1D linear advection →

$$\boxed{\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0}$$ → Propagation of wave with speed c without change of shape

initial condition —

$u(x, t=0) = u_0(x)$ → Initial shape of the wave front

Exact solution (for this initial condition) —

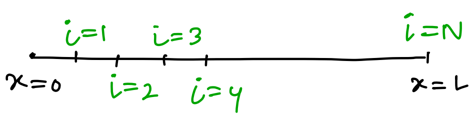
$$\boxed{u(x,t) = u_0(x - ct)}$$

* Numerical approach →

Discretization of time and shape

Discretization of time: forward difference scheme

Discretization of space: backward difference scheme



← Spatial discretization

discrete time interval: Δt

$$\frac{\partial u}{\partial x} \approx \frac{u(x+\Delta x) - u(x)}{\Delta x}$$ (derivative approximated as differences)

Governing equation: $\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$

n → time index
i → space index

$$\Rightarrow \frac{u_i^{n+1} - u_i^n}{\Delta t} + c \cdot \frac{u_i^n - u_{i-1}^n}{\Delta x} = 0$$

n & $n+1$ $\rightarrow$ are two consecutive steps in time

i & $i-1$ $\rightarrow$ Two neighbouring points of the discretized x co-ordinate.

When an initial condition is specified, the only unknown is u_i^{n+1}.

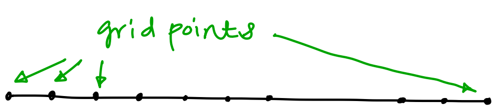
$$\therefore \frac{u_i^{n+1} - u_i^n}{\Delta t} + c \cdot \frac{u_i^n - u_{i-1}^n}{\Delta x} = 0$$

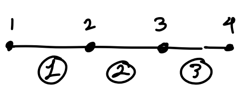
$$\Rightarrow \frac{u_i^{n+1}}{\Delta t} - \frac{u_i^n}{\Delta t} + c \cdot \frac{u_i^n - u_{i-1}^n}{\Delta x} = 0$$

$$\Rightarrow \frac{u_i^{n+1}}{\Delta t} = \frac{u_i^n}{\Delta t} - c \cdot \frac{u_i^n - u_{i-1}^n}{\Delta x} = 0$$

$$\Rightarrow u_i^{n+1} = u_i^n - c \frac{\Delta t}{\Delta x}\left(u_i^n - u_{i-1}^n\right)$$

* Computational domain —





{4 grid points
3 parts}

41

$\rightarrow$ nx: No of grid points

$a = 0$, $b = 2$

$dx = 2-0/(nx-1)$