1 ; $A2 < $A$1x$A$1-$A$1)
      ); 1 ; 0 )
```