Going Public over the Business Cycle[*]

Jisu Jeun [†]

Job Market Paper

October 27, 2024

[Click here for the latest version]

Abstract

This paper analyzes the role of Initial Public Offerings (IPOs) on employment over the business cycle. Empirical evidence shows that IPOs are procyclical and that the business cycle matters for post-IPO firm growth over several years. One factor driving this result is the lower amount of capital raised by firms going public during contraction periods. Building on this evidence, I develop a firm dynamics model in which private firms decide about going public, while public and private firms decide about exit, investment, and employment in the presence of borrowing constraints. The model is calibrated to match selected features of the U.S. non-financial firm sector and is able to replicate the procyclical number of IPOs, procyclical capital injections, and selection patterns observed in the data. Through the model, I find that IPO cyclicality amplifies the impact of negative aggregate productivity shocks on employment, delaying the recovery by 4-6 quarters. This amplification operates through two channels: first, a decline in the public firm share exacerbates capital misallocation; second, a lower propensity to go public reduces the number of new entrants, further delaying the recovery on the extensive margin. These findings suggest that policies mitigating IPO cyclicality during recessions could facilitate a faster recovery in aggregate employment.

Keywords: Firm dynamics; Growth; Business Cycle; Macroeconomics; IPO
JEL codes: D25, E32, G32, H32

---

[*]I am deeply indebted to Lidia Cruces, Georg Duernecker, Johannes Gönsch, Marek Ignaszak, Leo Kaas, Alexander Ludwig, and Nicoló Russo for their invaluable guidance and support. I also extend my thanks to Husnu C. Dalgic for a great discussion, and the audiences at FQMG brownbag seminar, Spanish Macroeconomics Network, and Frankfurt-Mannheim for their insightful comments and suggestions.

[†]Goethe University Frankfurt. Theodor-W.-Adorno-Platz 3, 60323 Frankfurt am Main, Germany. Email: jeun@wiwi.uni-frankfurt.de; Website: jisu-jeun.squarespace.com

1 Introduction

Labor markets tend to recover slowly after recessions, and research following the global financial crisis underscores financial frictions as a primary factor in this sluggish rebound. While studies frequently consider debt financing as the main channel for firms to secure external capital, equity financing also plays a critical role. In particular, an Initial Public Offering (IPO)—a firm's first equity issuance—may contribute to labor market recovery by promoting the growth of firms. Evidence indicates that IPOs significantly impact employment at the firm level, increasing it by an average of 37 percent in the IPO year (Borisov et al., 2021). More importantly, equity financing is distinct from debt in its susceptibility to excess volatility in stock prices (Shiller, 1981; Timmermann, 1993), a factor that can substantially amplify the effects of economic cycles. This volatility affects the capital firms can raise through IPOs, impacting employment dynamics in two main ways. First, firms going public during downturns may face limited capital injection, potentially missing a crucial opportunity to establish a robust foundation for sustained employment growth.[1] Second, at the aggregate level, such volatility may prompt firms to delay IPOs until expansions, driving procyclicality in IPO numbers and, in turn, amplifying aggregate employment dynamics over the business cycle.

This paper demonstrates that IPO firms exhibit significant post-IPO employment growth differences based on the timing of their IPO within the business cycle. Specifically, firms that go public during contraction periods experience slower employment growth than those that do so during expansion periods, with this growth disparity emerging in the second year post-IPO and reaching approximately 10 percentage points by the tenth year. Furthermore, I find that this post-IPO employment growth gap is linked to the amount of capital raised at the time of the IPO. Building on these findings, I develop a heterogeneous firm dynamics model with an endogenous IPO decision to explore the mechanisms driving IPO cyclicality in greater depth. The model incorporates aggregate productivity shocks that influence stochastic discount factors, interest rates, and wages, which in turn affect firms' IPO decisions by altering capital injection and demand for capital at the IPO stage. Through this model, I aim to address two core questions: what mechanisms drive the cyclical nature of IPOs, and how do IPOs

[1] Sedláček and Sterk (2017) document similar evidence in the context of firm entry, showing that firms entering during downturns experience persistently lower employment growth due to constrained growth potential at entry.

contribute to aggregate employment cyclicality?

Going public is an endogenous choice alongside other decisions of firms such as investment and exit. The transition from a private to a public firm through an IPO involves structurally irreversible changes induced by the heterogeneity between private and public firms.[2] Public firms, by definition, raise capital through IPOs by selling shares. Post-IPO, these firms are mandated to distribute a portion of future earnings to shareholders as dividends. Furthermore, investor protection regulations necessitate transparent information disclosure by public firms. While this transparency leads to higher regular operating costs (Breuer and Breuer, 2023), it also facilitates easier access to debt (Schenone, 2010). Weighing these costs and benefits, firms strategically determine the timing of their IPOs, which in turn affects their growth trajectory. Consequently, IPOs play a pivotal role in understanding firm dynamics. Despite this importance, the literature examining the impact of IPOs on firm dynamics remains sparse compared to the extensive research on the role of firm entry and exit.

The model identifies two primary channels through which the impact of aggregate productivity shocks on employment dynamics is amplified. First, a decreased share of public firms in the economy exacerbates capital misallocation, leading to higher inefficiency and a reduction in the average size of firms. This is because public firms face more relaxed borrowing constraints as they have greater transparency from regular disclosures than private firms. Therefore, the reallocation toward public firms imply a higher efficiency in the economy. Second, the lower propensity for firms to go public reduces the value of private firms, which, in turn, discourages potential new entrants, as their expected future value diminishes. Firms initially enter the market as private entities and face the option to make an IPO decision each period. When firms consider going public, they weigh the trade-offs between the benefits—such as access to capital and the relaxation of borrowing constraints—and the associated costs, including fixed IPO expenses and higher operational costs as a public firm. For financially constrained, small, and productive firms, the benefits of going public outweigh the costs, motivating them to pursue an IPO. As a result, potential entrants with high initial productivity expect the opportunity to conduct an IPO shortly after entering the market, which incentivizes their entry. In other words, when the likelihood of conducting an IPO diminishes, fewer firms choose to enter the market due to reduced expectations of

[2] Public firms may delist and return to private status, as seen with increased delistings after regulatory changes in the early 2000s. However, as delistings lack a clear cyclical pattern, this analysis focuses on IPO decisions over the business cycle and treats IPOs as irreversible.

future growth.

Related Literature. The first strand of literature examines heterogeneous firm dynamics models over the business cycle. Many studies focus on the cyclical entry and exit margins, building upon the model developed by Hopenhayn (1992). For example, Lee and Mukoyama (2015) highlights the significant role of cyclical entry costs in explaining the cyclical entry rates observed in the data. Clementi and Palazzo (2016) extend the model to include capital, finding that the cyclical entry margin is an important mechanism for the propagation of aggregate shocks. While the aforementioned papers are about the extensive margin, Moreira (2016) and Sedláček and Sterk (2017) shed light on the intensive margin of entrants, showing that entrants during adverse economic conditions are not only fewer but also characterized by lower growth potential. This paper extends Clementi and Palazzo (2016) by incorporating an endogenous IPO decision to explore its role in the propagation of aggregate shocks. By examining IPO decisions influenced by business cycles, this study addresses the transition of firms from private to public status and its impact on firm dynamics, a dimension not explored in previous research.

Another significant subfield within firm dynamics in business cycle literature addresses the role of financial choice in the presence of financial frictions. It is widely known that the impact of financial friction on firm dynamics is cyclical through the channel of firms' investment (Kiyotaki and Moore, 1997; Khan and Thomas, 2013), but most analyses are limited to debt financing. In practice, firms use both debt and equity financing, with the latter being procyclical, unlike debt finance (Jermann and Quadrini, 2012). Considering this, Di-Nola (2016) shows that models including equity financing may reveal that the impact of financial frictions on firm dynamics has been overestimated. The firm's external financing choice is thus crucial for understanding firm dynamics over the business cycle, which includes the endogenous IPO decision, choosing equity issuance over debt financing. A distinctive feature in this paper is the assumption that equity issuance is a one-time decision corresponding to the transition from private to public status, unlike the recurring debt-equity choice in other models.

The second strand of literature studies the firms' IPO decision. While many studies provide empirical insights into the firm's cyclical decision on IPOs (Jovanovic and Rousseau, 2004; Helwege and Liang, 2004; Tran and Jeon, 2011; Chemmanur and He, 2012; Angelini and Foglia, 2018), theoretical insights are comparatively scarce. Some papers offer theoretical models integrating the IPO decision to explore the long-term trend of IPOs rather than business cycles. González (2020) quantitatively analyze the differential impact of tax environments on private and public firms, explaining the surge in IPOs during the 1990s. This model shares similarities with Midrigan and Xu (2014), which examines capital misallocation through a one-time equity issuance during sector transitions from traditional to modern. Although they do not specifically refer to these sectors as private and public firms, the concept of irreversible transition through the one-time equity issuance is akin to the endogenous IPO choice. Casella et al. (2022) and Davydiuk et al. (2020) attribute the decline in IPOs to increased costs, such as operating expenses and mandatory disclosure. While this paper focuses on short-term fluctuations over the business cycle, it builds upon these quantitative firm dynamics models.

The remainder of the paper is organized as follows. Section 2 introduces data and the empirical evidence regarding the cyclical nature of going public. Section 3 presents the model, and section 4 documents the results of the quantitative analysis of the model.

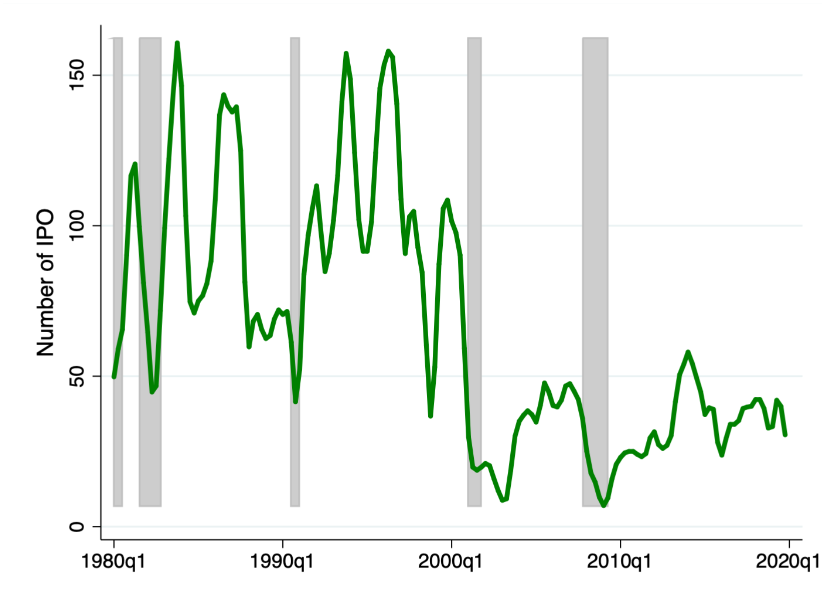
2 Empirical Evidence

In this section, I present empirical evidence on the cyclical patterns of IPOs to address the questions of how IPOs exhibit cyclical behavior and what factors may drive this cyclicality.

2.1 Data

I primarily utilize the Compustat/CRSP merged dataset, which includes stock data from CRSP and fundamental firm data from Compustat for U.S. publicly traded companies. The dataset provides IPO-related information such as the first effective date and IPO date, as well as capital injection at IPO, approximated by total stock sales around the IPO year. Following existing literature, this study excludes utilities (SIC 4900-4949), financial firms (SIC 6000-6999), and public administration (SIC 9000-9999), as well as firms headquartered outside the U.S. and subsidiaries. The dataset also omits firms that underwent leveraged buyouts or were delisted within three years of listing due to mergers and acquisitions. These criteria yield a panel dataset of 12,858 firms with 113,745 annual observations from 1980 to 2019. Financial values

Figure 1: Number of firms going public over the business cycle



Notes. TThe figure presents the number of IPOs in the U.S. based on quarterly IPO dates from Compustat/CRSP. Economic recessions, as defined by the National Bureau of Economic Research (NBER), are shaded in gray. The figure confirms that the number of IPOs is procyclical. For further details on structural changes in the early 2000s, see González (2020).

are in millions of 2012 U.S. dollars, adjusted using the CPI from the U.S. Bureau of Economic Analysis. Firm-level measures are winsorized at the 1st and 99th percentiles to mitigate outlier effects.

2.2 Cyclical Nature of Going Public

Number of IPOs. The procyclicality of IPOs is a well-established fact in the literature (Alti, 2005; Pástor and Veronesi, 2005; Chemmanur, 2010). Given the documented impact of IPOs on firm-level employment growth (Borisov et al., 2021), the cyclical nature of IPOs suggests that the number of firms realizing employment growth through IPOs may fluctuate in line with the business cycle, thereby affecting aggregate employment. In this context, understanding the procyclicality of IPOs is essential for this study. Figure 1 illustrates the number of IPOs on a quarterly basis, with economic downturns shaded in gray. The figure confirms a significant decline in IPO frequency during recessions.[3]

[3]The declines during other periods are associated with stock market performance. See Table 7 in the appendix.

The driving sources of the cyclical number of firms going public have been widely studied in the literature. At the micro-level, firms have a greater incentive to go public during expansionary periods when their productivity peaks over the cycle (Chemmanur et al., 2010) so that they can achieve higher returns from the IPO (Benninga et al., 2005). Additionally, based on evidence that the number of IPOs is clustered within industries and states, Alti (2005) explains the amplified response of IPO numbers to business cycles through information spillovers, which reduce IPO costs for other firms. Furthermore, several studies have identified the correlation between the frequency of IPOs and macroeconomic factors such as interest rates and stock market performance (Tran and Jeon, 2011; Angelini and Foglia, 2018). According to Jovanovic and Rousseau (2004), low interest rates make it easier to obtain financing through borrowing, reducing the pressure on firms to raise capital through an IPO, thus potentially leading them to delay it.

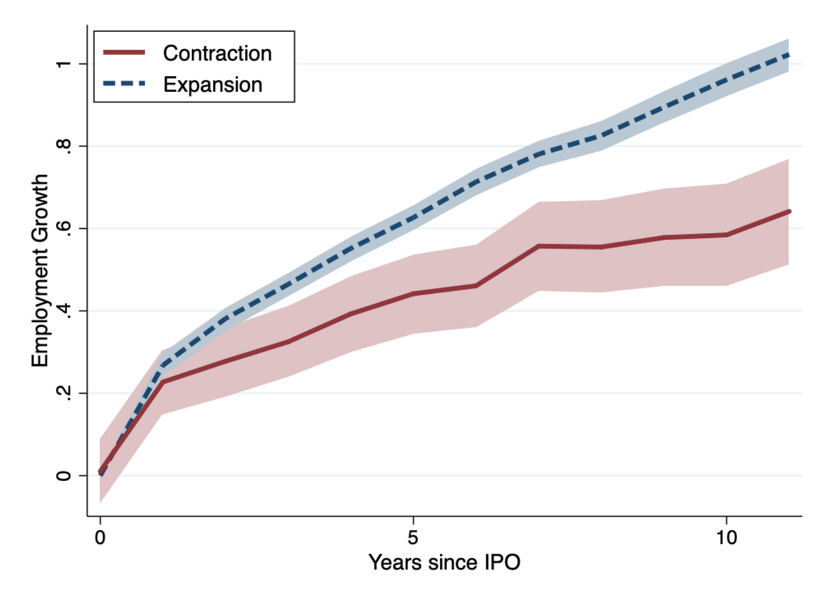
IPO Cohort. Beyond the number of firms realizing growth through IPOs, it is also crucial to understand the growth paths of these firms. Specifically, do firms that go public during recessions grow faster compared to those in expansions? To address this question, I define two separate IPO cohorts based on their quarterly IPO dates, categorizing them as either contractionary (periods from peak to trough) or expansionary (periods from trough to peak). Descriptive statistics indicate that, excluding the firms at the top right of the distribution, the contraction cohort is generally smaller and younger and receives a smaller amount of capital injection (see Table 8 in the Appendix).

To analyze post-IPO employment growth by age for each cohort, I run a non-parametric regression

$$\Delta^{ipo} N_{it} = \boldsymbol{\beta}_{a,c} Age_{it} \times Cohort_i^{ipo} + \Gamma \mathbf{X_i} + \gamma Y_{t-1} + \epsilon_{it}. \tag{1}$$

Here, $\Delta^{ipo} N_{it}$ denotes the cumulative employment growth rate from the year of IPO, calculated as $\ln(N_{it}) - \ln(N_{i0})$ where $t = 0$ represents the IPO year. The coefficient $\boldsymbol{\beta}_{a,c}$ captures the cohort effect, representing the average employment growth by IPO age (Age_{it}), defined as a dummy for the number of years since the IPO, and differentiated by the cohorts formed during economic expansions and contractions ($Cohort^{ipo}$). Control variables $\mathbf{X_i}$ include firm i's initial employment, sales per worker, pre-IPO capital intensity (gross value of property, plant and equipment), and liabilities (debt-to-asset

Figure 2: IPO cohort effect on post-IPO employment growth



Notes. The figures display cumulative post-IPO employment growth for cohorts classified by IPO timing within the business cycle. The 'Contraction' cohort includes firms that went public between a business cycle peak and trough, while the 'Expansion' cohort comprises firms that went public between a trough and peak. These results are based on the non-parametric regression specified in Equation (1), controlling for initial employment, sales per worker, pre-IPO capital intensity, and debt-to-asset ratio at the IPO year, as well as industry and regional fixed effects. The dataset is an unbalanced panel drawn from Compustat/CRSP.

ratio) at the year of IPO, as well as industry and regional fixed effects. Additionally, Y_t represents the detrended log of real GDP, accounting for varying economic conditions encountered by expansion and contraction cohorts post-IPO.

Figure 2 depicts the employment trend over the years since the IPO, highlighting the cohort effect. Firms that go public during contractions exhibit slower growth rates compared to those during expansions, with an average gap of at least 10 percentage points by the sixth year post-IPO. This gap widens until the eleventh year, reaching at least 13 percentage points, underscoring the significant impact of IPO timing on post-IPO growth.

Employment growth following an IPO is facilitated by increased internal finance (Tran and Jeon, 2011). The amount of capital injection at the time of the IPO is likely a critical factor influencing subsequent investment decisions and is closely related to the firm's growth. Therefore, I examine whether the heterogeneous amount of capital injection is related to the timing of the IPO, beyond selection effects. To address this

Table 1: Capital injection at IPO over the business cycle

	(1)	(2)	(3)
Log sale of stock at IPO			
IPO in Contraction	-0.378*		
	(-2.22)		
Detrended Log GDP		0.295***	0.195***
		(13.87)	(7.50)
Detrended PE Ratio			0.062***
			(6.67)
Firm Characteristics at IPO	✓	✓	✓
Industry FE	✓	✓	✓
State FE	✓	✓	✓
Observations	5,096	5,096	5,096
R^2	0.297	0.322	0.328

Notes. t-statistics in parentheses; $*$ $p < 0.05$, $**$ $p < 0.01$, $***$ $p < 0.001$. The table presents regression results of stock sales at IPO on the business cycle index using cross-sectional data of IPO firms. The cyclical components of log GDP and Shiller PE ratio are detrended using HP filter. 'Firm Characteristics at IPO' includes employment, sales per worker, capital intensity, and debt-to-asset ratio. For capital intensity and debt-to-asset ratio, data prior to IPO capital internalization are used. Firms exiting within two years post-IPO are excluded.

question, I run a cross-sectional Ordinary Least Squares (OLS) regression of capital injection—the sale of stock ($sstk$ in Compustat) during the IPO period—on the cyclical variable at the time of the IPO.[4]

$$\log(sstk_{it}) = \beta Cycle_{it} + \Gamma \mathbf{X_{it}} + \epsilon_{it} \tag{2}$$

The coefficient of interest, β, captures the relationship between capital injection and the state of the economy at the time of the IPO, measured either by a dummy variable for the contraction cohort or by the detrended log of real GDP. The control variables, $\mathbf{X_{it}}$, include employment, sales per worker, capital intensity, and debt-to-asset ratio in the year of the IPO[5], as well as industry and state fixed effects.

In Table 1, Column (1) shows that firms in the contraction cohort raise 38 percent

[4] A robustness check is conducted using the aggregate sale of stocks in the first three years post-IPO to account for potential time lags.

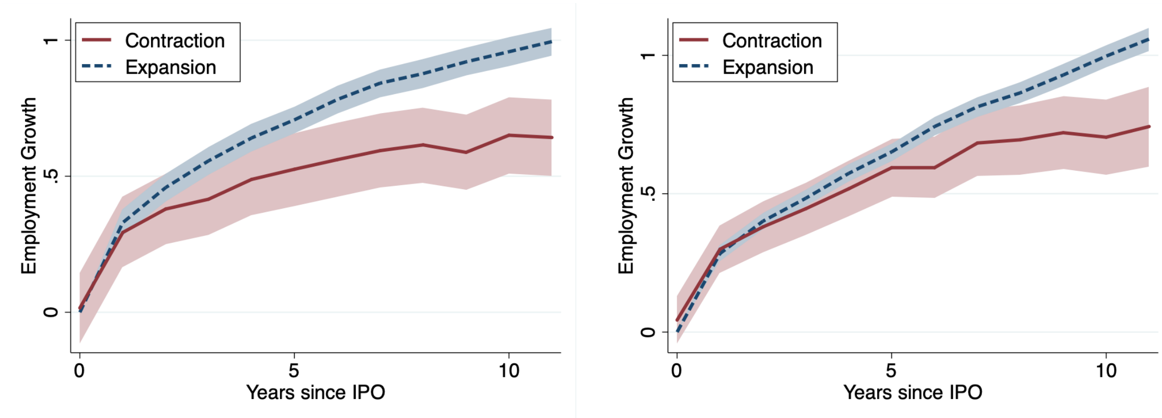
[5] For capital intensity and the debt-to-asset ratio, I specifically use data of the period only prior to the IPO, as the sale of stock is significantly collinear with these variables post-IPO.

less capital at IPO compared to those in the expansion cohort, holding other characteristics constant. To be specific, column (2) indicates that a 1 percentage point increase in GDP at the time of IPO results in a 30 percent greater amount of capital injection. Column (3) additionally considers the cyclicality of stock market performance represented by Shiller price-to-earnings (PE) ratio to confirm that the relationship with economic output cyclicality is not simply due to stock market fluctuations. This confirms that the amount of capital injection is indeed related to the economic state at the time of the IPO, consistent with existing literature (Tran and Jeon, 2011).

To support the proposed hypothesis, I revisit the non-parametric regression analysis specified in Equation (1), incorporating an additional control for the magnitude of capital injection at the time of the IPO. Figure 3 presents the growth gap between expansion and contraction IPO cohorts across different model specifications. In the left panel, which is based on a balanced panel excluding firms that exit within 10 years post-IPO, the analysis controls for heterogeneous exit rates between the cohorts. The results show that while the exit margin does not significantly affect the growth gap, the significance decreases as growth variability increases. In the right panel, the analysis incorporates differences in IPO capital injection, revealing that the growth gap substantially decreases. Up to the 8th year, the gap is no longer significant, and by the 10th year, its magnitude is nearly halved. This suggests that the growth gap between contraction and expansion cohorts is closely tied to differences in capital injection at the time of IPO.

One cannot exclude the possibility of omitted variable bias. For instance, the capital injection might reflect the firm's unobserved productivity, which could drive post-IPO employment growth. The results in this section may be influenced by the selection bias towards more productive IPO firms during expansionary periods. Are productive firms more likely to wait for favorable macroeconomic conditions before going public? Or, as previously suggested, does more discounted capital injection compress the growth of IPO cohorts during contraction periods? To address this, I propose and estimate a quantitative firm dynamics model incorporating endogenous IPO decisions.

Figure 3: IPO cohort effect on post-IPO employment growth: extended analysis



(a) Exit effect

(b) Capital injection effect

Notes. The figures extend the analysis from Figure 2 by displaying cumulative post-IPO employment growth. Panel (a) based on a balanced panel, examines the impact of heterogeneous exit rates between contraction and expansion cohorts. Panel (b) controls for the amount of capital raised at the IPO, measured by stock sales in the IPO year, to assess the influence of varying levels of capital injection.

3 The Model

Time is discrete, $t = 1, 2, ...$, and the horizon is infinite. There are two types of firms, privately held and publicly traded. Firms enter the market as private, and transitions to public occur via IPOs. For model tractability, the transition is assumed to be irreversible. Both type of firms produce a homogeneous good at time t through the production function

$$y_t = z_t s_t (k_t^\alpha l_t^{1-\alpha})^\eta \, . \tag{3}$$

Firms' productivity is the product of an aggregate and an idiosyncratic component each denoted by z and s. Given both components of productivity, firms use two inputs: labor denoted by l_t and predetermined capital denoted by k_t. The parameter $\alpha \in (0, 1)$ governs the share of capital in production and $\eta \in (0, 1)$ governs decreasing returns to scale.

The aggregate component of productivity z_t is driven by a persistent stochastic process

$$\log(z_{t+1}) = \rho_z \log(z_t) + \varepsilon_{z,t} \, , \tag{4}$$

with $\varepsilon_{z,t} \sim \mathcal{N}(0, \sigma_z)$. In both public and private firms, firm-specific productivity s_t evolves stochastically according to

$$\log(s_{t+1}) = \rho_s \log(s_t) + \varepsilon_{s,t} \; , \tag{5}$$

with $\varepsilon_{s,t} \sim \mathcal{N}(0, \sigma_s)$. Furthermore, the shocks in these two components $\varepsilon_{z,t}$ and $\varepsilon_{s,t}$ are assumed to be orthogonal to each other, ensuring independent variations in common and idiosyncratic productivity. The initial productivity of the entrant s_0 will be specified below.

Aggregate Fluctuation. Aggregate uncertainty affects firms' behavior. Following Clementi and Palazzo (2019), I assume that firms assess future cash flows using a stochastic discount factor, defined as

$$M(z_t, z_{t+1}) \equiv \beta \exp(\phi_0 \log(z_t) + \phi_1 \log(z_{t+1})) \; , \tag{6}$$

where the subjective discount factor, $\beta > 0$ is the time discount factor, and the parameters $\phi_0 > 0$ and $\phi_1 < 0$ represent the sensitivity to aggregate states. The risk-free rate is defined as the inverse of the expected value of the stochastic discount factor which reduces to[6]

$$R_t = \frac{1}{\beta} \exp\left(-z_t(\phi_0 + \rho_z \phi_1) - \frac{\phi_1^2 \sigma_z^2}{2}\right) \; . \tag{7}$$

Note that with the parameter restriction $\phi_1 \in \left(-\frac{\phi_0}{\rho_z}, 0\right)$, the risk-free rate is countercyclical.

IPO Decision and Financial Friction. Private firms decide to do IPO incurring a fixed cost κ. The firms going public offer a χ share of their future dividends to the public investors. The price of a share, denoted as $p(k_t, b_t, s_t, z_t)$, depends on the aggregate state z_t and firm's states including productivity s_t and capital structure k_t and b_t. The functional form of this price will be described further below. It is assumed that the demand for the shares of IPO firms is perfectly elastic at the offered price. That is, there is always a pool of investors who purchase all the shares, ensuring that no shares remain unsold in the market.

[6] Derivation is provided in the Appendix B.1.

By selling an equivalent fraction of future dividends at IPO, firms receive a capital injection of $\chi p(k_t, b_t, s_t, z_t)$. This implies that public firms distribute a χ share of their dividends to investors every period. This context implies an intertemporal substitution motive of IPO, where firms weigh the benefits of immediate capital injection against the cost of sharing future earnings with investors.

Firms are financially constrained. Firms issue risk-free debt b_t subject to a collateral constraint[7]

$$b_t \leq \theta k_t \, , \tag{8}$$

where $\theta = \{\theta^{pb}, \theta^{pr}\}$ differs between private and public firms. To avoid potentially binding financial constraints in the future, firms have a precautionary savings motive to increase net worth in advance. This implies that expanding net worth through capital injection at an IPO is particularly valuable for productive firms. In addition, financial frictions are assumed to be further relaxed for public firms. Public firms are required to make regular disclosures (e.g., SEC filings), which reduce information asymmetry for investors. Consequently, it is relatively easier for public firms to issue debt (Schenone, 2010), implying a more relaxed borrowing constraint $\theta^{pb} \geq \theta^{pr}$. At the same time, public firms incur higher operating costs due to additional expenses such as auditing costs and reporting costs.

In sum, IPOs involve two trade-offs. First, firms receive a capital injection but must distribute future dividends, allowing them to exploit the intertemporal substitution effect. Second, going public entails higher operating costs but also relaxes borrowing constraints, thereby enhancing precautionary savings.

Budget Constraints. Following the transition, public firms distribute dividends to shareholders each period, corresponding to the share sold at the IPO. Consequently, the public firm retains a $1 - \chi$ fraction of the total dividends in the post-IPO period. Additionally, operating as a public firm entails higher costs. Therefore, in each period t, the public firm aims to maximize the $1 - \chi$ share of the discounted value of its dividend stream subject to the following budget constraint

$$d_t + k_{t+1} - b_{t+1} = y_t - w_t l_t - (1 + r_t) b_t + (1 - \delta) k_t - f^{pb} \, . \tag{9}$$

[7]Firms are not allowed to save in the model, implying $b_t \geq 0$.

Here, d_t denotes dividends, which are subject to non-negativity constraints ($d_t \geq 0$). The variables w_t, r_t, and f^{pb} represent wage, interest rate, and operating cost, respectively.

Private firms maximize the entire share of the discounted value of its dividend stream, subject to budget constraint

$$d_t + k_{t+1} - b_{t+1} = y_t - w_t l_t - (1 + r_t)b_t + (1 - \delta)k_t - f^{pr} + \{\chi p(k_t, b_t, s_t, z_t) - \kappa\}\xi_{ipo} , \tag{10}$$

where $\xi_{ipo} \in \{0, 1\}$ is a binary indicator of the IPO decision. This equation elucidates that firms going public incur a fixed cost κ and receive capital injection $\chi p(k_t, b_t, s_t, z_t)$ by issuing equity. As above, f^{pr} represents the operating cost that is lower than that of public firm's ($f^{pr} \leq f^{pb}$).

Exit and Entry. Firms decide whether to exit after their production and before making investment. Upon exit, firms liquidate the internal cash flow and depreciated capital, net of outstanding debt, which reduces to

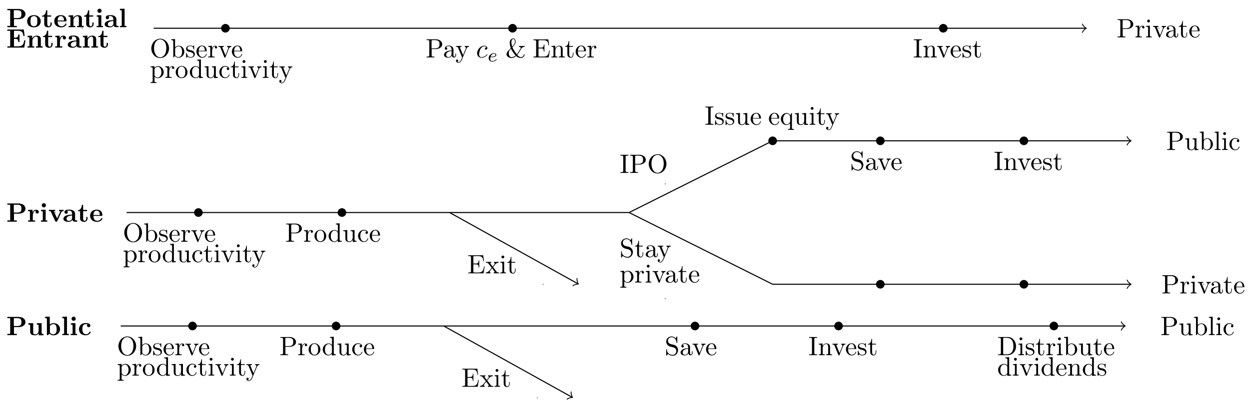
$$d_t^x = y_t - w_t l_t - (1 + r_t)b_t + (1 - \delta)k_t - f^{pr/pb} . \tag{11}$$

Both private and public firms endogenously exit. For public firms, as χ share of dividends belong to the investors, the equivalent share of liquidation upon exit is distributed to the investors.

There's a large number of potential entrants each period. They observe the aggregate productivity z and draw their initial idiosyncratic productivity s_0 from a log normal distribution. Potential entrants are endowed with an initial capital k_0. Since there is no debt, the initial net worth is $a_0 = k_0$. They can freely enter as a private firm by paying an entry cost $c_e > 0$. If the expected value of operating exceeds the entry cost, they decide to enter the market. Upon entry, they invest to start producing in the following period.

Price of share. Selling χ share of equity at IPO entails the distribution of the equivalent share of dividends in every subsequent period following the IPO, which pins

Figure 4: Timing of firms' problem



down the price of a share as

$$\chi p(k_t, b_t, s_t, z_t) = \mathbf{E_t}\Big[M(z_t, z_{t+1}) \max\{(\chi d_{t+1} + \chi p(k_{t+1}, b_{t+1}, s_{t+1}, z_{t+1})), \chi d^x_{t+1}\}\Big]. \tag{12}$$

As dividends are distributed starting from the period following the IPO, they are discounted by a factor $M(z_t, z_{t+1})$. Here, $d_{t+1} = d(k_{t+1}, b_{t+1}, s_{t+1}, z_{t+1})$ represents the dividends attainable post-IPO as a public firm, subject to the budget constraint outlined in (9). In addition, d^x_{t+1} denotes the liquidation value upon the firm's exit as described in (11). This setup reflects that, upon IPO, investors expect to receive a χ share of the dividends generated by the IPO firm as a public entity from the next period onwards, accounting for the possibility of the firm's exit. The equation thus poses a fixed-point problem, which will be reformulated below using the value function of public firms.

Recursive formulation. The time subscript is suppressed in the following recursive formulation. Firms discount future value by $M(z, z')$.

Figure 4 illustrates the timeline of the firms' problem. After observing the aggregate state z and drawing the initial productivity s_0, entrants pay the entry cost c_e, make an initial investment decision, and subsequently become a private firm. Similarly, incumbent firms begin with observing an aggregate shock z and idiosyncratic productivity s. Subsequently, firms hire labor maximizing profit given the state of productivity s and z and pre-determined capital k as

$$\begin{aligned} l(k,s,z) &= \arg\max \; sz(k^{\alpha}l^{1-\alpha})^{\eta} - w(z)l \\ &= \left(\frac{(1-\alpha)\eta}{w(z)} szk^{\alpha\eta}\right)^{\frac{1}{1-(1-\alpha)\eta}}. \end{aligned} \tag{13}$$

The period begins with the payment of relevant costs such as wages, interest, and operating expenses. Incumbents then decide whether to exit the market, followed by the decision of private firms to go public through an IPO. Firms undertaking an IPO issue equity, which involves selling shares and internalizing the capital injection. Subsequently, both private and public firms make investment and financing decisions, determining how to utilize internal and external capital for these investments. Finally, public firms distribute dividends from the residual profits.

In each period, firms decide on capital investments k' and how to allocate these investments between internal cash flows and debt b'. The internal cash flows are pre-determined by the productivity components s and z, realized at the beginning of the period, as well ass by k and b, which were determined in the previous period. Consequently, if the increase in k' exceeds the available internal cash flows, it necessarily results in an equivalent increase in b'. Thus, the decision on k' pins down b'. By introducing net worth, defined as capital net of debt ($a \equiv k - b$), the problem can be simplified to choosing the optimal future net worth a' in the firm's dynamic optimization. Then it entails the intra-period problem for k' and b' as

$$\begin{aligned} \pi' = \max_{k',b'} \int\int & s'z'(k'^{\alpha}l'^{1-\alpha})^{\eta} - w(z')l' - (1+r)b' + (1-\delta)k' \, dH(s'|s)dG(z'|z), \\ \text{s.t. } \; & k' - b' = a', \\ & b' \leq \theta k', \\ & l'(k',s',z') = \left(\frac{(1-\alpha)\eta}{w(z')} s'z'k'^{\alpha\eta}\right)^{\frac{1}{1-(1-\alpha)\eta}}. \end{aligned}$$

Therefore, given a', the firm chooses capital

$$k'^{*} = \min\left(\frac{a'}{1-\theta},\ \Phi\left(\frac{\int\int\left(\frac{s'z'}{w(z')^{(1-\alpha)\eta}}\right)^{\frac{1}{1-(1-\alpha)\eta}} dH(s'|s)dG(z'|z)}{\int r(z')dG(z'|z)+\delta}\right)^{\frac{1-(1-\alpha)\eta}{1-\eta}}\right), \quad (14)$$

where $\Phi = (\alpha\eta)^{\frac{1-(1-\alpha)\eta}{1-\eta}}((1-\alpha)\eta)^{\frac{(1-\alpha)\eta}{1-\eta}}$, and debt as $b'^{*} = k'^{*} - a'$. As a result, profit function can be expressed as a function of (a, s, z) as follows

$$\pi(a, s, z) = \Omega(szk^{*\alpha\eta})^{\frac{1}{1-(1-\alpha)\eta}} - (r(z)+\delta)k^{*} + (1+r(z))a, \quad (15)$$

where $\Omega = (1-(1-\alpha)\eta)\left(\frac{(1-\alpha)\eta}{w(z)}\right)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}}$. Hence, the forthcoming dynamic optimization problems are represented as state vector (a, s, z) and a control variable a' that maximize the value functions.

The public firms' problem can thus be recursively formulated as

$$V^{pb}(a, s, z) = \max\{V^{x}, \tilde{V}^{pb}\}, \quad (16)$$

$$\tilde{V}^{pb}(a, s, z) = \max_{a'}\ (1-\chi)d + \int_{s}\int_{z} M(z, z')V^{pb}(a', s', z')dH(s'|s)dG(z'|z) \quad (17)$$

$$\begin{aligned}\text{s.t.}\ \ & d + a' = \pi(a, s, z) - f^{pb},\\ & k' = k'(s, z, a', \theta^{pb}),\\ & d \geq 0,\end{aligned}$$

$$V^{x}(a, s, z) = (1-\chi)(\pi(a, s, z) - f^{pb}). \quad (18)$$

Public firms decide whether to exit based on the value functions specified in (16), with the value of operating and exiting detailed in (17) and (18), respectively. The value of an operating public firm is represented by the $(1-\chi)$ share of the stream of dividends as defined above. This value is subject to the budget constraints described in (9), the investment choices derived from the intra-period problem specified in (14), and the non-negativity condition for dividends.

The recursive formulation of public firms' value function helps to reformulate the price function as[8]

$$\chi p(a,s,z) = \frac{\chi}{1-\chi}\mathbf{E}\Big[M(z,z')V^{pb}(a',s',z')\Big]. \tag{19}$$

It is noteworthy that the variable a', the state variable of V^{pb}, is a policy function determined in the state of private at the time of its IPO.

Private firms solve following value functions:

$$V^{pr}(a,s,z) = \max\{V^{x}, V^{stay}, V^{ipo}\}, \tag{20}$$

$$\begin{aligned} V^{stay}(a,s,z) = \max_{a'} \ & d + \int_s\int_z M(z,z')V^{pr}(a',s',z')dH(s'|s)dG(z'|z) \\ \text{s.t.} \ \ & d + a' = \pi(a,s,z), \\ & k' = k'(s,z,a',\theta^{pr}), \\ & d \geq 0, \end{aligned} \tag{21}$$

$$\begin{aligned} V^{ipo}(a,s,z) = \max_{a'} \ & d + \int_s\int_z M(z,z')V^{pb}(a',s',z')dH(s'|s)dG(z'|z) \\ \text{s.t.} \ \ & d + a' = \pi(a,s,z) + \chi p(a,s,z) - \kappa, \\ & k' = k'(s,z,a',\theta^{pb}), \\ & d \geq 0, \\ & \chi p(a,s,z) = \frac{\chi}{1-\chi}\int_s\int_z M(z,z')V^{pb}(a',s',z')dH(s'|s)dG(z'|z), \end{aligned} \tag{22}$$

$$V^{x}(a,s,z) = \pi(a,s,z). \tag{23}$$

They make both exit and IPO decisions as specified in (20). Notably, the exit value for private firms described in (23) aligns with that of public firms when $\chi = 0$. Private firms decide to go public if the value of doing so, as given in (22), exceeds the value of remaining private, as given in (21). The value function for going public incorporates the future value of public firms, indicating the transition from private to public status occurring today. Under the assumption that the financing decision takes place after the IPO decision, IPO firms solve the intra-period problem subject to the collateral constraints of public firms, specified by θ^{pb}.

[8] Derivation is provided in the Appendix B.4.

Finally, potential entrants decide to enter if the value of entry

$$V^e(s_0, z) = \max_{a'} -a' + \int_s \int_z M(z, z')V^{pr}(a', s', z')dH(s'|s_0)dG(z'|z) \tag{24}$$
$$\text{s.t.} \quad k' = k'(s_0, z, a', \theta^{pr})$$

is greater than or equal to c_e.

Recursive Competitive Equilibrium For given initial distribution of Γ_0^{pr} and Γ_0^{pb}, a recursive competitive equilibrium consists of (i) value functions V^{pr}, V^{stay}, V^{ipo}, V^{pb}, $\tilde{V}^{pb}$, and $V^{x,j}$; (ii) policy functions $a^{j\prime}(a, s, z)$, $k^{j\prime}(a, s, z)$, $b^{j\prime}(a, s, z)$, $l^j(a, s, z)$ where $j \in \{pr, pb\}$, and IPO decision $\xi(a, s, z)$; and (iii) the measure of exiters $\{\Gamma_t^{x,j}\}_{t=1}^{\infty}$, incumbents $\{\Gamma_t^j\}_{t=1}^{\infty}$ where $j \in \{pr, pb\}$, and entrants $\{\Gamma_t^e\}_{t=1}^{\infty}$ such that, for all $t \geq 0$:

(i) V^{pb}, $\tilde{V}^{pb}$, $V^{pb,x}$, $a^{pb\prime}(a, s, z)$, $k^{pb\prime}(a, s, z)$, $b^{pb\prime}(a, s, z)$, $l^{pb}(a, s, z)$ solve the public firm's problem described in (16)-(18).

(ii) V^{pr}, V^{stay}, V^{ipo}, $V^{pr,x}$, $a^{pr\prime}(a, s, z)$, $k^{pr\prime}(a, s, z)$, $b^{pr\prime}(a, s, z)$, $l^{pr}(a, s, z)$, $\xi(a, s, z)$ solve the private firm's problem described in (20) - (23).

(iii) $V^e \geq c_e$ described in (24) solves the entrants' problem.

(iv) Measures evolve according to the law of motion:

$$\Gamma^{pb\prime} = \Gamma^{pb} - \Gamma^{pb,x} + \xi\Gamma^{pr},$$
$$\Gamma^{pr\prime} = (1 - \xi)(\Gamma^{pr} - \Gamma^{pr,x}) + \Gamma^e.$$

4 Quantitative Analysis

4.1 Calibration

The model is calibrated based on U.S. non-financial firm data from 2000Q1 to 2019Q4.[9] One period is equivalent to a quarter, and each production unit corresponds to a firm in the dataset.

Table 2 details the assigned parameter values. The returns to scale parameter η is set to 0.88, consistent with the range between 0.83 and 0.91 suggested by Lee (2005),

[9]The data from post-2000 is chosen due to the structural break in IPO dynamics observed in the late 1990s, illustrated in Figure 1.

Table 2: Assigned parameters

Parameter	Meaning	Value
η	Returns to scale	0.88
α	Capital share	0.3
δ	Depreciation rate	0.025
ρ_z	Persistence aggregate shock	0.95
σ_z	SD aggregate shock	0.007
χ	Equity share sold at IPO	0.1

ensuring decreasing returns to scale, a standard assumption in the firm dynamics literature. The capital share α is set at 0.3, resulting in a labor share of 0.609. The depreciation rate δ is assigned a value of 0.025, representing a quarter of the commonly used annual rate of 0.1 in the literature. Aggregate productivity shocks are assumed to evolve according to $\rho_z = 0.95$ and $\sigma_z = 0.007$, following Clementi and Palazzo (2019). The equity share sold at IPO is estimated using the dividend payout ratio from Compustat, defined as dividends per share divided by earnings per share. In the data, this ratio ranges from 0.05 to 0.15 and increases with firms' net worth, though it does not exhibit significant variation with the timing of the IPO. Therefore, the average value 0.1 is used to calibrate χ.

Table 3: Externally calibrated parameters

	Meaning	Value	Target	Data	Model
β	Time discount	0.965	Mean interest rate	0.004	0.004
ϕ_0	Stochastic discount factor	28.587	SD interest rate	0.009	0.008
ϕ_1	—"—	-30.903	Sharpe ratio	0.22	0.22

Table 3 presents the externally calibrated parameter values. The three parameters governing the stochastic discount factor are jointly determined to match the first and second moments of the risk-free rate and the mean Sharpe ratio of public firms. Specifically, the estimated mean Sharpe ratio is used to match its upper bound expressed as a function of the stochastic discount factor:

$$\overline{SR} = \frac{\sigma(M_{t+1})}{E_t[M_{t+1}]} = \sqrt{\exp(\phi_1^2 \sigma_z^2) - 1}.$$

Thus, ϕ_1 is set to match the target Sharpe ratio, while β and ϕ_0 are calibrated to align with the remaining moments of the risk-free rate, following Clementi and Palazzo

(2019). Due to nonlinearities in the mapping between parameters and moments, two distinct sets of values satisfy the targets: one produces a countercyclical risk-free rate, and the other a procyclical rate. I adopt the countercyclical specification, matching the target values estimated by Clementi and Palazzo (2019) using Compustat data.

Table 4: Targeted moments

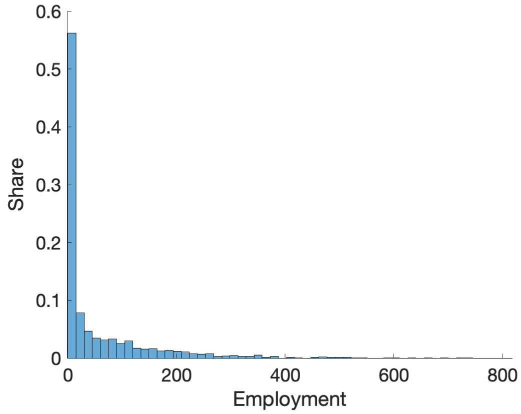
	Meaning	**Value**	**Target**	**Data**	**Model**
ρ_s	Persistence idio. shock	0.934	AC of log sales	0.628	0.41
σ_s	SD idiosyncratic shock	0.07	SD of log sales	0.352	0.35
θ^{pb}	Borrowing constraint	0.55	Debt-to-asset	0.774	0.77
θ^{pr}	—"—	0.35	Debt-to-asset	0.42	0.40
f^{pb}	Operating cost	5.2	Exit rate	0.02	0.03
f^{pr}	—"—	1.5	Exit rate	0.09	0.08
κ	IPO fixed cost	170	Emp. share of public	0.33	0.35

Notes. The first two parameters related to idiosyncratic productivity are calibrated using the simulated method of moments. The remaining parameters, which determine the costs and benefits of IPOs, are calibrated to align with observed moments for public and private firms, utilizing data from Compustat and aggregate sources such as Flow of Funds and BDS.

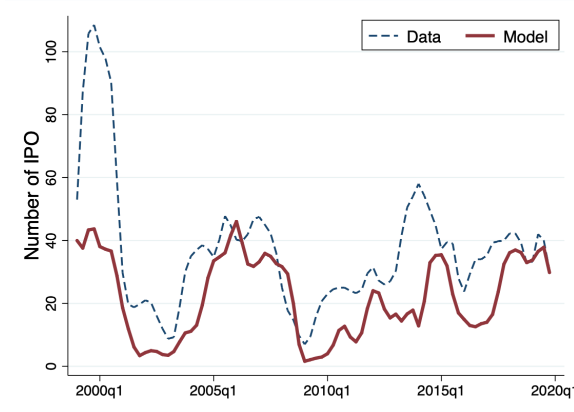
Table 4 presents the calibration of internally calibrated parameters. The idiosyncratic shock process is calibrated to match the autocorrelation and standard deviation of public firms' log output, estimated using the simulated method of moments to align with Compustat's quarterly sales data. To estimate the targets from Compustat, I restrict the sample to firms with at least 20 quarters of log sales data from the pooled sample. First-order autocorrelation of log sales is calculated to assess persistence, and cross-sectional variance is computed to capture volatility. Year-fixed effects are included to account for macroeconomic shocks, ensuring the estimates reflect firm-specific dynamics rather than time-specific disruptions. The same procedure is applied to simulated data from the stationary model to estimate the corresponding moments.

The remaining parameters govern IPO dynamics and the heterogeneity between private and public firms. The borrowing constraint parameter, θ, is calibrated by matching the debt-to-asset ratio, using Compustat data for public firms and aggregate data from the Flow of Funds for private firms. The borrowing constraint impacts IPO dynamics through two channels. First, the difference in collateral constraints between public and private firms affects the incentive to go public—greater disparity leads to larger benefits from an IPO, as public firms enjoy more relaxed borrowing constraints. Second, the level of borrowing constraint for private firms influences their

Figure 5: Size distribution and simulated number of IPOs



(a) Size distribution



(b) Number of IPOs

Notes. The figures illustrate two key features of the model. Panel (a) presents the distribution of firm sizes by employment, with the vertical axis representing the share of firms and the horizontal axis indicating the number of employees per firm. The numerical comparison with data is presented in the Appendix. Panel (b) compares the actual number of IPOs from Compustat/CRSP (blue dashed line) with the simulated IPOs generated by the model (red solid line), using detrended TFP as the input for aggregate productivity in each period.

growth trajectory. Tighter constraints slow their accumulation of capital, delaying the point at which they might benefit from an IPO. In contrast, firms with more relaxed constraints can grow more rapidly, reaching the IPO threshold sooner, which accelerates the overall transition from private to public status in the model. Operating costs are calibrated to match the exit rates of public firms using Compustat data, and private firms using BDS data. Despite the model's assumption of higher operating costs for public firms, the exit rate of public firms is only one-quarter that of private firms. This discrepancy arises from the tendency of public firms to be larger and more productive, making them less likely to exit. Nonetheless, the calibration confirms that public firms have higher operating costs than private firms, consistent with the model's assumptions. All of the parameters discussed thus far influence firms' IPO decisions. Finally, the IPO fixed cost κ is calibrated to pin down the share of public firms in the economy.

4.2 Untargeted Moment

The calibrated model replciates several patterns observed in the data. Panel (a) of Figure 5 illustrates the firm size distribution in terms of employment in the stationary

Table 5: Untargeted moments

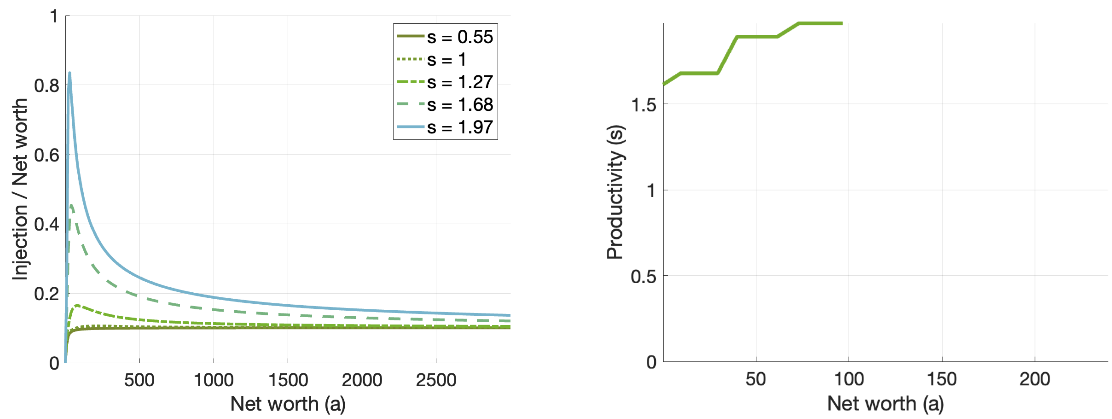
Moment	Model	Data	Source
Growth at IPO	0.82	0.87	Borisov et al. (2021)
IPO effect	0.42	0.37	Borisov et al. (2021)

Notes. The table presents untargeted moments that capture the impact of IPOs on firm-level employment. 'Growth at IPO' encompasses both the selection effect and the direct effect of the IPO. 'IPO effect' isolates the direct impact of going public, indicating that approximately half of the employment growth in the IPO year is directly attributable to the IPO.

case of the model, with a distinct right-skewed pattern. A significant proportion of firms are small, with over half employing fewer than 10 workers. As the number of employees increases, the share of firms decreases sharply. Beyond 200 employees, very few firms exist, highlighting the concentration of larger firms in a small portion of the distribution. This reflects the typical firm size distribution, where a small number of large firms dominate total employment, while the majority of firms remain relatively small. Panel (b) compares the number of IPOs from the data (blue dashed line) with the simulated number of IPOs generated by the model (red solid line). I simulated the model by inputting the detrended Total Factor Productivity (TFP) into the aggregate productivity process over the given period and recorded the number of firms going public each quarter. The model captures the procyclical pattern of IPOs, showing that the number of IPOs tends to increase during economic expansions and decline during recessions. While the model does not perfectly match the magnitude and timing of every peak and trough, it successfully replicates the overall trend, which is consistent with the procyclicality of IPOs observed in Figure 1.

Table 5 presents the model's untargeted moments. First, according to Clementi and Palazzo (2019), the average quarterly investment rate of public firms is 3.5 percent. In contrast, the model predicts an average investment rate of 11 percent for public firms, significantly higher than the value reported in the literature. This suggests that the model overestimates firms' investment behavior, likely due to the absence of adjustment costs in the model specification as pointed out by Cooper and Haltiwanger (2006). Second, to assess the model's performance in replicating IPO dynamics, I examine whether it reproduces the empirical evidence from Borisov et al. (2021). According to Borisov et al. (2021), IPO firms are typically fast-growing companies, resulting in two effects: a selection effect from firms with already high employment growth and an IPO effect from the capital raised through going public. To disentangle these two

Figure 6: Capital injection and IPO decision



(a) Capital injection

(b) IPO Threshold

Notes. The figure illustrates the the amount of capital injection and IPO threshold in a stationary case. Panel (a) displays capital injection normalized by net worth across idiosyncratic productivity (s). Panel (b) shows the IPO threshold in a stationary case ($z = 1$), identifying the region where firms, based on idiosyncratic productivity and net worth, choose to go public when these values exceed the threshold. The x-axis is truncated at a net worth of 250, out of a maximum of 3000.

effects, Borisov et al. (2021) compare the employment growth of firms that proceeded with their IPOs to those that withdrew. To replicate this in the model, I simulate a stationary case and compare the employment growth trajectories of firms that choose to go public with those of firms with similar characteristics that remain private. By comparing the employment growth of firms that undergo an IPO with firms of similar size and productivity that do not go public, I estimate the selection effect and the employment growth effect directly attributable to the IPO decision. As a result, the model successfully replicates both the selection effect and the IPO effect observed in the data.

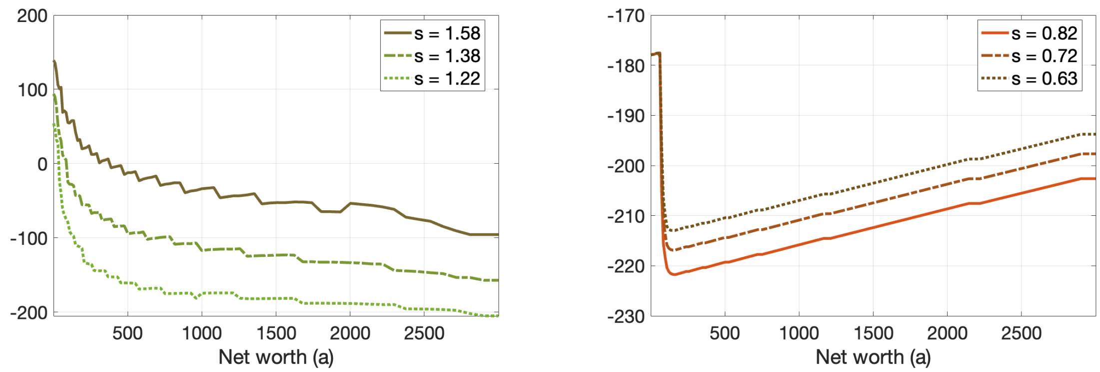
4.3 IPO Decision

Panel (a) of Figure 6 depicts the relative capital injection at IPO across productivity levels, normalized by net worth. While absolute capital injection increases with firm size, the figure emphasizes the impact of productivity on capital injection, particularly for smaller firms. Among small firms, capital injection varies significantly with productivity, whereas for larger firms, these differences become negligible. Additionally, there is an asymmetric pattern: unproductive firms (with productivity below 1) receive similar levels of capital, while productive firms (with productivity above 1) secure substantially larger injections with the increase in productivity. This asymmetry suggests that financially constrained firms, small and productive firms, can raise disproportionately more capital through IPOs. The expected future value of these firms rises significantly post-IPO, as they gain better access to external finance and face fewer borrowing constraints, prompting investors to anticipate higher growth and offer larger capital injections.

Panel (b) of Figure 6 presents the IPO decision threshold in the stationary case, based on logged net worth and idiosyncratic productivity. Firms above the threshold choose to go public, while those below remain private. The figure highlights that only productive firms opt for an IPO, as the associated costs and higher post-IPO operating expenses make it less advantageous for unproductive firms. As shown in Panel (a), these firms receive smaller capital injections, increasing their net worth by only around 10 percent. Moreover, because unproductive firms anticipate less positive productivity shocks and are less likely to face binding borrowing constraints, their incentive to raise external capital through an IPO is weaker. This also explains the upward-sloping threshold with net worth—larger firms are less likely to go public. Since collateral constraints are asset-based, firms with greater net worth are less likely to face binding borrowing limits, reducing their need to raise capital from the costly public stock market.

To better understand the IPO decision, Figure 7 illustrates the value differential between going public and remaining private. The two panels display the value differential for high-productivity firms (left) and low-productivity firms (right). Consistent with earlier observations, only small, high-productivity firms exhibit a positive value differential, indicating that they are the ones more likely to choose going public. In the left panel, for high-productivity firms, the value differential gradually decreases with firm size, suggesting that as firms grow larger and are less likely to face binding collateral constraints, the benefits of going public diminish. In contrast, the right panel shows that low-productivity firms exhibit a distinct "swoosh" pattern in their value differential. Initially, very small firms show a relatively higher value differential, as they are more financially constrained and, like in the earlier case, stand to gain significantly from capital injection and the relaxation of borrowing constraints. However, as firm size increases and these constraints are alleviated, the value differential drops sharply. Once net worth surpasses approximately 73, the value differential begins to

Figure 7: Private firms' value differential



(a) Productive firms

(b) Unproductive firms

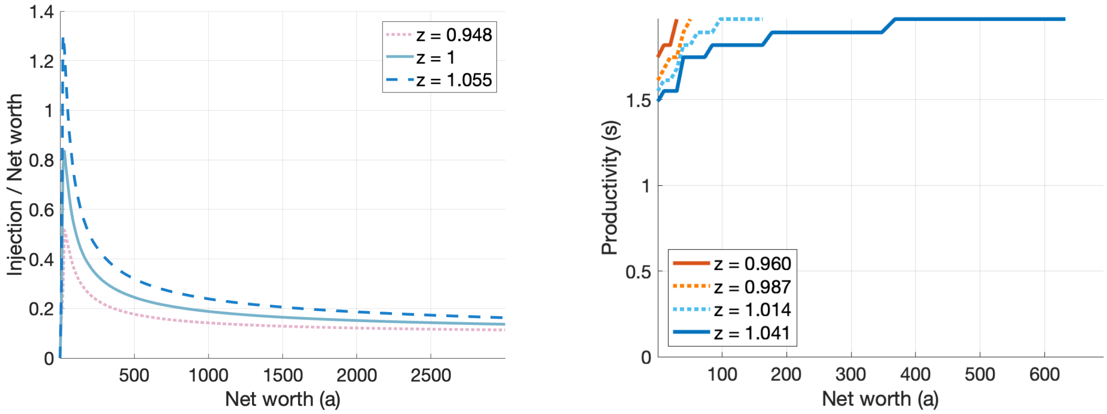
textbfNotes. The figure shows the value differential between firms going public versus staying private, where a positive differential indicates a higher value from going public. The left panel displays the value differential for productive firms ($s = 1.22$ to $s = 1.58$), while the right panel illustrates it for less productive firms ($s = 0.63$ to $s = 0.82$).

rise again. This pattern can be attributed to changes in exit risk and the firm's ability to bear the costs of going public. As low-productivity firms grow, they become more financially stable and less vulnerable to exit. Larger low-productivity firms accumulate enough internal resources to absorb the costs associated with going public, and the capital injection from an IPO can help offset the negative impact of low productivity. Additionally, the improved likelihood of survival post-IPO makes going public more attractive, leading to an increase in the value differential as firm size grows. Thus, if the fixed IPO cost were sufficiently low, even low-productivity firms with large net worth would find the benefits of raising capital from public stock markets more appealing. However, under the current calibration, the relatively high fixed IPO costs prevent these firms from pursuing an IPO.

Cyclicality of IPOs. The IPO decision follows a procyclical pattern. Panel (a) of Figure 8 illustrates how the potential capital injection for the most productive firm, normalized by net worth, changes in the nonstationary case. As shown in the figure, capital injections shift upward as aggregate productivity increases, indicating that firms receive larger capital injections. However, the increase is asymmetric: small firms can see an increase of up to 80 percentage points depending on the aggregate state, while for larger firms, the change is negligible.

Panel (b) demonstrates how the IPO threshold changes with aggregate states in the

Figure 8: Capital injection and IPO decision in a non-stationary case

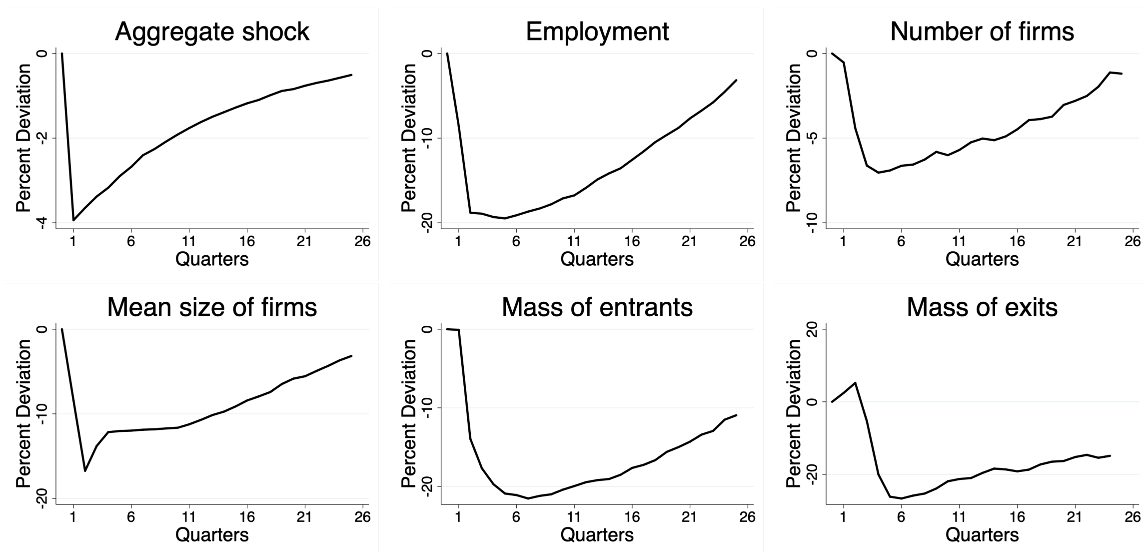


(a) Capital injection (b) IPO Threshold

Notes. The figure illustrates the IPO threshold and capital injection in a non-stationary case. Panel (a) shows the normalized potential capital injection at IPO, based on net worth for the most productive firm ($s = 1.97$) across varying aggregate states. In Panel (b), the x-axis is truncated at a net worth of 700, out of a maximum of 3000.

nonstationary case. As aggregate productivity shocks worsen, the IPO threshold shifts upward and to the left, leading to fewer IPOs. The firms that do go public tend to be smaller but more productive. This pattern is driven by the fact that smaller and more productive firms are more likely to face financial constraints, making the benefits of an IPO more pronounced for them. This is consistent with the selection effect observed in the data, where IPO firms tend to be smaller during contraction periods. However, the model suggests that post-IPO employment growth heterogeneity is not solely due to unobservable productivity. If the heterogeneity were purely driven by selection effects, post-IPO employment growth for contraction cohort firms should be higher, but this is not observed. The procyclical nature of the stochastic discount factor drives these dynamics. First, during recessions, countercyclical interest rates rise, reducing capital demand and diminishing firms' incentives to go public. Second, equity prices become more heavily discounted, which reduces the capital injection firms receive at IPO as observed in Panel (a). Consequently, firms may choose to delay their IPO to secure a larger capital injection in a more favorable economic environment. This creates a trade-off: firms must balance the immediate benefits of accessing capital and easing borrowing constraints against the potential advantage of waiting for a larger capital injection later. In practice, however, more financially constrained firms face higher

Figure 9: Response of firm dynamics to a negative productivity shock – Baseline



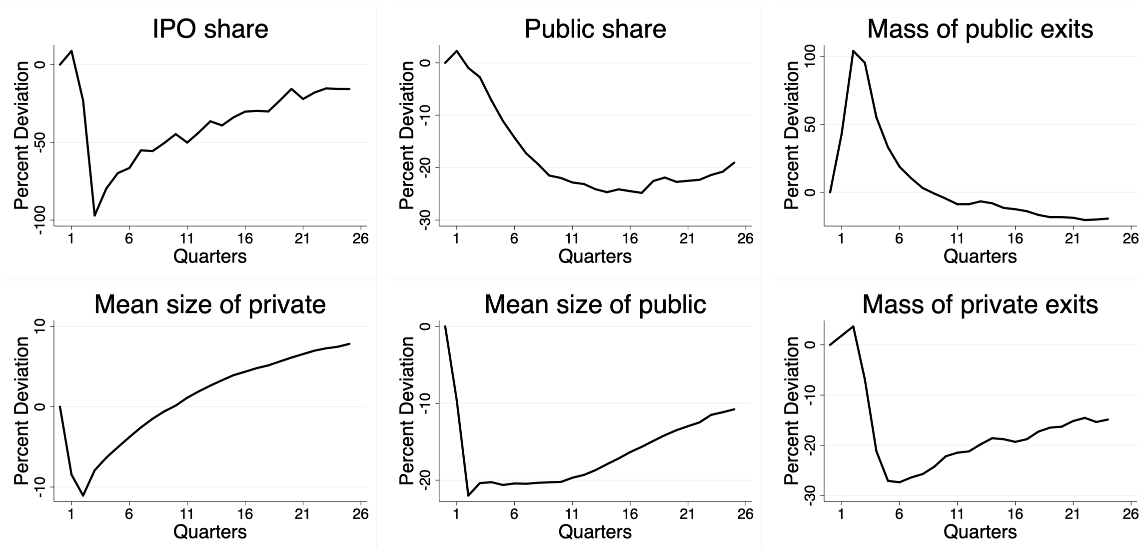
costs from delaying their IPO and are thus more likely to proceed with going public even in less favorable economic conditions.

4.4 Impulse Responses

To quantify the impact of the cyclical nature of IPOs on the propagation of aggregate shocks to employment, I compare the impulse response functions of the baseline model with those of counterfactual cases where the cyclicality is shut down. First, I examine the impulse response functions following a negative shock to the aggregate shock in the baseline model. For the system initialized at the steady state, I introduce a 4 percent drop in aggregate productivity z at $t = 1$ and track the evolution of private and public firm dynamics over the subsequent 26 quarters. This experiment is repeated 1,000 times, and the averages of selected variables are depicted in Figure 9 and 10.

Figure 9 shows the impulse response from the perspective of firm dynamics. A 4 percent drop in total factor productivity leads to a 20 percent decline in aggregate employment, which begins a slow recovery starting in the 6th quarter. This decline is driven by both extensive and intensive margins. First, the number of firms decreases by approximately 7 percent (extensive margin), primarily due to a sharp drop in the mass of entrants immediately following the aggregate shock. Over time, the number of firms gradually recovers as the mass of entrants increases and the mass of exits decreases. Second, a 16 percent reduction in average firm size (intensive margin) immediately

Figure 10: Response of IPO dynamics to a negative productivity shock – Baseline

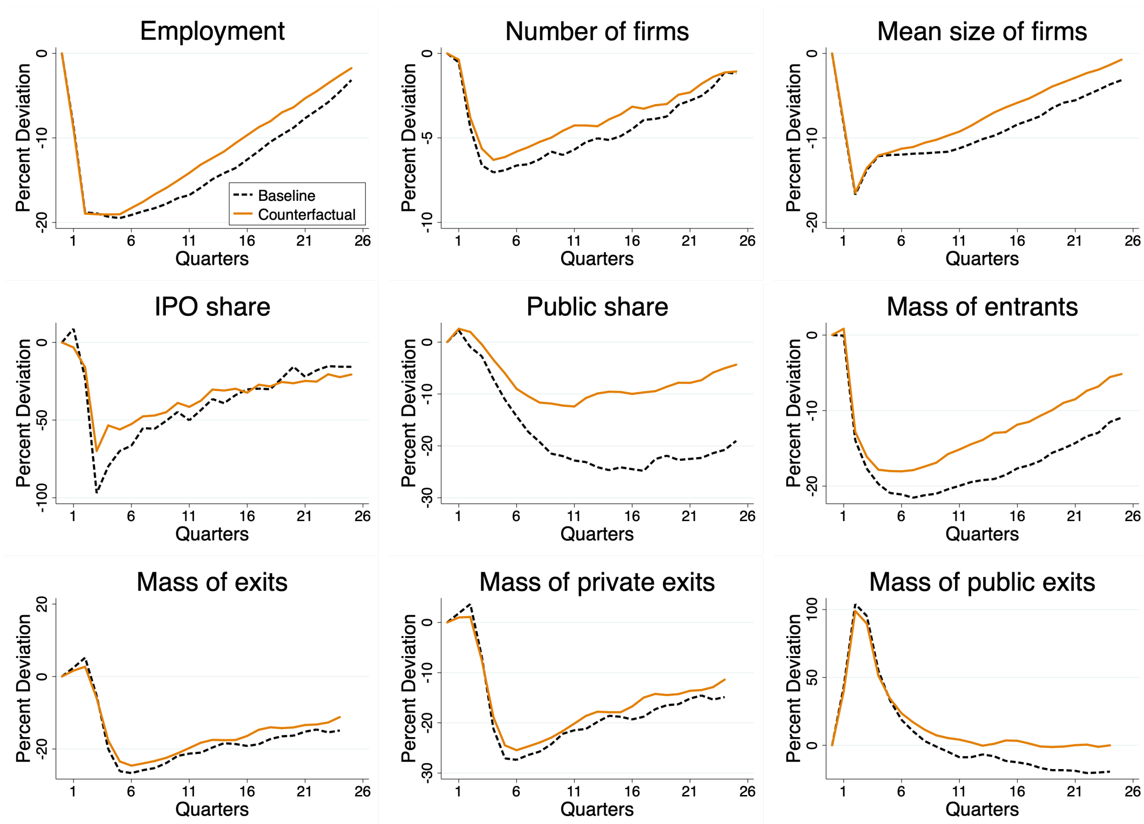


following the shock further contributes to the decline in aggregate employment. To fully understand this, I will explain the dynamics of IPOs in the following section.

Figure 10 presents the impulse response from the perspective of IPO dynamics, the transition from private to public. As examined earlier, a negative shock to aggregate productivity causes the IPO share to drop sharply, by as much as 100 percent. This leads to a decline in the entry margin of public firms, resulting in a lower share of public firms. This decline is further accelerated by a spike in the mass of public exits during the first five quarters. The share of public firms plays a critical role in determining the mean size of firms. In this model, public firms are more efficient due to their relaxed collateral constraints. Therefore, the overall efficiency of the economy is largely influenced by the share of public firms.

When decomposing the response of average firm size into private and public firms, an intriguing pattern emerges. The average size of private firms initially declines by up to 10 percent but quickly rebounds, surpassing its pre-shock steady state within 10 quarters. In contrast, the average size of public firms is more severely impacted, dropping by 20 percent and remaining at this lower level for 11 quarters before slowly recovering. Even after 25 quarters, the average size of public firms remains 10 percent below its steady state. This divergence reveals that while both private and public firms contract in response to adverse economic conditions, public firms experience a more substantial and persistent impact. The key driver of this pattern lies in the

Figure 11: Response to a negative productivity shock – Counterfactual I



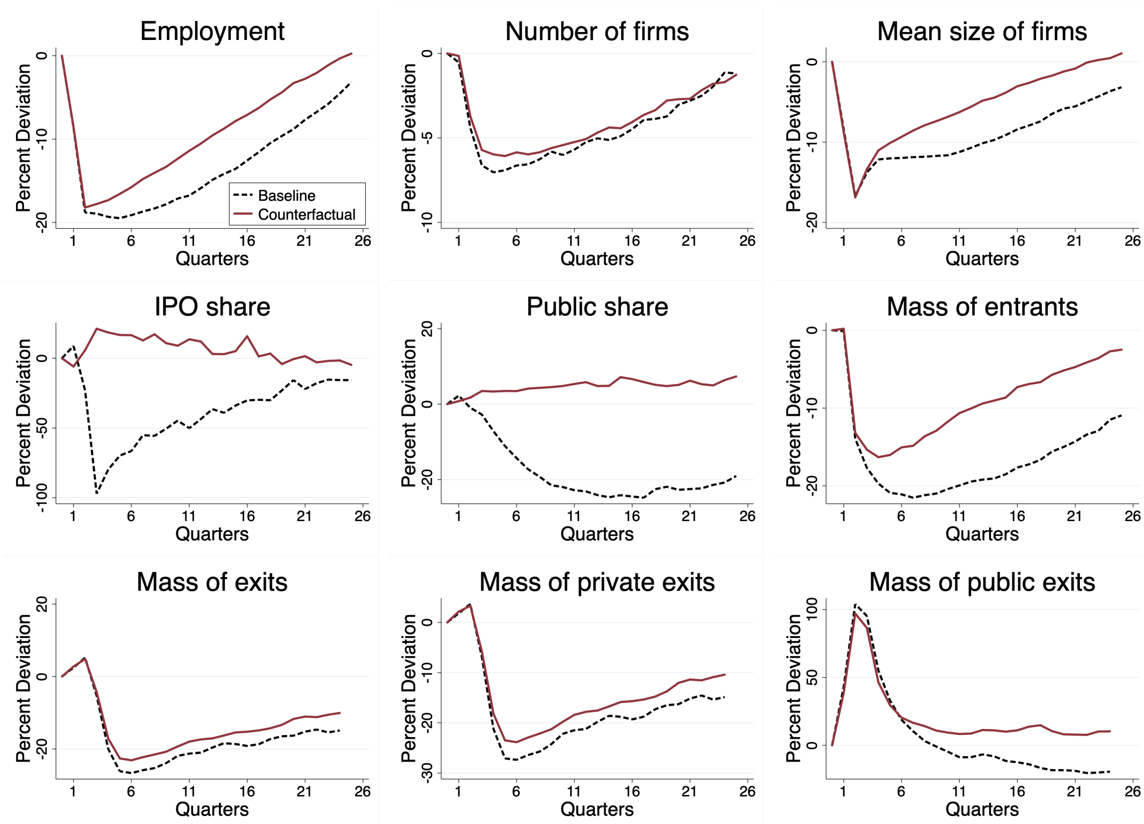
Notes. The black dashed line represents the response of the baseline model. The yellow solid line illustrates the response in the counterfactual scenario, in which IPO cyclicality related to the capital injection is shut down.

shifting IPO threshold, which moves upward and leftward, reducing both the number of firms transitioning to public status and their average size. As a result, larger firms are more likely to remain private, leading to a quicker recovery in the average size of private firms. Conversely, public firms see a slower and incomplete recovery, as fewer large firms transition into public status, leaving the average size of public firms at a persistently lower level. For the same reason, the mass of private exits decreases rather than increases.

The first counterfactual scenario compares against the baseline model by shutting down the procyclicality of capital injection. In this case, I assume that capital injection remains fixed at its average value, irrespective of aggregate productivity. The results are depicted in Figure 11, where the yellow solid line represents the response in the counterfactual scenario, and the black dashed line shows the baseline. In this counterfactual scenario, employment declines by the same magnitude as in the baseline but recovers more quickly by 2 percentage points. This faster recovery is driven by both extensive and intensive margins. The number of firms decreases by approximately 1 percentage point less, primarily because the drop in the mass of new entrants is smaller by around 8-10 percentage points compared to the baseline. This occurs because entrants' decisions to enter the market are influenced by the value function of private firms. Since entrants always enter the economy as private firms, their entry value is shaped by the likelihood of an IPO in the near future. In this scenario, the higher IPO share following the negative shock—driven by the constant capital injection—raises the value of private firms, thereby increasing the value of entry. Meanwhile, the average firm size, which initially decreases similarly to the baseline, begins to recover more rapidly starting from the 6th quarter. This is due to the higher proportion of public firms, which helps reduce capital misallocation in the economy. The public firm share decreases by a maximum of 12 percent, which is less than half the decline seen in the baseline. The constancy of capital injection reduces the volatility of the IPO share, leading to more firms transitioning to public status after a negative aggregate productivity shock.

The second counterfactual case examines the scenario in which the cyclicality of the number of IPOs is removed. In this scenario, I assign a stationary IPO threshold that remains exogenous across different aggregate states. Figure 12 presents the results. Similar to the baseline, employment initially drops by the same magnitude but recovers faster, avoiding the 2-3 quarter lag observed in the baseline case, leading to a recovery that is 5 percentage points faster. Interestingly, in this case, the extensive margin plays only a marginal role. The mechanism behind the faster recovery in the mean size of firms is similar to the previous scenario. The exogenous IPO threshold allows more firms to go public right after the negative shock, preventing the sharp decline in the IPO share. Since the IPO threshold remains constant, deviations in the IPO share from zero are minimal, resulting in a relatively stable public firm share. This stability enables the mean size of firms to recover much faster than in the baseline scenario. What stands out in this counterfactual case, unlike the previous one, is that the number of firms develops quite similarly to the baseline. This is because, unlike the earlier scenario, the exogenous IPO threshold increases the exit rate for both private and public firms, and this is offset by the increase in new entrants. The reason for the increased exit of public firms is that the exogenous IPO threshold forces relatively

Figure 12: Response to a negative productivity shock – Counterfactual II



Notes. The black dashed line represents the response of the baseline model. The red solid line illustrates the response in the counterfactual scenario, in which IPO cyclicality related to selection and the number of IPOs is shut down.

unproductive firms to go public, and these firms, unable to sustain the higher operating costs of being public, eventually exit the market. Similarly, private firm exits decrease by around 3-5 percentage points less in this counterfactual than in the baseline.

In sum, the two counterfactual exercises demonstrate that the cyclicality of IPOs amplifies the impact of a negative shock on aggregate employment, resulting in a slower recovery by 4-6 quarters. This amplification operates through two channels. First, the smaller number of IPOs following a negative productivity shock significantly reduces the public firm share, which exacerbates capital misallocation within the economy. Second, the lower propensity for firms to go public diminishes the value of private firms, which in turn discourages potential entrants from entering the market, further delaying the recovery. To quantify each margin and its impact, Table 6 presents the quantitative responses of key variables in the baseline and counterfactual scenarios three quarters

Table 6: Quantitative responses: Baseline vs. Counterfactual

	Baseline	**Counterfactual**	**Dev. (%)**
Employment	-6.98	-5.77	17.31
Mean size of firms	-3.55	-2.93	17.49
Public firm share	-0.67	-0.21	69.15
Number of firms	-2.23	-1.22	45.12
Entry rate	-10.14	-9.09	10.39

Notes. The table presents the quantitative responses to a -1 percent aggregate productivity shock in the third quarter, comparing the baseline scenario with the counterfactual scenario.

after a -1 percent aggregate productivity shock. In the baseline scenario, employment decreases by 6.98 percent, compared to a 5.77 percent decrease in the counterfactual, a difference of 17.31 percent. This smaller decline in employment is partly driven by a 17.49 percent smaller reduction in mean firm size and a 45.12 percent smaller decline in the number of firms. These changes are respectively associated with a 69.15 percent smaller reduction in the public firm share and a 10.39 percent smaller reduction in the entry rate, further supporting the consistency of the amplification mechanism.

5 Conclusion

This paper analyzes the impact of Initial Public Offerings (IPOs) on employment over the business cycle, building on the well-documented fact that IPO activity is procyclical. Empirical evidence not only confirms that the number of IPOs rises during economic expansions and falls during contractions, but also shows that post-IPO firm growth is strongly tied to the state of the economy at the time of the IPO. Specifically, firms that go public during economic downturns raise 37 percent less capital compared to those that do so in expansion periods, which has long-term implications for their growth. Contraction-period IPO firms grow approximately 17 percentage points slower in employment over the next decade.

To investigate the broader implications of this pattern, I develop a firm dynamics model where private firms make decisions about going public, and both private and public firms decide on exit, investment, and employment under borrowing constraints. The model incorporates a procyclical stochastic discount factor, replicating the procyclical nature of IPOs, the amount of capital raised at IPOs, and the selection patterns documented in the data. Through the lens of this model, I find that the cyclicality of IPO activity amplifies the impact of negative aggregate productivity shocks on employment results in a slower recovery by 4-6 quarters. This amplification arises through two primary channels. First, a reduced share of public firms in the economy during downturns exacerbates capital misallocation, as public firms tend to operate more efficiently due to relaxed borrowing constraints. Second, the lower propensity for firms to go public in recessions diminishes the value of private firms, reducing the incentive for new entrants, which slows recovery on the extensive margin.

The findings carry important policy implications. Encouraging more firms to go public during or immediately after recessions could help stabilize the labor market and accelerate recovery. Policies that reduce the fixed costs of IPOs (similar to JOBS act) or provide incentives for public listing in downturns could counteract the procyclicality of IPO activity, helping maintain a higher share of public firms, improving capital allocation efficiency, and encouraging more business entries. By promoting public listings during adverse economic conditions, policymakers could mitigate the employment gap that results from lower IPO activity and ensure a faster recovery from recessions.

References

Alti, A. (2005). IPO Market Timing. *Review of Financial Studies 18*(3), 1105–1138.

Angelini, E. and M. Foglia (2018). The relationship between ipo and macroeconomics factors: an empirical analysis from uk market. *Annals of Economics and Finance 19*, 319–336.

Benninga, S., M. Helmantel, and O. Sarig (2005). The timing of initial public offerings. *Journal of Financial Economics 75*(1), 115–132.

Borisov, A., A. Ellul, and M. Sevilir (2021). Access to public capital markets and employment growth. *Journal of Financial Economics 141*(3), 896–918.

Breuer, M. and P. Breuer (2023). Uneven regulation and economic reallocation: Evidence from transparency regulation. *SSRN Electronic Journal*.

Casella, S., H. Lee, and S. Villalvazo (2022). Rising intangible capital and the disappearance of public firms.

Chemmanur, T. J. (2010). Venture capital, private equity, IPOs, and banking: An introduction and agenda for future research. *Journal of Economics and Business 62*(6), 471–476.

Chemmanur, T. J. and J. He (2012). IPO waves, product market competition, and the going public decision: Theory and evidence. *SSRN Electronic Journal*.

Chemmanur, T. J., S. He, and D. K. Nandy (2010). The going-public decision and the product market. *The Review of Financial Studies 23*(5), 1855–1908.

Clementi, G. L. and B. Palazzo (2016). Entry, exit, firm dynamics, and aggregate fluctuations. *American Economic Journal: Macroeconomics 8*(3), 1–41.

Clementi, G. L. and B. Palazzo (2019). Investment and the cross-section of equity returns. *The Journal of Finance 74*(1), 281–321.

Cooper, R. W. and J. C. Haltiwanger (2006, 07). On the Nature of Capital Adjustment Costs. *The Review of Economic Studies 73*(3), 611–633.

Davydiuk, T., B. Glover, and R. Szymanski (2020). The decline in public firms.

Di-Nola, A. (2016). Capital misallocation during the great recession. *SSRN Electronic Journal*.

González, B. (2020). Macroeconomics, firm dynamics and IPOs. Working Papers 2030, Banco de España.

Helwege, J. and N. Liang (2004). Initial public offerings in hot and cold markets. *The Journal of Financial and Quantitative Analysis 39*(3), 541–569.

Hopenhayn, H. A. (1992, September). Entry, exit, and firm dynamics in long run equilibrium. *Econometrica 60*(5), 1127.

Jermann, U. and V. Quadrini (2012). Macroeconomic effects of financial shocks. *American Economic Review 102*(1), 238–71.

Jovanovic, B. and P. L. Rousseau (2004). Interest rates and initial public offerings. Working Paper 10298, National Bureau of Economic Research.

Khan, A. and J. K. Thomas (2013, December). Credit shocks and aggregate fluctuations in an economy with production heterogeneity. *Journal of Political Economy 121*(6), 1055–1107.

Kiyotaki, N. and J. Moore (1997). Credit cycles. *Journal of Political Economy 105*(2), 211–248.

Lee, Y. (2005). The importance of reallocations in cyclical productivity and returns to scale: evidence from plant-level data. Technical report.

Lee, Y. and T. Mukoyama (2015, July). Entry and exit of manufacturing plants over the business cycle. *European Economic Review 77*, 20–27.

Midrigan, V. and D. Y. Xu (2014). Finance and misallocation: Evidence from plant-level data. *American Economic Review 104*(2), 422–58.

Moreira, S. (2016). Firm dynamics, persistent effects of entry conditions, and business cycles. *SSRN Electronic Journal*.

Pástor, L. and P. Veronesi (2005). Rational ipo waves. *The Journal of Finance 60*(4), 1713–1757.

Schenone, C. (2010). Lending relationships and information rents: Do banks exploit their information advantages? *The Review of Financial Studies 23*(3), 1149–1199.

Sedláček, P. and V. Sterk (2017). The growth potential of startups over the business cycle. *American Economic Review 107*(10), 3182–3210.

Shiller, R. J. (1981, June). Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends? *American Economic Review 71*(3), 421–436.

Timmermann, A. G. (1993). How learning in financial markets generates excess volatility and predictability in stock prices. *The Quarterly Journal of Economics 108*(4), 1135–1145.

Tran, A. L. and B. N. Jeon (2011). The dynamic impact of macroeconomic factors on initial public offerings: evidence from time-series analysis. *Applied Economics 43*(23), 3187–3201.

Appendix

A Empirical Evidence

A.1 Cyclical Number of IPO

Table 7: Correlation Matrix of Number of IPO and Business Cycle Measures

	Number of IPOs	**PE Ratio**	**GDP**
***Panel A**. All (1980Q1-2021Q4)*			
Number of IPOs	1.00		
PE Ratio (detrended)	0.27***	1.00	
GDP (logged, detrended)	0.04	0.50***	1.00
***Panel B**. Post-2000Q1*			
Number of IPOs	1.00		
PE Ratio (detrended)	0.66***	1.00	
GDP (logged, detrended)	0.23**	0.63***	1.00

Figure 1 reveals fluctuations in other periods, which are likely to be related to the stock market performance (Tran and Jeon, 2011). Panel A of Table 7 provides a correlation analysis between IPO numbers and the cyclical components of output and stock market performance, represented by detrended log real GDP and the detrended Shiller price-to-earnings ratio. The results show a stronger correlation with the stock market's cyclical component (0.27) than with GDP (0.04), highlighting the significant impact of stock market conditions on IPO activities beyond recessionary periods. However, a structural shift around 2000 marked by a significant drop in IPO frequency and its variance alters this pattern. Post-2000 correlation analysis in Panel B reveals increased correlations between both stock market performance and GDP with the number of IPOs, with a notable rise in the correlation with GDP. This shift indicates not only a reduction in the number of IPOs but also an intensified cyclical pattern in the IPO market since 2000.

A.2 Descriptive Statistics

Table 8 presents the descriptive statistics in the year of IPO for the two IPO cohorts.[10] First, during contraction periods, the employment distribution of IPO firms exhibits greater variance. Although the average size for the contraction cohort is larger, this is likely driven by a few firms above the 90th percentile, as the average size up to the 75th percentile is consistently smaller compared to the expansion cohort. Similar patterns are observed in other dimensions of size, such as assets and sales, as well as the age, book value per share, and sale of common and preferred stock indicating the amount of capital injection raised at IPO. This suggests that, excluding the firms at the extreme right of the distribution, the contraction cohort is generally smaller and younger and raises less amount of capital compared to the expansion cohort.

Table 9 presents the exit rates for the IPO cohorts. The exit rate is defined as the likelihood of being delisted from the Compustat/CRSP dataset within 10 years. The data indicate that both cohorts exhibit a consistent exit rate of 56 percent within this period. However, the types of exits differ between the cohorts. For the expansion cohort, mergers and acquisitions constitute the majority of exits, whereas for the contraction cohort, closures or bankruptcies are more prevalent. The incidence of firms returning to private status is the least common for both cohorts but occurs at twice the rate in the expansion cohort compared to the contraction cohort. The five-year exit rates are 35 percent for both cohorts, with a consistent composition.

[10]Considering the possibility of a time lag, I also take the average up to a year after the IPO as a robustness check, which yields consistent results.

Table 8: Descriptive statistics of IPO firms

	N	Mean	Std. Dev.	p10	p25	p50	p75	p90
A. Expansion								
Employment (1K)	9,815	1.48	7.39	0.02	0.05	0.18	0.72	2.67
Asset (1M)	10,256	30,482	151,858	94.47	413.57	2,207	11,364	48,482
Sales (1M)	10,191	21,216	110,765	0.00	95.40	1,105	6,883	32,338
Age	4,346	14.69	19.49	2.00	4.00	8.00	16.00	34.00
Book value per share	10,225	184.61	761.56	2.17	35.37	118.82	284.23	670.80
Debt-to-asset ratio	9,565	0.52	0.75	0.08	0.17	0.35	0.63	1.02
Sale of stock (1M)	10,036	4,030	8,113	0.00	89.14	622.60	3,629	12,001
Exit age	7,814	7.88	6.43	2.00	3.00	6.00	11.00	17.00
B. Contraction								
Employment (1K)	609	2.18	14.35	0.01	0.03	0.12	0.62	2.70
Asset (1M)	651	48,206	249,736	44.02	85.99	369.40	4,018	45,856
Sales (1M)	649	31,711	187,398	2.37	35.14	246.81	1,788	24,247
Age	90	16.04	22.40	3.00	5.00	10.00	16.00	27.50
Book value per share	645	218.38	537.96	1.37	10.81	47.45	166.45	687.19
Debt-to-asset ratio	637	0.55	0.72	0.09	0.20	0.41	0.67	0.95
Sale of stock (1M)	642	3,455	9,564	0.00	3.76	54.41	338.89	10,444
Exit age	521	8.53	7.51	1.00	3.00	6.00	12.00	19.00

Notes. This table presents the summary statistics in the year of IPO for firms categorized as expansionary or contractionary based on the timing of the IPO. Employment is measured in thousands. Assets, sales, and stock sales are measured in millions of U.S. dollars, while book value per share is measured in U.S. dollars. All figures are adjusted to 2012 real terms. Age information, based on the year of foundation, is sourced from FactSet and the Field-Ritter dataset of company founding dates, covering 40 percent of IPO firms after excluding those with negative ages and discrepancies of more than 20 years between the two sources. The remaining data are sourced from Compustat/CRSP.

Table 9: Exit rates by IPO cohort

	Expansion	**Contraction**
Exit rate	0.564	0.565
Merger and acquisition	0.239	0.198
Bankruptcy or unknown	0.205	0.292
Delist (Back to private)	0.083	0.044

Table 10: Non-parametric regression of post-IPO employment growth

	(1)		(2)	
	Employment Growth		**Employment Growth**	
IPO Age	Expansion	Contraction	Expansion	Contraction
0	0	0.07	0	0.08
1	0.25***	0.29***	0.33***	0.36***
2	0.36***	0.35***	0.46***	0.43***
3	0.44***	0.39***	0.55***	0.46***
4	0.53***	0.44***	0.64***	0.53***
5	0.60***	0.49***	0.71***	0.58***
6	0.70***	0.50***	0.78***	0.61***
7	0.76***	0.60***	0.84***	0.64***
8	0.82***	0.62***	0.87***	0.68***
9	0.89***	0.63***	0.91***	0.65***
10	0.95***	0.64***	0.95***	0.72***
11	1.01***	0.71***	0.98***	0.72***
Balanced panel			✓	
N	68,444		35,009	
R^2	0.148		0.165	

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

B Numerical Appendix

B.1 Interest rate

$$
\begin{aligned}
\frac{1}{R_t} &= E[M(z_t, z_{t+1})] = E[\beta \exp(\phi_0 z_t + \phi_1 z_{t+1})] \\
&= \beta E[\exp(\phi_0 z_t + \phi_1 z_{t+1})] \\
&= \beta E[\exp(\phi_0 z_t + \phi_1 \rho_z z_t + \phi_1 \epsilon_{z,t})] \\
&= \beta \exp(\phi_0 z_t + \phi_1 \rho_z z_t) E[\exp(\phi_1 \epsilon_{z,t})] \\
&= \beta \exp(\phi_0 z_t + \phi_1 \rho_z z_t) \exp\left(0 + \frac{\phi_1^2 \sigma_z^2}{2}\right) \\
&= \beta \exp\left((\phi_0 + \phi_1 \rho_z) z_t + \frac{\phi_1^2 \sigma_z^2}{2}\right) \\
R_t &= \frac{1}{\beta} \exp\left(-(\phi_0 + \phi_1 \rho_z) z_t - \frac{\phi_1^2 \sigma_z^2}{2}\right)
\end{aligned}
$$

B.2 Labor decision

$$l = \text{argmax}_l sz(k^\alpha l^{1-\alpha})^\eta - wl$$

First order condition is

$$\begin{aligned}
&(sz)\eta(k^\alpha l^{1-\alpha})^{\eta-1}(1-\alpha)k^\alpha l^{-\alpha} - w = 0\\
&(sz)(1-\alpha)\eta k^{\alpha\eta} l^{(1-\alpha)\eta-1} = w\\
&l^{(1-\alpha)\eta-1} = \frac{w}{(1-\alpha)\eta(sz)} k^{-\alpha\eta}.
\end{aligned}$$

Therefore,

$$l^* = \left(\frac{w}{(1-\alpha)\eta(sz)} k^{-\alpha\eta}\right)^{\frac{1}{(1-\alpha)\eta-1}} = \left(\frac{(1-\alpha)\eta}{w(z)}\right)^{\frac{1}{1-(1-\alpha)\eta}} (sz)^{\frac{1}{1-(1-\alpha)\eta}} k^{\frac{\alpha\eta}{1-(1-\alpha)\eta}}.$$

Substituting the optimal labor into the profit function yields

$$\begin{aligned}
\pi &= sz(k^\alpha l^{1-\alpha})^\eta - wl - (1+r)(k-a) + (1-\delta)k\\
&= szk^{\alpha\eta} l^{(1-\alpha)\eta} - wl - (r+\delta)k + (1+r)a\\
&= szk^{\alpha\eta}\left(\frac{(1-\alpha)\eta}{w}\right)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}} (sz)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}} k^{\frac{(\alpha\eta)((1-\alpha)\eta)}{1-(1-\alpha)\eta}} - w\left(\frac{(1-\alpha)\eta}{w}\right)^{\frac{1}{1-(1-\alpha)\eta}} (sz)^{\frac{1}{1-(1-\alpha)\eta}} k^{\frac{\alpha\eta-1}{1-(1-\alpha)\eta}} - (r +\\
&= \left(\frac{(1-\alpha)\eta}{w}\right)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}} (sz)^{\frac{1}{1-(1-\alpha)\eta}} k^{\frac{(\alpha\eta)((1-\alpha)\eta)}{1-(1-\alpha)\eta}+\alpha\eta} - w\left(\frac{(1-\alpha)\eta}{w}\right)^{\frac{1}{1-(1-\alpha)\eta}} (sz)^{\frac{1}{1-(1-\alpha)\eta}} k^{\frac{\alpha\eta}{1-(1-\alpha)\eta}} - (r+\delta)\\
&= \left(\frac{(1-\alpha)\eta}{w}\right)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}} (sz)^{\frac{1}{1-(1-\alpha)\eta}} k^{\frac{\alpha\eta}{1-(1-\alpha)\eta}} - w\left(\frac{(1-\alpha)\eta}{w}\right)^{\frac{1}{1-(1-\alpha)\eta}} (sz)^{\frac{1}{1-(1-\alpha)\eta}} k^{\frac{\alpha\eta}{1-(1-\alpha)\eta}} - (r+\delta)k + (\\
&= \left(((1-\alpha)\eta)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}} - ((1-\alpha)\eta)^{\frac{1}{1-(1-\alpha)\eta}}\right) w^{\frac{-(1-\alpha)\eta}{1-(1-\alpha)\eta}} (sz)^{\frac{1}{1-(1-\alpha)\eta}} k^{\frac{\alpha\eta}{1-(1-\alpha)\eta}} - (r+\delta)k + (1+r)a\\
&= (1-(1-\alpha)\eta)\,((1-\alpha)\eta)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}} w^{-\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}} (sz)^{\frac{1}{1-(1-\alpha)\eta}} k^{\frac{\alpha\eta}{1-(1-\alpha)\eta}} - (r+\delta)k + (1+r)a\\
&= (1-(1-\alpha)\eta)\left(\frac{(1-\alpha)\eta}{w}\right)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}} (sz)^{\frac{1}{1-(1-\alpha)\eta}} k^{\frac{\alpha\eta}{1-(1-\alpha)\eta}} - (r+\delta)k + (1+r)a.
\end{aligned}$$

B.3 Capital allocation

$$k' = argmax \int \int \pi' dH(s'|s)dG(z'|z)$$
$$\text{s.t.} \quad k' \leq \frac{1}{1-\theta} a'$$

The first order condition is

$$\frac{\partial}{\partial k'} \int \int (1-(1-\alpha)\eta) \left(\frac{(1-\alpha)\eta}{w(z')} \right)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}} (s'z')^{\frac{1}{1-(1-\alpha)\eta}} k'^{\frac{\alpha\eta}{1-(1-\alpha)\eta}} - (r(z')+\delta)k' + (1+r)a' dH(s'|s)dG$$
$$= \int \int \frac{\partial}{\partial k'} \left((1-(1-\alpha)\eta) \left(\frac{(1-\alpha)\eta}{w(z')} \right)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}} (s'z')^{\frac{1}{1-(1-\alpha)\eta}} k'^{\frac{\alpha\eta}{1-(1-\alpha)\eta}} - (r(z')+\delta)k' + (1+r)a' \right) dH($$
$$= \int \int \alpha\eta \left(\frac{(1-\alpha)\eta}{w(z')} \right)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}} (s'z')^{\frac{1}{1-(1-\alpha)\eta}} k'^{\frac{\eta-1}{1-(1-\alpha)\eta}} - (r(z')+\delta) dH(s'|s)dG(z'|z) = 0$$
$$\alpha\eta((1-\alpha)\eta)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}} k'^{\frac{\eta-1}{1-(1-\alpha)\eta}} \int \int \left(\frac{s'z'}{w(z')^{(1-\alpha)\eta}} \right)^{\frac{1}{1-(1-\alpha)\eta}} dH(s'|s)dG(z'|z) = \int (r(z')+\delta)dG(z'|z)$$
$$k'^{\frac{\eta-1}{1-(1-\alpha)\eta}} = \frac{\int (r(z')+\delta)dG(z'|z)}{\alpha\eta((1-\alpha)\eta)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}} \int \int \left(\frac{s'z'}{w(z')^{(1-\alpha)\eta}} \right)^{\frac{1}{1-(1-\alpha)\eta}} dH(s'|s)dG(z'|z)}$$
$$k' = \left(\frac{\alpha\eta((1-\alpha)\eta)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}} \int \int \left(\frac{s'z'}{w(z')^{(1-\alpha)\eta}} \right)^{\frac{1}{1-(1-\alpha)\eta}} dH(s'|s)dG(z'|z)}{\int (r(z')+\delta)dG(z'|z)} \right)^{\frac{1-(1-\alpha)\eta}{1-\eta}}.$$

Therefore, the first-best capital investment is

$$k' = (\alpha\eta)^{\frac{1-(1-\alpha)\eta}{1-\eta}} ((1-\alpha)\eta)^{\frac{(1-\alpha)\eta}{1-\eta}} \left(\frac{\int \int \left(\frac{s'z'}{w(z')^{(1-\alpha)\eta}} \right)^{\frac{1}{1-(1-\alpha)\eta}} dH(s'|s)dG(z'|z)}{\int (r(z')+\delta)dG(z'|z)} \right)^{\frac{1-(1-\alpha)\eta}{1-\eta}},$$

which implies the optimal capital choice given a' as

$$k'^{*} = \max\left(\frac{a'}{1-\theta}, (\alpha\eta)^{\frac{1-(1-\alpha)\eta}{1-\eta}}((1-\alpha)\eta)^{\frac{(1-\alpha)\eta}{1-\eta}}\left(\frac{\int\int\left(\frac{s'z'}{w(z')^{(1-\alpha)\eta}}\right)^{\frac{1}{1-(1-\alpha)\eta}} dH(s'|s)dG(z'|z)}{\int(r(z')+\delta)dG(z'|z)}\right)^{\frac{1-(1-\alpha)\eta}{1-\eta}}\right)$$

B.4 Price of Share

The price of share described in (12) can be reformulated as

$$\chi p(a_t, k_t, s_t, z_t) = \mathbf{E_t}\Big[M(z_t, z_{t+1}) \max\{(\chi d_{t+1} + \chi p(a_{t+1}, k_{t+1}, s_{t+1}, z_{t+1})), \chi d^x_{t+1}\}\Big],$$

where $d_{t+1} = \pi_{t+1}(a_{t+1}, k_{t+1}, s_{t+1}, z_{t+1}) - f - a_{t+2}(a_{t+1}, k_{t+1}, s_{t+1}, z_{t+1})$ and π_{t+1} is as described in (15). By cancelling out χ, it is reduced to

$$p(a_t, k_t, s_t, z_t) = \mathbf{E_t}\Big[M(z_t, z_{t+1}) \max\{(d_{t+1} + p(a_{t+1}, k_{t+1}, s_{t+1}, z_{t+1})), d^x_{t+1}\}\Big]. \tag{25}$$

In the meantime,

$$\begin{aligned}
&\mathbf{E_t}\Big[M(z_t, z_{t+1})V^{pb}(a_{t+1}, k_{t+1}, s_{t+1}, z_{t+1})\Big] \\
&= \mathbf{E_t}\Big[M(z_t, z_{t+1}) \max\{\tilde{V}^{pb}(a_{t+1}, k_{t+1}, s_{t+1}, z_{t+1}), V^x(a_{t+1}, k_{t+1}, s_{t+1}, z_{t+1})\}\Big] \\
&= \mathbf{E_t}\Big[M(z_t, z_{t+1}) \max\{(1-\chi)d_{t+1} + \mathbf{E_{t+1}}\Big[M(z_{t+1}, z_{t+2})V^{pb}(a_{t+2}, k_{t+2}, s_{t+2}, z_{t+2})\Big], (1-\chi)d^x_{t+1}\}\Big] \\
&= (1-\chi)\mathbf{E_t}\Big[M(z_t, z_{t+1}) \max\{d_{t+1} + \frac{1}{1-\chi}\mathbf{E_{t+1}}\Big[M(z_{t+1}, z_{t+2})V^{pb}(a_{t+2}, k_{t+2}, s_{t+2}, z_{t+2})\Big], d^x_{t+1}\}\Big]
\end{aligned}$$

which can be reformulated as

$$\begin{aligned}
&\frac{1}{1-\chi}\mathbf{E_t}\Big[M(z_t, z_{t+1})V^{pb}(a_{t+1}, k_{t+1}, s_{t+1}, z_{t+1})\Big] \\
&= \mathbf{E_t}\Big[M(z_t, z_{t+1}) \max\{d_{t+1} + \frac{1}{1-\chi}\mathbf{E_{t+1}}\Big[M(z_{t+1}, z_{t+2})V^{pb}(a_{t+2}, k_{t+2}, s_{t+2}, z_{t+2})\Big], d^x_{t+1}\}\Big].
\end{aligned} \tag{26}$$

The condition $p(a_t, k_t, s_t, z_t) = \frac{1}{1-\chi}\mathbf{E_t}\Big[M(z_t, z_{t+1})V^{pb}(a_{t+1}, k_{t+1}, s_{t+1}, z_{t+1})\Big]$ makes the Equation (25) and (26) equivalent. Therefore,

$$\chi p(a, s, z) = \frac{\chi}{1-\chi}\mathbf{E_t}\Big[M(z, z')V^{pb}(a', s', z')\Big].$$

C Quantitative Analysis

C.1 Model propoerties

Table 11: Size (employment) distribution of firms

Size	**1-9**	**10-19**	**20-99**	**100+**
Model	0.61	0.24	0.12	0.03
Data	0.77	0.12	0.10	0.02