Synthetic Likelihoods for Parameter Inference in Electricity Spot Price Models

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Outline

- Part I: Features of Spot Electricity Prices and a Model
- Part II: What is a Synthetic Likelihood?
- Part III: Fitting & Results

Part I: Features of Electricity Spot Prices

Features of Electricity Prices:

- Large Volatility, Mean Reversion, Seasonality, Price Spikes

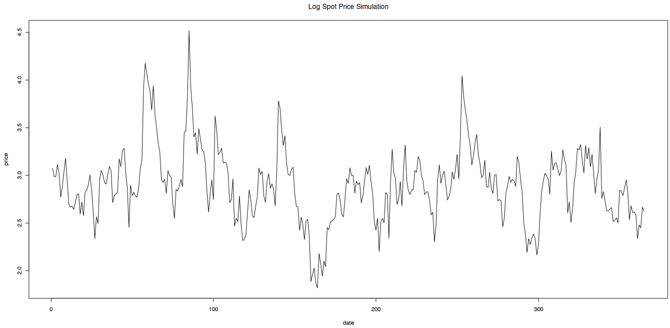


Figure: Example Trajectory from Model

Part I: Electricity Spot Price Model

We can account for these features by using a regime-switching model (Huisman & Mahieu 2003). We need three regimes:

- Regime 0: Default Price Levels.
- Regime 1: A price Spike.
- Regime -1: Reverting after a price spike.

Switching between regimes is given by the Markov process below where p is the probability of remaining in Regime 0.

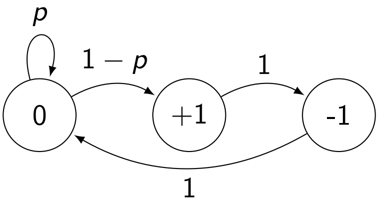


Figure: State Transition Diagram

Part I: Mathematical Specification of the Model

Let $s(t)$ be the natural log of the spot price on day t. Let $(\epsilon(t))$ be a sequence of iid standard Normal random variables.

- $s(t) = f(t) + x(t)$
- $f(t) = \mu_0 + \beta_1 D_1(t) + \beta_2 D_2(t)$
- $\mathrm{d}x(t) := x(t) - x(t-1)$, $x(0) = 0$
- $\mathrm{d}x(t) = \begin{cases} -\alpha_0 x(t-1) + \sigma_0 \epsilon(t) & \text{Day } t \text{ in Regime } 0 \\ \mu_1 + \sigma_1 \epsilon(t) & \text{Day } t \text{ in Regime } 1 \\ -\alpha_{-1} x(t-1) + \sigma_{-1} \epsilon(t) & \text{Day } t \text{ in Regime } -1 \end{cases}$

Part II: What is a Synthetic Likelihood?

- Model: $f(\boldsymbol{y}|\boldsymbol{\theta})$, Observed trajectory: $\boldsymbol{y}_0$
- Idea:
  - MLE: choose parameters that maximise the chance of seeing the observed trajectory
  - SLE: choose parameters that maximise the chance of a trajectory presenting some observed summary statistics.
- Issue: Computationally intensive (and choosing good statistics)
- Solution: Robust Covariance Matrix
- Issue: Need M-Statistics (we can write each statistic as the minimisation of a loss function)

Part II: Technical Details

Synthetic Likelihood Estimation (Wood 2010)

- Choose statistics: $\boldsymbol{s}(\cdot) = (s_1(\cdot), \dots, s_M(\cdot)) \sim \mathcal{N}(\boldsymbol{\mu}, \Sigma)$
- For some given parameters $\boldsymbol{\theta}$:
  - Generate samples: $\boldsymbol{y}_1, \dots \boldsymbol{y}_N$
  - Transform samples to statistics: $\boldsymbol{s}(\boldsymbol{y}_1), \dots, \boldsymbol{s}(\boldsymbol{y}_N)$
  - Calculate sample mean $\hat{\boldsymbol{\mu}}$ and covariance $\hat{\Sigma}$ of statistics
  - Compute Synthetic Likelihood $\log \phi(\boldsymbol{s}(\boldsymbol{y}_0) | \hat{\boldsymbol{\mu}}, \hat{\Sigma})$
- $\boldsymbol{\theta}_{\text{SLE}}$ chosen as the $\boldsymbol{\theta}$ to maximise $\log \phi(\boldsymbol{s}(\boldsymbol{y}_0) | \hat{\boldsymbol{\mu}}, \hat{\Sigma})$

Robust Covariance Matrix (Huber 1967)

- Write each statistic $s_i(\cdot)$ as the minimisation of loss function L_i. Define $L := \sum_{i=1}^{M} L_i$.
- Compute the hessian H_L of L and V, the sample covariance of the gradient of L (taken over each element of the trajectory).
- $\Sigma \approx \tilde{\Sigma} := H_L^{-1} V_L H_L^{-1}$

Part II: What statistics can we include?

M-Estimators

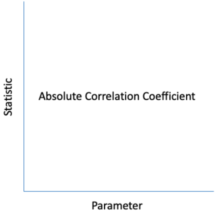
- ML estimators (minimum of negative log-likelihood)
  - mean, standard deviation, gamma shape parameter
- Regression Coefficients (minimum of squared residuals sum)
  - auto-linear regression - useful for those pesky mean-reversion parameters
  - polynomial regression of ordered simulation values on ordered observation values

Not M-Estimators

- Autocorrelation coefficients
- mean - median
- quartiles (min, Q1, median, Q3, max)
- more complex statistics

Part II: What is a useful statistic?

- Good statistics demonstrate high correlation with model parameters and low correlation with other statistics
- We can conduct simulation studies to compute correlation matrices to evaluate a set of parameters



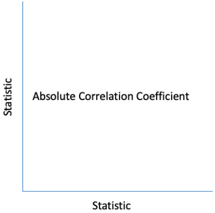


Figure: Correlation Diagrams

Part III: Fitting and Results

- Fitting the simplified model
- Computational efficiency of the RCM
- Full model correlation matrices
- Next steps

Part III(a): Simplified Model (Set-up)

Simplified Model: Only default regime (mean reversion)

- $s(t) = f(t) + x(t)$
- $f(t) = \mu_0 + \beta_1 D_1(t) + \beta_2 D_2(t)$
- $\mathrm{d}x(t) = -\alpha_0 x(t-1) + \sigma_0 \epsilon(t)$

Statistics

- We can infer μ_0 and σ_0 by including the maximum likelihood estimators of normal distribution
- For β_1 and β_2 we can use the mean taken over Saturdays and Sundays (with a quadratic loss)
- For α_0, an autoregression up to lag 1 makes sense.
- All *M*-Estimators!

Part III(a): Simplified Model (Statistics Analysis)

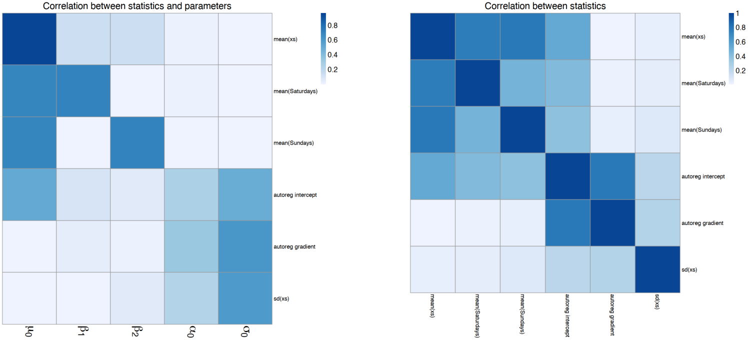


Figure: Correlation plots

Part III(a): Simplified Model (Simulation Study)

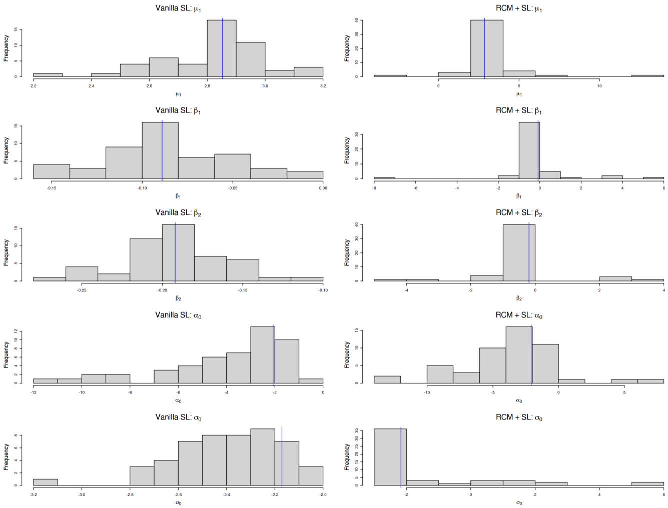
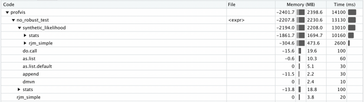


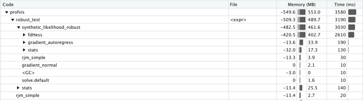
Figure: Simple Model Simulation Study

Part III(b): Computational Efficiency of RCM

- Used the **profvis** R library
- Wrote functions that:
  - generated an ‘observed trajectory’ using the true parameters
  - evaluated the observed statistics
  - calculated the synthetic likelihood of the true parameters
- Evaluated the above 1000 times each



Code	File	Memory (MB)		Time (ms)
▾ profvis		-2401.7	2398.6	14100
▾ no_robust_test	<expr>	-2207.8	2230.6	13130
▾ synthetic_likelihood		-2194.0	2208.0	13010
▸ stats		-1861.7	1694.7	10160
▸ rjm_simple		-304.6	473.6	2600
do.call		-15.6	19.6	100
as.list		-0.6	10.3	60
as.list.default		0	5.1	30
append		-11.5	2.2	30
dmvn		0	2.4	10
▸ stats		-13.8	18.8	100
rjm_simple		0	3.8	20

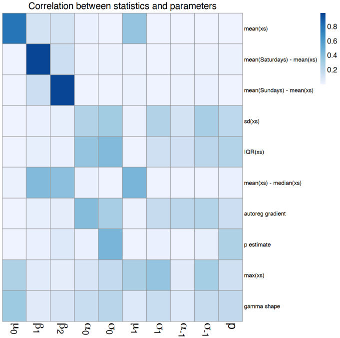


Code	File	Memory (MB)		Time (ms)
▾ profvis		-549.6	553.0	3580
▾ robust_test	<expr>	-509.3	489.7	3190
▾ synthetic_likelihood_robust		-482.5	461.6	3030
▸ fdHess		-420.5	402.7	2610
▸ gradient_autoregress		-13.6	33.9	190
▸ stats		-32.0	17.3	130
rjm_simple		-13.3	3.9	30
gradient_normal		0	2.1	10
<GC>		-3.0	0	10
solve.default		0	1.6	10
▸ stats		-13.4	25.5	140
rjm_simple		-13.4	2.7	20

Figure: **profvis** results

- RCM is $\approx 4\times$ faster.
- Comes from needing $100\times$ fewer samples (observation length kept the same)

Part III(c): Full model correlation matrices



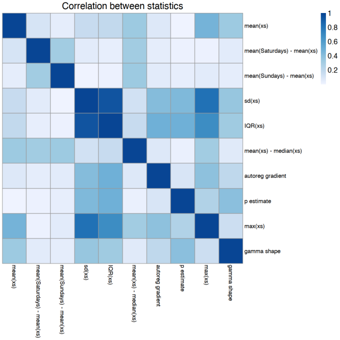


Figure: Correlation plots

Next Steps & References

Next Steps

- Finish fitting the full model
- See if non *M*-estimators can be replaced with *M*-estimators
- Fit actual data, use this to price financial options

References

- Huisman, R. and Mahieu, R., 2003. Regime jumps in electricity prices. Energy Economics, 25(5), pp.425-434.
- Wood, S., 2010. Statistical inference for noisy nonlinear ecological dynamic systems. Nature, 466(7310), pp.1102-1104.
- Huber, P., 1967. The behavior of maximum likelihood estimates under nonstandard conditions. Proceedings of the 5th Berkeley symposium on mathematical statistics and probability, 1, pp.221-233.