Minimum headway constraint for dispatching adjacent vehicles:

|  |  |
| --- | --- |
|  | (5) |

In the sub-problem , the minimum headway constraint (5) is a hard constraint, which imposes conflicts for trains choosing arcs in each set . We use slack variables , , to convert the inequality constraint (5) into the equality constraint as listed in Eq. (D-1).

|  |  |
| --- | --- |
|  | (D-1) |

Then, we adopt the ADMM to dualize constraint (D-1) into the objective function (12) by introducing additional Lagrangian multipliers and quadratic penalty terms. Then, the sub-problem can be transformed to sub-problem as follow.

* M7: ADMM formulation of model M5:

**Objective function:**

|  |  |
| --- | --- |
|  | (D-2) |

**Subject to:** Eqs. (3) and (7).

The objective function (D-2) of model M7 has a quadratic penalty terms, which leads the problem difficult to be directly solved. However, the sub-problem can be decomposed into the sub-problem (Model M6), which is a standard time-dependent shortest path problem. We design an iterative framework to solve the sub-problem by solving the problem focusing on each train sequentially in the TSTN as shown in Appendix B. The detailed decomposition process of M7 to M6 is stated in the follows.

To divide sub-problem into several independent problems for each train and solve the sub-problem sequentially, we need to separate out the variables related with train from other trains, which is a simple process in linear addition terms but difficult in quadratic terms in Eq. (D-2). To handle this issue, we define to represent the total number of trains (except train ) who occupy . The is calculated as Eq. (D-3).

|  |  |
| --- | --- |
|  | (D-3) |

Then, the quadratic term in sub-problem can be transformed into Eq. (D-4):

|  |  |
| --- | --- |
|  | (D-4) |

In order to discuss the value of we first express as a function of as Eq. (D-5):

|  |  |
| --- | --- |
|  | (D-5) |

where is independent on . As is a quadratic function, the optimal value of can be represented by Eq. (D-6).

|  |  |
| --- | --- |
|  | (D-6) |

As shown in Eq. (D-6), the best value of depends on and . Thus, we discuss the value of and the corresponding calculation method of the quadratic terms based on following two **s**cenarios.

1. **Scenario 1:**

In this scenario, the function is shown in Fig.. D.1, in which we can see that , thus the optimal value of is .



Fig. D.1 The value of varying with

Based on this, the quadratic terms in sub-problem can be represented by Eq. (D-7):

|  |  |
| --- | --- |
|  | (D-7) |

where is a constant value in the sub-problem for train *q*. And can only be 0 or 1 as that each train can occupy once at most. Thus, we have

Further, the quadratic terms can be expressed as a linear function about represented by Eq. (D-8):

|  |  |
| --- | --- |
|  | (D-8) |

Then, the objective function (Eq. (D-9)) for each train *q* becomes a time-dependent shortest path finding problem with general arc costs represented by Eq. (D-10).

|  |  |
| --- | --- |
|  | (D-9) |
|  | (D-10) |

1. **Scenario 2:**

In this situation, is in the range of 0 to 1, so we have Eq. (D-11):

|  |  |
| --- | --- |
|  | (D-11) |

Put Eq. (D-11) into the quadratic terms, we have Eq. (D-12):

|  |  |
| --- | --- |
|  | (D-12) |

Similarly with that in scenario 1, the objective function (Eq. (D-13)) for each train *q* is a time-dependent shortest path finding problem with general arc costs represented by Eq. (D-14).

|  |  |
| --- | --- |
|  | (D-13) |
|  | (D-14) |

According to the above two scenarios, the value of can be represented by Eq. (D-15):

|  |  |
| --- | --- |
|  | (D-15) |

The sub-problem (Eq. (D-16)) for each vehicle is reformulated as a time-dependent shortest path problem,

|  |  |
| --- | --- |
|  | (D-16) |

where is an constant value, and the arc costs can be expressed as Eq. (D-17).

|  |  |
| --- | --- |
|  | (D-17) |