Multi-Factor Models for Forward Curve Analysis: An Introduction to Principal Component Analysis

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A model of the dynamics of the evolution of the forward curve is an essential building block of any energy trading and risk management methodology. But it is also one of the most challenging to construct. Several market practitioners have learned the hard way that using a pricing or risk management model that does not appropriately capture the evolution through time of the full forward curve often results in severe mis-pricing of certain instruments or large errors in risk estimations.

In this article we describe a multi-factor model for the evolution of the forward price curve in energy markets. The article is based on development work being undertaken by FEA's Financial Engineers using Seasonal Principal Component Analysis methods. This new approach captures the evolution of forward prices in a more realistic fashion than either the Black-Scholes type, or the mean reverting one-factor models used by most practitioners. Additionally, it is more accurate than other Principal Component Analysis techniques that use the full sample data without taking into account the seasonal nature of that data. In a later article we will describe some practical applications of the technique.

Traders and portfolio managers often deal with instruments or contracts that depend on the evolution of the forward curve through time. Common examples are OTC derivatives such as Swaptions. Other contracts that depend on multiple forward prices are certain average price and average strike options, swing and take-or-pay contracts, and gas storage contracts. Any attempt to price or manage the risk of a contract that depends on the specific forward price at more than one moment in time requires a model that describes the joint evolution of the forward prices in the curve.

Risk managers also face the challenge of modeling forward curves for Earnings or Value at Risk calculations. This is particularly important over long time horizons when seasonality and forward curve dynamics are crucial ingredients of these calculations.

Principal Component Analysis and Forward Curves

Some methodologies used to capture the evolution of the forward curve require information about the volatilities and correlations of each of the different points in the forward curve. A significant problem with this type of approach is that, as the number of forward prices in the curve increases, so does the complexity of the analysis. For example, NYMEX Natural Gas futures contracts trade for 72 consecutive months. If we have a forward curve with 72 maturities then we need to model 72 correlated variables. We can do the analysis by estimating volatilities for each variable and a correlation between every pair of variables. With so many parameters to estimate, however, the calculation is difficult to manage and the result is liable to be non-intuitive.

Fortunately there is an alternative to this "brute-force" approach, and this is where Principal Component Analysis (PCA) comes to the rescue. Using PCA, we can reduce the number of dimensions of the problem considerably. We can end up with only 2 or 3 significant uncorrelated sources of risk, while retaining most of the information contained in the original data set. In our NYMEX example, the PCA will still yield 72 dimensions (called components), but generally only a very small number of them (the principal ones) will have a significant contribution to the variability of the data.

Another advantage of the PCA process is that it yields information on the importance of each component: the first principal component contributes more to the variability of the data than the second, and so on. We can also be confident that the components are independent, whereas our original data was correlated.

PCA is widely used in many fields, including finance, to perform analysis on complex sets of data. However, it is important to understand that this type of analysis does not result in discovering the underlying economic drivers of data changes. For example, using PCA on electricity prices would not "discover" weather and coal prices as important factors in electricity price evolution. Rather PCA uses abstract mathematical techniques to reveal the underlying structure of the data.

PCA and Energy Forward Price Curves

In the context of energy forward curves, PCA enables us to gain a much richer and more intuitive understanding of the way in which the forward curve changes. By observing historical forward curve movements, PCA can identify a set of principal components that describe those movements mathematically and rank their importance according to their contribution to the total variability of the data.

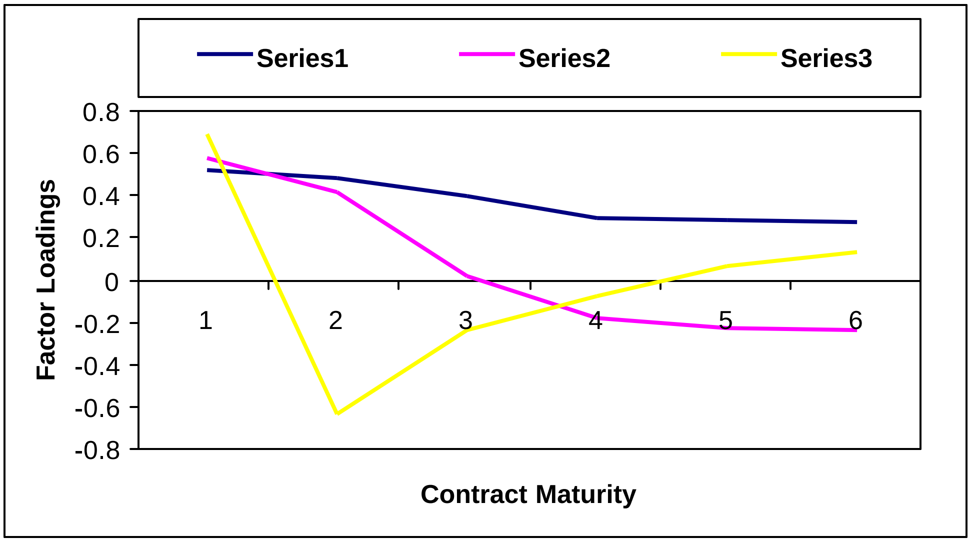
Each of the principal components can be interpreted as a source of risk, and the importance of that component (its "score") is an expression of the volatility of that risk source. Associated with each principal component is a set of "factor loadings", which can be interpreted as the volatilities for each maturity forward in the original dataset corresponding to the source of risk. For example, the factor loadings for the first principal component define how the price of each of the original contracts moves in response to a change in that component.

For energy forward price curves (and in financial markets as well), these uncorrelated sources of risk are highly abstract and usually take the form of:

- 1st Factor: Parallel Shift – this factor governs changes in the overall level of prices. The factor loadings usually have the same sign but slightly different magnitudes.
- 2nd Factor: Slope – this factor governs the steepness of the curve: it may result in the front of the curve going up and the back going down, or vice versa. It can be interpreted as a change in the overall level of the term structure of convenience yields.

- 3rd Factor: Curvature – this relates to the possibility of introducing a bend in the curve, that is the front and back go up and the middle goes down, or vice-versa.

The chart below illustrates the three different components and their effect on the forward curve.



Simulating Forward Curve Changes with Seasonal Principal Component Analysis

Some authors have suggested the use of PCA to model energy forward curves. Analysis of this type that uses the full set of sample data will produce forward curve scenarios that will capture the average variability throughout the year. However, without taking into account the seasonal dependence of those factors, the resulting curve may not be realistic. For example, the results may characterize price behavior in a “typical month”, but that month will never exist in reality because it is a composite average of 12 real months, none of which exhibit the “typical” behavior.

In order to simulate movements in the forward curve through time, we need a dynamic framework that captures the most characteristic patterns that drive forward curve changes. FEA’s approach allows the data to be viewed in seasonal slices so that we only compare like time periods with like.

In our next column, we will present a specific example of this type of seasonal analysis to value natural gas storage contracts using the NYMEX Natural Gas forward price curve series. In addition, we will also present a way to simulate spot price changes jointly with forward curve fluctuations, allowing for different correlations between those two sources of risk. Finally, instead of using historical volatilities, we will show how the model can be calibrated to implied volatilities from option prices observed in the market.

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