PS8

Thursday, 3 December 2020 21:54

1) $H_0: \lambda \leq 0.5$
$H_1: \lambda > 0.5$

The equivilent test statistic $T(\underline{X}) = \sum_{i=1}^{n} X_i \sim \text{Poisson}(n\lambda)$

Now T does not dependent on λ so it is the UMP test statistic for the situation above.

We have

$$T(\underline{x}) = 0 \times 109 + 1 \times 65 + 2 \times 22 + 3 \times 3 + 4 \times 1 = 122$$

$$p(x) = \mathbb{P}\left(\sum_{i=1}^{n} X_i \geq 122; \lambda = 0.5\right)$$
$$= 1 - \text{ppois}(122, 0.5 \times 200)$$
$$= 0.0143 \quad (3\text{s.f})$$

2a) $T(\underline{X}) \sim \text{Poisson}(n\lambda) \simeq \text{Normal}(n\lambda, n\lambda)$

$$\Rightarrow P(x) = \mathbb{P}\left(Z \geq \frac{122 - 100}{\sqrt{100}}\right)$$
$$= \mathbb{P}(Z \geq 2.2)$$
$$= 1 - \text{pnorm}(2.2)$$
$$= 0.0139 \quad (3\text{s.f})$$

b) Monte Carlo Integration:

$$F = \int_a^b f(x)\, dx$$

$$\{\hat{F}_i\} = (b-a)\frac{1}{N}\sum_{j=0}^{N-1} f(a + \xi_j(b-a))$$

where ξ_j = random number uniformly distributed between $[0,1]$

[From Internet]

In this case

$$F = \int_{-\infty}^{2.2} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}\, dx$$

Make substitution $U = e^x$ to get rid of $-\infty$.

$$dU = e^x\, dx = U\, dx$$

$$F = \int_0^{e^{2.2}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(\ln U)^2} \frac{1}{U}\, dU$$

Approx p-value = 0.132. See R-code.

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3a) $H_0: \lambda = 0.55$
$H_1: \lambda \neq 0.55$

$$\Lambda_n(\underline{x}) = \frac{\sup_{\theta \in \{0.55\}} f_n(\underline{x};\theta)}{\sup_{\theta \in \mathbb{R}_{\geq 0}} f_n(\underline{x};\theta)}$$

$$= \frac{f_n(\underline{x}; 0.55)}{f_n(\underline{x}; \hat{\lambda}_n)}$$

$\hat{\lambda}_n$ is ML Estimate.

$T_n(\underline{x}) = -2\log \Lambda_n$

$$= -2\left[\ell(0.55;\underline{x}) - \ell(\hat{\lambda}_n; \underline{x})\right] \xrightarrow{\mathcal{D}} \chi^2_1$$

So $T_n(\underline{x}) = 1.263926$ from R.

$p(\underline{x}) = 1 - \text{pchisq}(T_n(\underline{x}), df=1) = 0.2609...$

So Retain H_0.

As $T_n \xrightarrow{\mathcal{D}} \chi^2_1 = Z^2$ by Rule of Thumb our 95% CI is

$$\{\theta \in \mathbb{R}_{>0} : \ell(\theta;\underline{x}) \geq \ell(\hat{\lambda}_n;\underline{x}) - 2\}$$

$$= \{\theta \in \mathbb{R}_{>0} : \ell(\theta;\underline{x}) \geq -184.304\}$$

By R we get $CI = \{\theta \in \mathbb{R}_{>0} : 0.51 \leq \theta \leq 0.72\}$

b) At 10% we still retain H_0 as our p-value is $0.26... > 0.1$.

Problem Sheet 8

Problem Sheet 8

Question 1

```
p = 1 - ppois(122,0.5*200)
p
```

```
## [1] 0.01430626
```

Question 2a

```
1 - pnorm(2.2)
```

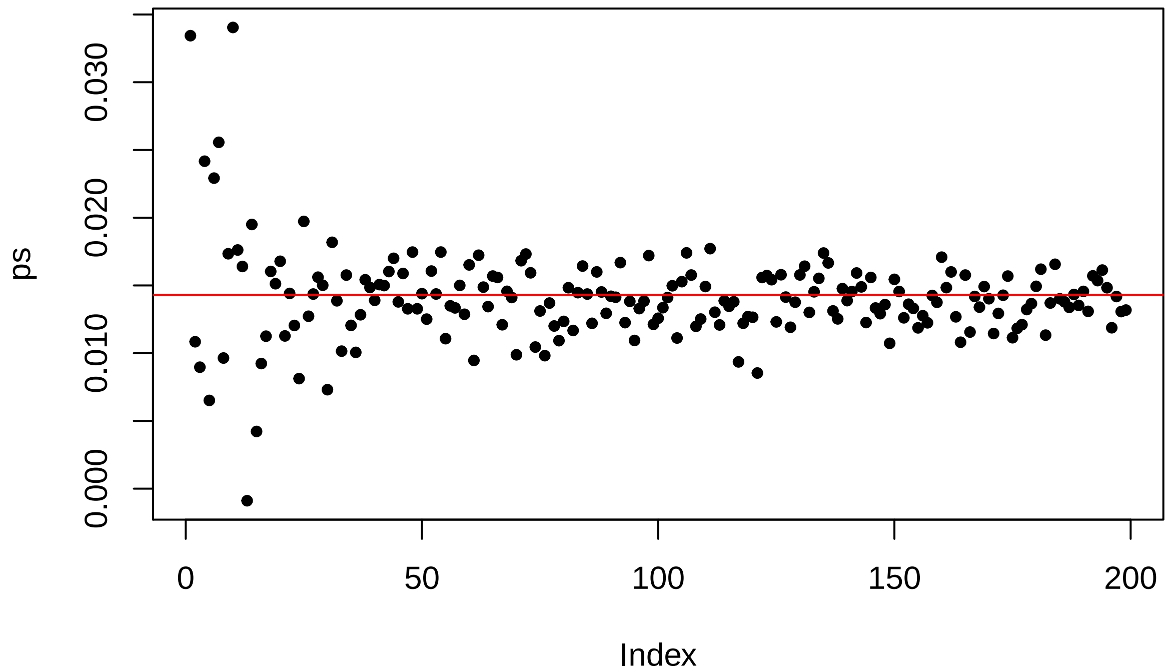
```
## [1] 0.01390345
```

Question 2b

```
f <- function(u) (1/sqrt(2*pi)) * exp(-(log(u)^2) / 2) / u
a <- 0
b <- exp(2.2)

mc.int <- function(f, a, b, N) {
  F <- 0
  for (i in 0:(N-1)) {
    xi <- runif(1, 0, 1)

    F <- F + f(a + xi*(b - a))
  }
  F <- F * (b - a) / N

  1-F
}

ps <- vector()
for (N in seq(10000, 1000000, 5000)) {
  ps <- c(ps, mc.int(f, a, b, N))
}

tail(ps, 1)
```

```
## [1] 0.01318355
```

```
plot(ps, pch = 20)
abline(h=p, col="red")
```



Question 3a

```
ell <- function(lambda, xobs) {
  stopifnot(all(lambda > 0))
  n <- length(xobs)
  n * (-lambda + mean(xobs) * log(lambda))
}

xobs <- c(rep(0, 109), rep(1,65), rep(2, 22), rep(3, 3), rep(4, 1))

lambda.hat <- mean(xobs)

Tn <- -2*(ell(0.55, xobs) - ell(lambda.hat, xobs))

Tn
```

```
## [1] 1.263926
```

```r
1-pchisq(Tn, df=1)
```

```
## [1] 0.2609093
```

```r
l = ell(lambda.hat, xobs)-2

C <- vector()

for (i in seq(0.01,1,0.01)) {
  if (ell(i, xobs) >= l) {
    C <- c(C, i)
  }
}

c(min(C), max(C))
```

```
## [1] 0.51 0.72
```