Honors Contract: Modeling the Motion of a Knuckleball

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Submitted: May 5, 2023

Abstract

Accurately predicting the motion of a knuckleball is a difficult feat due to the many forces that contribute to its motion. Recent experimental data has suggested that the motion is influenced by the positions of the seam's relative to the ball's forward direction of motion. In this work, this proposed equation of motion is studied in two orientations and compared to the motion of a simple baseball pitch subject to the same initial conditions. The results of this analysis show that the knuckleball follows a distinctly different trajectory depending on its orientation and the initial angle of attack, and leads to a final displacement in the range of 0.1–0.2 m compared to the simple baseball.These results highlight the power of computational methods to test the validity of the equations of motions for systems that are very sensitive to their initial conditions.

Introduction

One of the challenging aspects of a baseball game lie in the batter's ability to predict the motion of an incoming baseball. Pitches have a wide repertoire of pitches available to them: in addition to non-rotating balls, they can employ spinning pitches such as the curveball and fastball. One particularly difficult pitch is the knuckleball, which has no initial spin but is characterized by its erratic motion.

This report aims to study what forces contribute to the complicated trajectories of knuckleball pitches. For simple baseball throws, the corresponding motion can be accurately modeled by accounting for lift, drag, and Magnus forces. These simple and spinning pitches have been studied extensively; by contrast, the knuckleball's motion is notoriously hard to predict. With these pitches, the approximation of the baseball as a spherical object fails due to the laminar effects of the seams on the ball's motion. By comparing data from recent wind tunnel experiments [2] to the known forces acting on the ball, recent work has revealed a new model to predict the knuckleball's equation of motion. This motion is dependent upon an angle of attack of the ball θ (the orientation of the ball relative to the wind direction), and has the form [3]:

$$C_L(\theta) = a_0 \sin(4\theta - \pi) + a_1 \sum_{i=1}^{n} \Big[\sin(\frac{||\mathbf{s_i} - \mathbf{p}||\pi}{2d} + \pi/2) \cdot \mathrm{sgn}(p^* - s_i^*) \Big]. \tag{1}$$

Here, the first term accounts for vortex shedding and the second term accounts for the effects of the seams on the ball's motion. In particular, $\mathbf{s}_i$ is the position of the ith seam relative to the stagnation point $\mathbf{p}$, and the starred terms indicate the z-component of the position of the stagnation point and seam. The coefficients a_0 and a_1 are constants determined by comparison with experimental data. Physically, this means that each stitch produces an independent force with a higher magnitude closer to the stagnation point and that accounts for the z-axis symmetry through the use of the sign function. Summing all of these contributions gives the total force on the ball.

For the simple baseball, the lift force depends on the spin parameter $S = \frac{r\omega}{v}$, where r is the radius of the ball, ω is the ball's angular velocity, and v its velocity. Experimental data has shown that the lift force has a power law dependence on the spin parameter:

$$C_L(S) = 0.62 \times S^{0.7}. \tag{2}$$

These lift forces contribute to the overall Magnus force, which has the form

$$\boldsymbol{F}_M = \alpha \boldsymbol{\omega} \times \boldsymbol{v} = \frac{1}{2} C_L \frac{\rho A r}{S} \boldsymbol{\omega} \times \mathbf{v}, \tag{3}$$

where α is a proportionality constant that depends on the lift term. This force, in addition to the contributions from gravity and quadratic drag, describe the motion of the baseball.

Methods

The equations of motion for the baseball have the form

$$\frac{d\mathbf{r}}{dt} = \mathbf{v}, \qquad \frac{d\mathbf{v}}{dt} = -g\hat{\mathbf{e}}_y - \frac{b_2}{m} v\mathbf{v} + \frac{\alpha}{m}\, \boldsymbol{\omega} \times \mathbf{v}. \tag{4}$$

Code can be found at https://github.com/jistanto/honors-contract.git.

Here, the first term in the acceleration comes from gravity, and the second accounts for quadratic air resistance, where $b_2 = \frac{1}{2} C_D \rho A$ depends on the drag coefficient C_D, the density of air ρ, and the cross-sectional area of the baseball A. These equations of motion can be used with an RK4 integrator to model the baseball's trajectory. This integrator models projectile motion well, since the system need not conserve energy.

To determine the lift term C_L for the knuckleball, one must obtain an accurate picture of the baseball's geometry—the position of the seams is of particular importance here. In this work, two orientations of the baseball were studied: the four seam (2S) and two seam (4S) models are defined by the number of seams that are facing the ball's direction of motion. The values for the ball's radius, mass, and number of seams were taken from [1]. The resulting baseball orientations are shown in Figure 1.

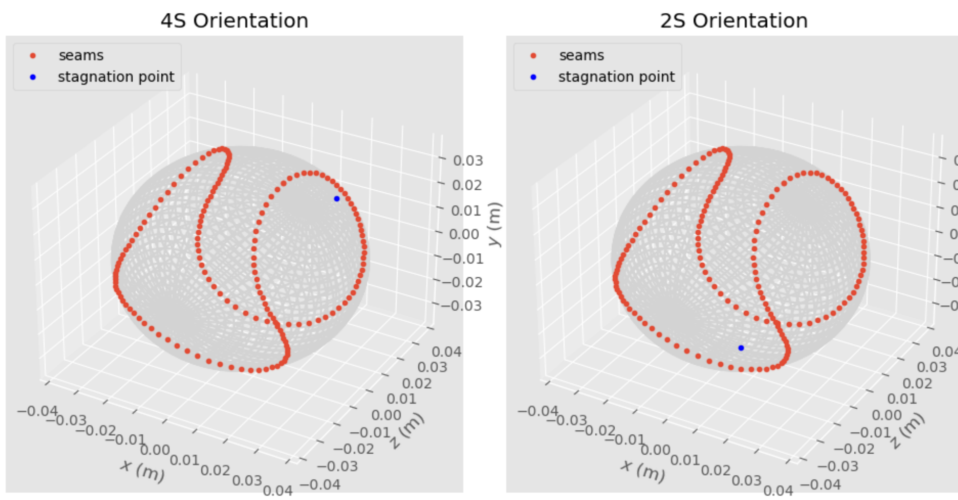

Fig. 1. Visualization of baseball and seams relative to the stagnation point for both the 2S and 4S orientations. Tools to construct the 3D sphere and the seams were taken from [4] and [5], respectively.

In Figure 1, the stagnation point (shown in blue) shows the forward direction of the ball's motion in flight. More specifically, the stagnation point is the point where the ball experiences zero fluid velocity from air. The lift force can be found using the difference between the individual seam positions and the stagnation point as explained by Equation 1. Because the ball is subject to rotation while it travels through the air, C_L must be recalculated at every time step using a new θ value.

Results

Before modelling the motion of the baseball itself, it's worth studying how the lift term changes as a function of the attack angle. Figure 2 shows the results of this analysis—C_L was shown to have a sinusoidal dependence upon the angle of attack on the ball. These results agree reasonably well with experimental data, although the amplitude remains relatively consistent in this model compared to the choppier motion seen in experiments [2].

This term can be used with the equations of motion from Equation 4 in the RK4 integrator to obtain the baseball's trajectory. (The only difference between the implementation of the simple baseball and the knuckleball comes from the dependence of C_L on the attack angle and spin

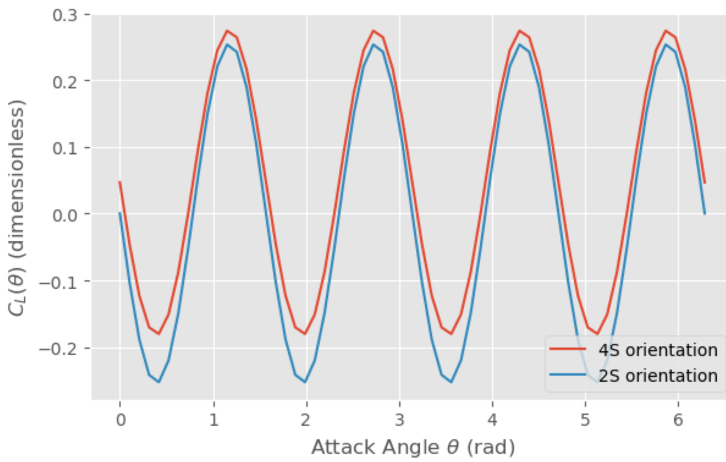

Fig. 2. Lift term C_L for different attack angles on the baseball. Note that this dependence is very different from the power law dependence for the simple baseball pitch described in Equation 2. To obtain amplitudes that matched those found from experimental data, the coefficients were defined as follows: in the 4S orientation $a_0 = 0.22824$ and $a1 = 0.2232$, while in the 2S orientation $a_0 = 0.25416$ and $a_1 = 0.01278$.

parameter, respectively.) An exmaple of the trajectories for the simple baseball versus the two orientations of the knuckleball can be seen in Figure 3.

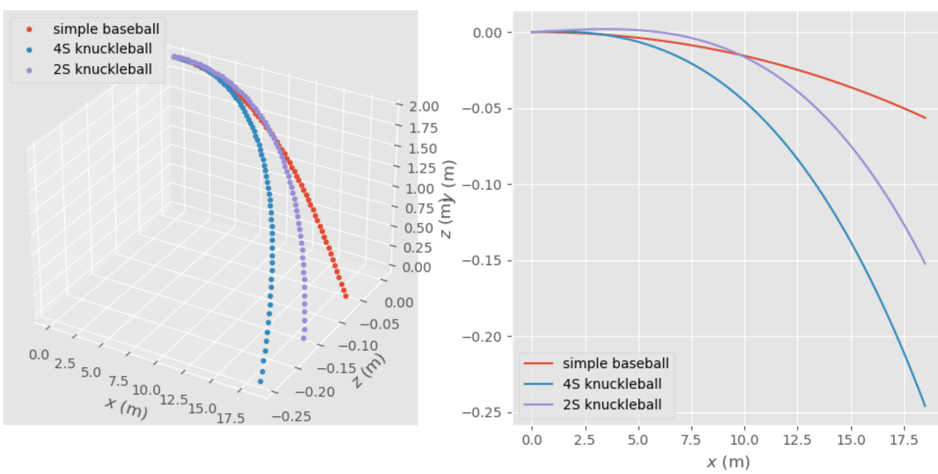

Fig. 3. Trajectories of simple baseball and knuckleball pitches for 4S and 2S orientations. Here, the initial velocity of the pitch was 30 m/s in the x-direction, and the spin $\omega = 50$ rpm only in the xy plane. Both of these values were chosen to match the experimental conditions used in [2].

The difference in the final position for each of the pitches is striking—the position of the seam's relative to the stagnation point changes the ball's final position by nearly 0.2 meters. To gain a better idea of how the initial attack angle changes the final position of the ball, this simulation can be run for a range of different initial theta values. The final z-positions of the ball can then be compared to that of the simple baseball subject to the same initial conditions, these results are summarized in Figure 4.

These results are consistent with experimental data [1]: the final displacement is sinusoidal for

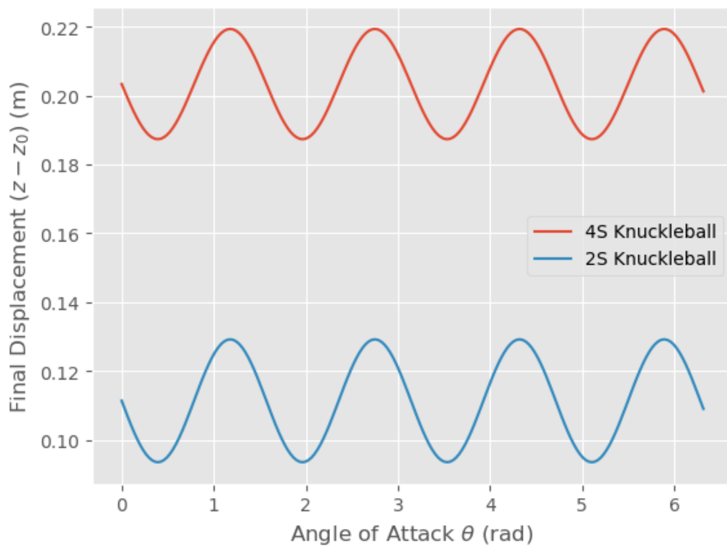

Fig. 4. Difference in the final z position of the simple baseball pitch compared to the knuckleball's final z position.

both orientations, with varying amplitudes between the 2S and 4S orientations. The maximum displacement in shown to occur around $\theta = 0.5$ radians, and has a period of 1.5 radians. These results differ from experimental data in that the knuckleball trajectory never matches the simple baseball's trajectory. This is likely due to the ω term that was introduced to the model. These differences could also be due to the differences in the coefficients a_0 and a_1 in this analysis.

Discussion

Modeling the trajectory of a knuckleball provides a valuable example of the ways in which computational methods can study sensitive physical systems. The proposed equation for a knuckleball is very sensitive to a number of factors, including the position of the stagnation point relative to the ball's seams, as well as the weight given to the vortex shedding and seam terms by the coefficients a_0 and a_1. In general, the contribution of the seams to the ball's motion causes deviations up to 0.22 meters to the final position of the ball when it reaches home plate. These deviations contribute to the unpredictable nature of the knuckleball. Looking at the 2S and 4S orientations show similar oscillating relationships for both the lift force and the final displacement, although the amplitudes of these sinusoids varies between the two orientations.

The motion of knuckleball's is by definition erratic and difficult to predict—computational methods provide a promising avenue to test the validity of predicted equations of motion for this and other similar systems. Further studies on the introduction of spin and the relationship between the spin parameter and angle of attack in the lift force would serve to further elucidate the studies of this and other complicated physical systems.

References

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