

1 QuickSort

See the source code file `quickSort.cpp` and the tests given in `tests.cpp`.

2 Big-O Proofs

Problem 1. Show that $5n^3 + n^2 + 4$ is $O(n^3)$.

$$c = 10$$

$$n_0 = 1$$

$$\text{for } n \geq 1 = n_0$$

$$5n^3 + n^2 + 4 \leq 10n^3$$

$$5n^3 + n^2 + 4 \leq 5n^3 + n^3 + 4n^3 \leq 10n^3 \text{ since } n \geq 1$$

Problem 2. Show that $2n^4 - 3n^2 + n$ is $O(n^4)$.

$$c = 3$$

$$n_0 = 1$$

$$\text{for } n \geq 1 = n_0$$

$$2n^4 - 3n^2 + n \leq 3n^4$$

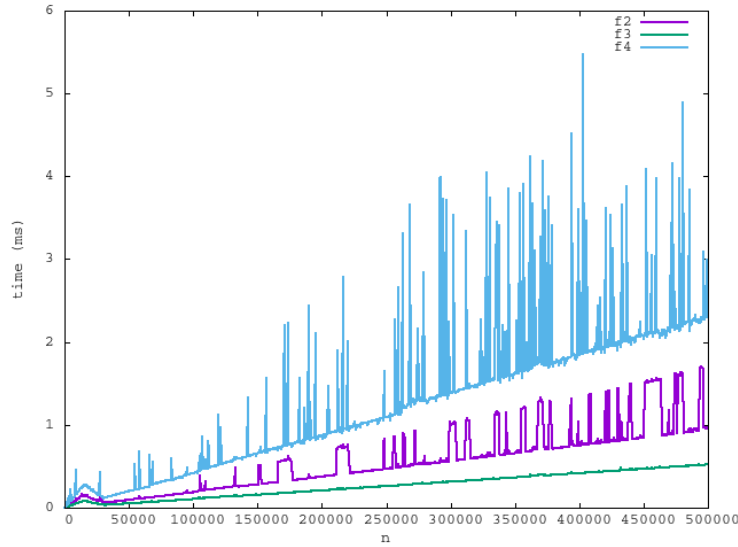
$$2n^4 - 3n^2 + n \leq 2n^4 + n^4 \leq 3n^4 \text{ since } n \geq 1$$

3 Mystery Functions

fnA (f3) is linear $O(n)$ because for any n the number of steps will be $n/2$ where 2 is a constant that does not affect the big O. fnA is f3, because fnA is the fastest function, and the slope of f3 was lower than all the others.

fnB (f5) is quadratic $O(n^2)$ because for any n the amount of steps taken will be n^2 . fnB is f5, because fnB is the third slowest function, after fnD, which is $O(n^4)$, and fnF, which is $O(n^3)$.

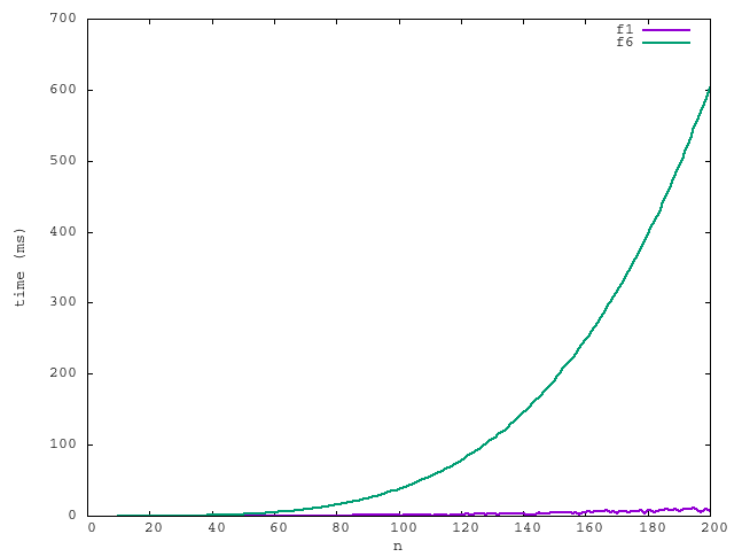
fnC (f4) is $O(n \log n)$ because the outer for loop runs n times. And each time the while loop runs the number of steps will be cut in half, thus making it $\log n$. fnC is f4 because the $O(n \log n)$ function is slower than the two linear functions, fnA and fnE (as shown below), and is faster than the other functions, which f4 is.



fnD (f6) is $O(n^4)$ because for any n the amount of steps taken for the function to finish is $(n * n)(n * n)$. fnD is f6 because fnD is the slowest function, and f6 has the highest slope.

fnE (f2) is linear $O(n)$ because for any n the number of steps will be $4n$ where 4 is a constant that does not affect the big O. fnC is f4 because fnE is the second fastest function, after fnA (which is also linear, but has a smaller constant), and f4 has the second lowest slope.

fnF (f1) is $O(n^3)$ because for any n the amount of steps taken for the function to finish is n -cubed. fnF is f1, because it is the second slowest function after fnD, which is $O(n^4)$, and f1 has the second highest slope (see figure below).



Functions in sorted order from fastest to slowest:

fnA
fnE
fnC
fnB
fnF
fnD