The effects of exoplanet's shape on its transit light curve

https://github.com/jiriurbanek/SFG

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1 Introduction

Several decades have passed since the discovery of the first exoplanet. To this day, several thousand exoplanets have been confirmed and many more candidates are yet to be confirmed. Transit photometry is the most common way of detecting these planets. Transit photometry relies on the fact, that when a planet passes in front of its host star, the star's brightness decreases. The relation between brightness and time can tell us the ratio of the planet's and star's projected areas, the length of the orbit can tell us their respetive masses and then we can calculate these parameters absolutely thanks to the star's luminosity and spectrum.

Of course, planets aren't always strictly spherical and thus more variables come into play. Namely, the planet can be ellipsoidal in shape with 3 distinct semi-axes. The aim of this project was to determine the planet's transit light curve based on its shape.

2 Methods

2.1 Angular size

First, we need to determine how much the apparent size of the planet changes. We can directly compare the difference between angular diameter of the planet at the point when it is closest to earth (in the middle of the transit) and at the point when it just crosses its host star (the beginning/end of the transit).We can approximate the angular size as

$$\delta = \frac{d}{D} \tag{2.1.1}$$

where d is the diameter of the planet and D is its distance to observer. Let's assume that the planet orbits a circular orbit and that its distance from its host star is $2 \cdot 10^8$ km. Next, we assume that the star is 10^{13} km far away and its diameter is 10^5 km. Let's be very generous and assume that the transit takes place between $-\frac{\pi}{20}$ and $\frac{\pi}{20}$ radians from when the planet is closest to the observer. Applying basic trigonometry, we get the equation

$$D_2 = \sqrt{(D_1 + o)^2 + 1 - 2\cos(\sigma)(o^2 + D_1 o)} \tag{2.1.2}$$

where D_2 is distance when the planet is the farthest away during its transit and D_1 is distance when the planet is the nearest, o is the distance between the planet and the star and 2σ is the angle of the whole transit. Calculating the difference between the angular size for D_1 and D_2 with above mentioned values we get $\Delta\delta = 2.02 \cdot 10^{-13}$. This difference only becomes smaller for larger distances and smaller angles (we were very generous in both cases), so we we will neglect all effects of angular size variation.

2.2 Planet shape

To some degree, all planets are non-spherical but the degree of oblateneness varies. Oblateness depends on rotation, external gravitational tides and material rigidity. [1] Close-in planets undergo strong tidal effects due to their parent star. Some of the consequences are that their spins and orbits change until equilibrium is reached, their orbits become circular and synchronous and planetary bodies less spherical. [2] The shape of such a planet can generally be described by a 3-axis ellipsoid. Such an ellipsoid, when projected to a plane, can be thought of as an ellipse with varying semi-axes a' and b'. Semi-axes a' and b' are described by the equation [2] [thanks to Dr. Walterová]

$$a', b' = \frac{-\sqrt{-2(C^2 - 4AB)\left(A + B \mp \sqrt{(A-B)^2 + C^2}\right)}}{C^2 - 4AB} \tag{2.2.1}$$

where

$$A = \frac{\sin^2 \alpha}{a^2} + \frac{\cos^2 \alpha}{b^2}, B = \frac{(\cos\alpha \cos i)^2}{a^2} + \frac{(\sin\alpha \cos i)^2}{b^2} + \frac{\sin^2 i}{c^2}, C = \left(\frac{1}{a^2} - \frac{1}{b^2}\right)\sin 2\alpha \cos i \tag{2.2.2}$$

Parameters a, b and c are lengths of ellipsoid semi-axes such that a and b are on the plane of planet's orbit and c is perpendicular. For the sake of simplicity, this is indeed the case as tidal effects tend to align the rotation axis of the planet to its orbital axis. α is the phase of the planet . The inclination i is 0 when the plane of orbit is perpendicular to line of sight and $\frac{\pi}{2}$ when the planet passes directly across the centre of the star.

2.3 Relative positions of a circle and an ellipse, their overlap area

From now on, we will describe the planet as a 3-axes ellipsoid and the star as a uniform sphere with their projections being an ellipse and a circle respectively. Relative positions of an ellipse and a circle, can be described in several parameters, namely the circle radius r and centre $[h_1, k_1]$ and the ellipse semi-axes a and b, centre $[h_2, k_2]$ and angle ϕ against x-axis. For our purposes, ϕ is described by equation [2] [thanks to Dr. Walterová]

$$\phi = \frac{1}{2}\arctan 2(-C, B - A) \tag{2.3.1}$$

There exist semi-analytical methods for calculating ellipse-ellipse intersection area (analytical on a case-by-case basis) [3] but in our case, due to higher precision, we use a numerical method for calculating the overlap. We calculate the relative positions on-the-fly by projecting the orbits of the planet and the star to a plane tangent to the sky

$$\begin{bmatrix} h_1 \\ k_1 \end{bmatrix} = \frac{rm_2}{m_2 + m_1}\begin{bmatrix} -\cos(\omega t) \\ \cos(i)\cdot\sin(\omega t)\end{bmatrix}, \begin{bmatrix} h_2 \\ k_2 \end{bmatrix} = \frac{rm_1}{m_2 + m_1}\begin{bmatrix} \cos(\omega t) \\ -\cos(i)\cdot\sin(\omega t)\end{bmatrix} \tag{2.3.2}$$

where

$$\omega = \frac{1}{r}\sqrt{G\frac{m_1 + m_2}{r}} \tag{2.3.3}$$

Here, r is the distance between the star and the planet, m_1 and m_2 are the masses of the star and the planet respectively, i is the inclination of the orbit, G is the gravitational constant (in our case set to 1 for the sake of simplicity) and t is the time parameter. Because we are taking only tidally locked planets into account, the general formula for α is

$$\alpha = \omega t - \frac{\pi}{2} \tag{2.3.4}$$

For all purposes, t starts at the point when the planet is exactly perpendicular to the line connecting the observer and its host star. Our coordinate system is set in such a way that the planet starts on the left side of the star and moves to the right side, the centre is at the barycentre. For calculating ellipse-ellipse overlap we use `https://github.com/chraibi/EEOver`

2.4 Realistic transit light curves

Realistically, the transit light curve of a transiting planet isn't defined only by the mass and shape of the star and the planet but depends on limb darkening of the star and stellar activity too. Predicting stellar activity or accounting for limb darkening is however beyond the scope of this project so we will only take mass and shape into account.

3 Results

Now that we have all necessary equations, we can calculate the transit light curves. In the following figures we will assume that the star is always the same size ($r_1 = 2$), masses of the star and the planet remain the same ($m_1 = 8$, $m_2 = 1$), their relative distance remains the same ($r = 10$) and G is equal to 1. One orbit is signified by t. For demonstration purposees, the parameters are very unrealistic.

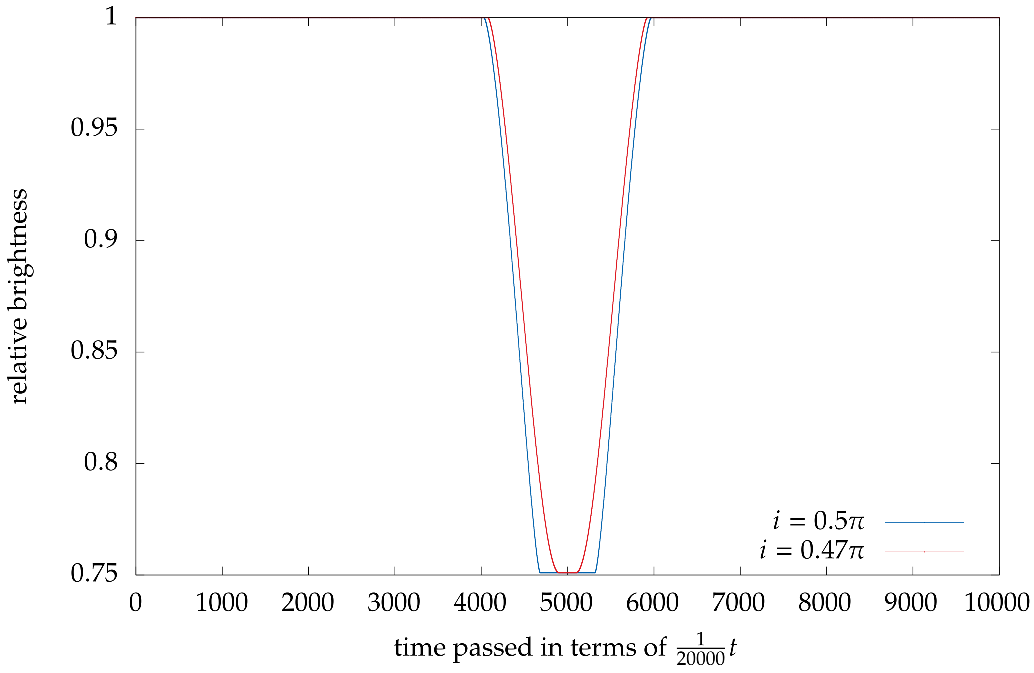

Figure 1: Two spheres of the same size ($r_2 = 1$) but of different inclination i, the planet with inclnation closer to 0.5π crosses the star as much as possible but the star with smaller inclination passes across a smaller portion of the star's surface, resulting in a different transit light curve while retaining the same peak dimming.

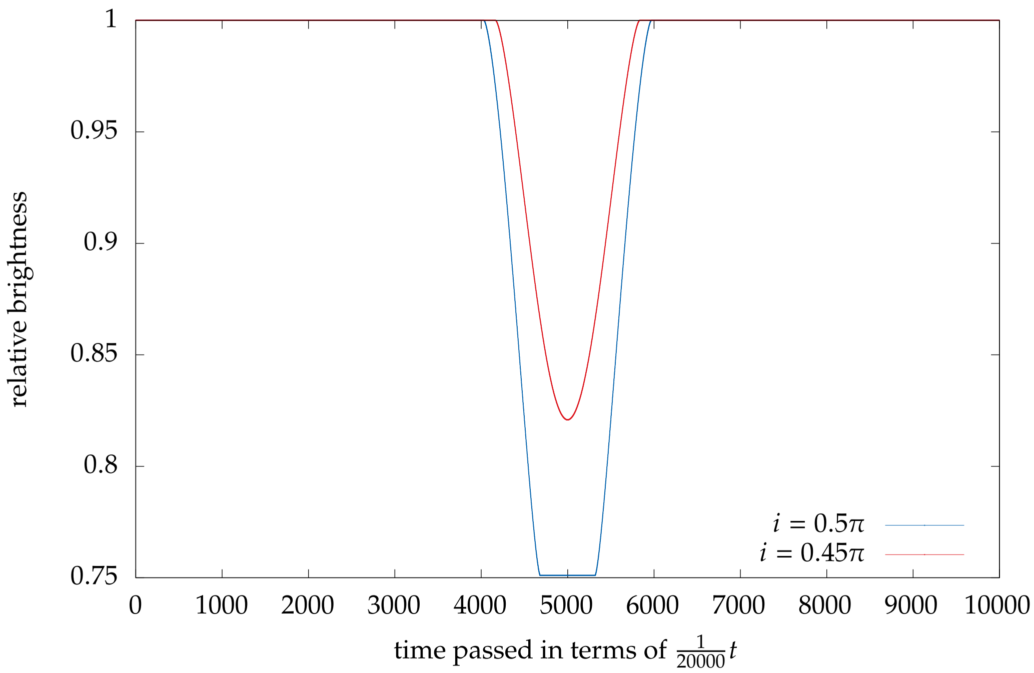

Figure 2: Two spheres of the same size ($r_2 = 1$) but of different inclination i. The sphere with inclination 0.45π doesn't cross the star's surface with its whole projected area and as a result, the largest dip in the star's relative brightness isn't as pronounced and the transit light curve has a different shape altogether.

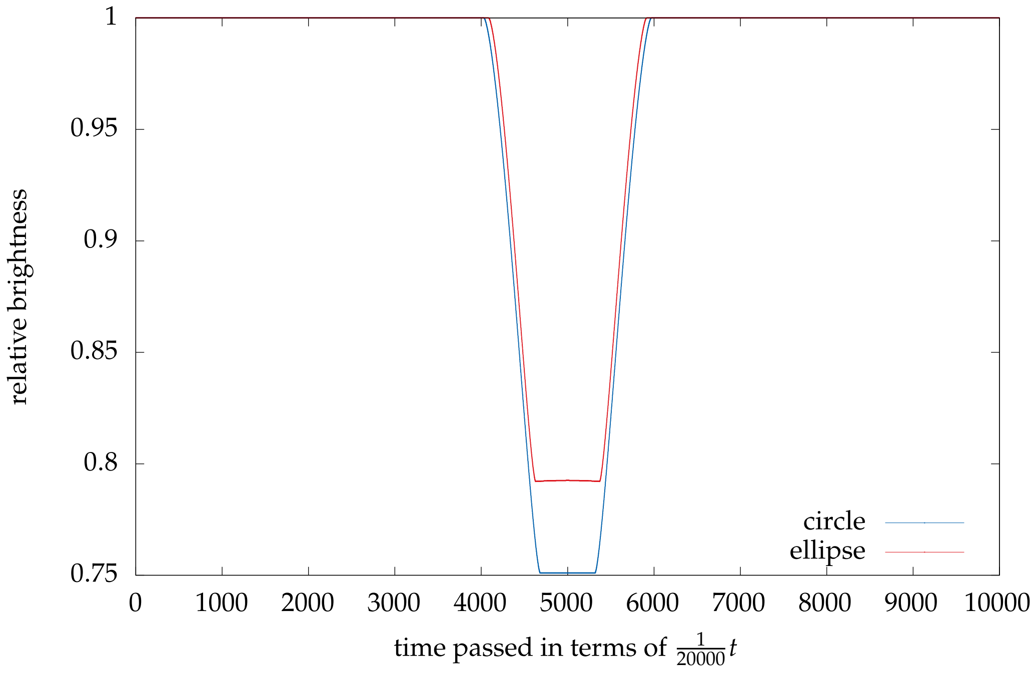

Figure 3: Sphere ($r_2 = 1$) and ellipsoid ($a = 1.2$, $b = 1$, $c = 0.8333333$) of effectively the same volume and same $i = 0.5\pi$. As can be seen, the ellipsoidal planet faces the observer directly when its projected area is the smallest and its projected area is always smaller than the projected area of the spherical planet.

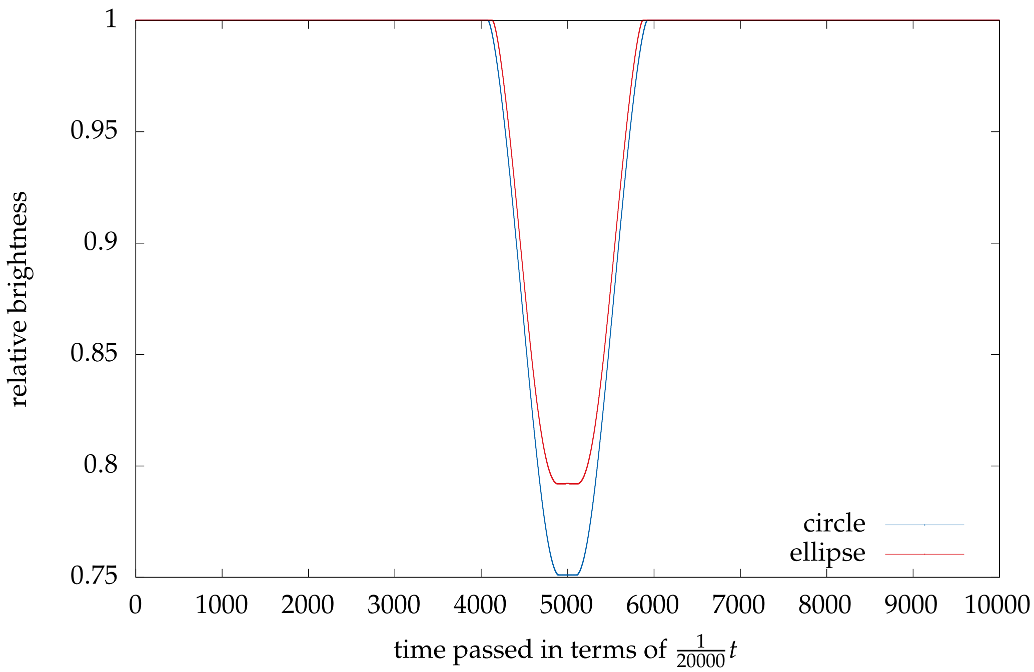

Figure 4: Sphere ($r_2 = 1$) and ellipsoid ($a = 1.2$, $b = 1$, $c = 0.8333333$) with $i = 0.47\pi$. As can be seen, the ellipsoid blocks less light for the same reason as above.

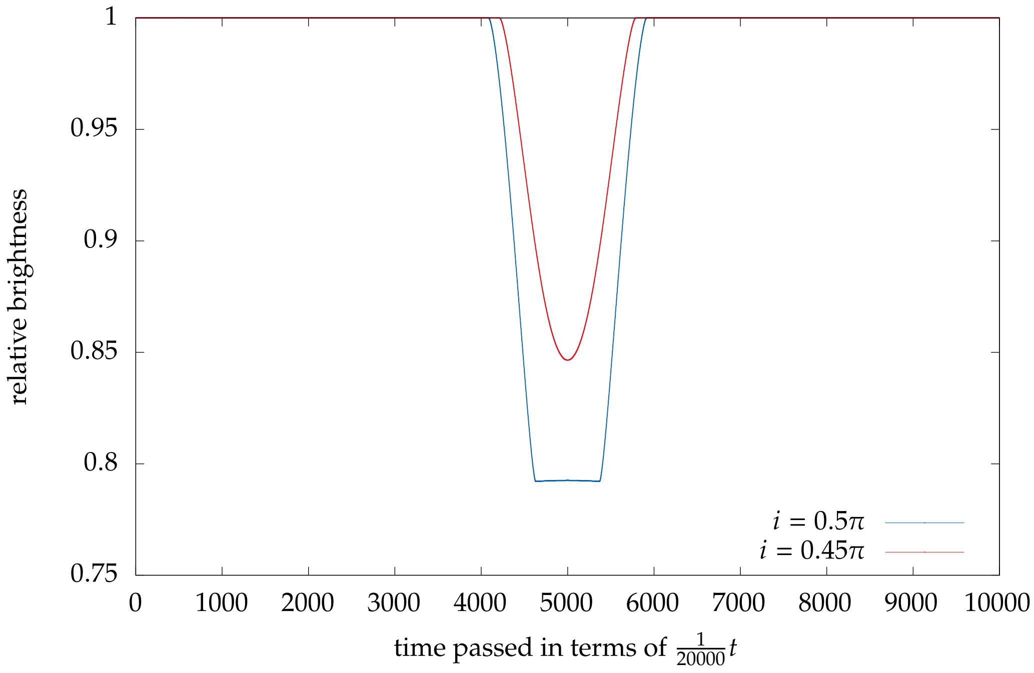

Figure 5: Two ellipsoids of the same size ($a = 1.2$, $b = 1$, $c = 0.8333333$) but of different inclination i. The ellipsoid with $i = 0.45\pi$ doesn't block the star with its whole projected area. In this case, the effect of the ellipsoid's changing area can be seen (just barely), the ellipsoid with inclination $i = 0.5\pi$ doesn't produce a curve with a flat bottom (as with the sphere) but rather a curve with a slightly concave bottom.

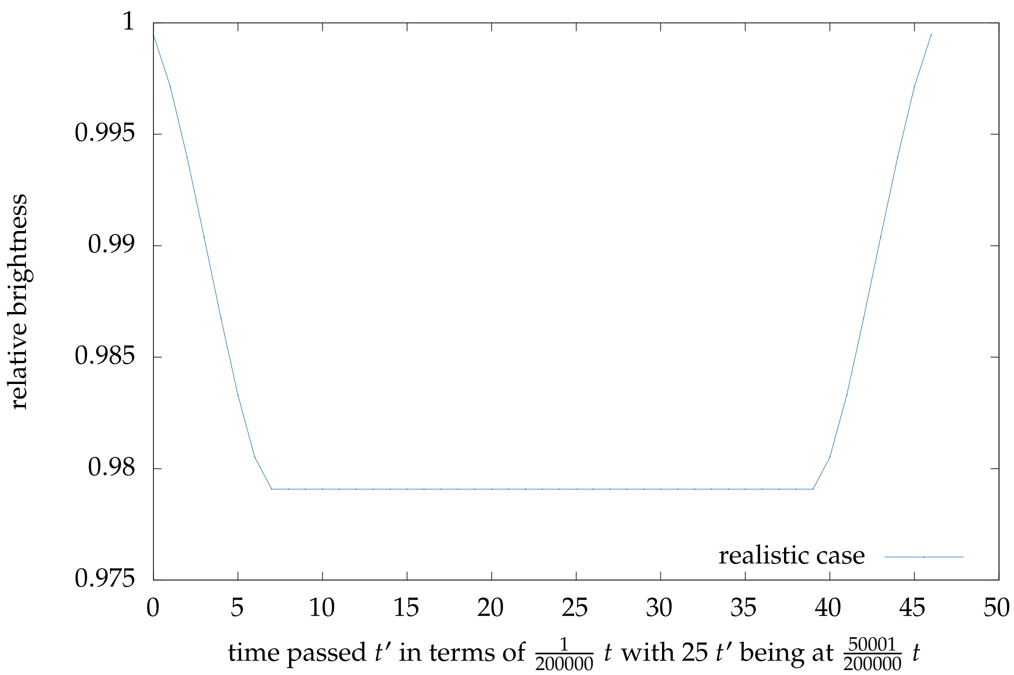

Figure 6: for the sake of argument, a more "realistic" transit light curve with the parameters: $G = 3.9362{\cdot}10^{-7}M_\odot R_\odot^3 s^{-2}$, $m_1 = 0.4M_\odot$, $m_2 = 0.002M_\odot$, $r_1 = 0.7R_\odot$, $a = 0.1100R_\odot$, $b = 0.1R_\odot$, $c = 0.1028R\odot$, $i = 0.4999\pi$, $r = 1000R_\odot$, t is roughly 16 years and the transit takes less than 2 days.

4 Conclusion

As can be seen above, the transit light curve indeed does depend on the shape of the planet quite strongly *if the planet is oblate enough*. In our special cases, the planets were singificantly more oblate than real planets, but as can be seen in figure 6, detecting any signal at all is difficult enough, moreover, the signal is full of noise and it becomes increasingly difficult to distinguish a transit from variations in the star's brightness. In real scenarios, it might be difficult to for example distinguish between a slightly inclined transit and a transit of an ellipsoidal planet. In a more general scenario, speed of rotation differs from the speed of orbit and two new parameters need to be added - n for the ratio between those speeds and α_0 for the initial phase of the planet. For those scenarios, only a slight modification of the project is needed.

References

[1] Joshua A. Carter and Joshua N. Winn. EMPIRICAL CONSTRAINTS ON THE OBLATENESS OF AN EXOPLANET. 2010 http://dx.doi.org/10.1088/0004-637X/709/2/1219

[2] Alexandre C. M. Correia. Transit light curve and inner structure of close-in planets. 2014. https://doi.org/10.48550/arXiv.0912.1594

[3] Hughes, G.B., Chraibi, M. Calculating ellipse overlap areas. Comput. Visual Sci. 15, 291–301 (2012). https://doi.org/10.1007/s00791-013-0214-3