University of California, Santa Barbara

XYZ Financial Portfolio Proposal and Analysis

for the period from January 1, 2015 to December 31, 2020

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1 Executive Summary

The XYZ wealth management firm has prepared this report to demonstrate our investment strategy and fund management capabilities. Financial portfolios with an initial capital of $10 million USD are simulated for three risk profiles between 1st January 2015 and 31st December 2020. We first form investment theses for each risk profile, then backtest each strategy. The three risk profiles analyzed are as follows: high-risk high-return aggressive growth, moderate growth, and defensive capital preservation. The aggressive growth portfolio aims to perform 300 basis points higher than inflation, the moderate growth portfolio aims to perform 100 basis points higher than inflation, and the defensive portfolio aims to match inflation. In order to meet the aggressive growth targets, investments that gain exposure to growth in the healthcare sector are chosen, especially targeting post-epidemic growth. For the moderate growth portfolio, tech companies with large market capitalization are chosen in less volatile markets such as hardware and software, rather than a more cyclical market such as semiconductors. As for the defensive portfolio, investments in utilities and treasury bill ETFs are chosen to attain overall steady growth with the market, as well as investments as gold to hedge against an economic or market downturn. For all portfolios, even assets are first selected based on their asset returns. Then, the portfolio weights are calculated using the Max-Sharpe method for the aggressive growth portfolio, the maximum expected return method for the moderate growth portfolio, and the global minimum variance method for the defensive portfolio. The Max-Sharpe method is used only for the aggressive growth portfolio so the 120/20 long-short strategy could be employed, whereas the other portfolios use a long-only strategy. This is done to mitigate strong downward movements in the market, since the aggressive growth portfolio takes on assets with high risk. The moderate growth and defensive portfolios are chosen with strong diversification and stable assets with lower volatility, so it is not necessary to incur the risk of using short-selling. Through the use of various visualizations and analysis of a few selected performance metrics, we ultimately find that all three investment strategies perform particularly well starting 2018 to the end of 2020. Though there are points in time where returns and covariance are influenced by larger market disruptions and global news, the exceptional performance and exponential growth in returns of these three investment portfolios justify our overall investment strategies and portfolio decisions.

2 Investment Strategy

2.1 Aggressive Growth Portfolio

The investment strategy for high-risk high-return aggressive growth is to profit off of health services and technology sector growth, spurred by an increasing and aging population as well as the rise of epidemics and other large-scale health crises. The latter is especially relevant as seen in the past decade, with the SARS epidemic in 2003, H1N1 flu in 2009 and 2010, Ebola outbreak from 2014 to 2016, and Covid-19 pandemic as prime examples that have both influenced the incorporation of technology and software as well as triggered research and development.[1]

Changes in healthcare policies such as the expansion of coverage from the Affordable Care Act in 2014

[1] "The Future of US Healthcare: What's next for the Industry Post-COVID-19" (McKinsey & Company, July 19, 2022), https://www.mckinsey.com/industries/healthcare/our-insights/the-future-of-us-healthcare-whats-next-for-the-industry-post-covid-19.

has also helped increase health spending.[2] Overall, this strategy consists of three main considerations: the asset type for selection, diversification of the portfolio, and growth potential of the firms. First, stocks are chosen as the only asset type for this portfolio as they have more potential for earning capital gains on large upsides than ETFs, which are more diversified and are typically less volatile. Then, since this risk profile focuses on high-risk and high-return, the second and third considerations are implemented by prioritizing companies with large market capitalization from a variety of industries for selection. The healthcare sector comprises numerous industries due to the constant evolution of technology and research, which are generalized into pharmaceuticals, healthcare insurance, and healthcare facilities. In order to minimize correlation between assets and hedge against the risk of compounding losses within our portfolio, the number of industries within the healthcare sector is leveraged such that the portfolio is diversified across the healthcare sector. Additionally, the leading firms within each industry are selected through the criterion of relatively large market capitalization, so that return is maximized. Though market capitalization in other sectors typically indicate a stable but not necessarily high returns, the healthcare sector has relatively more breadth of industries than depth and thus the firms with large market capitalization are more likely to have larger growth potential and higher returns than their peers in the same industry. As such, the market capitalization of firms are considered in conjunction with further research into the companies' financial stability and overall competitive advantage. A company possesses advantage over its rival when it earns or has the potential to earn a consistently higher rate of profit, which allows for reinvestment and maintenance of their higher performance. By choosing the leading firms from different industries, the risk of correlation within the portfolio is hence minimized while maximizing potential returns.

2.2 Moderate Growth Portfolio

The information technology sector has historically demonstrated perpetual growth due to the widespread incorporation of technology into society and the evolving nature of technology. It has the second best record for annual returns out of all sectors at 61.7%, in comparison to the S&P 500's best record of 32.4%.[3] (The first best record was in energy, but the energy sector includes oil which is highly volatile and thus not considered). As such, the investment strategy for moderate growth leverages the less volatile markets in the technology sector to achieve stable but moderate returns. Akin to the aggressive growth investment strategy, the asset type is first considered before examining market growth potential and the stability. Stocks are again chosen as the only asset type for this portfolio as they have more potential for earning capital gains on large upsides than ETFs, though there are now increased considerations to counteract the volatility of prices. The technology sector encompasses the hardware, software, networking, and semiconductor markets. The increasing pervasiveness of technologies such as smartphones and tablets, accompanied with the inevitable need to upgrade and replace the hardware over time, signal an increasing demand and potential continued growth within the hardware, software, and networking markets. By diversifying within the hardware, software, and networking markets, the risk of correlation within markets is mitigated. Additionally, this portfolio is more risk-averse than the aggressive growth portfolio, so non-cyclical markets with less price volatility and no periods of low to no returns are preferred. As such, these three markets are prioritized while the semiconductor market, which

[2] Anne B. Martin et al., "National Health Spending: Faster Growth In 2015 As Coverage Expands And Utilization Increases," Health Affairs 36, no. 1 (January 2017), https://www.healthaffairs.org/doi/10.1377/hlthaff.2016.1330

[3] "Annual S&P Sector Performance," Novel Investor, March 31, 2023, https://novelinvestor.com/sector-performance/.

has historically been cyclical with the short lifespan and constant evolution of its products, is excluded.[4] Furthermore, firms with large market capitalization are selected to gain exposure to the market growth within its corresponding industry while maintaining a stable investment. Companies with large market capitalization are more likely to have the resources to navigate a volatile market (in other words, they have a larger risk threshold) and attain stable growth, which make these companies a relatively less risky and thus more desirable investment. Overall, this strategy focuses on attaining moderate growth by gaining exposure to tech firms with relatively lower volatility and higher stability.

2.3 Defensive Portfolio

The defensive portfolio investment strategy consists of minimizing risk as much as possible by investing in the least volatile sectors and hedging against inflation and economic downturns. First of all, the ETF asset type is more diversified than investing in individual stock and consequently reduces the effect of price fluctuations, resulting in a relatively safer investment vehicle. As such, ETFs have been chosen to gain exposure to the United States' broad utilities sector, the consumer staples sector, and the gold commodity market. The broad utilities ETFs track electricity, water, gas, and other alternative power source markets, whereas the consumer staples ETFs track food, beverages, tobacco, household products, and personal products. The utilities and consumer staples sectors are two of the least volatile sectors within the economy and have had overall steady growth over the years. This is due to the inelastic demand for utilities and consumer staples, as both essential services and goods, coupled with the continual growth of the United States population. As of March 31, 2023, the S&P 500 Consumer Staples Index's worst return is -15.4%, whereas the S&P 500 Utilities Index's worst return is -29.0%, in comparison with the S&P 500 Index's worst return of 37.0%.[5] This demonstrates the relatively lower correlation of the utilities and consumer staples sectors with the market than other sectors. Accordingly, these sectors are relatively less risky than others in the case of economic downturns. However, since the utilities and consumer staples sectors still correlate with the market, assets in the gold commodities market are included to hedge against an economic or market downturn. Historically, the gold market has been resistant to the market even during some of the largest market crises.[6] The gold market is also typically backed by actual gold that retains its value regardless of the economy, and as such is a suitable hedge against inflation. Through this strategy, the initial capital is preserved by ensuring that the portfolio returns will match inflation with low volatility, and by diversifying into markets that are less affected by market conditions.

[4] "The Semiconductor Cycle" (Regions Asset Management, February 2019), https://www.regions.com/-/media/pdfs/AssetManagement-The-Semiconductor-Cycle.pdf.

[5] "Annual S&P Sector Performance."

[6] Guglielmo Maria Caporale and Gil-Alana, Luis Alberiko, "Gold and Silver as Safe Havens: A Fractional Integration and Cointegration Analysis," PLoS ONE 18, no. 3 (March 3, 2023), https://doi.org/10.1371/journal.pone.0282631.

3 Asset Selection

After forming the above investment theses, individual assets are selected accordingly. For each portfolio, assets are selected by examining several metrics along with background information about the firm as needed. Under each risk profile, a pool of at least ten assets is compiled.

3.1 Aggressive Growth Portfolio

The aggressive portfolio investment strategy is ultimately determined by the growth potential of firms and diversification within our portfolio. In alignment with the investment strategy, we first sort the assets by highest market capitalization and revenue to measure the growth potential. Then, the industry and target market of each firm is considered. As discussed previously, there are a breadth of industries in the health sector, and the firms' target markets even within an industry may differ greatly. One such industry is Medical Specialization. For example, Metter-Toledo International (MTD) specializes in balance scales and analytical equipment, while Haemonetics (HAE) specializes in providing blood and plasma supplies and services. Since the target markets within this industry can differ greatly from firm to firm, assets from different target markets should be less correlated to each other since each firm has its own specialization. As such, firms with different target markets were chosen within the Medical Specialization industry. Ultimately, some of the companies selected are included in the S&P 500 index, which tracks firms with the largest market capitalization and further justifies these investments. Below is the total pool of eighteen assets considered for the aggressive growth portfolio.

Ticker	Company Name	Industry	Market Cap (billions)	Exchange Listed
AMGN	Amgen, Inc.	Biotechnology	$125.262	NASDAQ
IQV	IQVIA Holdings, Inc.	Health Services	$24.898	NYSE
NRC	National Research Corporation	Health Services	$1.083	NASDAQ
CHE	Chemed Corporation	Hospital/Nursing Management	$8.287	NYSE
JNJ	Johnson & Johnson	Major Pharmaceuticals	$418.526	NYSE
MRK	Emerck & Company, Inc.	Major Pharmaceuticals	$299.164	NYSE
ABBV	ABBVIE, Inc.	Major Pharmaceuticals	$258.327	NYSE
PFE	Pfizer, Inc.	Major Pharmaceuticals	$211.165	NYSE
JAZZ	Jazz Pharmaceuticals	Major Pharmaceuticals	$8.508	NASDAQ
RVNC	Revance Therapeutics, Inc.	Major Pharmaceuticals	$2.732	NASDAQ
ELV	Elevance Health, Inc.	Managed Health Care	$108.344	HYSE
DXCM	Dexcom, Inc.	Medical Specialties	$46.241	NASDAQ
IDXX	IDEXX Laboratories, Inc.	Medical Specialties	$40.361	NASDAQ
MTD	Metter-Toledo International, Inc.	Medical Specialties	$30.800	NYSE
PODD	Insulet Corporation	Medical Specialties	$21.365	NASDAQ
STE	Steris PLC	Medical Specialties	$20.544	NYSE
HAE	Haemonetics Corporation	Medical Specialties	$4.211	NYSE
USPH	U.S. Physical Therapy, Inc.	Medical/Nursing Services	$1.434	NYSE

We employ a 120/20 long-short strategy when investing in the aggressive growth portfolio, since the strategy is to use high-risk high-return assets which will have more volatility. Though short-selling is risky, this risk is appropriate for the aggressive growth strategy to help hedge against losses due to market downturns.

3.2 Moderate Growth Portfolio

According to the moderate growth investment strategy, assets are selected by focusing on tech firms with relatively lower volatility and higher stability in markets that are less correlated. As such, we first prioritize firms with the highest market capitalization, and then consider the industries to which the firm belongs.

Ticker	Company Name	Industry	Market Cap (billions)	Exchange Listed
NFLX	Netflix, Inc.	Data Processing Services	$161.371	NASDAQ
SCX	L.S. Starrett Company	Hardware	$0.082	NYSE
GOOG	Alphabet Inc. Class C	Internet Software/Services	$1,370.000	NASDAQ
META	Meta Platforms	Internet Software/Services	$601.000	NASDAQ
DXCM	Dexcom, Inc.	Medical Specialties (Hardware)	$46.241	NASDAQ
WST	West Pharmaceutical Services, Inc.	Medical Specialties (Hardware)	$26.053	NYSE
PODD	Insulet Corporation	Medical Specialties (Hardware)	$21.365	NASDAQ
MSFT	Microsoft Corporation	Packaged Software	$2,388.000	NASDAQ
CRM	Salesforce, Inc.	Packaged Software	$210.260	NYSE
ADBE	Adobe, Inc.	Packaged Software	$170.659	NASDAQ
FICO	Fair Isaac Corporation	Packaged Software	$19.413	NYSE
AAPL	Apple, Inc.	Telecommunications Equipment	$2,700.000	NASDAQ
CSCO	Cisco Systems, Inc.	Telecommunications Equipment	$190.000	NASDAQ
MSI	Motorola Solutions, Inc.	Telecommunications Equipment	$48.100	NYSE

The pool above includes firms with relatively lower market capitalization, such as SCX. This is due to minor readjustments to our asset pool after running the full analysis in order to meet performance targets. We use a long-only strategy in the moderate growth profile as short-selling would introduce even higher risk to our portfolio. This increased risk is unnecessary as our moderate growth strategy leverages a more risky sector.

3.3 Defensive Portfolio

The defensive portfolio strategy selects ETFs rather than stocks, so the asset selection considerations differ from the other two portfolios. Within the three target markets of gold, consumer staples, and utilities as previously discussed, we first sort by the assets under management. The higher the assets under management by the index fund managers, the more likely it is that the fund managers are experienced and capable. This helps us differentiate between and choose funds that are better managed and thus a more stable and secure investment. The index weighting method and expense ratio of each ETF is also taken into account, as we want to minimize the expense ratio and capture the upside movements in each market. There are a total of three gold ETFs weighted by the single asset, three consumer staples ETFs weighted by market capitalization, and

five utilities ETFs with different weighting methods. Of the five utilities ETFs, three are weighted by market capitalization, one is weighted equally across the utilities, and one is tiered by market capitalization and then weighted equally across each tier. The latter two are included in the case that the firms with mid-to-low market capitalization perform better in the market. Overall, the expense ratio holds the least weight in determining our asset pool.

Ticker	Fund Name	Index Weighting Method	Assets Under Management (billions)	Expense Ratio	Exchange Listed
GLD	SPDR Gold Trust	Single Asset	$60.210	0.40%	NYSEArca
IAU	iShares Gold Trust	Single Asset	$29.300	0.25%	NYSEArca
SGOL	abrdn Physical Gold Shares ETF	Single Asset	$2.850	0.17%	NYSEArca
XLP	Consumer Staples Select Sector SPDR Fund	Market Cap	$18.620	0.10%	NYSEArca
VDC	Vanguard Consumer Staples Index Fund ETF	Market Cap	$6.880	0.10%	NYSEArca
KXI	iShares Global Consumer Staples ETF	Market Cap	$1.610	0.40%	NYSEArca
XLU	Utilities Select Sector SPDR Fund	Market Cap	$16.096	0.10%	NYSEArca
VPU	Vanguard Utilities Index Fund	Market Cap	$5.330	0.10%	NYSEArca
IDU	iShares U.S. Utilities ETF	Market Cap	$0.992	0.39%	NYSEArca
RYU	Invesco S&P 500 Equal Weight Utilities ETF	Equal	$0.371	0.40%	NYSEArca
FXU	First Trust Utilities AlphaDEX Fund	Tiered	$0.314	0.64%	NYSEArca

As in the moderate growth portfolio, we use a long-only strategy in the defensive portfolio as the goal is to minimize volatility and risk. Using short-selling would only introduce unnecessary risk.

4 Portfolio Selection

The Markowitz mean-variance approach to portfolio optimization is used in this simulation, and it is a risk-based approach rather than a utility-based selection. This is also a key factor in why the strategies for the aggressive and moderate growth risk profiles prioritize diversification in order to decrease correlation between assets. By doing this, the covariance of the portfolio will decrease and the overall risk of price fluctuations is decreased. This risk-based approach thus assumes that investors are never satiated and are risk averse, which means the portfolio with the higher expected return or lower variance will be preferred when holding the other fixed.

Within all three risk profiles, we first calculate the minimum covariance determinant estimator, and then calculate portfolio weights using the overall pool of assets in order to select seven using the appropriate criterion. Finally, we re-calculate the weights of the portfolio using the seven selected assets for our portfolio. This process is repeated each time the portfolio is rebalanced. A rebalancing period of one month is used as we expect monthly trading fees to provide the highest risk-responsivity with relatively less trading fees than rebalancing weekly. Thus, we are able to reduce losses by rebalancing faster with a smaller detriment on portfolio profit.

4.1 Aggressive Growth Portfolio

To find the initial investment portfolio for aggressive growth, the expected return are first calculated for the pool of eighteen assets using the minimum covariance determinant matrix. Then, the assets with the seven highest returns are chosen for initial investment and the Max-Sharpe portfolio is calculated using only these seven assets. This is done such that at each rebalancing period, the assets with the maximized returns are chosen and the portfolio for investment maximizes the return in terms of risk.

4.2 Moderate Growth Portfolio

The moderate growth portfolio is chosen in a similar fashion to the aggressive growth portfolio. The asset returns are first calculated for the pool of fourteen assets to choose seven assets. However, in this portfolio the seven asset portfolio weights are calculated using the maximum expected return portfolio rather than the Max-Sharpe portfolio since we are using a long-only strategy.

4.3 Defensive Portfolio

Since the defensive portfolio seeks to preserve the initial capital, we again first used the asset returns to select seven assets out of a pool of eleven, and then the minimum variance method is used to calculate the weights of the seven assets. This method is used as it minimizes the risk for the portfolio and thus decreases the chance of the investment suffering from large losses, while still maintaining a decent return.

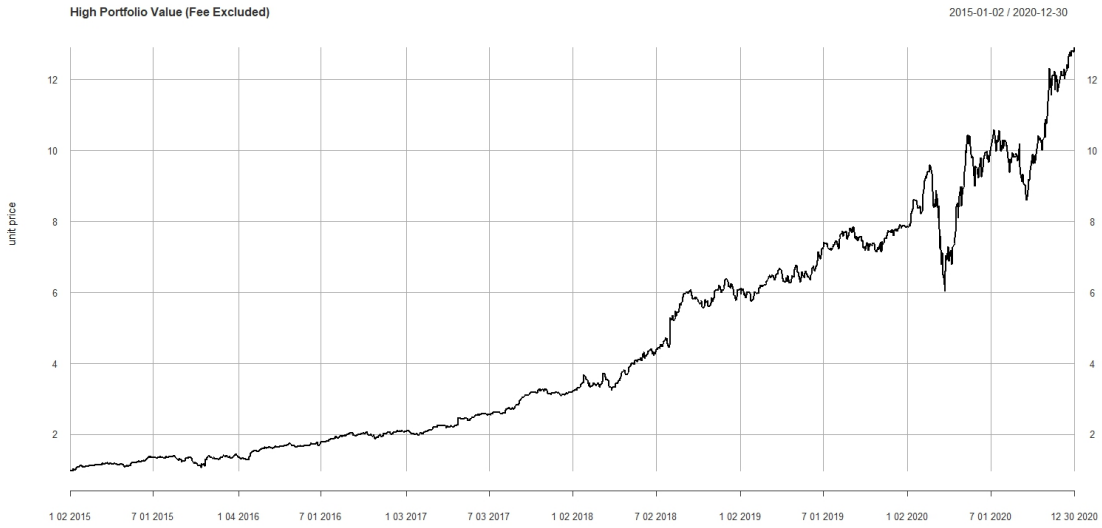
5 Performance Analysis

5.1 Aggressive Growth Portfolio

We conduct a portfolio return analysis by looking at the overall portfolio value, as well as the empirical distribution of the portfolio returns.

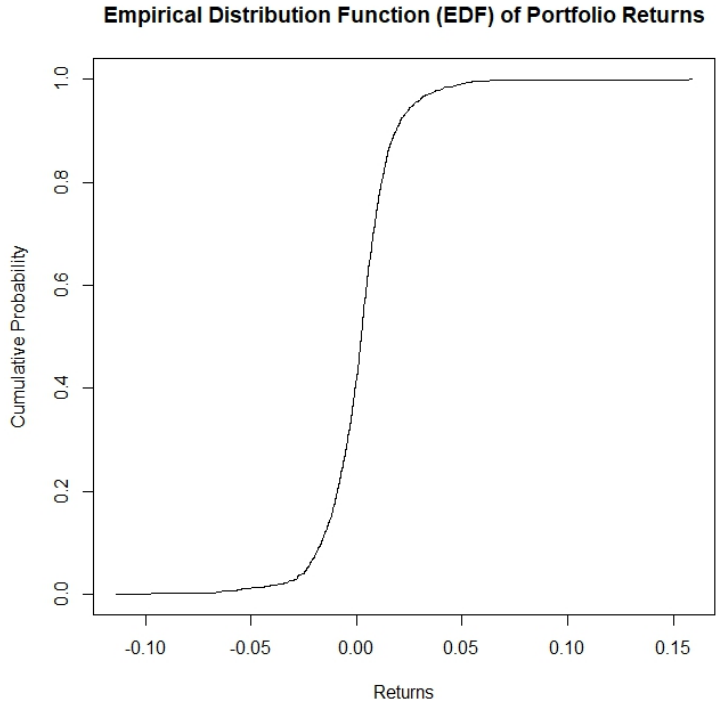
5.1.1 Portfolio Return

Portfolio Value



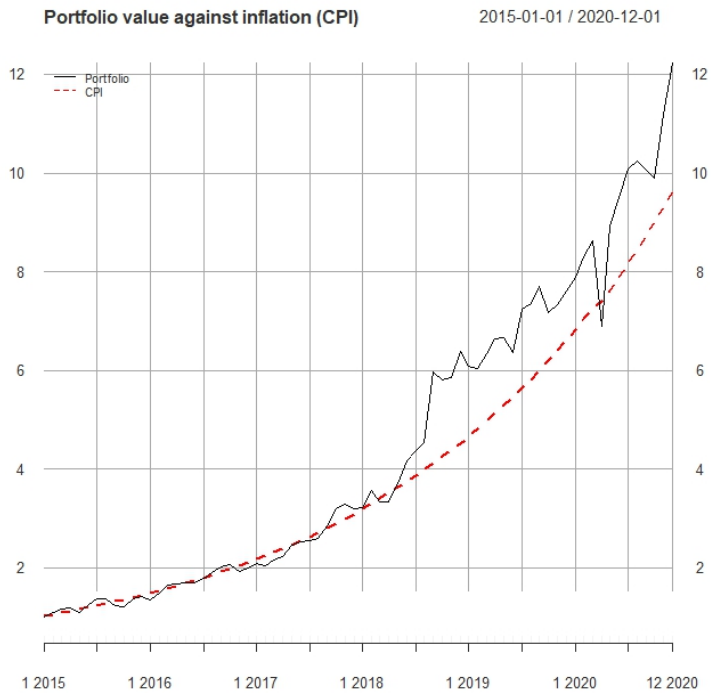
The above graph plots our portfolio value across the six year period with monthly rebalancing. The portfolio value is computed from the exponential of its cumulative log return, which is further computed as the logarithm of the weighted return of the individual assets. For the six year period, the portfolio value has an overall gain of more than 12%. In the first three years (from 2015 to 2017), the value grows steadily to around 4-times of the initial capital and rises drastically up to 12-times the initial capital in the latter three years. An increase trend in volatility is seen starting July 2018 to the end of the period. There is a remarkable drop in the first half of 2020 from 10 to 6 units, along with an immediate bounce-back to 10 in the same year. This fluctuation is likely a result of the COVID-19 pandemic's effect on the market. Similar fluctuations are noted in all three of the portfolios, though the especially high degree of change in this case is in line with the high-risk, high-return nature of this portfolio.

Empirical Distribution



The empirical distribution function approximates the cumulative distribution of our daily portfolio log return. The cumulative probability of negative to zero returns is around 0.3, which is less than 0.5 thus indicating a higher frequency of positive return. Meanwhile, there is a stark increase in cumulative probability when the return is between -0.03 to $+0.05$, so returns for this profile is highly likely to be between 0 and 0.05. The low cumulative probability of negative to zero returns and the large increase in cumulative probability between 0 and 0.05 hint at a high likelihood of positive returns in our portfolio.

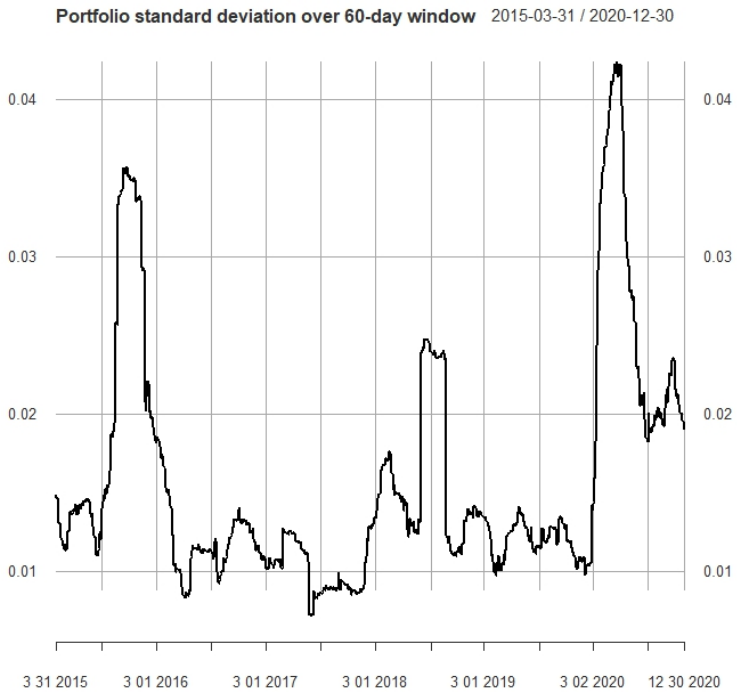
Performance Target



We compare our portfolio value against the monthly +0.03 Consumer Price Index (CPI) target. When comparing over the six year period, our portfolio beats the CPI. Our portfolio also generally meets the CPI target in terms of yearly performance, though in the first three years (from 2015 to 2017) we are slightly behind the target. Nonetheless, we have a drastic positive difference at the end of the last three years (from 2018 to 2020). Overall, we consider our portfolio to be able to beat the inflation and meet the performance target of 300 basis points over inflation.

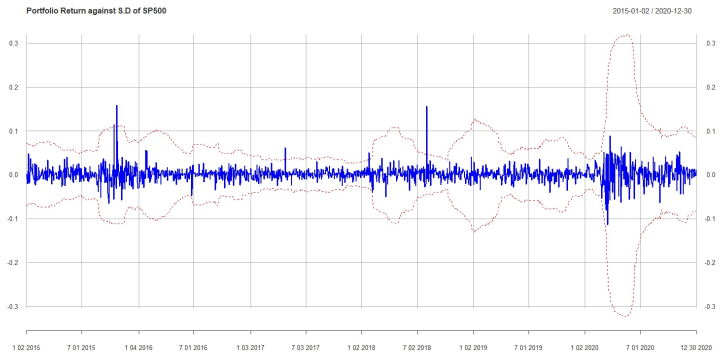
5.1.2 Portfolio Risks

Portfolio Standard Deviation



The above graph shows the portfolio standard deviation over a sixty day window - which is why we start in 31 March 2015 instead of 1 January 2015 - across the six year period. The risky nature of our portfolio is observed from the fact that the graph hovers above 0.01 for the majority of time. In addition, three sharp peaks could be observed in the middle of $2015, 2018, 2020$ respectively. This may be attributed to the aftereffects of the Ebola outbreak from 2014 to 2016 and 2018 to 2020, as well as the COVID-19 pandemic. Since we expect higher risk in the aggressive growth strategy, the high volatility in this plot is justified and not a cause for concern.

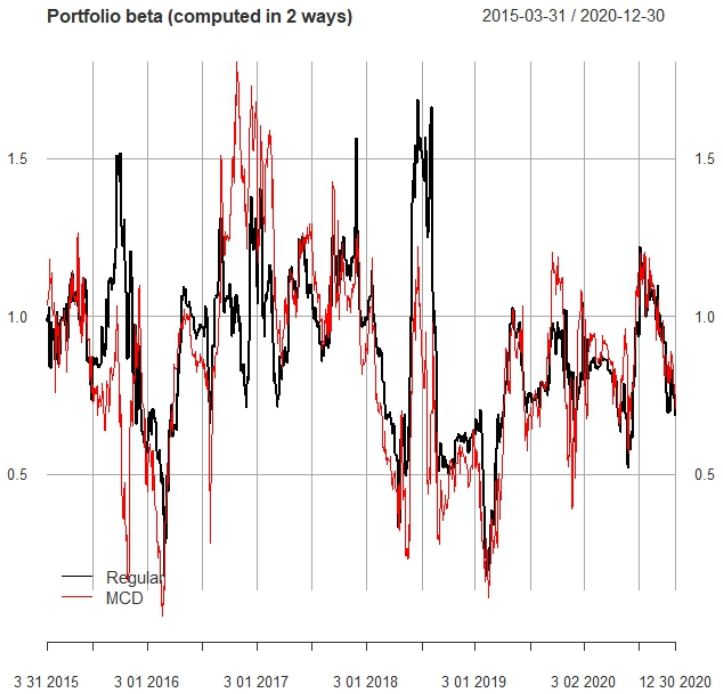
Portfolio Return against SP500 Standard Deviation



Next we test our risk against the market risk, as modelled by the S&P 500 fund. As seen from the graph our portfolio return never reaches the -8 standard deviation line and so satisfies the risk threshold condition

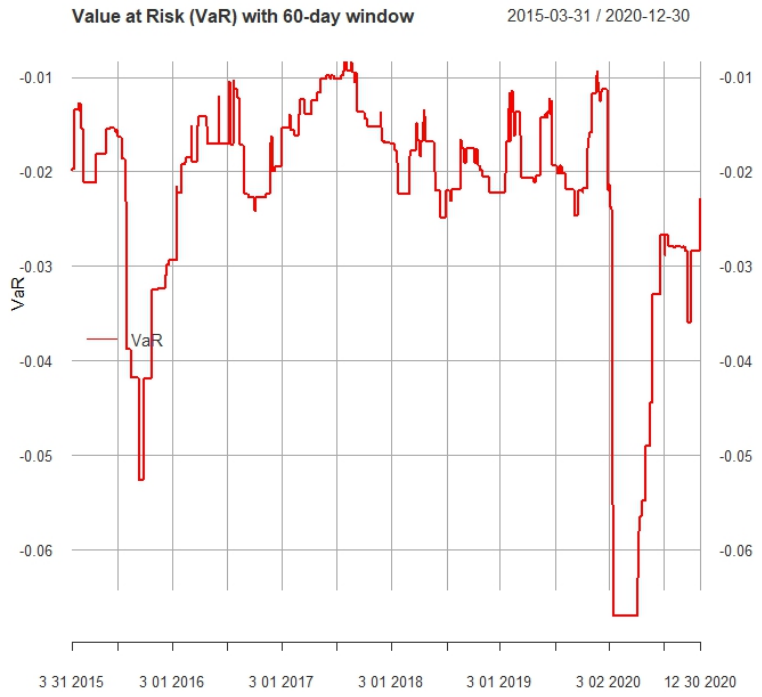
required by the client. It can also be observed from the chart that the portfolio returns had sharp changes in 2015, 2018, which is in line with the peaks of the portfolio standard deviation. Fortunately, these are positive changes, making our portfolio benefit from the risk. This also further justifies our investment in the healthcare sector as it demonstrates the market growth especially after a large-scale health crisis.

Portfolio Beta



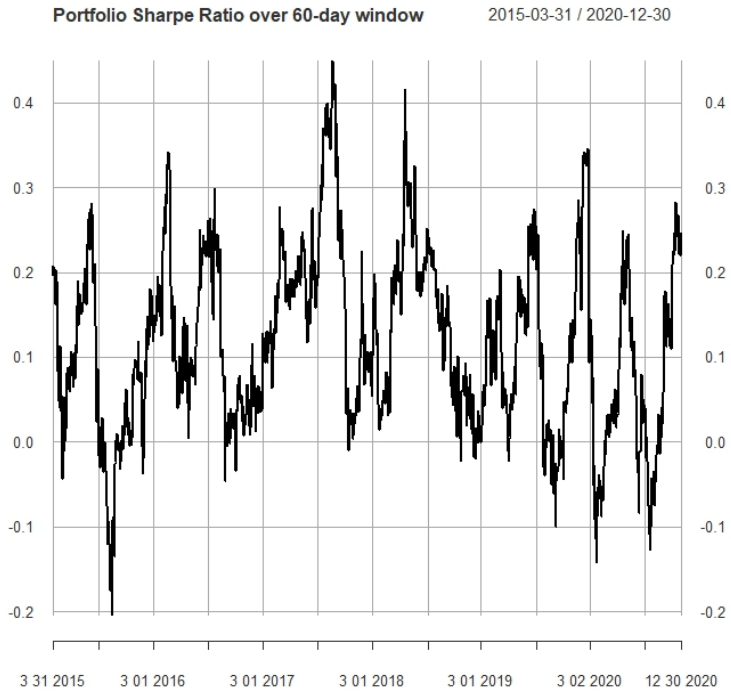
We compute the portfolio beta in two ways using different estimation to the covariance matrix of the underlying asset, which are by the sample covariance and the maximum covariance determinant. We could see from the plots that the two methods give similar results. The trend of the beta can be divided at the mid-year of 2018, before which the beta stays above 1 most of the time while after which the beta stays below 1 most of the times. This implies that our portfolio is quite correlated with the market in the beginning but becomes less correlated to it in the later period.

Portfolio Value at Risk



It is also important to consider the value at risk (VaR) in a high-risk portfolio, which provides a measure for the degree of loss we could have in extreme cases. The graph shows at value at risk at the 95th percentile of negative return. The portfolio VaR was lying between -0.01 and -0.02 in majority of the time, except near the two peaks in mid 2015 and 2020. This shows that while the portfolio has a non-trivial change during losses ($-0.01 = 1\%$ loss per day is non-negligible), the losses are also bounded below by the $-0.02 = 2\%$ threshold most of the time. Hence, the portfolio is risky but still in an acceptable range most of the time. It can also be observed that the portfolio value at risk is relatively volatile, which is again acceptable since the strategy takes on a high-risk.

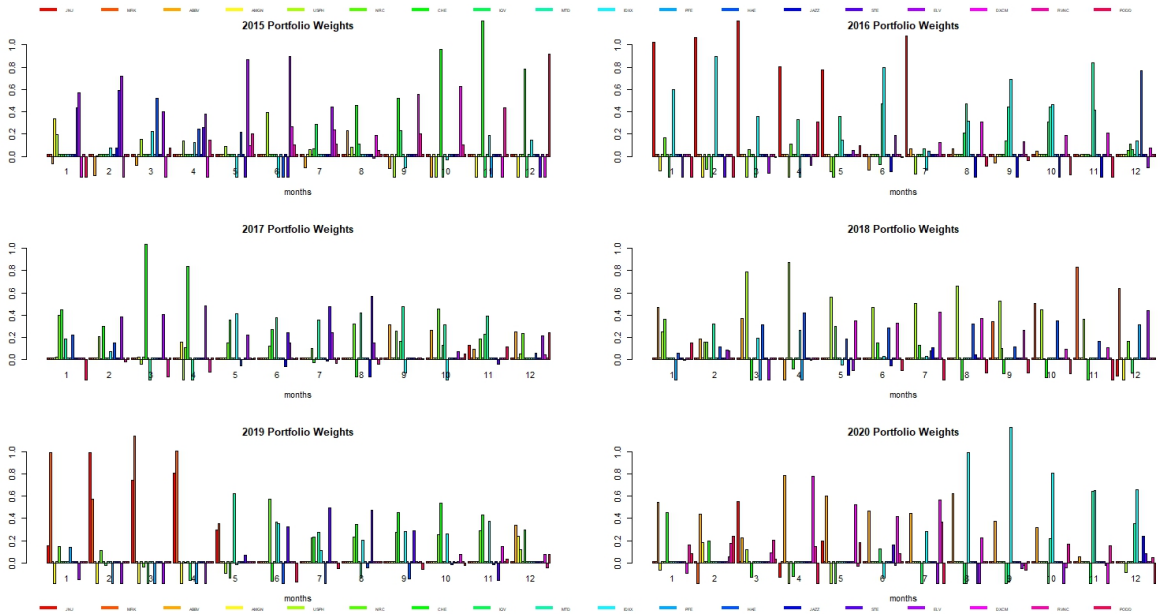
Sharpe Ratio



The Sharpe ratio measures the portfolio return per unit risk. One can see that it ranges in the interval $[-0.2, 0.4]$. In addition, even when the ratio goes below zero it usually stays above -0.1. Hence the portfolio has relatively fewer losses per unit risk during losses but higher gain per unit risk during gains. The gives another justification that our high-risk high return portfolio is quite optimal.

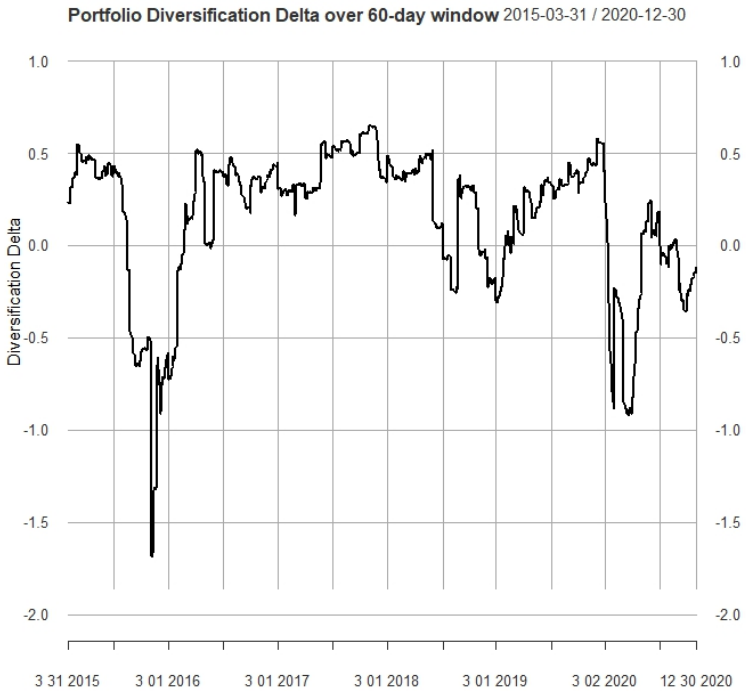
5.1.3 Portfolio Diversification

Weight barplot



The barplot above graphs our monthly portfolio weights. Considering the highest-weighted asset in each monthly portfolio, we can see that they spread cross the color spectrum and so across portfolios we have a good amount of diversification. Within portfolios, we can also see that the seven different assets usually have nontrivial weights ($> 0.1 = 10\%$). This also gives a good amount of diversification. One thing to remark is that our allowance of short-selling is in fact partly responsible for the non-trivial weights; we can see that in some portfolios, despite having assets with weight 1, we have other assets with non-trivial but negative weights.

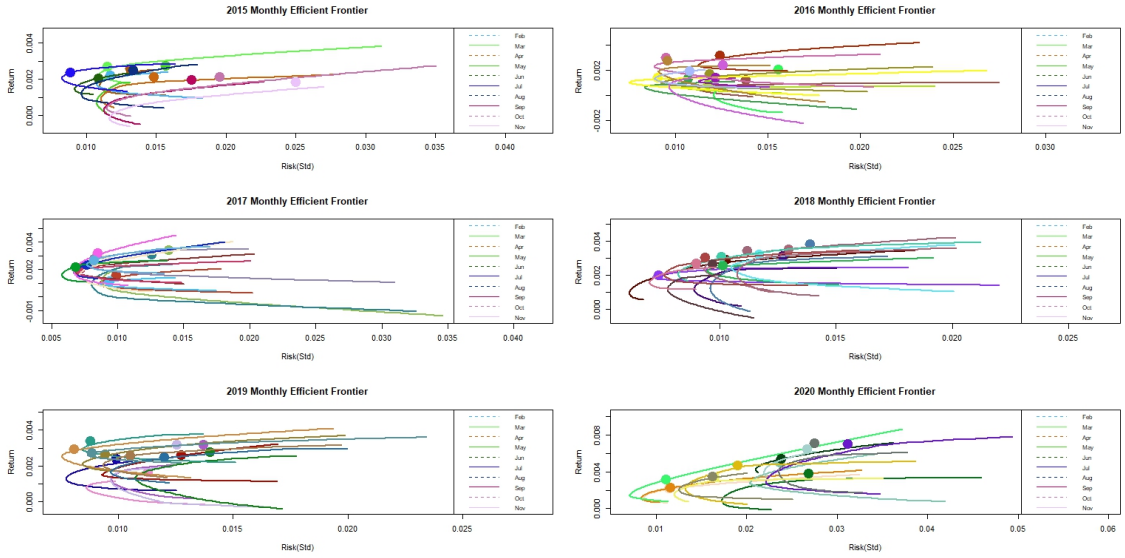
Diversification Delta



The diversification delta measures the gain based on diversification of the portfolio. A smaller delta means a higher diversification gain. With short-selling, the diversification delta can become negative. In the above plot, the delta stays low, below 0.5 most of the time. Meanwhile, it reaches two negative peaks in mid 2015 as well as mid 2020, which is in line with the peaks in the portfolio standard portfolio. This shows that our portfolio has enough diversification to deal with the risk, especially in the short positions.

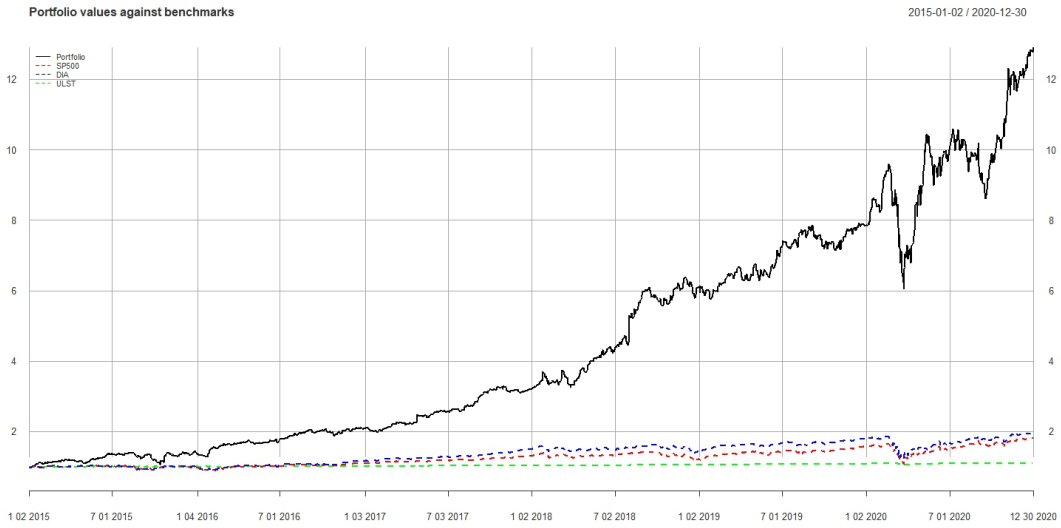
5.1.4 Portfolio Visualizations

Yearly Comparison of Efficient Frontier Plots



First, we consider our portfolio positions. Since we are using the Max-Sharpe portfolio, our portfolio (represented by dots) is the point of intersection of the risk-free line with the efficient frontier. Next, we can see that the portfolios are generally lying on a horizontal line in the plot, which means that we have quite steady portfolio returns during the whole time. Furthermore, we could see that our efficient frontiers have a certain amount of steepness with those in 2020 and 2017 being the steepest. This reflects that we have a high-return-to-risk ratio along the frontier most of the time, which justifies are portfolio choices.

Comparison with SPDR Benchmarks

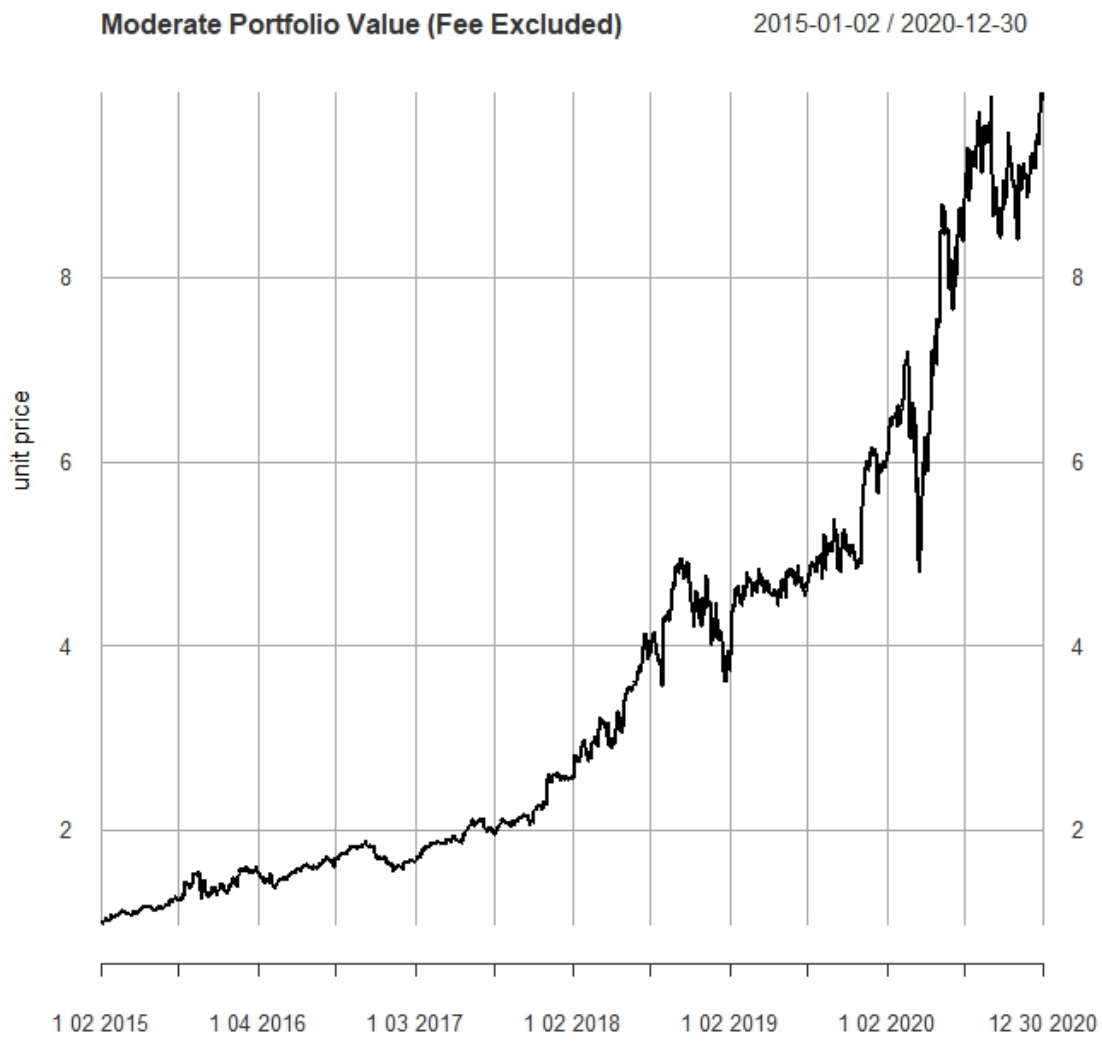


The return is increasingly more than the benchmarks throughout the year.

5.2 Moderate Return Portfolio

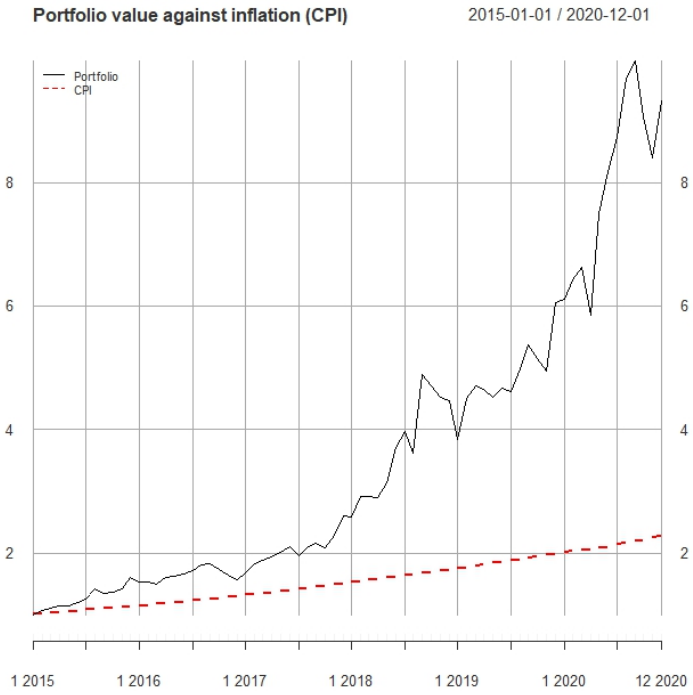
5.2.1 Portfolio Return

Portfolio Value



The above graph plots our portfolio value across the six year period with monthly rebalancing. Again, the portfolio value is computed from the exponential of its cumulative log return, which is further computed as the logarithm of the weighted return of the individual assets. For the six year period, the portfolio value has an overall gain of 9%. There are two main large downward fluctuations in the second half of 2018 as well as the first half of 2020. The 2018 dip is likely due to the trade-wars between China and the USA, which disrupted manufacturing in the tech sector especially, as well as wide-scale data breaches at Google. The 2020 dip correlates with the overall economic downturn due to COVID-19, though the portfolio value quickly recovers and improves still within the first half of 2020. This high recovery is likely due to the increased utilization and quick evolution of technology services to fulfill needs that were identified during COVID-19. For example, utilization of delivery apps, gaming software, FinTech software, and cloud services greatly increased because of the lockdowns. Though COVID-19 spurred exponential growth in the overarching tech sector and thus our portfolio value, the overall trend of our portfolio value is positive.

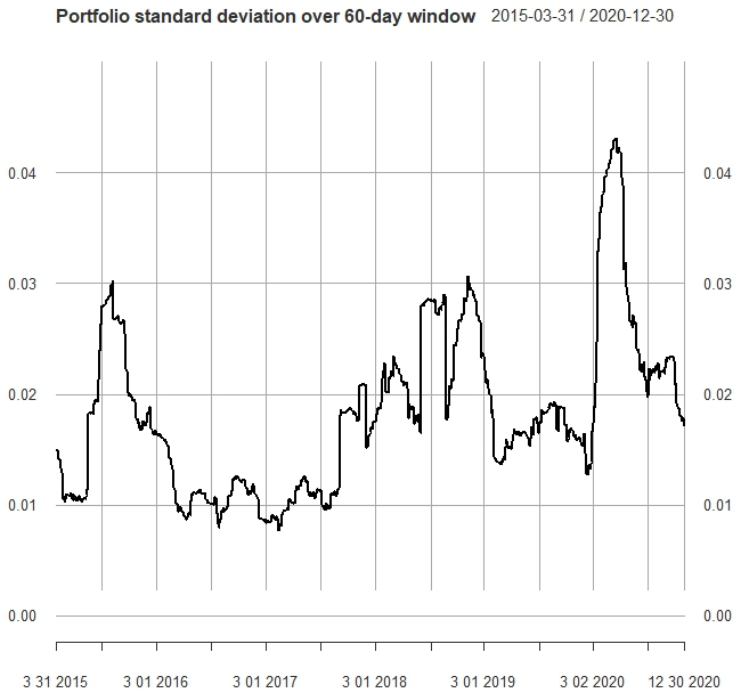
Performance Target



We compare our portfolio value against the monthly +0.01 CPI index target. Overall, our portfolio beats the CPI as well as the CPI target of 100 basis points above inflation. This supports our strategy of investing in firms with high market capitalization. We also observe that the gap between the portfolio value and the performance target widens at an quickly increasing rate, which means that the underlying assets are still growing quickly. We can conclude that our moderate growth portfolio is able to beat the inflation and meet performance targets.

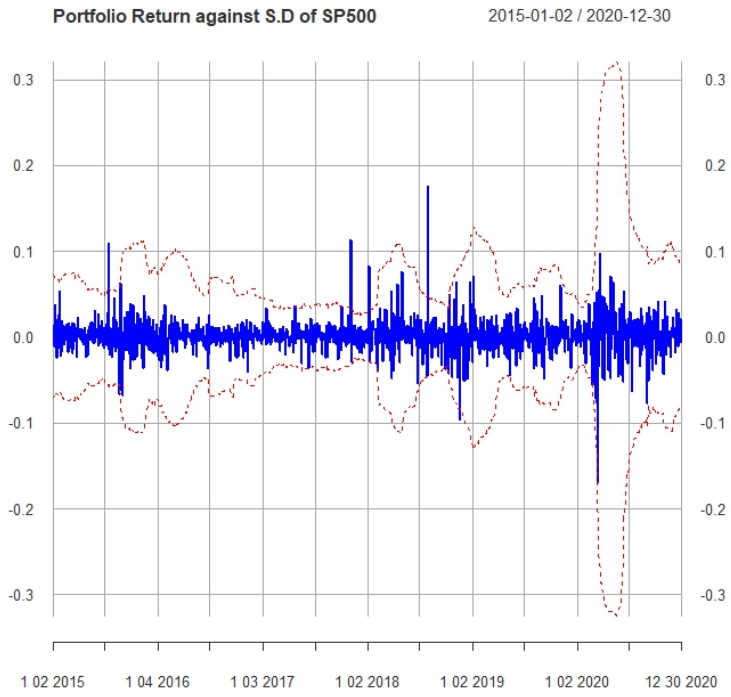
5.2.2 Portfolio Risks

Portfolio Standard Deviation



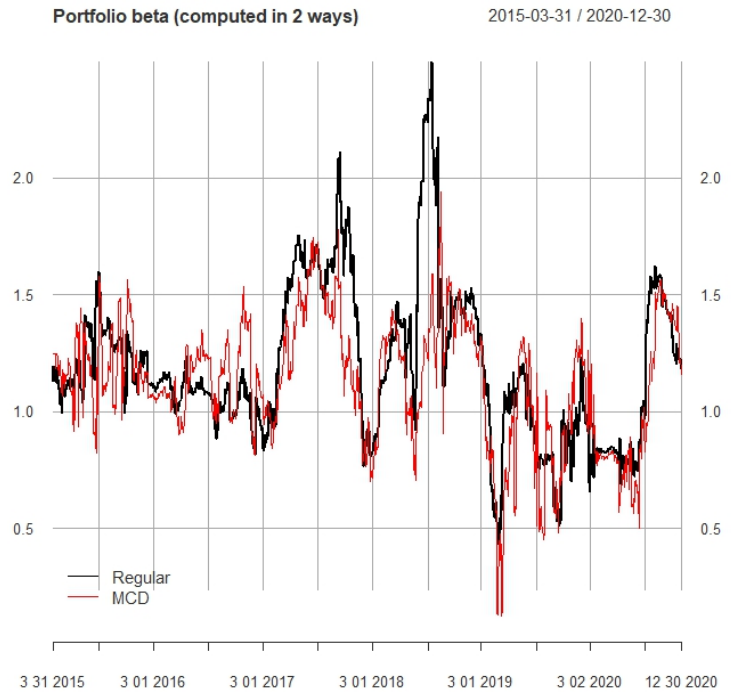
The above graph shows the portfolio standard deviation over a sixty-day window - that is why we start in 31 March 2015 instead of 1 January 2015 - across the six-year period. Three sharp peaks are observed in the middle of 2015, 2018, and 2020 respectively, which correspond with the dips in portfolio value as discussed previously. The Dow Jones decreased by 2.2% in 2015 as a result of the Chinese market crash of 2015, which is likely to be the cause of the 2015 dip observed as well.

Portfolio Return against SP500 Standard Deviation



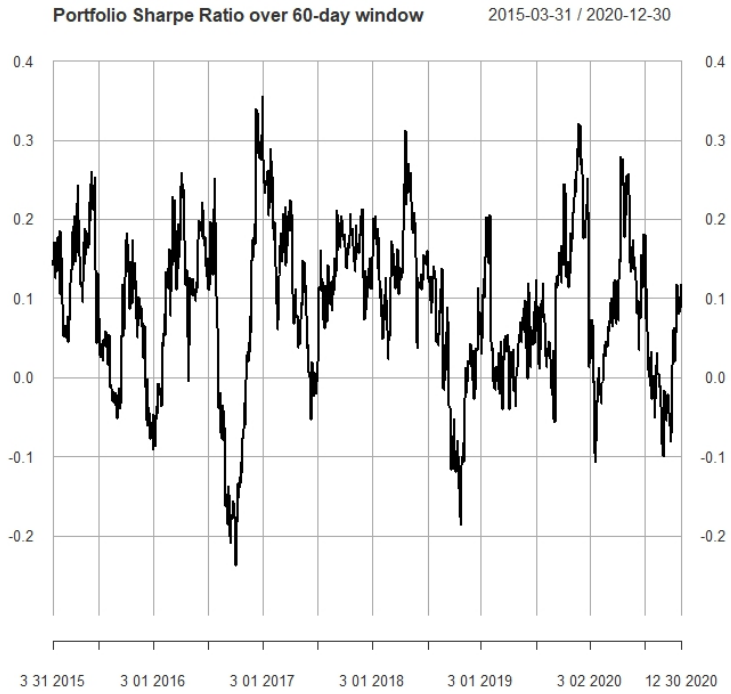
Next, we test our risk against the market risk, modelled by the S&P 500 fund. As seen from the graph, our portfolio return generally stays within the -8 standard deviation line and satisfies the risk threshold required by the client. However, the standard deviation seems to almost cross the risk threshold in $2015, 2018, 2020$, which are in line with the peaks of the portfolio standard deviation.

Portfolio Beta



We compute the portfolio beta in two ways using different estimation to the covariance matrix of the underlying asset, which are by the sample covariance and the maximum covariance determinant. Overall, the beta oscillates around value of 1, which also signifies that there is correlation between the underlying assets and the portfolio. This is likely due to the fact that we mainly chose two to three industries within a portfolio, which increases the likelihood of correlation within our portfolio.

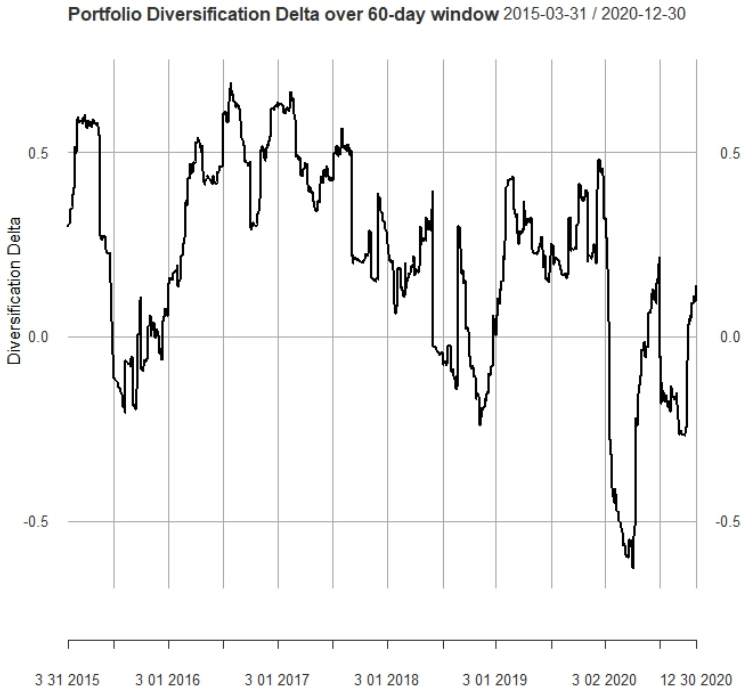
Sharpe Ratio



The Sharpe ratio measures the portfolio return per unit risk. In comparison to the high-risk high-return portfolio, there are fewer and less extreme peaks and fluctuations. One can see that it ranges in the interval $[-0.2, 0.4]$, same as the high-risk high-return portfolio. Since the Sharpe ratio is less volatile than the high-risk portfolio, we can conclude that the moderate growth portfolio is less risky.

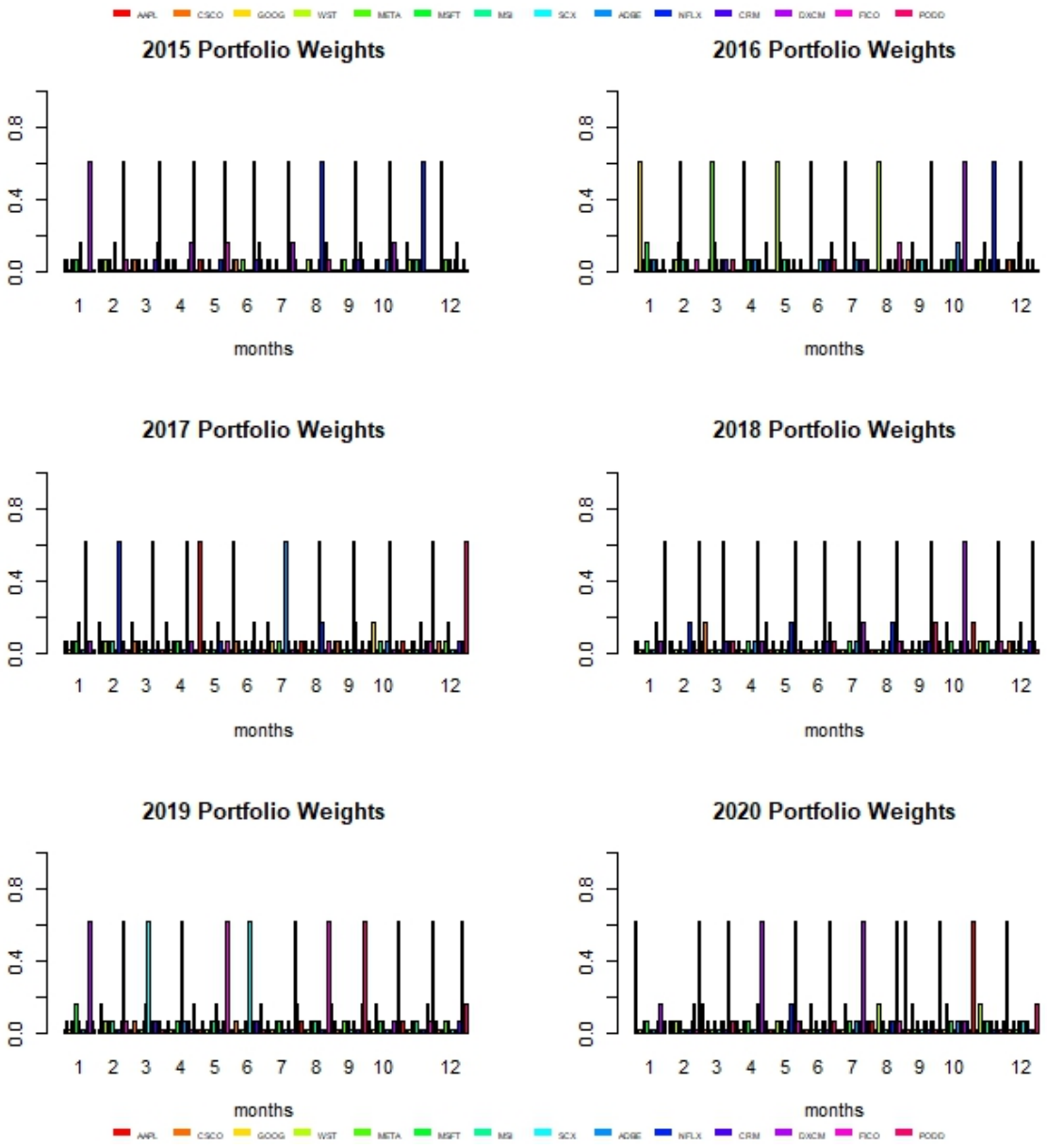
5.2.3 Portfolio Diversification

Diversification Delta



The diversification delta measures the gain based on diversification of the portfolio. Compared to the high-risk high-return portfolio, the diversification delta in the moderate portfolio is more volatile and fluctuates more. When viewed in conjunction with the weights at each monthly rebalancing, we observe that one asset has the maximum weight of 0.6 with a different asset chosen at each rebalancing period. The other six stocks generally each have less than 0.1 weights, which leads to an increased diversification. In the above plot, the delta also trends a bit higher than the high-risk high-return portfolio. This indicates that the high-risk high-return diversification was more effective at reducing risk than the moderate portfolio.

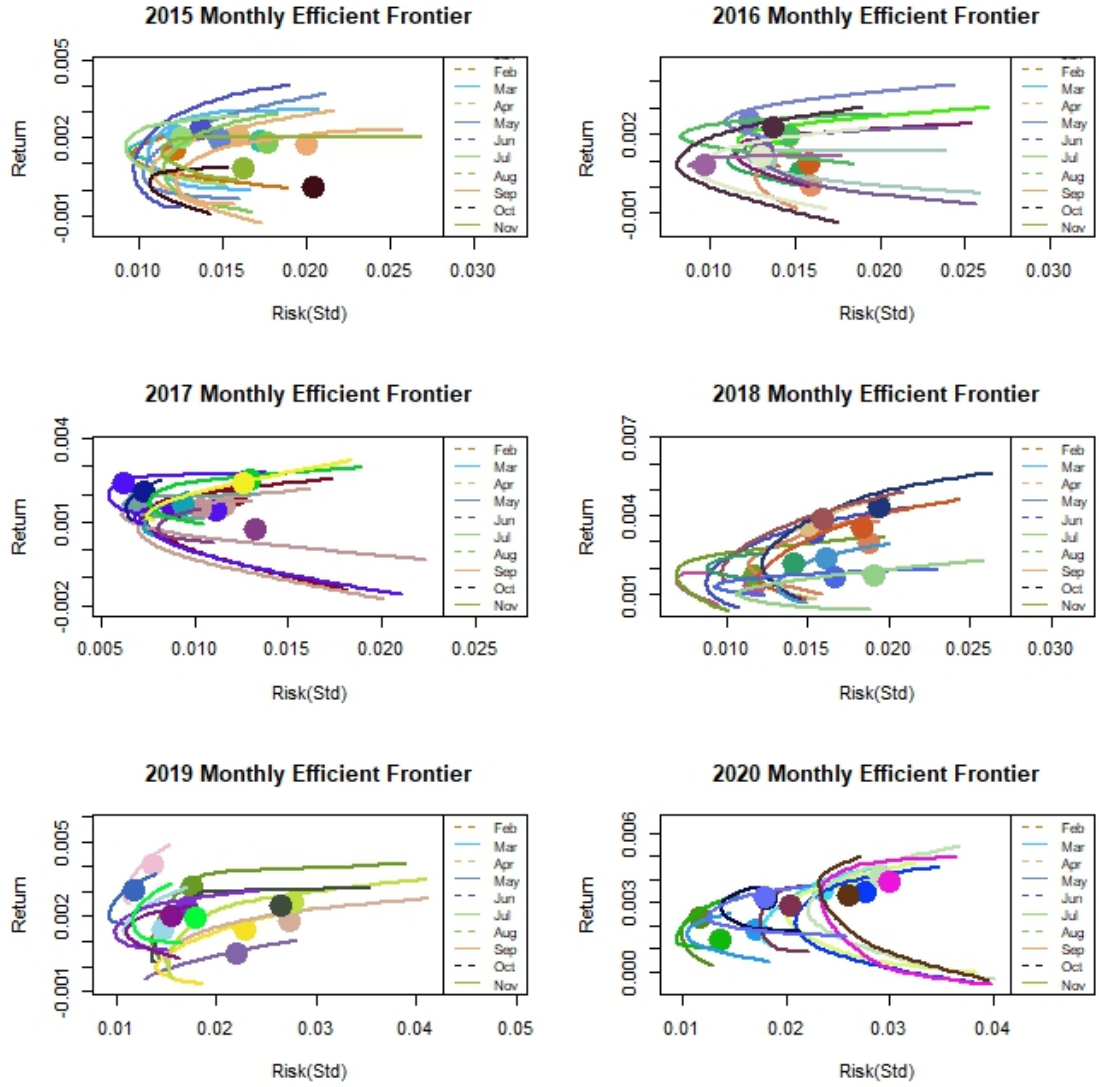
Weight barplot



Through further analysis of our methods, we observe that the calculated seven-asset portfolio takes the constraint with a maximum weight of the stocks of 0.6 and a minimum weight of 0.05. Each time we rebalance, we are thus effectively choosing one stock to maximize the expected return.

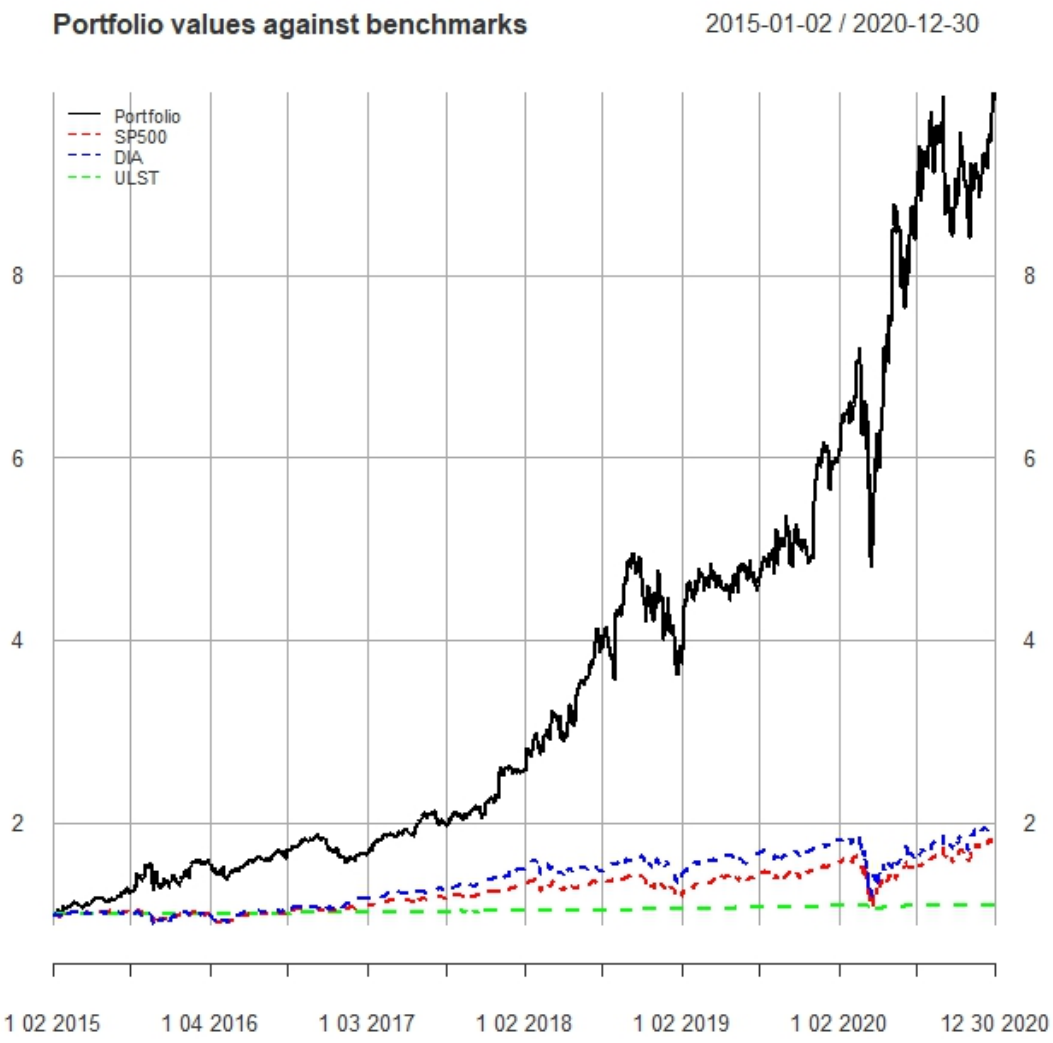
5.2.4 Portfolio Visualizations

Yearly Comparison of Efficient Frontier Plots



Because we use the maximum expected return method, our portfolio is always above the expected return for the global minimal variance point.

Comparison with SPDR Benchmarks

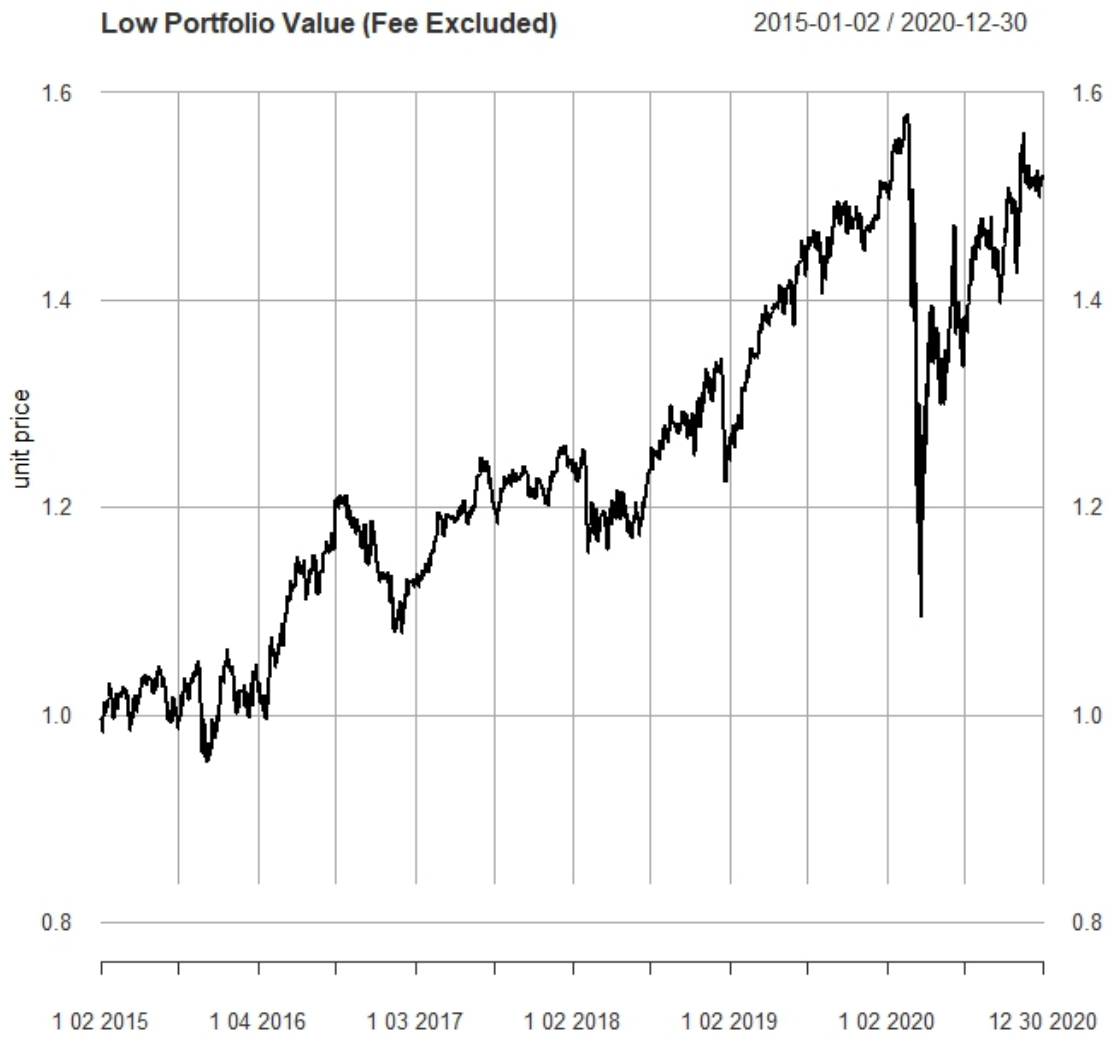


The return is increasingly more than the benchmarks throughout all six years.

5.3 Defensive Portfolio

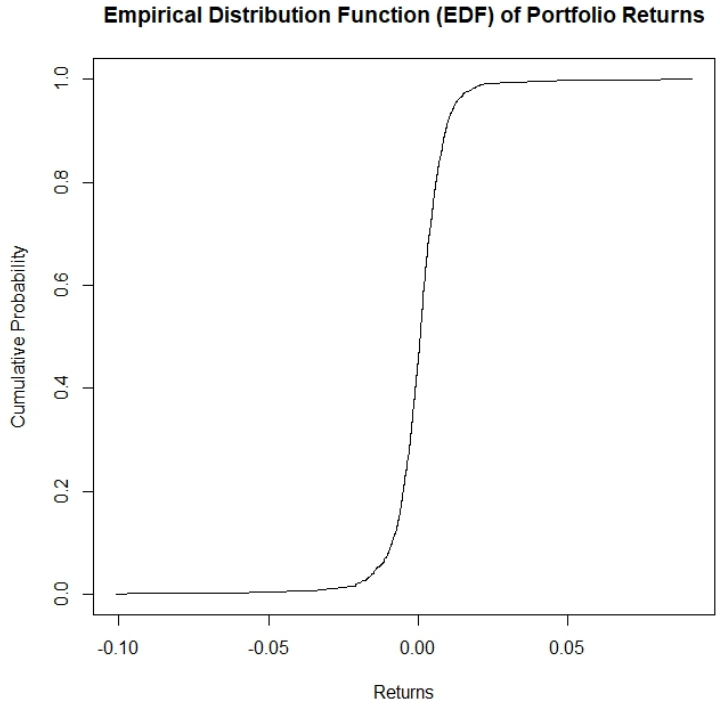
5.3.1 Portfolio Return

Portfolio Value



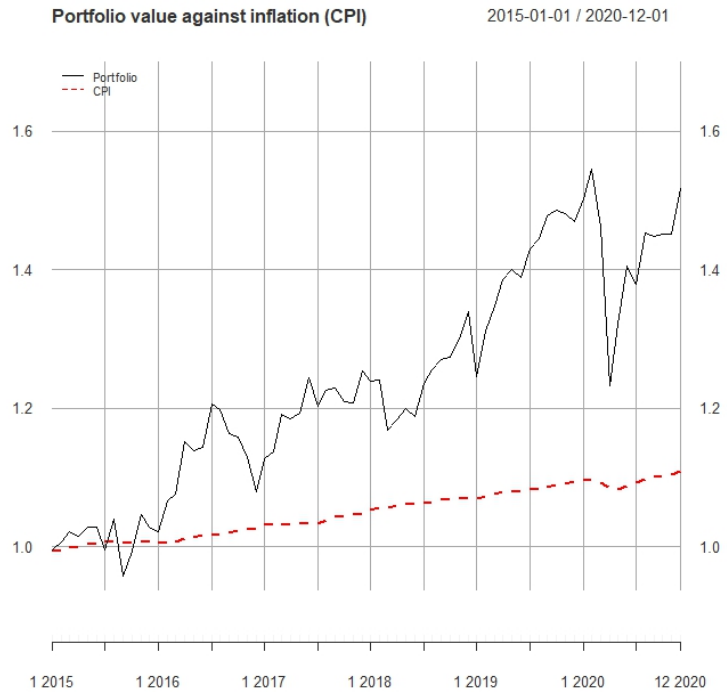
The above graph plots our portfolio value across the six year period with monthly rebalancing. The portfolio value is computed from the exponential of its cumulative log return, which is further computed as the logarithm of the weighted return of the individual assets. For the six year period, the portfolio value has an overall gain of around 1.6 times. The grow was quite steady throughout the six years with a relatively higher growth in late 2016 and early 2019. There was also a remarkable drop in early 2020. The steady grow and relatively low return is in line with our defensive strategy.

Empirical Distribution



The empirical distribution function approximates the cumulative distribution of our daily portfolio log return. We could see the support is approximately in the interval $[-0.1, 0.1]$, which reflects a low volatility. We could also see that probability of negative return is around 0.6, which is less that 0.5, indicating a high frequency of decrease. Nonetheless, the decrease is not too high due to the low volatility.

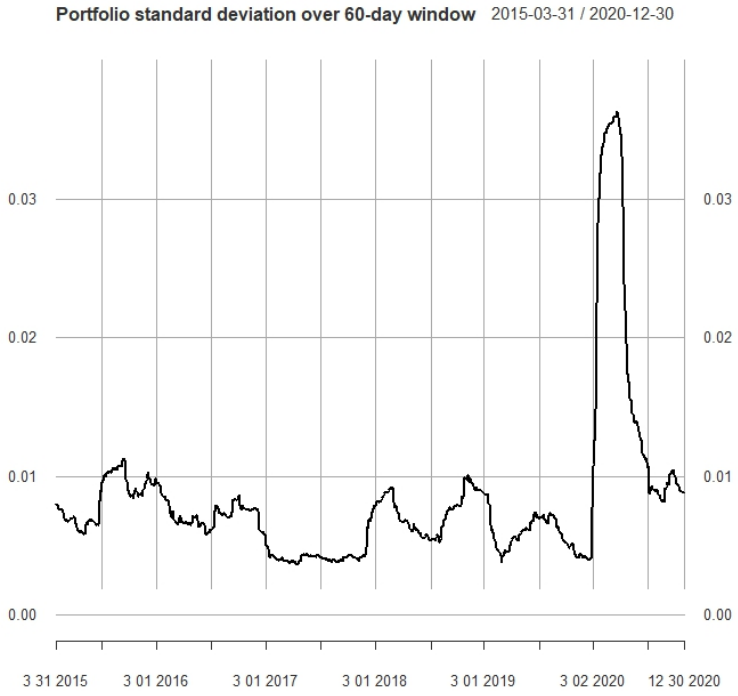
Return against CPI



We compare our portfolio value against the monthly CPI index. Overall, our portfolio beats the CPI after the six year period. In terms of yearly performance, our portfolio in general won over the CPI target except for a little fallback in late 2015. This dip may be due to crash in Chinese markets around 2015 to 2015. Considering that the gap between the portfolio value and CPI target has quickly widened, this is unlikely to occur again. As such, we accept this portfolio that is able to preserve the initial capital and even increase returns beyond it.

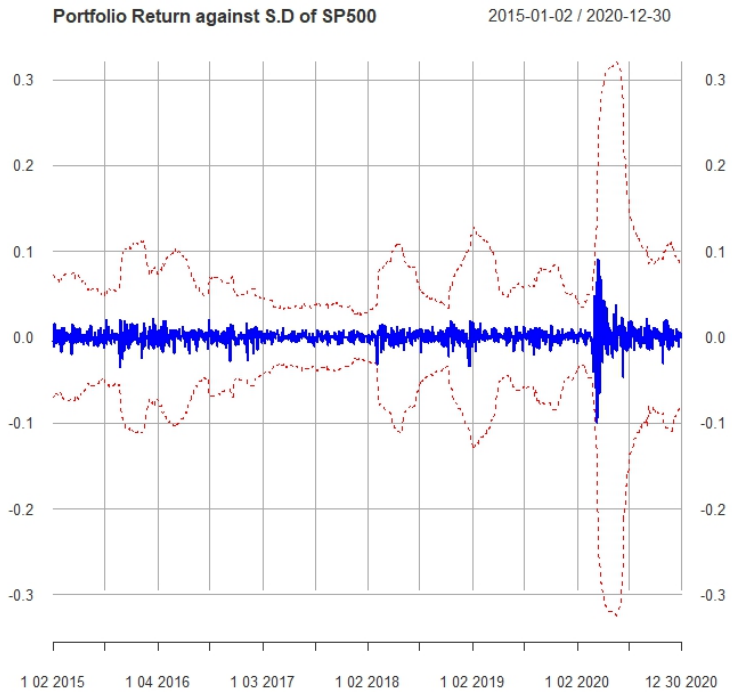
5.3.2 Portfolio Risks

Portfolio Standard Deviation



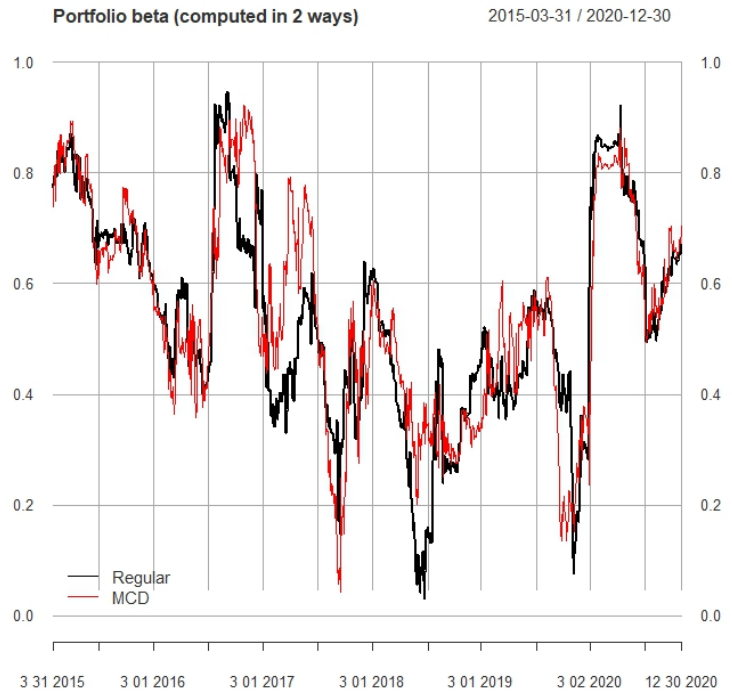
The above graph shows the portfolio standard deviation over a sixty day window - that is why we start in 31 March 2015 instead of 1 January 2015 - across the six year period. The low-risk nature of our portfolio could be seen from the fact that the graph lies under 0.01 in majority of the time. An exception to this trend happens in early 2020 where the standard deviation goes over 0.03, three times more than where it usually lies.

Portfolio Return against SP500 Standard Deviation



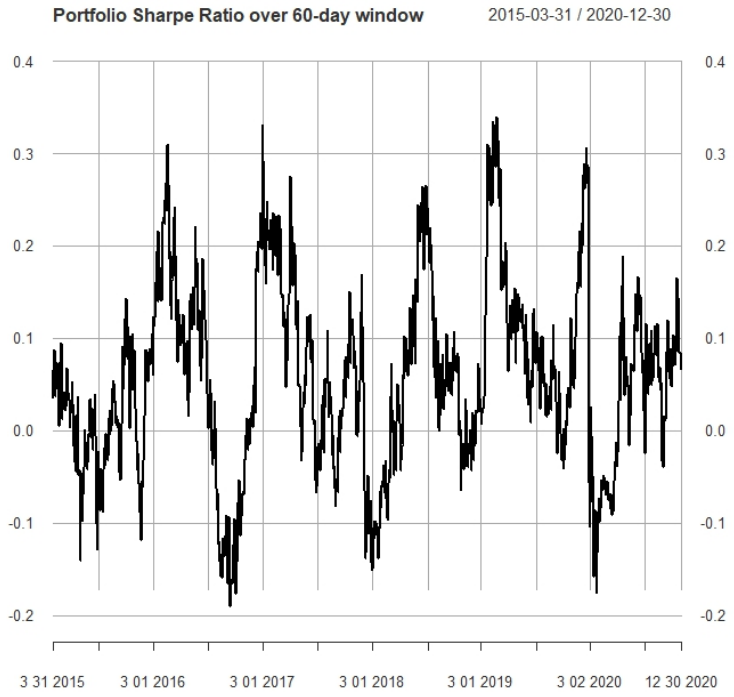
Next, we test our risk against the market risk, modelled by the S&P 500 fund. As seen from the graph our portfolio return never reaches the -8 standard deviation line and so satisfies the risk condition required by the client. A noteworthy observation is that the return almost touches the low threshold in early 2020, which is likely due to the global pandemic.

Portfolio Beta



We similarly compute the portfolio beta in two ways using different estimation to the covariance matrix of the underlying asset, which are by the sample covariance and the maximum covariance determinant. We could see from the plots that the two methods give similar result. Throughout the whole period, our portfolio beta fell below the 1.0 threshold, indicating that it generally does not correlate with the market. More than 50% of the time, the beta falls below 0.6. This indicates that the portfolio is less volatile than the market to a high degree, and thus justifies the low-risk strategy for this portfolio.

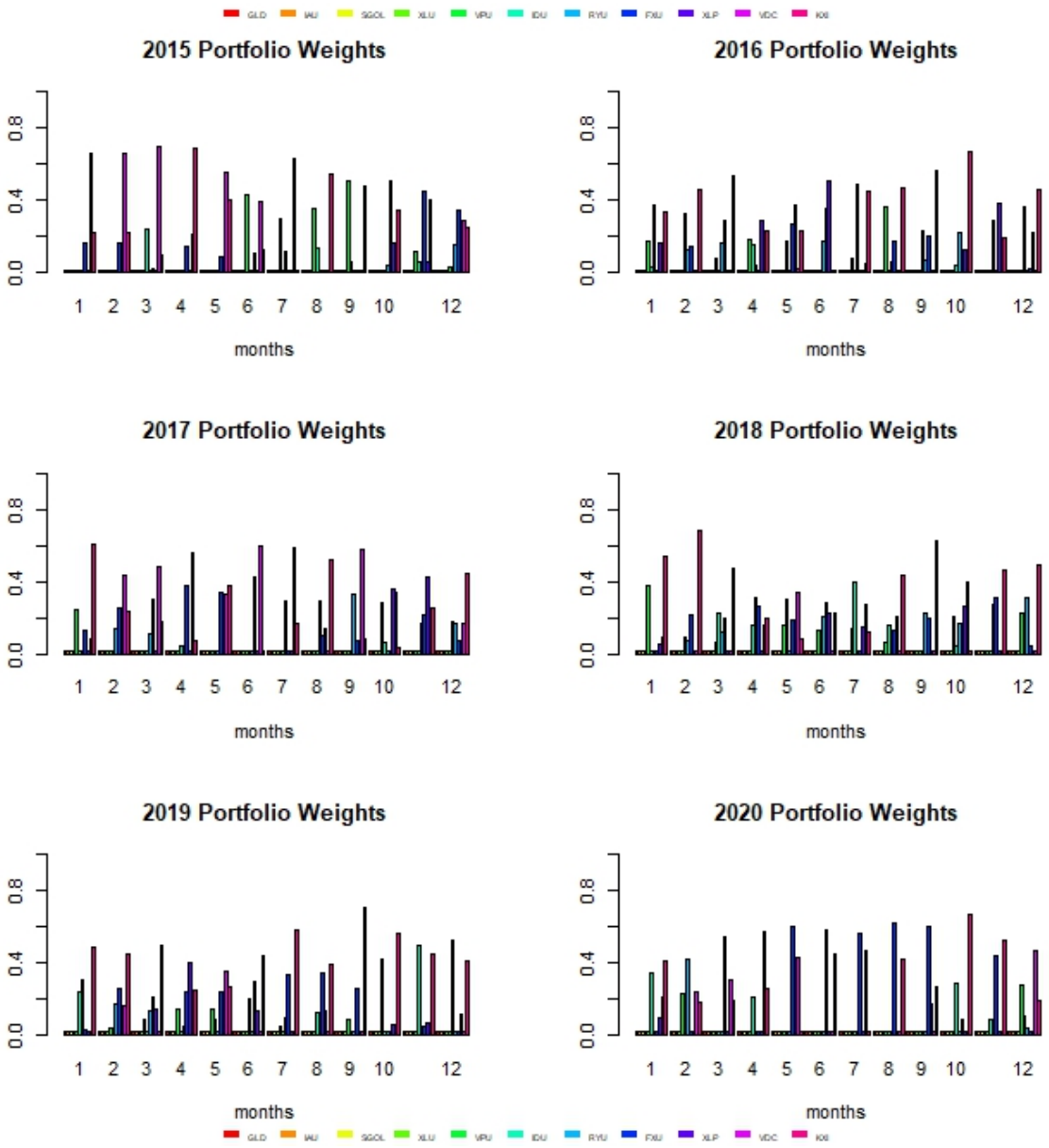
Sharpe Ratio



The Sharpe ratio measures the portfolio return per unit risk. One can see that it ranges in the interval $[-0.2, 0.3]$. When it does go below zero, it usually stays above -0.1. Hence, the portfolio has relatively fewer losses per unit risk during periods of loss and also higher gains per unit risk during periods of gains. This gives another justification that our low-risk, low-return portfolio is quite optimal.

5.3.3 Portfolio Diversification

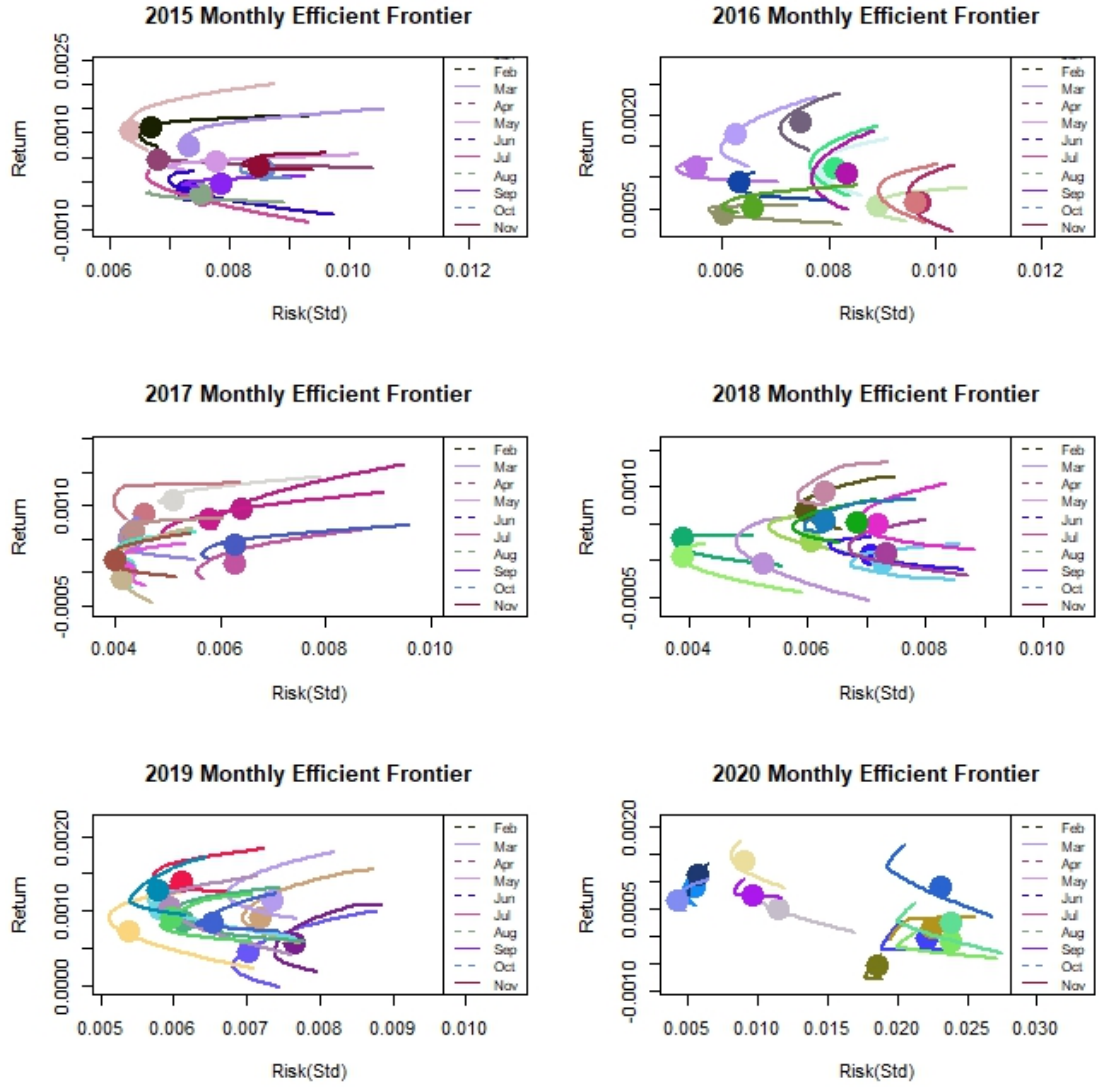
Weight barplot



The barplot above graphs our monthly portfolio weights. We can see that the portfolio shares a similar color spectrum over the six year period. This shows that there are not much diversification across portfolios, which is reasonable since we do not need much diversification across assets to hedge against volatility and risk. Nonetheless, there is still a good amount of diversification existing within portfolios, as shown from the changing color of the longest bar. In addition, the other bars mostly consist of non-trivial weights of more than 0.1.

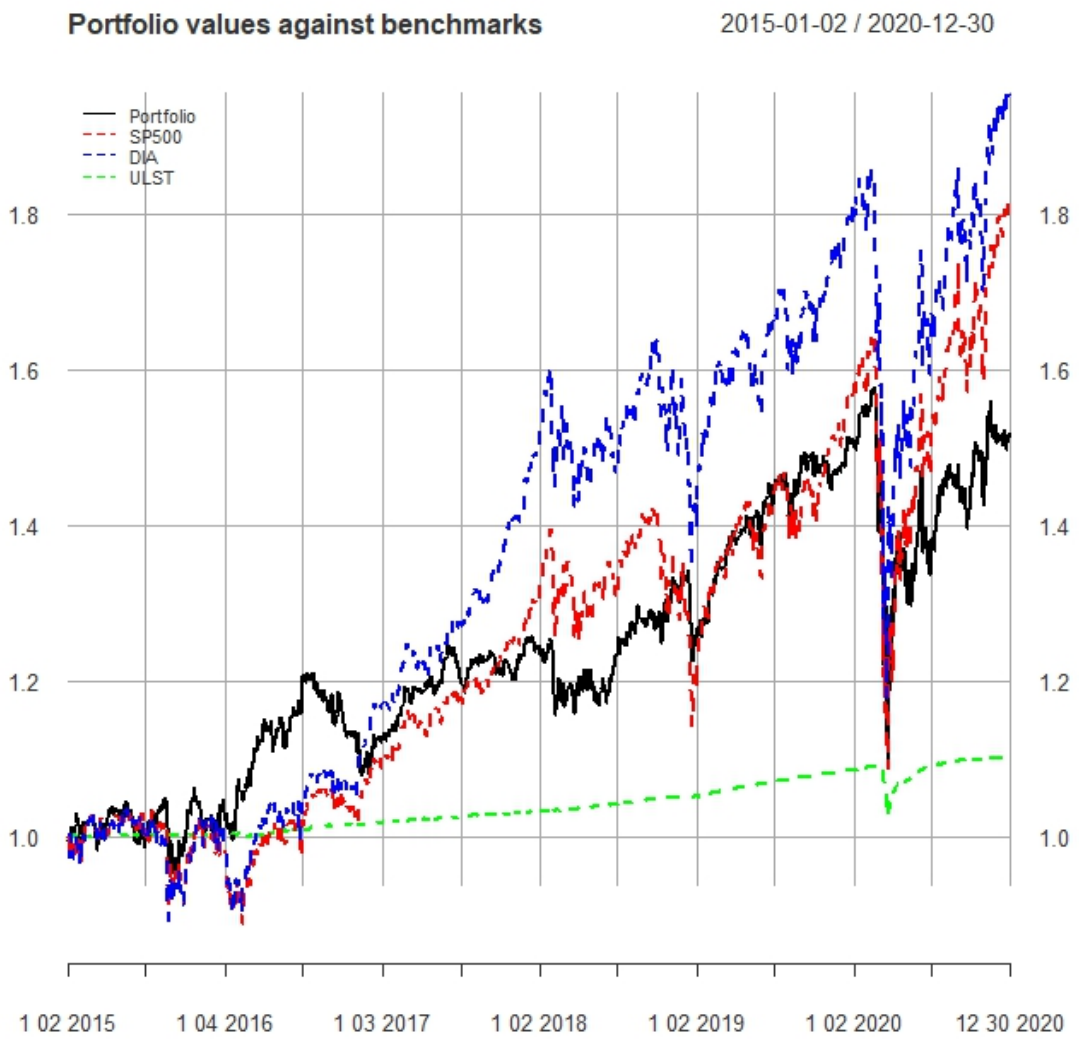
5.3.4 Portfolio Visualizations

Yearly Comparison of Efficient Frontier Plots



Since we are using the portfolio with global minimum variance ratio, our portfolio is at around the vertex of the efficient frontiers.

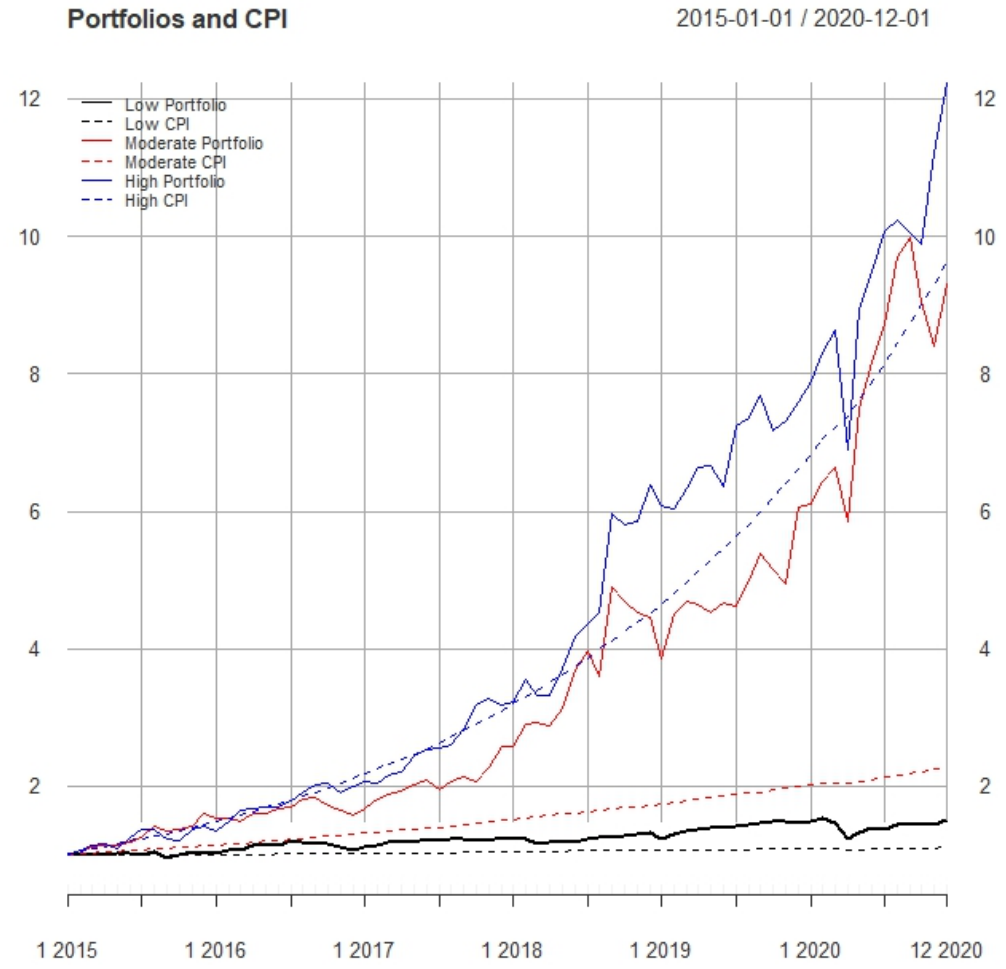
Comparison with SPDR Benchmarks



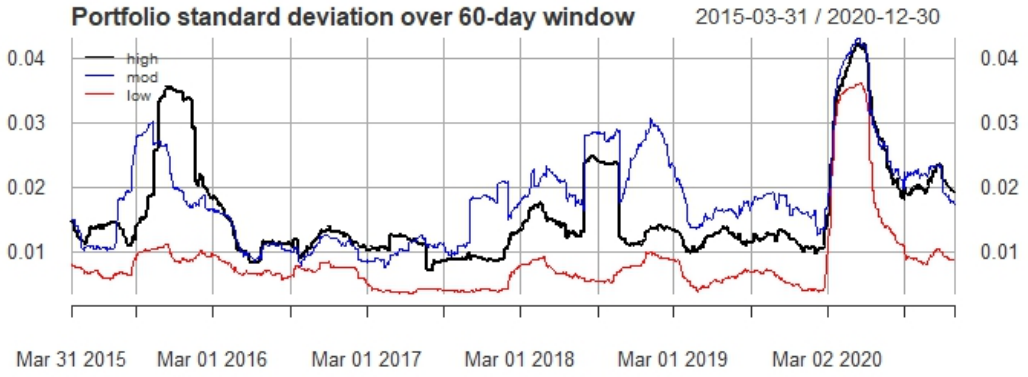
Our portfolio cannot beat S&P 500 as well as the DIA. This makes sense since the low-risk portfolio mainly consists of ETFs, which are less riskier than individual stocks and thus have lower returns than both S&P 500 and DIA. Nonetheless, the portfolio still beats the ULST, which represents the growth of a risk-free asset. This is acceptable as the priority of the defensive portfolio is to preserve capital rather than achieve higher returns.

6 Findings and Recommendations

To summarize, we present here the value and standard deviation of our three portfolios on the same plot.



Performance of Three Portfolios



Risk of Three Portfolios

From the above risk and return analysis, we can observe that all three investment strategies perform particularly well starting 2018 to the end of 2020 and generally meet the performance targets otherwise. However, throughout all three portfolios, we identify instances of especially high covariance and strong fluctuations in

prices and returns in 2015, 2018, and 2020. By conducting further research into the economy during these years, we find that the returns and covariance are influenced by larger market disruptions and global news. Within the high-risk high-return portfolio, we identify the Ebola outbreak of 2014 to 2016 and 2018 to 2020 and the COVID-19 pandemic as sources of market disruption and downturn for the health sector especially. Within the moderate growth portfolio, we identify the Chinese stock market crash of 2015, the China-US trade wars and Google data breaches in 2018, and the COVID-19 pandemic as the issues influencing the volatility in the tech sector. Finally, the defensive portfolio is likely to have been impacted by the same noteworthy events. Despite of the market disruption caused by these events, all three investment portfolios showcased a quickly widening gap between portfolio value and the original performance targets set using the CPI. In conclusion, the overall investment strategies and portfolio decisions outlined in this report are reliable and will result in a strong investment performance. The healthcare sector and the tech sector are strong options for attaining aggressive and moderate growth respectively, while gold, consumer staples, and utilities are appropriate for capital preservation.