The effects of exoplanet's shape on its transit light curve

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1 Introduction

Several decades have passed since the discovery of the first exoplanet. To this day, several thousand exoplanets have been confirmed and many more candidates are yet to be confirmed. Transit photometry is the most common way of detecting these planets. Transit photometry relies on the fact, that when a planet passes in front of its host star, the star's brightness decreases. The relation between brightness and time can tell us the ratio of the planet's and star's projected areas, the length of the orbit can tell us their respetive masses and then we can calculate these parameters absolutely thanks to the star's luminosity and spectrum.

Of course, planets aren't always strictly spherical and thus more variables come into play. Namely, the planet can be ellipsoidal in shape with 3 distinct semi-axes. The aim of this project was to determine the planet's transit light curve based on its shape.

2 Methods

2.1 Angular size

First, we need to determine how much the apparent size of the planet changes. We can directly compare the difference between angular diameter of the planet at the point when it is closest to earth and at the point when it just crosses its host star, and the difference between the angular diameter of two planets - one spherical and the other ellipsoidal, both projecting to a circle and ellipse of the same respective area. We can approximate the angular size as

$$\delta = \frac{d}{D} \tag{2.1.1}$$

where d is the diameter of the planet and D is its distance to observer. Let's assume that the planet orbits a circular orbit and that its distance from its host star is $2 \cdot 10^8$ km. Next, we assume that the star is 10^{13} km far away and its diameter is 10^5 km. Let's be very generous and assume that the transit takes place between $-\frac{\pi}{20}$ and $\frac{\pi}{20}$ radians from when the planet is closest to the observer. Applying basic trigonometry, we get the equation

$$D_2 = \sqrt{(D_1 + o)^2 + 1 - 2\cos(\sigma)(o^2 + D_1 o)} \tag{2.1.2}$$

where D_2 is distance when the planet is the farthest away during its transit and D_1 is distance when the planet is the nearest, o is the distance between the planet and the star and 2σ is the angle of the whole transit. Calculating the difference between the angular size for D_1 and D_2 with above mentioned values we get $\Delta\delta = 2.02 \cdot 10^{-13}$. This difference only becomes smaller for larger distances and smaller angles (we were very generous in both cases), so we we will neglect all effects of angular size.

2.2 Planet shape

To some degree, all planets are non-spherical but the degree of oblateneness varies. Oblateness depends on rotation, external gracitational tides and material rigidity. [1] Close-in planets undergo strong tidal effects due to their parent star. Some of the concequences are that their spins and orbits change until equilibrium is reached, their orbits become circular and synchronous and planetary bodies less spherical. [2] The shape of such a planet can generally be described by a 3-axis ellipsoid. Such an ellipsoid, when projected to a plane, can be thought of as an ellipse with varying semi-axes a' and b'. Semi-axes a' and b' are described by the equation [2] [thanks to Dr. Walterová]

$$a', b' = \frac{-\sqrt{-2(C^2 - 4AB)\left(A + B \mp \sqrt{(A-B)^2 + C^2}\right)}}{C^2 - 4AB} \tag{2.2.1}$$

where

$$A = \frac{\sin^2\alpha}{a^2} + \frac{\cos^2\alpha}{b^2}, B = \frac{(\cos\alpha\cos i)^2}{a^2} + \frac{(\sin\alpha\cos i)^2}{b^2} + \frac{\sin^2 i}{c^2}, C = \left(\frac{1}{a^2} - \frac{1}{b^2}\right)\sin 2\alpha\cos i \tag{2.2.2}$$

Parameters a, b and c are lengths of ellipsoid semi-axes such that a and b are on the plane of planet's orbit and c is perpendicular. α is the phase of the planet (in our case it is also the angle of the planet's orbit + some initial phase). The inclination i is 0 when the plane of orbit is perpendicular to line of sight and $\frac{\pi}{2}$ when the planet passes directly across the centre of the star.

2.3 Relative positions of a circle and an allipse, their overlap area

Relative positions of an ellipse and a circle, can be described in several parameters, namely the circle radius r and centre $[h_1, k_1]$ and the ellipse semi-axes a and b, centre $[h_2, k_2]$ and angle ϕ against x-axis. For our purposes, ϕ is described by equation [2] [thanks to Dr. Walterová]

$$\phi = \frac{1}{2}\arctan 2(-C, B - A) \tag{2.3.1}$$

There exist semi-analytical methods for calculating ellipse-ellipse intersection area (analytical on a case-by-case basis) [3] but in our case, due to higher precision, we use a numerical method for calculating the overlap. We calculate the relative positions on-the-fly by projecting the orbits of the planet and the star to a plane tangent to the sky

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \frac{rm_2}{m_2 + m_1}\begin{bmatrix} -\cos(\omega t) \\ \cos(i)\cdot\sin(\omega t) \end{bmatrix}, \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \frac{rm_1}{m_2 + m_1}\begin{bmatrix} \cos(\omega t) \\ -\cos(i)\cdot\sin(\omega t) \end{bmatrix} \tag{2.3.2}$$

where

$$\omega = \frac{1}{r}\sqrt{G\frac{m_1 + m_2}{r}} \tag{2.3.3}$$

Here, r is the distance between the star and the planet, m_1 and m_2 are the masses of the star and the planet respectively, i is the inclination of the orbit, G is the gravitational constant (in our case set to 1 for the sake of simplicity) and t is the time parameter.

2.4 realistic transit light curves

Realistically, the transit light curve of a transiting planet isn't defined only by the mass and shape of the star and the planet but depends on limb darkening of the star and stellar activity too. Predicting stellar activity or accounting for limb darkening is however beyond the scope of this project so we will only take mass and shape into account.

3 Results

Now that we have all necessary equations, we can calculate the transit light curves. In the following figures we will assume that the star is always the same size ($r_1 = 2$), masses of the star and the planet remain the same ($m_1 = 8$, $m_2 = 1$), their relative distance remains the same ($r = 10$) and G is equal to 1. One orbit is signified by t. For demonstration purposees, the parameters are very unrealistic.

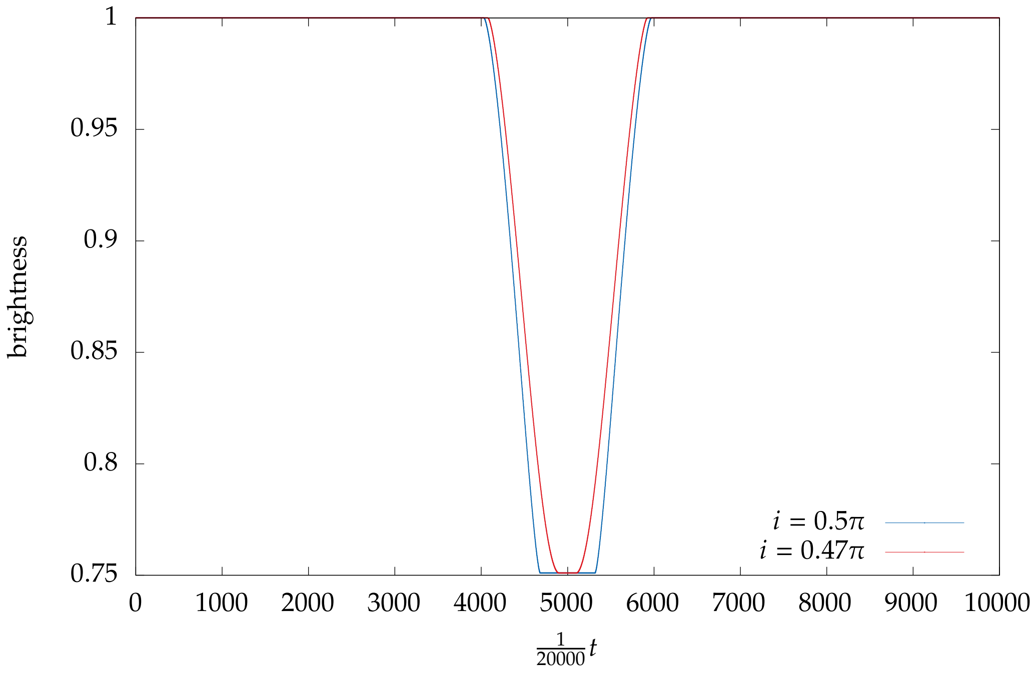

Figure 1: Two spheres of the same size ($r_2 = 1$) but of different inclination i

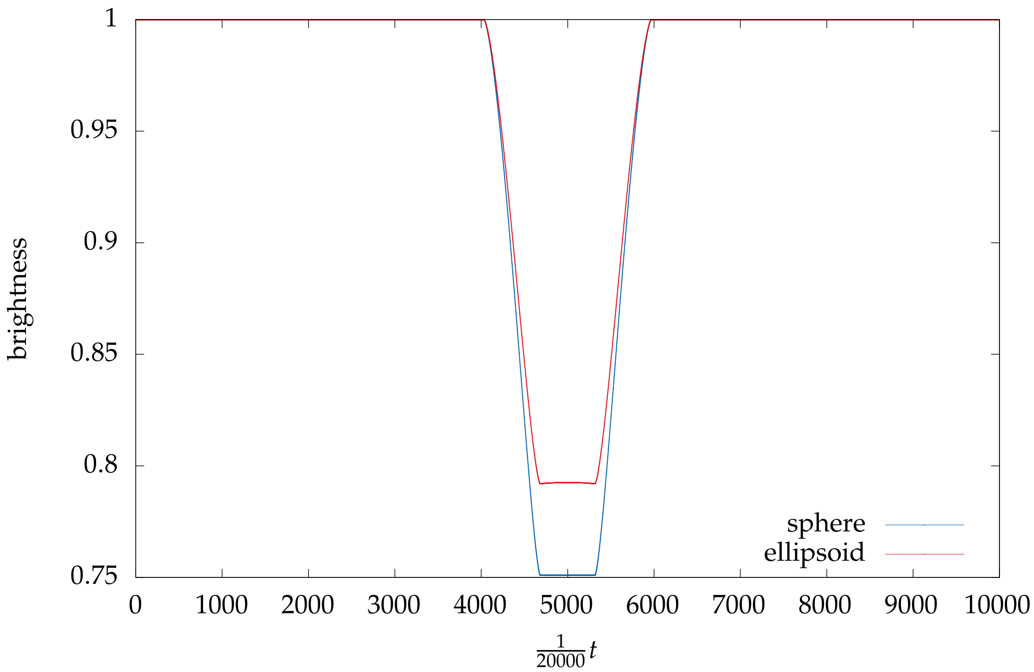

Figure 2: Ellipsoid and sphere of the same volume ($r_2 = 1$, $a = 1.2$, $b = 0.8333333$, $c = 1$), inclination $i = 0.5\pi$ and phase $\alpha_0 = 0$

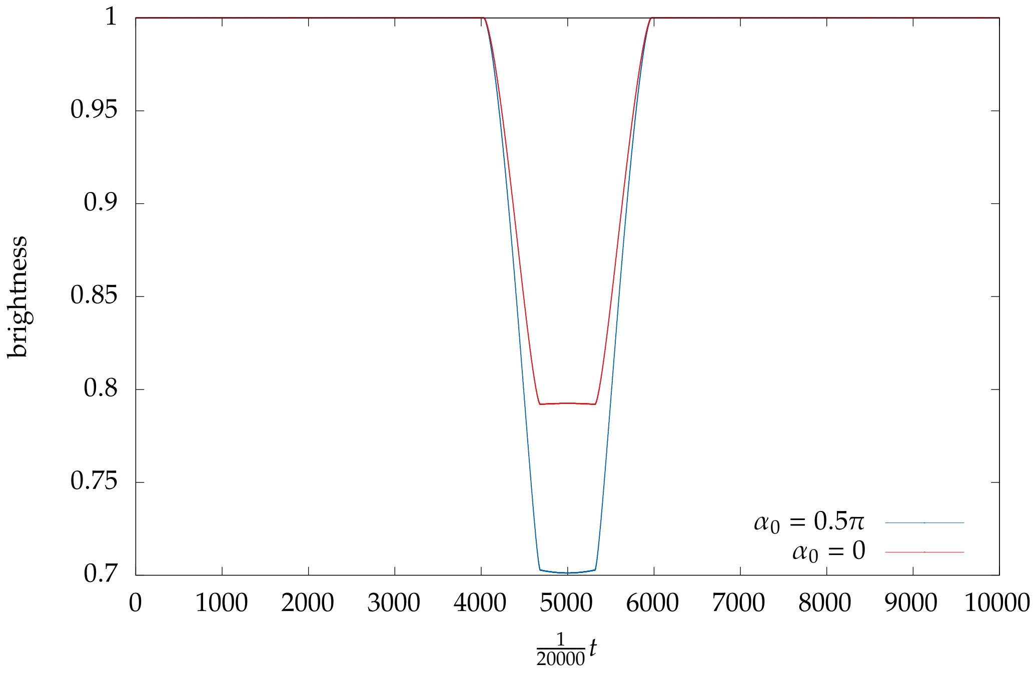

Figure 3: Two ellipsoids with the same $i = 0.5\pi$ and size ($a = 1.2$, $b = 0.8333333$, $c = 1$) but different phase α_0

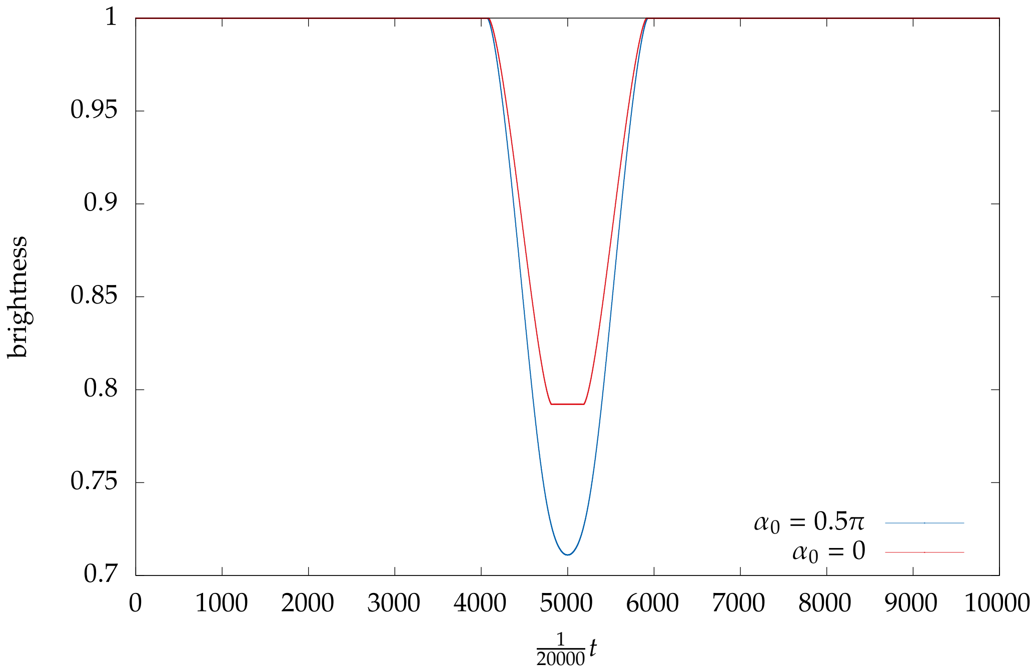

Figure 4: Two ellipsoids with the same $i = 0.5\pi$ and size ($a = 1.2$, $b = 0.8333333$, $c = 1$), but different phase α_0

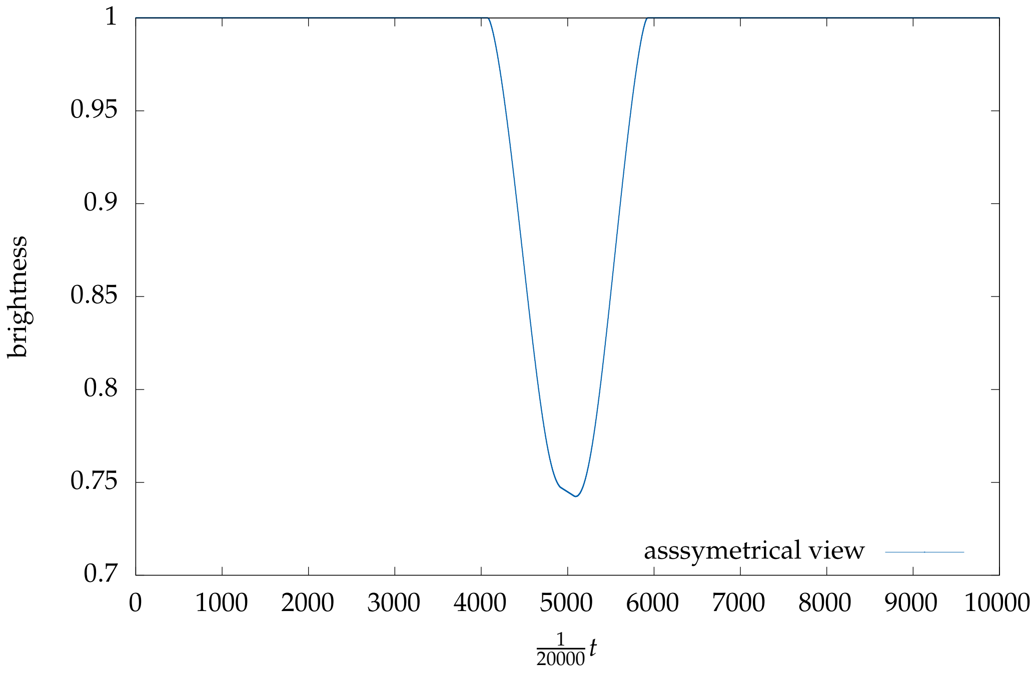

Figure 5: Ellipsoid in a more "general" configuration, $a = 1.2$, $b = 0.833333$, c = 1, $i = 0.47\pi$, $\alpha_0 = 0.3\pi$

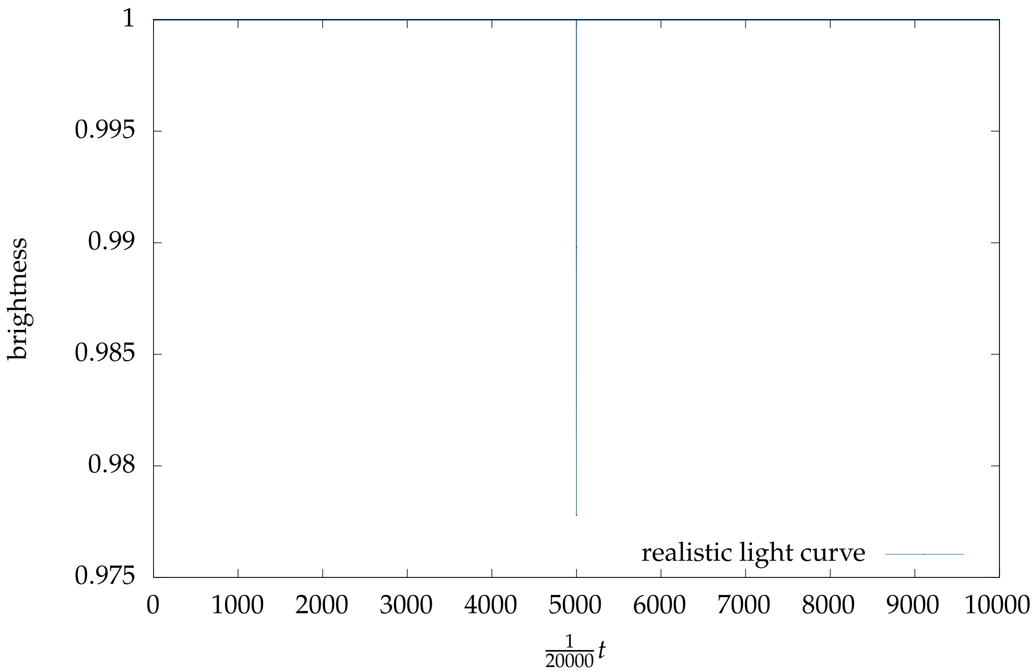

Figure 6: for the sake of argument, a more "realistic" transit light curve with the parameters: $G = 3.0362 M_\odot R_\odot^3 s^{-2}$, $m_1 = 0.4 M_\odot$, $m_2 = 0.002 M_\odot$, $r_1 = 0.7 R_\odot$, $a = 0.1100 R_\odot$, $b = 0.1 R_\odot$, $c = 0.1028\odot$, $\alpha_0 = 0.3\pi$, $i = 0.4999\pi$, $r = 1000 R_\odot$

4 Conclusion

As can be seen above, the transit light curve indeed does depend on the shape of the planet quite strongly *if the planet is oblate enough*. In our special cases, the planets were singificantly more oblate than real planets, but as can be seen in figure 6, detecting any signal at all is difficult enough, moreover, the signal is full of noise and it becomes increasingly difficult to distinguish a transit from variations in the star's brightness. In real scenarios, it might be difficult to for example distinguish between a slightly inclined transit and a transit of an ellipsoidal planet.

References

[1] Joshua A. Carter and Joshua N. Winn. EMPIRICAL CONSTRAINTS ON THE OBLATENESS OF AN EXOPLANET. 2010 `http://dx.doi.org/10.1088/0004-637X/709/2/1219`

[2] Alexandre C. M. Correia. Transit light curve and inner structure of close-in planets. 2014. `https://doi.org/10.48550/arXiv.0912.1594`

[3] Hughes, G.B., Chraibi, M. Calculating ellipse overlap areas. Comput. Visual Sci. 15, 291–301 (2012). `https://doi.org/10.1007/s00791-013-0214-3`