PHZ4151C, Fall 2019
Homework 3
Due Friday Oct 4th, 11.59PM

Instructions: Solve all four problems. Every problem is worth 10 points. Write your codes following the strategy discussed in the "Structure of the program and program design" video and the format used in the sample programs discussed in the class and lecture videos. Make sure that your code is clear, readable for users other than yourself, and properly commented. Once finished, include all your program files and figures into a zip file named after your last name and homework number (e.g. "Ullah_HW3.zip") and submit it through canvas. Submitting individual files for each problem or part of the problem will not be accepted and disregarded. There is no need to include big data files generated by your programs unless the data files are read by your programs (that is the program reads data from the file and uses that data). Email me any questions or concerns about the homework either through canvas or directly my email address.
Note: You are allowed to submit incomplete codes for partial credit.

Problem 1: Numerically, the differential of a sampled function is given as

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{\Delta x} \qquad (1)$$

Equation (1) is called forward difference formula where $\Delta x = x_{i+1} - x_i$. Generate a vector containing 100 values of the function sin(x) starting at x = 0 and using a step size Δx = 0.05. Find the differential of your vector at each x value using equation (1).
Since the differential at the last x (100^{th}) value cannot be calculated using equation (1) (because you will need the 101^{th} value; notice x_{i+1}), use the following backward difference formula to calculate the differential at the last x value.

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{\Delta x} \qquad (2)$$

Problem 2: The probability of Normalized Gaussian Distribution is given by the following function

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \qquad (3)$$

Integrate *f(x)* using Trapezoid Rule over the range $-4 \le x \le 4$. Plot the absolute error (|approximate value – theoretical value|) versus N where N runs from 11 to 101 and can take only odd values.

Problem 3: Repeat **Problem 2** using Simpson Rule and Gaussian Quadrature.

Problem 4: Write a single program that uses Simpson's rule to evaluate the following integrals.

$$\int_0^{\pi} x^2 \cos(x)\, dx$$

$$\int_0^{\pi} x^2 \sin(x)\, dx \qquad (4)$$

Increase N from 3 in units of 2 until the results from Simpson's rule agree to the exact values of the integrals within 10^{-5}.