Going Public over the Business Cycle*

Jisu Jeun †

Job Market Paper

November 1, 2024

[Click here for the latest version]

Abstract

This paper analyzes the role of Initial Public Offerings (IPOs) on employment cyclicality. Using Compustat data, I show that IPOs are cyclical in numbers and compositions and that the economic conditions at the time of IPO have long-lasting effects on subsequent firm growth. An important factor driving this result is the procyclical amount of capital raised from IPOs. Building on this evidence, I develop a firm dynamics model in which private firms decide whether to go public, while public and private firms decide about employment, investment, and exit in the presence of borrowing constraints. Crucially for the model mechanism, public firms face a looser borrowing constraint. The model is calibrated to match selected features of the U.S. non-financial firm sector and replicates the procyclical number of IPOs, selection patterns, and procyclical capital injections observed in the data. Using my model, I find that if IPOs were acyclical, the volatility of employment would be reduced by 17%. This amplification operates through two channels: first, a decline in the public firm share exacerbates capital misallocation; second, a lower propensity to go public reduces potential entrants' expected future value, decreasing entry rates. These findings suggest that policies mitigating IPO cyclicality during recessions could facilitate a faster recovery in aggregate employment.

Keywords: Firm dynamics; Growth; Business Cycle; Macroeconomics; IPO
JEL codes: D25, E32, G32, H32

*I am deeply indebted to Lidia Cruces, Georg Duernecker, Johannes Gönsch, Marek Ignaszak, Leo Kaas, Alexander Ludwig, and Nicolò Russo for their invaluable guidance and support. I also extend my thanks to Husnu C. Dalgic for an insightful discussion, and to the audiences at FQMG brown bag seminar, Spanish Macroeconomics Network, and Frankfurt-Mannheim Macro Workshop for their comments and suggestions.

†Goethe University Frankfurt. Theodor-W.-Adorno-Platz 3, 60323 Frankfurt am Main, Germany. Email: jeun@wiwi.uni-frankfurt.de; Website: jisu-jeun.squarespace.com

1 Introduction

Labor markets tend to recover slowly after recessions, and financial frictions are often considered as an important driver of this sluggish rebound. While debt financing is a fundamental channel for firms to secure external capital, equity financing also plays a critical role. In particular, an Initial Public Offering (IPO)—a firm's first equity issuance—may contribute to labor market recovery by promoting the growth of firms. Evidence indicates that IPOs significantly impact employment at the firm level, increasing it by an average of 37 percent in the IPO year (Borisov et al., 2021). More importantly, equity financing is distinct from debt in its susceptibility to excess volatility in stock prices (Shiller, 1981; Timmermann, 1993), a factor that can substantially amplify the effects of economic cycles. This volatility affects the capital that firms can raise through IPOs, impacting employment dynamics in two main ways. First, firms going public during downturns may face limited capital injection, potentially missing a crucial opportunity to establish a robust foundation for sustained employment growth.[1] Second, at the aggregate level, such volatility may prompt firms to delay IPOs until expansions, driving procyclicality in IPO numbers and, in turn, amplifying aggregate employment dynamics over the business cycle.

In this paper, I investigate the mechanisms that drive the cyclical nature of IPOs, and how IPOs contribute to aggregate employment cyclicality. I start by documenting the cyclical patterns of IPOs in the United States using Compustat data. I find that firms going public at different moments of the business cycle exhibit different employment growth trajectories. Specifically, firms that go public during contraction periods experience slower employment growth than those that do so during expansion periods, with this growth disparity emerging in the second year post-IPO and reaching approximately 20 percentage points by the tenth year. Furthermore, I find that the post-IPO employment growth gap is associated with the amount of capital raised at the time of the IPO. Building on my empirical findings, I develop a heterogeneous firm dynamics model with an endogenous IPO decision to explore the mechanisms driving IPO cyclicality. My model features aggregate productivity shocks that influence stochastic discount factors, interest rates, and wages, which in turn affect firms' IPO decisions by altering capital injection and demand for capital at the IPO stage. I calibrate my

[1] Sedláček and Sterk (2017) document similar evidence in the context of firm entry, showing that firms entering during downturns experience persistently lower employment growth due to constrained growth potential at entry.

model to the U.S. economy between 2000 and 2019 and use my calibrated model to quantify the effect of IPO cyclicality on aggregate employment.

Going public is an endogenous choice, alongside other firm decisions like investment and exit. The IPO process entails substational costs such as underwriting fees and expenses for legal and accounting services. After paying these fixed costs, during an IPO, firms issue shares to public investors in exchange for capital, which becomes part of the firm's equity. This capital influx allows firms to finance new projects, reduce debt, or invest in expanding their operations. These now-public firms are required to distribute a portion of future earnings to shareholders, including new investors, in the form of dividends. The transition from a private to a public firm through an IPO involves structurally irreversible changes induced by the heterogeneity between private and public firms.[2] Furthermore, investor protection regulations mandate transparent information disclosure by public firms. While this transparency leads to higher regular operating costs (Breuer and Breuer, 2023), it also facilitates easier access to debt (Schenone, 2010). Weighing these costs and benefits, firms strategically determine the timing of their IPOs, which in turn affects their growth trajectory.

I find that the cyclicality of IPOs amplifies the impact of aggregate productivity shocks on employment dynamics. The model identifies two main channels. First, a lower share of public firms exacerbates capital misallocation, increases inefficiency, and reduces the average size of firms. This is because public firms face more relaxed borrowing constraints as they have greater transparency through regular disclosure than private firms. Thus, a shift towards a higher share of public firms implies that resources are allocated more efficiently across the economy. Second, a lower propensity for firms to go public reduces the value of private firms, which in turn discourages potential entrants by reducing their expected future value due to the lower probability of access to capital after entry.

Related Literature. My paper connects to three branches of the literature. First, my paper relates to the literature on examining heterogeneous firm dynamics models over the business cycle. Many studies focus on the cyclical entry and exit margins, building upon the model developed by Hopenhayn (1992). For example, Lee and Mukoyama (2015) highlights the significant role of cyclical entry costs in explaining the

[2]Public firms may delist and return to private status, as seen with increased delistings after regulatory changes in the early 2000s. However, as delistings lack a clear cyclical pattern, this analysis focuses on IPO decisions over the business cycle and treats IPOs as irreversible.

cyclical entry rates observed in the data. Clementi and Palazzo (2016) extend the model to include capital, finding that the cyclical entry margin is an important mechanism for the propagation of aggregate shocks. While the aforementioned papers are about the extensive margin, Moreira (2016) and Sedláček and Sterk (2017) shed light on the intensive margin of entrants, showing that entrants during adverse economic conditions are not only fewer but also characterized by lower growth potential. This paper extends Clementi and Palazzo (2016) by incorporating an endogenous IPO decision to explore its role in the propagation of aggregate shocks. By examining IPO decisions influenced by business cycles, this study addresses the transition of firms from private to public status and its impact on firm dynamics, a dimension not explored in previous research.

Second, my paper connects to the literature on firms' financial choices in the presence of financial frictions. The impact of financial friction on firm dynamics is cyclical through the channel of firms' investment (Kiyotaki and Moore, 1997; Khan and Thomas, 2013), but most analyses are limited to debt financing. In practice, firms use both debt and equity financing, with the latter being procyclical, unlike debt finance (Jermann and Quadrini, 2012). Considering this, Nola (2016) shows that models including equity financing may reveal that the impact of financial frictions on firm dynamics has been overestimated. The firm's external financing choice is thus crucial for understanding firm dynamics over the business cycle, which includes the endogenous IPO decision, choosing equity issuance over debt financing. A distinctive feature in this paper is the assumption that equity issuance is a one-time decision corresponding to the transition from private to public status, unlike the recurring debt-equity choice in other models.

Third, my paper relates to the literature studies the firms' IPO decisions. While many studies provide empirical insights into the firm's cyclical decision on IPOs (Jovanovic and Rousseau, 2004; Helwege and Liang, 2004; Tran and Jeon, 2011; Chemmanur and He, 2012; Angelini and Foglia, 2018), theoretical insights are comparatively scarce. Some papers offer theoretical models integrating the IPO decision to explore the long-term trend of IPOs rather than business cycles. González (2020) quantitatively analyze the differential impact of tax environments on private and public firms, explaining the surge in IPOs during the 1990s. This model shares similarities with Midrigan and Xu (2014), which examines capital misallocation through a one-time equity issuance during sector transitions from traditional to modern. Although they do not specifically refer to these sectors as private and public firms, the concept of irreversible transition through the one-time equity issuance is akin to the endogenous IPO choice. Casella et al. (2022) and Davydiuk et al. (2020) attribute the decline in IPOs to increased costs, such as operating expenses and mandatory disclosure. While this paper focuses on short-term fluctuations over the business cycle, it builds upon these quantitative firm dynamics models.

The remainder of the paper is organized as follows. Section 2 introduces data and the empirical evidence regarding the cyclical nature of going public. Section 3 presents the model, and section 4 documents the results of the quantitative analysis of the model.

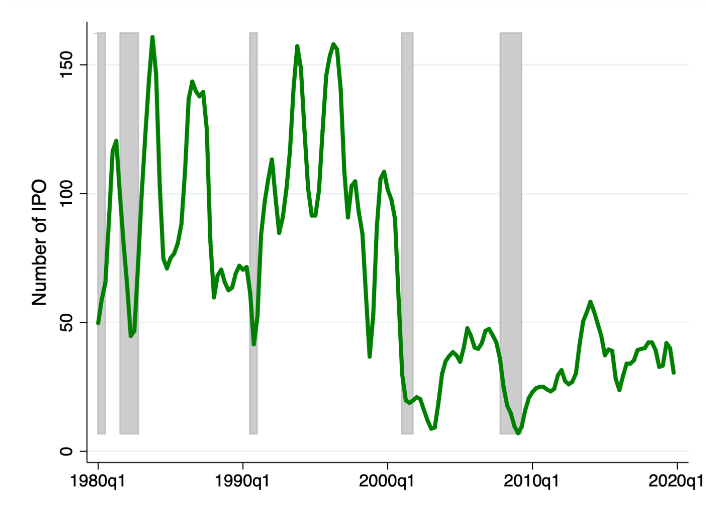
2 Empirical Evidence

In this section, I present empirical evidence on IPO cyclicality, examining IPO frequency over the business cycle, firm composition, and post-IPO performance, including exit patterns and employment growth.

2.1 Data

I primarily utilize the Compustat/CRSP merged dataset, which includes stock data from CRSP and fundamental firm data from Compustat for U.S. publicly traded companies. The dataset provides IPO-related information such as the first effective date and IPO date, as well as capital injection at IPO, approximated by total stock sales around the IPO year. Following existing literature, this study excludes utilities (SIC 4900-4949), financial firms (SIC 6000-6999), and public administration (SIC 9000-9999), as well as firms headquartered outside the U.S. and subsidiaries. The dataset also omits firms that underwent leveraged buyouts or were delisted within three years of listing due to mergers and acquisitions. These criteria yield a panel dataset of 12,858 firms with 113,745 annual observations from 1980 to 2019. Financial values are in millions of 2012 U.S. dollars, adjusted using the CPI from the U.S. Bureau of Economic Analysis. Firm-level measures are winsorized at the 1st and 99th percentiles to mitigate outlier effects.

Figure 1: Number of firms going public over the business cycle



Notes. The figure presents the number of IPOs in the U.S. based on quarterly IPO dates from Compustat/CRSP. Economic recessions, as defined by the National Bureau of Economic Research (NBER), are shaded in gray.

2.2 Cyclicality of Going Public

Number of IPOs. The procyclicality of IPOs is a well-established fact in the literature (Alti, 2005; Pástor and Veronesi, 2005; Chemmanur, 2010). Given the documented impact of IPOs on firm-level employment growth (Borisov et al., 2021), the cyclical nature of IPOs suggests that the number of firms realizing employment growth through IPOs may fluctuate in line with the business cycle, thereby affecting aggregate employment. In this context, understanding the procyclicality of IPOs is essential for this study. Figure 1 illustrates the number of IPOs on a quarterly basis, with economic downturns shaded in gray. The figure confirms a significant decline in IPO frequency during recessions.[3]

The driving sources of the cyclical number of firms going public have been widely studied in the literature. At the micro-level, firms have a greater incentive to go public during expansionary periods when their productivity peaks over the cycle (Chemmanur et al., 2010) so that they can achieve higher returns from the IPO (Benninga et al., 2005). Additionally, based on evidence that the number of IPOs is clustered within industries and states, Alti (2005) explains the amplified response of IPO num-

[3]The declines during other periods are associated with stock market performance. See Table 8 in Appendix A.1.

Table 1: Descriptive statistics of firms at IPO

	Expansion	**Contraction**
Employment (1K)	0.20	0.13
Asset (1M, $)	2,575	470
Debt-to-asset ratio	0.41	0.44
Sale of stock (1M, $)	622.6	54.4
relative to Asset	0.35	0.12

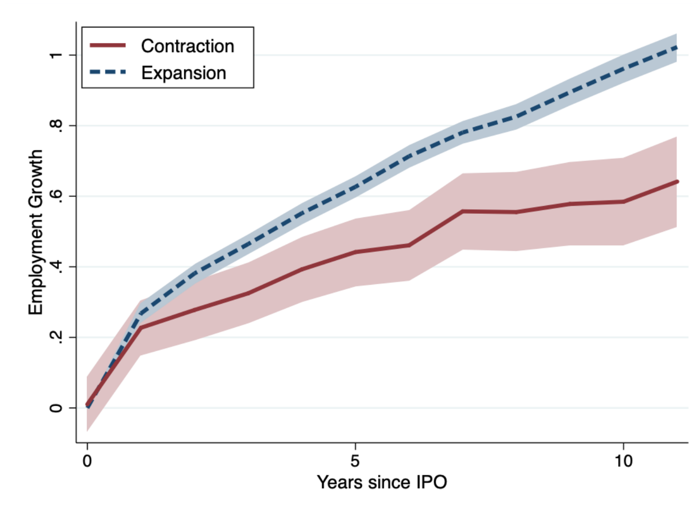
Notes. This table presents the median statistics for IPO firms categorized by IPO timing. The 'Contraction' ('Expansion') cohort includes firms that went public between a business cycle peak and trough (trough and peak). The dataset is pooled cross-sectional data of IPO firms from Compustat/CRSP. For more detailed summary statistics, see Table 9 in Appendix A.2.

bers to business cycles through information spillovers, which reduce IPO costs for other firms. Furthermore, several studies have identified the correlation between the frequency of IPOs and macroeconomic factors such as interest rates and stock market performance (Tran and Jeon, 2011; Angelini and Foglia, 2018). According to Jovanovic and Rousseau (2004), low interest rates make it easier to obtain financing through borrowing, reducing the pressure on firms to raise capital through an IPO, thus potentially leading them to delay it.

IPO cohort. I examine the extent to which firms that go public during different phases of the business cycle are distinct in terms of their composition and whether they demonstrate heterogeneous post-IPO growth patterns. To investigate these questions, I categorize firms into two cohorts based on their IPO timing, distinguishing between those that occurred during a contraction period (from peak to trough) and those that occurred during an expansionary period (from trough to peak). Descriptive statistics in Table 1 show that, on average, contraction IPO firms are smaller in both employment and assets. Both cohorts exhibit similar leverage, with debt-to-asset ratios around 0.4, indicating marginal differences in capital structure. Nonetheless, contraction IPO firms raise less capital from public stock markets than expansionary IPO firms, with the latter raising 35 percent of their assets, compared to only 12 percent for the contraction cohort. This discrepancy remains even after controlling for firm-specific characteristics. See Appendix A.3 for details.

To analyze post-IPO employment growth by age for each cohort, I run a non-

Figure 2: IPO cohort effect on post-IPO employment growth



Notes. The figures display cumulative post-IPO employment growth for cohorts classified by IPO timing within the business cycle. The 'Contraction' ('Expansion') cohort includes firms that went public between a business cycle peak and trough (trough and peak). These results are based on the non-parametric regression specified in Equation (1), controlling for initial employment, sales per worker, pre-IPO capital intensity, and debt-to-asset ratio at the IPO year, as well as industry and regional fixed effects. The dataset is an unbalanced panel drawn from Compustat/CRSP.

parametric regression

$$\Delta^{ipo} N_{it} = \boldsymbol{\beta}_{a,c} Age_{it} \times Cohort_i^{ipo} + \Gamma \mathbf{X_i} + \gamma Y_{t-1} + \epsilon_{it}. \tag{1}$$

Here, $\Delta^{ipo} N_{it}$ denotes the cumulative employment growth rate from the year of IPO, calculated as $\ln(N_{it}) - \ln(N_{i0})$ where $t = 0$ represents the IPO year. The coefficient $\boldsymbol{\beta}_{a,c}$ captures the cohort effect, representing the average employment growth by IPO age (Age_{it}), defined as a dummy for the number of years since the IPO, and differentiated by the cohorts formed during economic expansions and contractions ($Cohort^{ipo}$). Control variables $\mathbf{X_i}$ include firm i's initial employment, sales per worker, pre-IPO capital intensity (gross value of property, plant and equipment), and liabilities (debt-to-asset ratio) at the year of IPO, as well as industry and regional fixed effects. Additionally, Y_t represents the detrended log of real GDP, accounting for varying economic conditions encountered by expansion and contraction cohorts post-IPO.

Figure 2 depicts the employment trend over the years since the IPO, highlighting the cohort effect. Firms that go public during contractions exhibit slower growth rates compared to those during expansions, with an average gap of at least 10 percentage points by the sixth year post-IPO. This gap widens until the tenth year, reaching at least 20 percentage points, underscoring the significant impact of IPO timing on post-IPO growth.

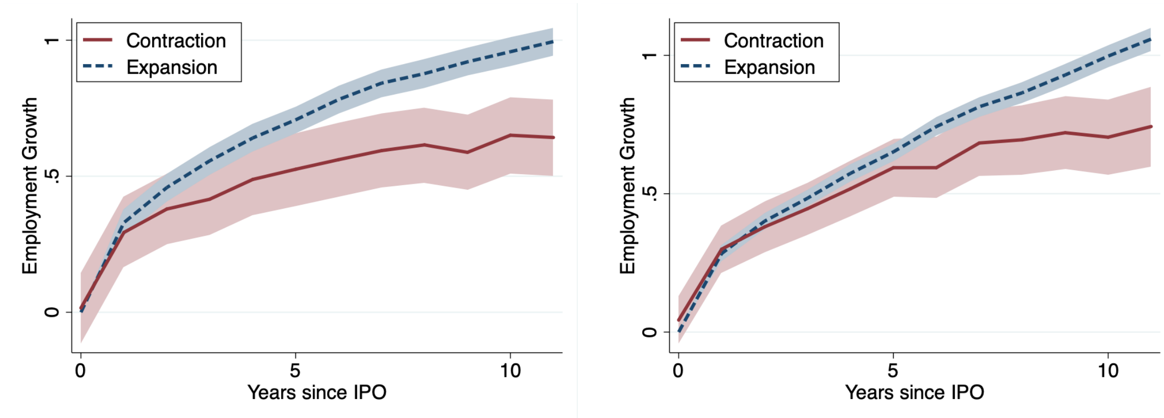
Employment growth following an IPO is facilitated by increased internal finance (Tran and Jeon, 2011). The amount of capital injection at the time of the IPO is likely a critical factor influencing subsequent investment decisions and is closely related to the firm's growth. Therefore, I examine whether the heterogeneous amount of capital injection is related to the timing of the IPO, beyond selection effects. To support the proposed hypothesis, I revisit the non-parametric regression analysis specified in Equation (1), incorporating an additional control for the magnitude of capital injection at the time of the IPO. Figure 3 presents the growth gap between expansion and contraction IPO cohorts across different model specifications. In the left panel, which is based on a balanced panel excluding firms that exit within 10 years post-IPO, the analysis controls for heterogeneous exit rates between the cohorts. The results show that while the exit margin does not significantly affect the growth gap, the significance decreases as growth variability increases. In the right panel, the analysis incorporates differences in IPO capital injection, revealing that the growth gap substantially decreases. Up to the 8th year, the gap is no longer significant, and by the 10th year, its magnitude is nearly halved. This suggests that the growth gap between contraction and expansion cohorts is closely tied to differences in capital injection at the time of IPO.

One cannot exclude the possibility of omitted variable bias. For instance, the capital injection might reflect the firm's unobserved productivity, which could drive post-IPO employment growth. The results in this section may be influenced by the selection bias towards more productive IPO firms during expansionary periods. Are productive firms more likely to wait for favorable macroeconomic conditions before going public? Or, as previously suggested, does more discounted capital injection compress the growth of IPO cohorts during contraction periods? To address this, I propose and estimate a quantitative firm dynamics model incorporating endogenous IPO decisions.

3 The Model

Time is discrete, $t = 1, 2, ...$, and the horizon is infinite. There are two types of firms, privately held and publicly traded. Firms enter the market as private, and transition

Figure 3: IPO cohort effect on post-IPO employment growth: extended analysis



(a) Exit effect

(b) Capital injection effect

Notes. The figures extend the analysis from Figure 2 displaying cumulative post-IPO employment growth. Panel (a) based on a balanced panel, examines the impact of heterogeneous exit patterns between contraction and expansion cohorts. Panel (b) controls for the amount of capital raised at the IPO, measured by the sale of stock in the IPO year, to assess the influence of varying levels of capital injection at IPO.

to public occur via IPOs. For model tractability, the transition is assumed to be irreversible. Both type of firms produce a homogeneous good at time t through the production function

$$y_t = z_t s_t (k_t^\alpha l_t^{1-\alpha})^\eta \, . \tag{2}$$

Firms' productivity is the product of an aggregate and an idiosyncratic component each denoted by z and s. Given both components of productivity, firms use two inputs: labor denoted by l_t and predetermined capital denoted by k_t. The parameter $\alpha \in (0, 1)$ governs the share of capital in production and $\eta \in (0, 1)$ governs decreasing returns to scale.

The aggregate component of productivity z_t is driven by a persistent stochastic process

$$\log(z_{t+1}) = \rho_z \log(z_t) + \varepsilon_{z,t} \, , \tag{3}$$

with $\varepsilon_{z,t} \sim \mathcal{N}(0, \sigma_z)$. In both public and private firms, firm-specific productivity s_t

evolves stochastically according to

$$\log(s_{t+1}) = \rho_s \log(s_t) + \varepsilon_{s,t} \ , \tag{4}$$

with $\varepsilon_{s,t} \sim \mathcal{N}(0, \sigma_s)$. Furthermore, the shocks in these two components $\varepsilon_{z,t}$ and $\varepsilon_{s,t}$ are assumed to be orthogonal to each other, ensuring independent variations in common and idiosyncratic productivity. The initial productivity of the entrant s_0 will be specified below.

Aggregate Fluctuation. Aggregate uncertainty affects firms' behavior. Following Clementi and Palazzo (2019), I assume that firms assess future cash flows using a stochastic discount factor, defined as

$$M(z_t, z_{t+1}) \equiv \beta \exp(\phi_0 \log(z_t) + \phi_1 \log(z_{t+1})) \ , \tag{5}$$

where the subjective discount factor, $\beta > 0$ is the time discount factor, and the parameters ϕ_0 and ϕ_1 represent the sensitivity to aggregate states. The risk-free rate is defined as the inverse of the expected value of the stochastic discount factor which reduces to[4]

$$R_t = \frac{1}{\beta} \exp\left(-z_t(\phi_0 + \rho_z \phi_1) - \frac{\phi_1^2 \sigma_z^2}{2} \right) \ . \tag{6}$$

With the parameter restriction $\phi_1 \in \left(-\frac{\phi_0}{\rho_z}, 0 \right)$, the risk-free rate is countercyclical.

IPO Decision and Financial Friction. Private firms decide whether to go public, incurring a fixed cost κ associated with the IPO process. Through an IPO, firms offer a χ share of their future dividends to public investors, with the price of each share, denoted $p(k_t, b_t, s_t, z_t)$, determined by demand from a large pool of investors who apply the same stochastic discount factor $M(z_t, z_{t+1})$ as the firms. The price depends on the aggregate state z_t and firm-specific factors, including productivity s_t and capital structure k_t and b_t. This share price, whose functional form is detailed below, allows firms to receive a capital injection of $\chi p(k_t, b_t, s_t, z_t)$ in exchange for selling a portion of their future earnings. In turn, public firms distribute a χ share of their dividends to investors each period post-IPO. Therefore, firms face an intertemporal trade-off,

[4] Derivation is provided in Appendix B.1.

weighing the benefit of immediate capital injection against the cost of sharing future earnings with investors.

Firms operate under financial constraints, issuing risk-free debt b_t subject to a collateral constraint

$$b_t \leq \theta k_t \,, \tag{7}$$

where $\theta = \{\theta^{pb}, \theta^{pr}\}$ differs between public and private firms. This difference reflects the varying degrees of transparency and access to credit markets. For public firms, financial frictions are further relaxed due to mandatory disclosures (e.g., SEC filings), which enhance transparency and facilitate easier access to debt financing (Schenone, 2010). As a result, public firms face a more relaxed borrowing constraint, represented by $\theta^{pb} \geq \theta^{pr}$. However, this increased transparency comes with higher operating costs, including expenses such as auditing and reporting fees, adding to the financial burden of public firms.

To mitigate potential future constraints, firms are incentivized to build net worth as a form of precautionary savings, especially in adverse economic conditions. This dynamic highlights the value of capital raised at IPOs, particularly for highly productive firms that benefit most from reducing financial constraints.

Incumbents' Problem. Following the transition, public firms distribute dividends to shareholders each period, corresponding to the share sold at the IPO. Consequently, the public firm retains a $1-\chi$ fraction of the total dividends in the post-IPO period. Additionally, operating as a public firm entails higher costs. Therefore, in each period t, the public firm aims to maximize the $1-\chi$ share of the discounted value of its dividend stream[5] subject to the following budget constraint

$$d_t + k_{t+1} - b_{t+1} = y_t - w_t l_t - (1+r_t) b_t + (1-\delta) k_t - f^{pb} \,. \tag{8}$$

Here, d_t denotes dividends, which are subject to non-negativity constraints ($d_t \geq 0$). The variables w_t, r_t, and f^{pb} represent wage, interest rate, and operating cost, respectively.

Private firms maximize the entire share of the discounted value of its dividend

[5]For the public firm's decision, it is irrelevant whether they maximize the dividend stream or its discounted value by $1-\chi$. However, this distinction is crucial for private firms, which compare the value function of remaining private with that of becoming a public firm.

stream, subject to budget constraint

$$d_t + k_{t+1} - b_{t+1} = y_t - w_t l_t - (1+r_t)b_t + (1-\delta)k_t - f^{pr} + \{\chi p(k_t, b_t, s_t, z_t) - \kappa\}\xi_{ipo} \,, \tag{9}$$

where $\xi_{ipo} \in \{0, 1\}$ is a binary indicator of the IPO decision. This equation elucidates that firms going public incur a fixed cost κ and receive capital injection $\chi p(k_t, b_t, s_t, z_t)$ by issuing equity. As above, f^{pr} represents the operating cost that is lower than that of public firm's ($f^{pr} \leq f^{pb}$).

Exit and Entry. Both private and public firms decide whether to exit after their production and prior to making investment. Upon exit, firms liquidate the internal cash flow and depreciated capital, net of outstanding debt, which reduces to

$$d_t^{x,pr/pb} = y_t - w_t l_t - (1+r_t)b_t + (1-\delta)k_t - f^{pr/pb} \,. \tag{10}$$

For public firms, a χ share of dividends is allocated to investors, and accordingly, an equivalent share of liquidation proceeds is distributed to them upon exit. Consequently, exiting public firms retain only $(1-\chi)\, d_t^{x,pb}$.

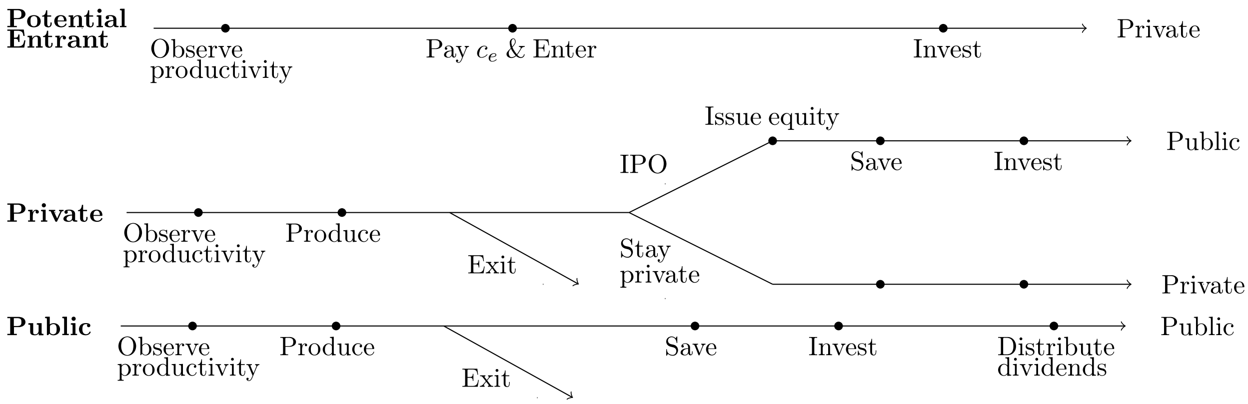
There's a large number of potential entrants each period. They observe the aggregate productivity z and draw their initial idiosyncratic productivity s_0 from a log normal distribution. Potential entrants are endowed with an initial capital k_0. Since there is no debt, the initial net worth is $a_0 = k_0$. They can freely enter as a private firm by paying an entry cost $c_e > 0$. If the expected value of operating exceeds the entry cost, they decide to enter the market. Upon entry, they invest to start producing in the following period.

Price of share. Selling χ share of equity at IPO entails the distribution of the equivalent share of dividends in every subsequent period following the IPO, which pins down the price of a share as

$$\chi p(k_t, b_t, s_t, z_t) = \mathbb{E}_t \Big[M(z_t, z_{t+1}) \max\{(\chi d_{t+1} + \chi p(k_{t+1}, b_{t+1}, s_{t+1}, z_{t+1}))\,, \chi d_{t+1}^{x,pb}\} \Big] \,. \tag{11}$$

As dividends are distributed beginning in the period following the IPO, they are discounted by a factor $M(z_t, z_{t+1})$. Here, $d_{t+1} = d(k_{t+1}, b_{t+1}, s_{t+1}, z_{t+1})$ represents the

Figure 4: Timing of firms' problem



dividends generated as a public firm post-IPO, subject to the budget constraint specified in (8). Additionally, $d_{t+1}^{x,pb}$ denotes the liquidation value upon firm exit, as defined in (10). This setup reflects that, following the IPO, investors expect a χ share of dividends from the IPO firm in its public phase, accounting for the possibility of firm exit, in which case their interests align with the owners regarding liquidation. This formulation presents a fixed-point problem, which will be further specified below using the value function of public firms.

Recursive formulation. The time subscript is suppressed in the following recursive formulation. Firms discount future value by $M(z, z')$.

Figure 4 illustrates the timeline of the firms' problem. After observing the aggregate state z and drawing the initial productivity s_0, entrants pay the entry cost c_e, make an initial investment decision, and subsequently become a private firm. Similarly, incumbent firms begin with observing an aggregate shock z and idiosyncratic productivity s. Subsequently, firms hire labor maximizing profit given the state of productivity s and z and pre-determined capital k as

$$\begin{aligned} l(k, s, z) &= \arg\max \; sz(k^{\alpha} l^{1-\alpha})^{\eta} - w(z)l \\ &= \left(\frac{(1-\alpha)\eta}{w(z)} szk^{\alpha\eta} \right)^{\frac{1}{1-(1-\alpha)\eta}}. \end{aligned} \tag{12}$$

The period begins with the payment of relevant costs such as wages, interest, and operating expenses. Incumbents then decide whether to exit the market, followed by the decision of private firms to go public through an IPO. Firms undertaking an

IPO issue equity, which involves selling shares and internalizing the capital injection. Subsequently, both private and public firms make investment and financing decisions, determining how to utilize internal and external capital for these investments. Finally, public firms distribute dividends from the residual profits.

In each period, firms decide on capital investments k' and how to allocate these investments between internal cash flows and debt b'. The internal cash flows are predetermined by the productivity components s and z, realized at the beginning of the period, as well ass by k and b, which were determined in the previous period. Consequently, if the increase in k' exceeds the available internal cash flows, it necessarily results in an equivalent increase in b'. Thus, the decision on k' pins down b'. By introducing net worth, defined as capital net of debt ($a \equiv k - b$), the problem can be simplified to choosing the optimal future net worth a' in the firm's dynamic optimization. Then it entails the intra-period problem for k' and b' as

$$
\begin{aligned}
\pi' = \max_{k',b'} & \int\int s'z'(k'^{\alpha}l'^{1-\alpha})^{\eta} - w(z')l' - (1+r)b' + (1-\delta)k' \, dH(s'|s)dG(z'|z), \\
\text{s.t. } & k' - b' = a', \\
& b' \leq \theta k', \\
& l'(k',s',z') = \left(\frac{(1-\alpha)\eta}{w(z')} s'z'k'^{\alpha\eta}\right)^{\frac{1}{1-(1-\alpha)\eta}}.
\end{aligned}
$$

Therefore, given a', the firm chooses capital

$$
k'^{*} = \min\left(\frac{a'}{1-\theta}, \ \Phi\left(\frac{\int\int\left(\frac{s'z'}{w(z')^{(1-\alpha)\eta}}\right)^{\frac{1}{1-(1-\alpha)\eta}} dH(s'|s)dG(z'|z)}{\int r(z')dG(z'|z) + \delta}\right)^{\frac{1-(1-\alpha)\eta}{1-\eta}}\right), \quad (13)
$$

where $\Phi = (\alpha\eta)^{\frac{1-(1-\alpha)\eta}{1-\eta}}((1-\alpha)\eta)^{\frac{(1-\alpha)\eta}{1-\eta}}$, and debt as $b'^{*} = k'^{*} - a'$. As a result, profit function can be expressed as a function of (a, s, z) as follows

$$
\pi(a,s,z) = \Omega(szk^{*\alpha\eta})^{\frac{1}{1-(1-\alpha)\eta}} - (r(z)+\delta)k^{*} + (1+r(z))a, \quad (14)
$$

where $\Omega = (1-(1-\alpha)\eta)\left(\frac{(1-\alpha)\eta}{w(z)}\right)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}}$. Hence, the forthcoming dynamic optimization problems are represented as state vector $\mathcal{S} = (s, z, a)$ and a control variable a'

that maximizes the value functions.

The public firms' problem can thus be recursively formulated as

$$V^{pb}(\mathcal{S}) = \max\{\tilde{V}^{pb}(\mathcal{S}), V^{x,pb}(\mathcal{S})\} , \tag{15}$$

$$\tilde{V}^{pb}(\mathcal{S}) = \max_{a'} \ (1-\chi)d + \mathbb{E}[M(z,z')V^{pb}(\mathcal{S}')|\mathcal{S}] , \tag{16}$$

$$\begin{aligned} \text{s.t.} \ \ & d + a' = \overbrace{y - wl - (r+\delta)k - f^{pb}}^{\pi(\mathcal{S})} + (1+r)a , \\ & k \leq \frac{a}{1-\theta^{pb}} , \\ & d \geq 0 , \end{aligned}$$

$$V^{x,pb}(\mathcal{S}) = (1-\chi)[\pi(\mathcal{S}) + (1+r)a] . \tag{17}$$

Public firms decide whether to exit based on the value functions specified in (15), with the value of operating and exiting detailed in (16) and (17), respectively. The value of an operating public firm is represented by the $(1-\chi)$ share of the stream of dividends as defined above. This value is subject to the budget constraints described in (8), the investment choices derived from the intra-period problem specified in (13), and the non-negativity condition for dividends.

Private firms solve following value functions:

$$
\begin{aligned}
V^{pr}(\mathcal{S}) &= \max\{V^{x,pr}, \tilde{V}^{pr}, \tilde{V}^{ipo}\} , && (18)\\
\tilde{V}^{pr}(\mathcal{S}) &= \max_{a'} \; d + \mathbb{E}[M(z,z')V^{pr}(\mathcal{S}')|\mathcal{S}] , && (19)\\
&\quad \text{s.t.} \;\; d + a' = \overbrace{y - wl - (r+\delta)k - f^{pr}}^{\pi(\mathcal{S})} + (1+r)a ,\\
&\qquad\quad k \leq \frac{a}{1-\theta^{pr}} ,\\
&\qquad\quad d \geq 0 ,\\
\tilde{V}^{ipo}(\mathcal{S}) &= \max_{a'} \; d + \mathbb{E}[M(z,z')V^{pb}(\mathcal{S}')|\mathcal{S}] , && (20)\\
&\quad \text{s.t.} \;\; d + a' = y - wl - (r+\delta)k - f^{pr} + (1+r)a + \chi p(\mathcal{S}) - \kappa ,\\
&\qquad\quad k \leq \frac{a}{1-\theta^{pb}} ,\\
&\qquad\quad d \geq 0 ,\\
&\qquad\quad \chi p(\mathcal{S}) = \frac{\chi}{1-\chi}\mathbb{E}[M(z,z')V^{pb}(\mathcal{S}')|\mathcal{S}] , && (21)\\
V^{x,pr}(\mathcal{S}) &= \pi(\mathcal{S}) + (1+r)a . && (22)
\end{aligned}
$$

Private firms make both exit and IPO decisions as specified in (18). Notably, the exit value for private firms described in (22) aligns with that of public firms when $\chi = 0$. Private firms decide to go public if the value of doing so, as given in (20), exceeds the value of remaining private, as given in (19). The value function for going public incorporates the future value of public firms, indicating the transition from private to public status occurring today. Under the assumption that the financing decision takes place after the IPO decision, IPO firms solve the intra-period problem subject to the collateral constraints of public firms, specified by θ^{pb}. Equation (22) represents the price function specified in (11), reformulated using the recursive formulation of public firms' value function.[6]

Finally, potential entrants decide to enter if the value of entry

$$
V^{e}(\mathcal{S}_0) = \max_{a'} -a' + \mathbb{E}[M(z,z')V^{pr}(\mathcal{S}')|\mathcal{S}_0] \tag{23}
$$

[6] This formulation indicates that at the time of IPO, as a private firm, the full value of the firm remains with the original owner. However, beginning in the subsequent period, once the firm transitions to public status, the owner's stake is reduced to $1-\chi$, with the remaining χ share allocated to investors. Derivation is provided in Appendix B.4.

is greater than or equal to the entry cost c_e.

Recursive Competitive Equilibrium For given initial distribution of Γ_0^{pr} and Γ_0^{pb}, a recursive competitive equilibrium consists of (i) value functions V^{pr}, V^{stay}, V^{ipo}, V^{pb}, $\tilde{V}^{pb}$, and $V^{x,j}$; (ii) policy functions $a^{j\prime}(a,s,z)$, $k^{j\prime}(a,s,z)$, $b^{j\prime}(a,s,z)$, $l^{j}(a,s,z)$ where $j \in \{pr, pb\}$, and IPO decision $\xi(a,s,z)$; and (iii) the measure of exiters $\{\Gamma_t^{x,j}\}_{t=1}^{\infty}$, incumbents $\{\Gamma_t^{j}\}_{t=1}^{\infty}$ where $j \in \{pr, pb\}$, and entrants $\{\Gamma_t^{e}\}_{t=1}^{\infty}$ such that, for all $t \geq 0$:

(i) V^{pb}, $\tilde{V}^{pb}$, $V^{pb,x}$, $a^{pb\prime}(a,s,z)$, $k^{pb\prime}(a,s,z)$, $b^{pb\prime}(a,s,z)$, $l^{pb}(a,s,z)$ solve the public firm's problem described in (15)-(17).

(ii) V^{pr}, V^{stay}, V^{ipo}, $V^{pr,x}$, $a^{pr\prime}(a,s,z)$, $k^{pr\prime}(a,s,z)$, $b^{pr\prime}(a,s,z)$, $l^{pr}(a,s,z)$, $\xi(a,s,z)$ solve the private firm's problem described in (18) - (22).

(iii) $V^{e} \geq c_e$ described in (23) solves the entrants' problem.

(iv) Measures evolve according to the law of motion:

$$\Gamma^{pb\prime} = \Gamma^{pb} - \Gamma^{pb,x} + \xi \Gamma^{pr},$$
$$\Gamma^{pr\prime} = (1-\xi)(\Gamma^{pr} - \Gamma^{pr,x}) + \Gamma^{e}.$$

4 Quantitative Analysis

4.1 Calibration

The model is calibrated based on U.S. non-financial firm data from the first quarter of 2000 to the fourth quarter of 2019.[7] One period is equivalent to a quarter, and each production unit corresponds to a firm in the dataset.

Table 2 presents the assigned parameter values. The returns to scale parameter η is set to 0.88. This value is consistent with the range between 0.83 and 0.91 that was proposed by Lee (2005). The value of 0.88 ensures that the returns to scale are decreasing, which is a standard assumption in the firm dynamics literature. The capital share α is set at 0.3, which results in a labor share of 0.609. The depreciation rate δ is assigned a value of 0.025, which is equivalent to one-quarter of the commonly utilized

[7]The data from the period after 2000 is chosen due to the structural break in IPO dynamics observed in the late 1990s.

Table 2: Assigned parameters

Parameter	Meaning	Value
η	Returns to scale	0.88
α	Capital share	0.3
δ	Depreciation rate	0.025
ρ_z	Persistence aggregate shock	0.95
σ_z	SD aggregate shock	0.007
χ	Equity share sold at IPO	0.1

annual rate of 0.1, as observed in the existing literature. Aggregate productivity shocks are assumed to evolve according to $\rho_z = 0.95$ and $\sigma_z = 0.007$, following Clementi and Palazzo (2019). The estimated equity share sold at IPO is calculated using the dividend payout ratio from Compustat, defined as dividends per share divided by earnings per share. The ratio ranges from 0.05 to 0.15 in the data set, exhibiting an increase with firms' net worth. However, there is no notable variation in this ratio with regard to IPO timing. Thus, the mean value of 0.1 is employed to calibrate χ.

Table 3: Externally calibrated parameters

	Meaning	Value	Target	Data	Model
β	Time discount	0.965	Mean interest rate	0.004	0.004
ϕ_0	Stochastic discount factor	28.587	SD interest rate	0.009	0.008
ϕ_1	Stochastic discount factor	-30.903	Sharpe ratio	0.22	0.22

Table 3 presents the externally calibrated parameter values. The three parameters governing the stochastic discount factor are jointly determined in order to match the first and second moments of the risk-free rate and the mean Sharpe ratio of public firms. In particular, the estimated mean Sharpe ratio is used to match its upper bound, which is expressed as a function of the stochastic discount factor:

$$\overline{SR} = \frac{\sigma(M_{t+1})}{E_t[M_{t+1}]} = \sqrt{\exp(\phi_1^2 \sigma_z^2) - 1}.$$

Thus, ϕ_1 is set to match the target Sharpe ratio, while β and ϕ_0 are calibrated to align with the remaining moments of the risk-free rate, following Clementi and Palazzo (2019). Due to the nonlinearities inherent in the mapping between parameters and moments, two distinct sets of values can be identified that satisfy the specified targets. One set produces a countercyclical risk-free rate, while the other produces a procyclical rate. I adopt the countercyclical specification, matching the target values estimated by Clementi and Palazzo (2019) using Compustat data.

Table 4: Targeted moments

	Meaning	Value	Target	Data	Model
ρ_s	Persistence idio. shock	0.934	AC of log sales	0.628	0.41
σ_s	SD idiosyncratic shock	0.07	SD of log sales	0.352	0.35
θ^{pb}	Borrowing constraint	0.55	Debt-to-asset	0.774	0.77
θ^{pr}	Borrowing constraint	0.35	Debt-to-asset	0.42	0.40
f^{pb}	Operating cost	5.2	Exit rate	0.02	0.03
f^{pr}	Operating cost	1.5	Exit rate	0.09	0.08
κ	IPO fixed cost	170	Emp. share of public	0.33	0.35

Notes. The first two parameters related to idiosyncratic productivity are calibrated using the simulated method of moments. The remaining parameters, which determine the costs and benefits of IPOs, are calibrated to align with observed moments for public and private firms, utilizing data from Compustat and aggregate sources such as Flow of Funds and BDS.

Table 4 presents the calibration of internally calibrated parameters. The idiosyncratic shock process is calibrated to match the autocorrelation and standard deviation of public firms' log output, estimated using the simulated method of moments to align with Compustat's quarterly sales data. To estimate the targets from Compustat, I restrict the sample to firms with at least 20 quarters of log sales data from the pooled sample. First-order autocorrelation of log sales is calculated to assess persistence, and cross-sectional variance is computed to capture volatility. Year-fixed effects are included to account for macroeconomic shocks, ensuring the estimates reflect firm-specific dynamics rather than time-specific disruptions. The same procedure is applied to simulated data from the stationary model to estimate the corresponding moments.

The remaining parameters govern the IPO dynamics and the heterogeneity between private and public firms. The parameter determining the borrowing constraint, θ, is calibrated by matching the debt-to-asset ratio. This is achieved by utilising data from Compustat for public firms and aggregate data from the Flow of Funds for private firms. The impact of the borrowing constraint on IPO dynamics is exerted through two distinct channels. First, the discrepancy in collateral constraints between public and private firms influences the incentive to pursue an IPO. A greater disparity in constraints results in greater benefits from an IPO, as public the degree of borrowing constraints for private firms affects their growth trajectory. Constraints that are more stringent result in a slower accumulation of capital, which in turn delays the point at which the firm might benefit from an IPO. In contrast, firms with more lenient constraints can expand at a faster rate, reaching the IPO threshold at an earlier stage, which in turn hastens the overall transition from private to public status in the model. Operating costs are calibrated to match the exit rates of public firms using Compustat data, and private firms using Business Dynamics Statistics (BDS) data. The exit rate for public firms is even more modest than that observed for private firms. This is due to the fact that public firms are larger and more productive, which makes them less likely to exit. Nevertheless, the calibration results confirm that public firms have higher operating costs than private firms, which is consistent with the assumptions of the model. All of the aforementioned variables affect a firm's decision to go public. All of the parameters discussed thus far influence firms' IPO decisions. Finally, the IPO fixed cost κ is calibrated to pin down the employment share of public firms in the economy.

4.2 Model Performance

The calibrated model successfully replicates several observed patterns in the data. Table 5 presents the firm size distribution by employment in the model's stationary

Table 5: Size distribution of firms

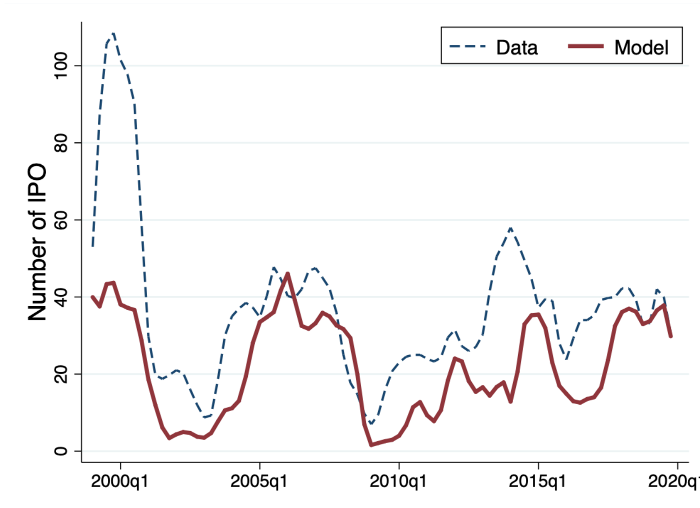
Size	**1 to 9**	**10 to 19**	**20 to 99**	**More than 100**
Model	0.61	0.24	0.12	0.03
Data	0.77	0.12	0.10	0.02

Notes. This table presents the firm size distribution by number of employees. The model captures the stationary distribution of firms, while the data shows the average firm share within each category based on BDS.

case, which displays a distinct right-skewed pattern consistent with the data. Over 80 percent of firms are small, with fewer than 19 employees, while only about 2–3 percent of firms have more than 100 employees. This pattern aligns with typical firm size distributions, where a small number of large firms have a dominant influence on total employment, while the majority of firms remain relatively small.

Figure 5 presents a comparison between the number of IPOs observed in the data set (represented by the blue dashed line) and the number of IPOs generated by the model (represented by the red solid line). The number of IPO firms is simulated by feeding the detrended total factor productivity (TFP) into the aggregate productivity

Figure 5: Simulated number of IPOs



Notes. This figure presents the actual number of IPOs from Compustat/CRSP (blue dashed line) with the simulated IPOs generated by the model (red solid line), using detrended TFP fed in as the input for aggregate productivity in each period.

process over the specified period. The model demonstrates a procyclical pattern of IPOs, indicating that the number of IPOs tends to increase during economic expansions and decline during recessions. Although the model does not perfectly align with the magnitude and timing of every peak and trough, it effectively replicates the overall trend of procyclicality of IPOs observed in the data.

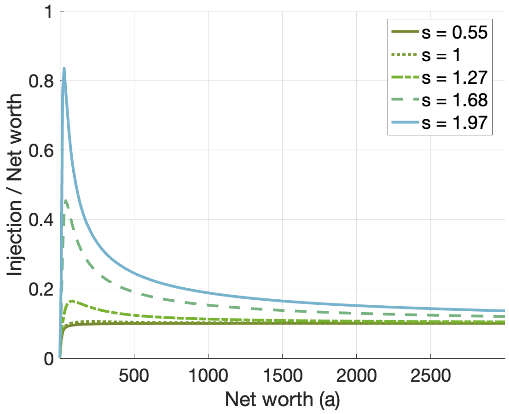
Table 6: Untargeted moments

Moment	**Model**	**Data**	**Source**
Growth at IPO	0.82	0.87	Borisov et al. (2021)
IPO effect	0.42	0.37	Borisov et al. (2021)

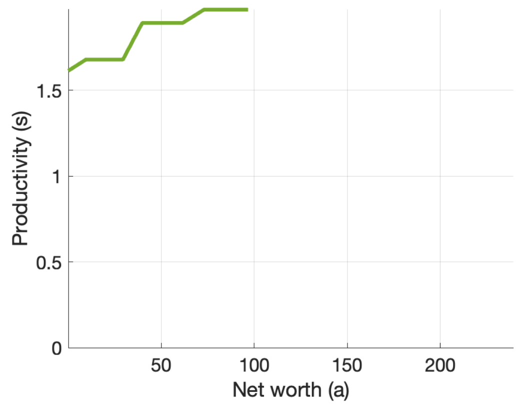
Notes. In this table, 'Growth at IPO' encompasses both the selection effect and the direct effect of the IPO. 'IPO effect' isolates the direct impact of going public.

Furthermore, I examine whether it reproduces the empirical evidence regarding the contribution of IPOs to employment growth at the individual firm level. According to Borisov et al. (2021), IPO firms tend to be fast-growing firms, which leads to two effects: a selection effect of firms that already have high employment growth and an IPO effect due to the capital raised by going public. To separate these two effects, Borisov et al. (2021) compares the employment growth rates of firms that conduct an IPO and those that withdraw. To reproduce this in the model, I simulate the model in

Figure 6: Capital injection and IPO decision



(a) Capital injection



(b) IPO Threshold

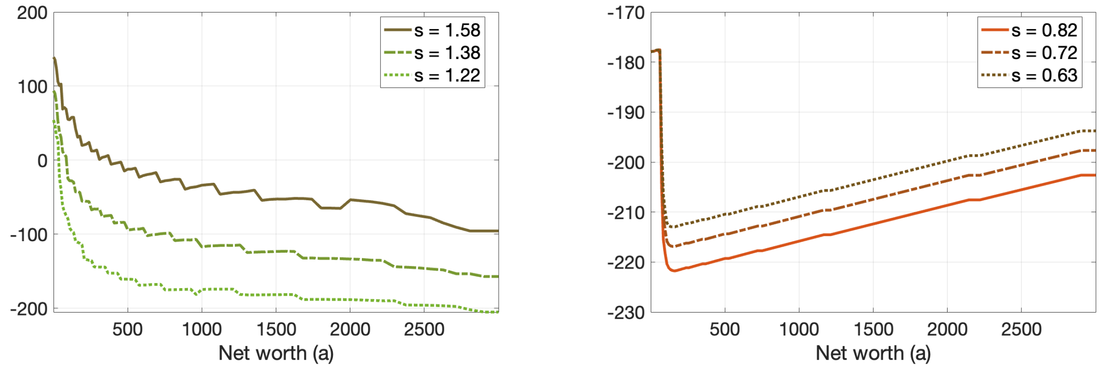
Notes. The figure illustrates the amount of capital injection and IPO threshold in a stationary case. Panel (a) displays capital injection normalized by net worth across idiosyncratic productivity (s). Panel (b) shows the IPO threshold in a stationary case ($z = 1$), identifying the region where firms, based on idiosyncratic productivity and net worth, choose to go public when these values exceed the threshold.

a stationary setup and compare the employment growth trajectory of firms that choose to go public with the employment growth trajectory of firms that remain private with similar characteristics. By comparing the employment growth of firms that go public with firms of similar size and productivity but do not go public, I disentangle the selection effect and the employment growth that is directly attributable to the IPO. As a result, as described in Table 6, the model successfully replicates both the selection effect and the IPO effect observed in the data.

4.3 IPO Decision

Panel (a) of figure 6 shows capital injections at IPO by productivity level, normalized by net worth. It shows different characteristics by size and by productivity. First, by size, capital injections vary significantly with productivity for small firms, while for large firms this difference becomes negligible. By productivity, there is an asymmetric pattern where unproductive firms (productivity below 1) raise similar amounts of capital, while productive firms (productivity above 1) raise significantly more capital as productivity increases. This asymmetry suggests that financially constrained firms, i.e. small and productive firms, may raise disproportionately more capital through

Figure 7: Private firms' value differential



(a) Productive firms

(b) Unproductive firms

Notes. The figure shows the value differential between firms going public versus staying private, where a positive differential indicates a higher value from going public. The left panel displays the value differential for productive firms ($s = 1.22$ to $s = 1.58$), while the right panel illustrates it for less productive firms ($s = 0.63$ to $s = 0.82$).

IPOs. The future value of these firms increases significantly after an IPO because they have better access to external financing and fewer borrowing constraints, and investors are able to inject more capital in anticipation of higher growth. The expected future value of these firms rises significantly after the IPO as they gain better access to external financing, leading investors to anticipate higher growth and provide larger capital injections.

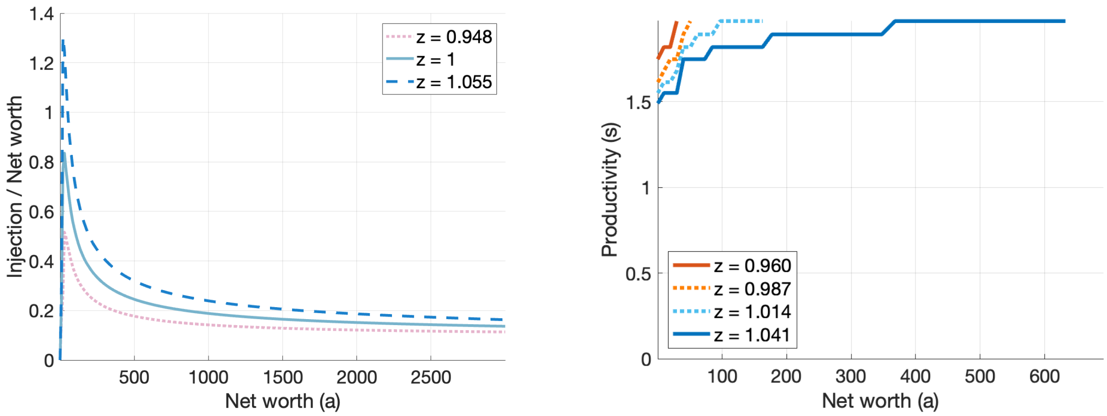
Panel (b) of Figure 6 depicts the IPO decision thresholds in the stationary case, based on recorded net worth and idiosyncratic productivity. Firms that exceed the specified threshold opt to pursue an IPO, whereas those that fall below it maintain their private status. The figure illustrates that only firms with high productivity levels opt to become publicly traded. This is because, first, as illustrated in Panel A, firms with low productivity can only increase their net worth by approximately 10 percent by raising capital at IPO. Secondly, unproductive firms have a diminished incentive to raise external capital through an IPO, as productivity shocks are less favourable and they are less likely to encounter binding borrowing constraints in the first place. Finally, productive but large firms are also less likely to opt for an IPO, as illustrated by the upward-sloping threshold. This is because large firms are less likely to face binding asset-based borrowing constraints, reducing the necessity to raise capital in costly public equity markets.

Is the incentive to go public always decreasing in net worth and increasing in idiosyncratic productivity? To better understand the IPO decision, Figure 7 illustrates the value differential between going public and remaining private. The two panels show the value differential for productive firms in Panel (a) and unproductive firms in Panel (b). First, consistent with earlier observations, only small productive firms have a positive value differential, indicating that they are the ones that go public. In terms of the relationship with productivity, both panels show that the higher the productivity, the higher the incentive to go public, as the curves with higher productivity are always above the lower ones. Regarding the relationship with size, in panel (a), the value differential gradually decreases with firm size, suggesting that the benefits of going public diminish as firms become larger and less likely to face binding collateral constraints. In contrast, panel (b) shows that the value differential for unproductive firms exhibits a distinct "swoosh" pattern. Initially, very small firms have relatively higher value differentials because they are financially constrained and benefit greatly from the relaxation of capital input and borrowing constraints. Similarly, as firms grow larger, constraints are relaxed and value differentials decline. However, once net worth exceeds about 73, value differentials begin to rise again. This pattern is due to the fact that as unproductive firms grow, they become more financially stable and less vulnerable to exit risk. Large unproductive firms accumulate enough internal resources to absorb the costs associated with going public, and the capital injection from going public can help offset the negative effects of low productivity. Thus, if the fixed costs of going public were sufficiently low, it would be more attractive for unproductive firms with high net worth to raise capital in the stock market. However, under the current calibration, the relatively high fixed IPO costs discourage these firms from going public.

Cyclicality of IPOs. The IPO decision follows a procyclical pattern. Panel (a) of Figure 8 illustrates how the potential capital injection for the most productive firm ($s = 1.97$), normalized by net worth, changes in the nonstationary case. As shown in the figure, capital injections shift upward as aggregate productivity increases, indicating that firms receive larger capital injections. However, the increase is asymmetric: small firms can see an increase of up to 80 percentage points depending on the aggregate state, while for larger firms, the change is negligible.

Panel (b) of Figure 8 shows how the IPO threshold changes with the aggregate state in the nonstationary case: as aggregate productivity shocks worsen, the IPO threshold

Figure 8: Capital injection and IPO decision in a non-stationary case



(a) Capital injection

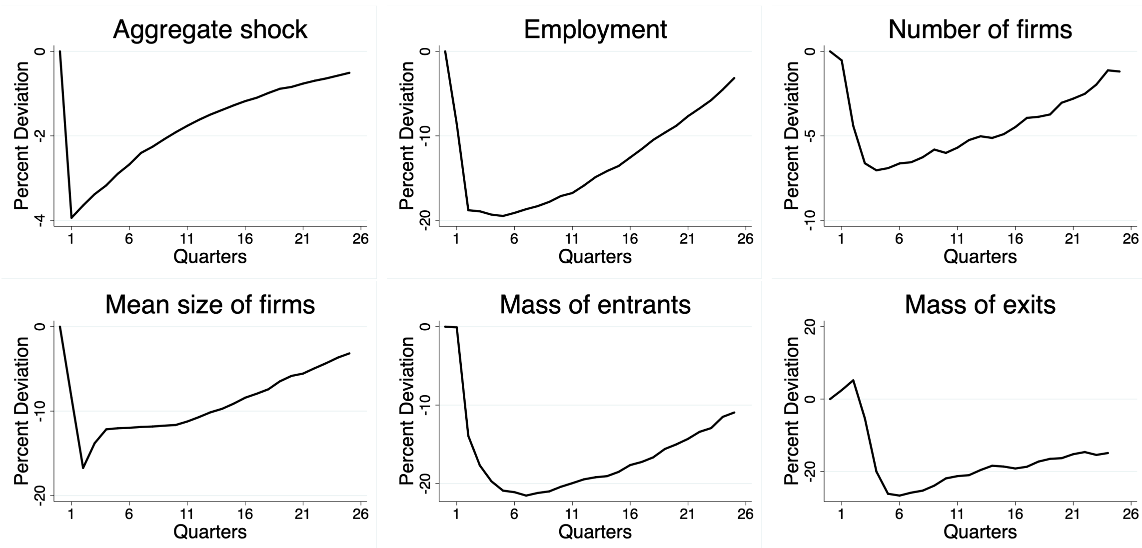
(b) IPO Threshold

Notes. The figure illustrates the IPO threshold and capital injection in a non-stationary case. Panel (a) shows the normalized potential capital injection at IPO, based on net worth for the most productive firm ($s = 1.97$) across varying aggregate states. In Panel (b), the x-axis is truncated at a net worth of 700, out of a maximum of 3000.

shifts upward and to the left, resulting in fewer IPOs, and the firms that go public tend to be smaller but more productive. This pattern is due to the fact that smaller and more productive firms are more likely to face financial constraints, making the benefits of going public more pronounced. This is consistent with the selection effect observed in the data, where IPO firms tend to be smaller during economic contractions. Of course, selection on productivity is not observable in the data. However, based on the lower growth of the contraction cohort, I had raised the possibility that the growth gap could simply be the result of selection, and that the different capital injections reflect ex-ante differences in productivity rather than ex-post capital injections. However, the model suggests that this is unlikely. If the heterogeneity were purely due to selection effects, firms in the contraction cohort should have higher post-IPO employment growth, but this is not observed.

What drives this IPO cyclicality? The procyclical nature of the stochastic discount rate drives this dynamic. First, during recessions, countercyclical interest rates rise, reducing the demand for capital and the incentive for firms to go public. Second, stock prices are more heavily discounted, reducing the capital injection that firms receive through IPOs, as observed in panel (a). As a result, firms may postpone IPOs in order to secure a larger capital injection in a more favorable economic environment. Firms

Figure 9: Response of firm dynamics to a negative productivity shock – Baseline



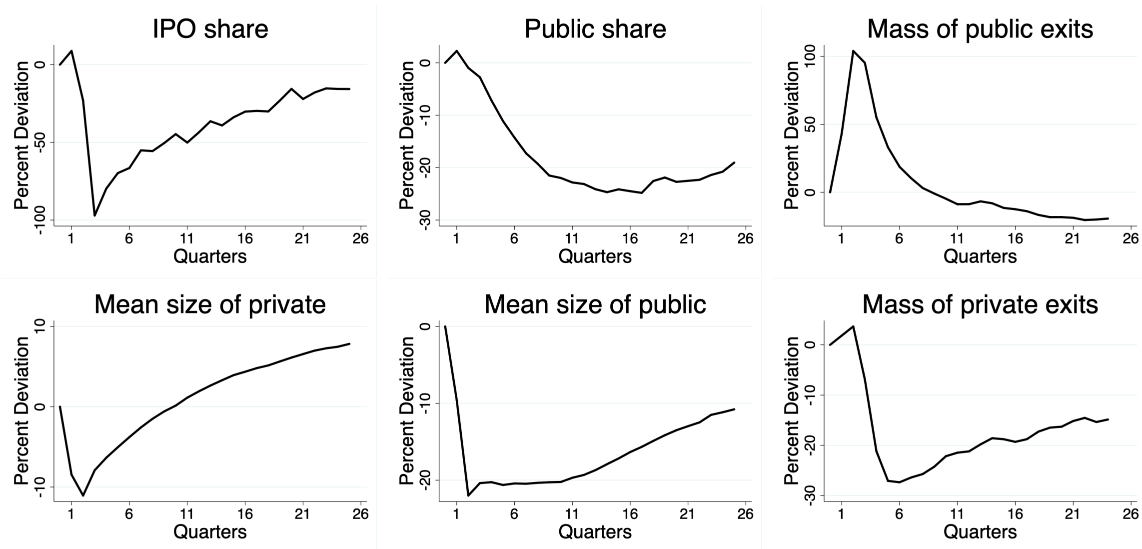
must weigh the immediate benefits of raising capital and easing borrowing constraints against the potential benefits of waiting for a larger capital injection at a later date. In practice, however, the more financially constrained firms face higher costs from delaying an IPO and are therefore more likely to go public even in adverse economic conditions.

4.4 Impulse Responses

To analyze the impact of the cyclical nature of IPOs on the propagation of aggregate shocks to employment, I compare the impulse response functions of the baseline model with those of counterfactual cases where cyclicality is shut down. First, I examine the impulse response functions following a negative shock to the aggregate shock in the baseline model. For the system initialized at the steady state, I introduce a 4 percent drop in aggregate productivity z at $t = 1$ and track the evolution of private and public firm dynamics over the subsequent 26 quarters. This experiment is repeated 1,000 times, and the averages of selected variables are depicted in Figure 9 and 10.

Figure 9 shows the impulse response from a firm dynamics perspective. A 4 percent drop in aggregate productivity leads to a 20 percent decline in employment, which begins a slow recovery from the 6th quarter. This decline is driven by both extensive and intensive margins. First, the number of firms declines by about 7 percent, mainly due to a sharp drop in the mass of entrants immediately following the aggregate shock.

Figure 10: Response of IPO dynamics to a negative productivity shock – Baseline

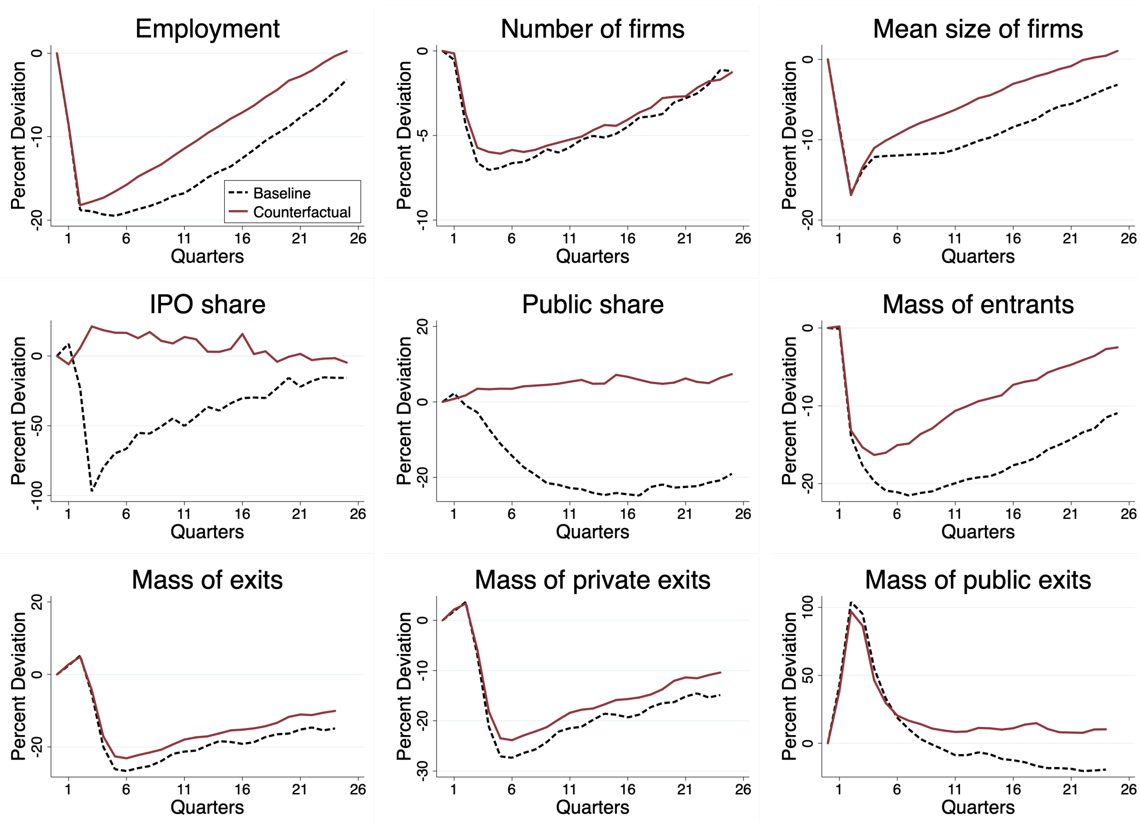


Over time, the number of firms gradually recovers as the mass of entrants increases and the mass of exits decreases. Second, a 16 percent reduction in the average firm size immediately following the shock further contributes to the decline in aggregate employment, which is associated with IPO dynamics.

Figure 10 presents the impulse response from the perspective of IPO dynamics, the transition from private to public. As examined earlier, a negative shock to aggregate productivity causes the IPO share to drop sharply, by as much as 100 percent. This leads to a decline in the entry margin of public firms, resulting in a lower share of public firms. This decline is further accelerated by a spike in the mass of public exits during the first five quarters. The share of public firms plays a critical role in determining the mean size of firms. In this model, public firms are more efficient due to their relaxed collateral constraints. Therefore, the overall efficiency of the economy is largely influenced by the share of public firms.

Private and public firms show quite different patterns of average firm size. The average size of private firms initially declines by up to 10 percent, but rebounds quickly to exceed its pre-shock steady state within 10 quarters. In contrast, the average size of public firms is more severely affected, falling by 20 percent and remaining at this lower level for 11 quarters before slowly recovering. Even after 25 quarters, the average size of public firms remains 10 percent below its steady state. This divergence shows that while both private and public firms contract in response to adverse economic

Figure 11: Response to a negative productivity shock – Counterfactual I



Notes. The black dashed line represents the response of the baseline model. The red solid line illustrates the response in the counterfactual scenario, in which IPO cyclicality related to selection and the number of IPOs is shut down.

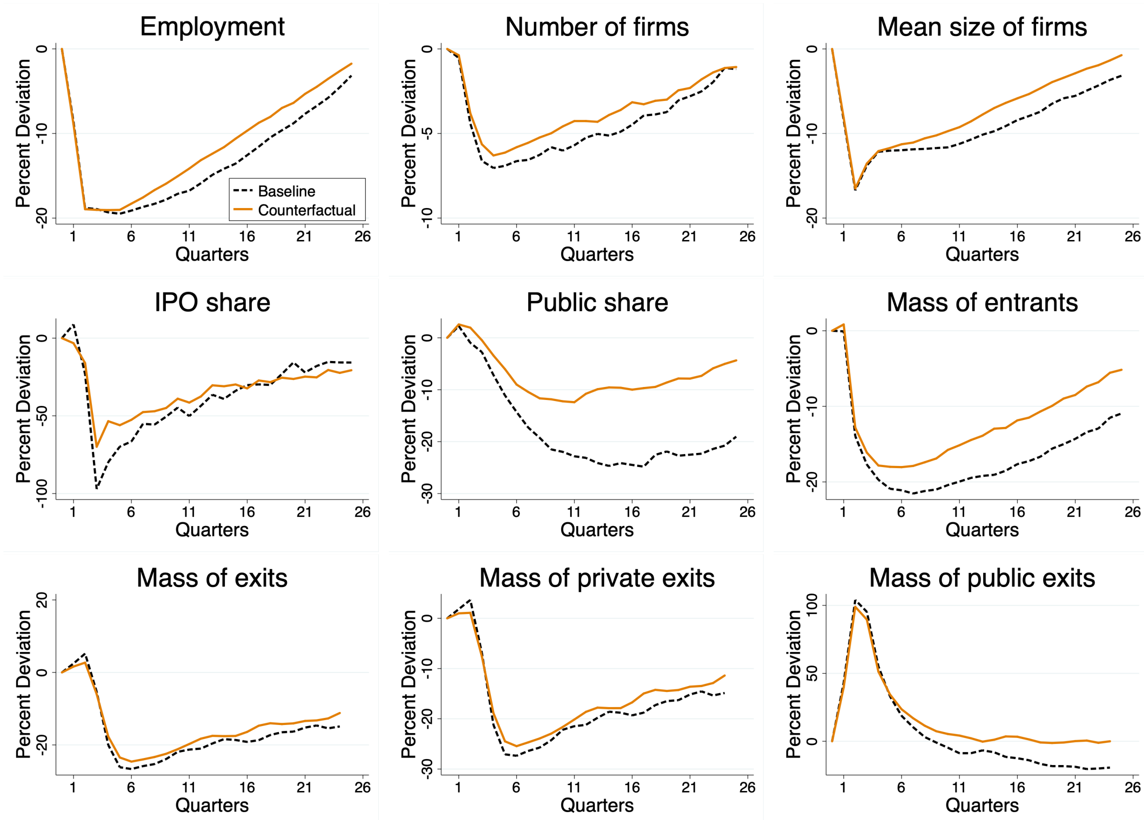
conditions, public firms experience a more substantial and persistent impact. The key driver of this pattern is the shifting IPO threshold, which moves upward and to the left, reducing both the number of firms that transition to public status and their average size. As a result, larger firms are more likely to remain private, leading to a faster recovery in the average size of private firms. Conversely, public firms experience a slower and incomplete recovery, as fewer large firms transition to public status, leaving the average size of public firms at a persistently lower level. For the same reason, the mass of private exits decreases rather than increases.

The first counterfactual assigns a fixed IPO threshold that remains exogenous across aggregate states. In other words, the threshold outlined in Figure 6 applies uniformly across aggregate states, effectively shutting down both the cyclicality of IPO numbers and selection. While IPO decisions in this setup are no longer driven by profit maximization, but are instead exogenously determined, this exercise allows to understand the mechanisms behind the effects of IPO cyclicality on aggregate employment. The results, shown in Figure 11, indicate that, similar to the baseline case, employment initially declines by the same amount but recovers more quickly, avoiding the 2-3 quarter lag observed in the baseline case. By the fourth quarter, employment is about 10 percent higher than in the baseline. This accelerated recovery is driven by both intensive and extensive margins. First, average firm size, which initially declines similarly to the baseline, begins to recover more rapidly from the fourth quarter onward, as public firms, facing relaxed borrowing constraints due to greater transparency, are able to allocate resources more efficiently than private firms. Therefore, with a relatively stable share of public firms in the economy, capital misallocation is reduced, improving overall allocative efficiency in the counterfactual. This improvement in allocative efficiency leads to higher economic productivity, which is reflected in an increase in average firm size. Second, the number of firms declines by about 13 percent less in the first year after the shock, with this difference diminishing over time. The smaller reduction along the extensive margin appears to be driven primarily by new entrants. In the counterfactual, the number of entrants is about 5-17 percent lower in the first year after the shock than in the baseline. However, this increase in the number of firms due to entry is offset to some extent by the increase in the number of exits.

How does IPO cyclicality affect entry and exit dynamics? A fixed IPO threshold provides firms meeting this criterion with guaranteed access to public markets, independent of aggregate economic conditions. For potential entrants, the increased predictability of raising capital from going public increases their expected future value, enhancing their value function and making entry more attractive. Although the timing of IPOs may not be profit-maximizing under this scenario, the certainty of public market access adds value by easing financial constraints earlier in the firm's life cycle. Firms no longer need to await favorable conditions; by meeting the exogenous threshold, they can pursue an IPO, lowering barriers to growth. As the perceived opportunity to go public incentivizes more firms to enter, some entrants may lack the productivity or resilience. While these firms are initially attracted by the prospect of an IPO, they may exit sooner if they struggle to maintain profitability in a challenging economic environment. Thus, higher entry rates may be accompanied by increased exit rates, as weaker firms find it difficult to sustain operations.

In the second counterfactual scenario, I examine a more realistic case where firms

Figure 12: Response to a negative productivity shock – Counterfactual II



Notes. The black dashed line represents the response of the baseline model. The yellow solid line illustrates the response in the counterfactual scenario, in which IPO cyclicality related to the capital injection is shut down.

make endogenous IPO decisions based on profit maximization while removing the procyclicality of equity prices. In this framework, capital injection remains fixed at its average level, influenced only by firm-specific states, such as idiosyncratic productivity and net worth, and is not affected by aggregate productivity. Figure 12 illustrates the results, with the yellow solid line showing the response in the counterfactual scenario and the black dashed line representing the baseline. Under this counterfactual, employment initially declines by the same amount as in the baseline but recovers more rapidly, reaching a 1.3 percent higher employment level by the fourth quarter post-shock. Similarly, this accelerated recovery is driven by both intensive and extensive margins.

First, the initial decrease in the average firm size, as observed in the baseline scenario, is followed by a more rapid recovery beginning in the sixth quarter. In the

Table 7: Quantitative responses: Baseline vs. Counterfactual

	Baseline	**Counterfactual**	**Dev. (%)**
Employment	-6.98	-5.77	17.31
Mean size of firms	-3.55	-2.93	17.49
Number of firms	-2.23	-1.22	45.12
Public firm share	-0.67	-0.21	69.15
Entry rate	-10.14	-9.09	10.39

Notes. The table presents the quantitative responses to a percent negative aggregate productivity shock in the fourth quarter, comparing the baseline scenario with the counterfactual scenario, in which IPO cyclicality—both selection effects and the number of IPOs—is shut down.

baseline scenario, a negative productivity shock leads to a reduction in capital injection, which in turn further discourages IPOs. However, in this counterfactual case, stable capital injections maintain firms' incentives to go public even after such a shock. This stability serves to mitigate the decline in the IPO share, which in turn serves to mitigate the reduction in the number of public firms. The allocation of resources by public firms is more efficient, and thus a smaller decline in the public firm share contributes to a reduction in capital misallocation and an increase in the average firm size. The number of firms declines by approximately 10-14 percent within a year following the shock. In the year following the shock, the number of firms declines by approximately 10–14 percent less than in the baseline scenario. As in previous cases, this outcome is closely associated with entry and exit dynamics. First, the stable equity price allows potential entrants to perceive a more predictable and secure opportunity to go public, even during periods of economic downturn. The steady expectation of public market access raises the value of entry for private firms, increasing their willingness to enter. This effect results in a decline in the number of entrants that is approximately 15 percent less than would otherwise be expected. In comparison to the preceding counterfactual scenario, the discrepancy in the number of new entrants is less pronounced. However, the comparatively greater discrepancy in the total number of firms can be attributed to exit dynamics. Due to the constant equity price, which facilitates IPO access, there is a reduction in the number of firms exiting the market during downturns. A successful IPO provides firms with access to greater capital, enabling them to maintain operations even during periods of low productivity.

In sum, the two counterfactual results demonstrate that the cyclicality of IPOs amplifies the effect of a negative shock on aggregate employment. This amplification operates through two primary channels. First, a reduced number of IPOs following a negative productivity shock significantly decreases the public firm share, exacerbating capital misallocation within the economy. Second, a lower propensity for firms to go public diminishes the value of private firms, which discourages potential entrants from entering the market, further delaying recovery. Table 6 quantifies the impact of each margin, presenting the responses of key variables in both the baseline and counterfactual scenarios three quarters after a -1 percent aggregate productivity shock. In the baseline scenario, employment declines by 6.98 percent, compared to a 5.77 percent decline in the counterfactual, representing a difference of 17.31 percent. This smaller decline in employment is partly driven by a 17.49 percent smaller reduction in mean firm size and a 45.12 percent smaller reduction in the number of firms. These changes align with a 69.15 percent smaller reduction in the public firm share and a 10.39 percent smaller reduction in the entry rate, further validating the amplification mechanism.

5 Conclusions

This paper examines the contribution of IPOs on employment cyclicality, building on the well-established observation that IPO activity is procyclical. Empirical evidence shows that not only is the number of IPOs procyclical, but there is also a selection effect in terms of firm size and differential post-IPO employment growth patterns that vary according to IPO timing within the business cycle. Specifically, firms going public during downturns experience employment growth that is at least 10 percentage points slower over several years post-IPO compared to firms going public during expansions. This growth disparity is associated with the smaller capital raised at IPO during contraction periods.

To investigate the broader implications of this pattern, I develop a firm dynamics model in which private firms decide on going public, while both private and public firms make decisions regarding exit, investment, and employment under borrowing constraints. The model incorporates a procyclical stochastic discount factor, capturing the observed procyclicality in IPO numbers, capital raised at IPOs, and selection patterns documented in the data. In counterfactual analyses where I shut down the cyclicality in IPO composition and frequency, results indicate that IPO cyclicality amplifies the impact of aggregate productivity shocks on employment. In particular, without IPO cyclicality, aggregate employment would decrease by 5.8 percent, compared to a 7 percent decrease in the baseline model. This amplification operates through two primary channels. First, a lower share of public firms during downturns exacerbates capital misallocation, as public firms typically operate more efficiently due to relaxed borrowing constraints. Second, the lower propensity for firms to go public in recessions diminishes the value of private firms, reducing the incentive for new entrants and thereby slowing recovery on the extensive margin. An alternative counterfactual analysis, in which the cyclicality of stock prices is eliminated, shows consistent results with regard to the underlying mechanism.

These findings carry important policy implications. Encouraging more firms to go public during or shortly after recessions could stabilize the labor market and accelerate recovery. Policies that reduce IPO fixed costs (as with the JOBS Act), provide incentives for public listing during downturns, or mitigate stock market volatility could counteract IPO cyclicality, maintaining a higher share of public firms, improving capital allocation efficiency, and encouraging new market entries.

References

Alti, A. (2005). IPO Market Timing. *Review of Financial Studies 18*(3), 1105–1138.

Angelini, E. and M. Foglia (2018). The relationship between ipo and macroeconomics factors: an empirical analysis from uk market. *Annals of Economics and Finance 19*, 319–336.

Benninga, S., M. Helmantel, and O. Sarig (2005). The timing of initial public offerings. *Journal of Financial Economics 75*(1), 115–132.

Borisov, A., A. Ellul, and M. Sevilir (2021). Access to public capital markets and employment growth. *Journal of Financial Economics 141*(3), 896–918.

Breuer, M. and P. Breuer (2023). Uneven regulation and economic reallocation: Evidence from transparency regulation. *SSRN Electronic Journal*.

Casella, S., H. Lee, and S. Villalvazo (2022). Rising intangible capital and the disappearance of public firms.

Chemmanur, T. J. (2010). Venture capital, private equity, IPOs, and banking: An introduction and agenda for future research. *Journal of Economics and Business 62*(6), 471–476.

Chemmanur, T. J. and J. He (2012). IPO waves, product market competition, and the going public decision: Theory and evidence. *SSRN Electronic Journal*.

Chemmanur, T. J., S. He, and D. K. Nandy (2010). The going-public decision and the product market. *The Review of Financial Studies 23*(5), 1855–1908.

Clementi, G. L. and B. Palazzo (2016). Entry, exit, firm dynamics, and aggregate fluctuations. *American Economic Journal: Macroeconomics 8*(3), 1–41.

Clementi, G. L. and B. Palazzo (2019). Investment and the cross-section of equity returns. *The Journal of Finance 74*(1), 281–321.

Davydiuk, T., B. Glover, and R. Szymanski (2020). The decline in public firms.

González, B. (2020). Macroeconomics, firm dynamics and IPOs. Working Papers 2030, Banco de España.

Helwege, J. and N. Liang (2004). Initial public offerings in hot and cold markets. *The Journal of Financial and Quantitative Analysis 39*(3), 541–569.

Hopenhayn, H. A. (1992, September). Entry, exit, and firm dynamics in long run equilibrium. *Econometrica 60*(5), 1127.

Jermann, U. and V. Quadrini (2012). Macroeconomic effects of financial shocks. *American Economic Review 102*(1), 238–71.

Jovanovic, B. and P. L. Rousseau (2004). Interest rates and initial public offerings. Working Paper 10298, National Bureau of Economic Research.

Khan, A. and J. K. Thomas (2013, December). Credit shocks and aggregate fluctuations in an economy with production heterogeneity. *Journal of Political Economy 121*(6), 1055–1107.

Kiyotaki, N. and J. Moore (1997). Credit cycles. *Journal of Political Economy 105*(2), 211–248.

Lee, Y. (2005). The importance of reallocations in cyclical productivity and returns to scale: evidence from plant-level data. Technical report.

Lee, Y. and T. Mukoyama (2015, July). Entry and exit of manufacturing plants over the business cycle. *European Economic Review 77*, 20–27.

Midrigan, V. and D. Y. Xu (2014). Finance and misallocation: Evidence from plant-level data. *American Economic Review 104*(2), 422–58.

Moreira, S. (2016). Firm dynamics, persistent effects of entry conditions, and business cycles. *SSRN Electronic Journal*.

Nola, A. D. (2016). Capital misallocation during the great recession. *SSRN Electronic Journal*.

Pástor, L. and P. Veronesi (2005). Rational ipo waves. *The Journal of Finance 60*(4), 1713–1757.

Schenone, C. (2010). Lending relationships and information rents: Do banks exploit their information advantages? *The Review of Financial Studies 23*(3), 1149–1199.

Sedláček, P. and V. Sterk (2017). The growth potential of startups over the business cycle. *American Economic Review 107*(10), 3182–3210.

Shiller, R. J. (1981, June). Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends? *American Economic Review 71*(3), 421–436.

Timmermann, A. G. (1993). How learning in financial markets generates excess volatility and predictability in stock prices. *The Quarterly Journal of Economics 108*(4), 1135–1145.

Tran, A. L. and B. N. Jeon (2011). The dynamic impact of macroeconomic factors on initial public offerings: evidence from time-series analysis. *Applied Economics 43*(23), 3187–3201.

Appendix

A Empirical Evidence

A.1 Cyclical Number of IPO

Table 8: Correlation Matrix of Number of IPO and Business Cycle Measures

	Number of IPOs	**PE Ratio**	**GDP**
***Panel A**. All (1980Q1-2021Q4)*			
Number of IPOs	1.00		
PE Ratio (detrended)	0.27***	1.00	
GDP (logged, detrended)	0.04	0.50***	1.00
***Panel B**. Post-2000Q1*			
Number of IPOs	1.00		
PE Ratio (detrended)	0.66***	1.00	
GDP (logged, detrended)	0.23**	0.63***	1.00

Figure 1 reveals fluctuations in other periods, which are likely to be related to the stock market performance (Tran and Jeon, 2011). Panel A of Table 8 provides a correlation analysis between IPO numbers and the cyclical components of output and stock market performance, represented by detrended log real GDP and the detrended Shiller price-to-earnings ratio. The results show a stronger correlation with the stock market's cyclical component (0.27) than with GDP (0.04), highlighting the significant impact of stock market conditions on IPO activities beyond recessionary periods. However, a structural shift around 2000 marked by a significant drop in IPO frequency and its variance alters this pattern. Post-2000 correlation analysis in Panel B reveals increased correlations between both stock market performance and GDP with the number of IPOs, with a notable rise in the correlation with GDP. This shift indicates not only a reduction in the number of IPOs but also an intensified cyclical pattern in the IPO market since 2000.

Table 9: Descriptive statistics of IPO firms

	N	Mean	Std. Dev.	p10	p25	p50	p75	p90
A. Expansion								
Employment (1K)	9,815	1.48	7.39	0.02	0.05	0.18	0.72	2.67
Asset (1M)	10,256	30,482	151,858	94.47	413.57	2,207	11,364	48,482
Sales (1M)	10,191	21,216	110,765	0.00	95.40	1,105	6,883	32,338
Age	4,346	14.69	19.49	2.00	4.00	8.00	16.00	34.00
Book value per share	10,225	184.61	761.56	2.17	35.37	118.82	284.23	670.80
Debt-to-asset ratio	9,565	0.52	0.75	0.08	0.17	0.35	0.63	1.02
Sale of stock (1M)	10,036	4,030	8,113	0.00	89.14	622.60	3,629	12,001
Exit age	7,814	7.88	6.43	2.00	3.00	6.00	11.00	17.00
B. Contraction								
Employment (1K)	609	2.18	14.35	0.01	0.03	0.12	0.62	2.70
Asset (1M)	651	48,206	249,736	44.02	85.99	369.40	4,018	45,856
Sales (1M)	649	31,711	187,398	2.37	35.14	246.81	1,788	24,247
Age	90	16.04	22.40	3.00	5.00	10.00	16.00	27.50
Book value per share	645	218.38	537.96	1.37	10.81	47.45	166.45	687.19
Debt-to-asset ratio	637	0.55	0.72	0.09	0.20	0.41	0.67	0.95
Sale of stock (1M)	642	3,455	9,564	0.00	3.76	54.41	338.89	10,444
Exit age	521	8.53	7.51	1.00	3.00	6.00	12.00	19.00

Notes. This table presents the summary statistics in the year of IPO for firms categorized as expansionary or contractionary based on the timing of the IPO. Employment is measured in thousands. Assets, sales, and stock sales are measured in millions of U.S. dollars, while book value per share is measured in U.S. dollars. All figures are adjusted to 2012 real terms. Age information, based on the year of foundation, is sourced from FactSet and the Field-Ritter dataset of company founding dates, covering 40 percent of IPO firms after excluding those with negative ages and discrepancies of more than 20 years between the two sources. The remaining data are sourced from Compustat/CRSP.

A.2 Descriptive Statistics

Table 9 presents the descriptive statistics in the year of IPO for the two IPO cohorts.[8] First, during contraction periods, the employment distribution of IPO firms exhibits greater variance. Although the average size for the contraction cohort is larger, this is likely driven by a few firms above the 90th percentile, as the average size up to the 75th percentile is consistently smaller compared to the expansion cohort. Similar patterns are observed in other dimensions of size, such as assets and sales, as well as the age, book value per share, and sale of common and preferred stock indicating the amount of capital injection raised at IPO. This suggests that, excluding the firms at the extreme right of the distribution, the contraction cohort is generally smaller and younger and

[8] Considering the possibility of a time lag, I also take the average up to a year after the IPO as a robustness check, which yields consistent results.

raises less amount of capital compared to the expansion cohort.

Table 10: Exit rates by IPO cohort

	Expansion	**Contraction**
Exit rate	0.564	0.565
Merger and acquisition	0.239	0.198
Bankruptcy or unknown	0.205	0.292
Delist (Back to private)	0.083	0.044

Table 10 presents the exit rates for the IPO cohorts. The exit rate is defined as the likelihood of being delisted from the Compustat/CRSP dataset within 10 years. The data indicate that both cohorts exhibit a consistent exit rate of 56 percent within this period. However, the types of exits differ between the cohorts. For the expansion cohort, mergers and acquisitions constitute the majority of exits, whereas for the contraction cohort, closures or bankruptcies are more prevalent. The incidence of firms returning to private status is the least common for both cohorts but occurs at twice the rate in the expansion cohort compared to the contraction cohort. The five-year exit rates are 35 percent for both cohorts, with a consistent composition.

A.3 Procyclical Capital Injection

I run a cross-sectional Ordinary Least Squares (OLS) regression of capital injection—the sale of stock (*sstk* in Compustat) during the IPO period—on the cyclical variable at the time of the IPO.[9]

$$\log(sstk_{it}) = \beta Cycle_{it} + \Gamma \mathbf{X_{it}} + \epsilon_{it} \tag{24}$$

The coefficient of interest, β, captures the relationship between capital injection and the state of the economy at the time of the IPO, measured either by a dummy variable for the contraction cohort or by the detrended log of real GDP. The control variables, $\mathbf{X_{it}}$, include employment, sales per worker, capital intensity, and debt-to-asset ratio in the year of the IPO, as well as industry and state fixed effects.[10]

[9] A robustness check is conducted using the aggregate sale of stocks in the first three years post-IPO to account for potential time lags.

[10] For capital intensity and the debt-to-asset ratio, I specifically use data of the period only prior to the IPO, as the sale of stock is significantly collinear with these variables post-IPO.

Table 11: Capital injection at IPO over the business cycle

	(1)	(2)	(3)
Log sale of stock at IPO			
IPO in Contraction	-0.378*		
	(-2.22)		
Detrended Log GDP		0.295***	0.195***
		(13.87)	(7.50)
Detrended PE Ratio			0.062***
			(6.67)
Firm Characteristics at IPO	✓	✓	✓
Industry FE	✓	✓	✓
State FE	✓	✓	✓
Observations	5,096	5,096	5,096
R^2	0.297	0.322	0.328

Notes. t-statistics in parentheses; $*\ p < 0.05$, $**\ p < 0.01$, $***\ p < 0.001$. The table presents regression results of stock sales at IPO on the business cycle index using cross-sectional data of IPO firms. The cyclical components of log GDP and Shiller PE ratio are detrended using HP filter. 'Firm Characteristics at IPO' includes employment, sales per worker, capital intensity, and debt-to-asset ratio. For capital intensity and debt-to-asset ratio, data prior to IPO capital internalization are used. Firms exiting within two years post-IPO are excluded.

In Table 11, Column (1) shows that firms in the contraction cohort raise 38 percent less capital at IPO compared to those in the expansion cohort, holding other characteristics constant. To be specific, column (2) indicates that a 1 percentage point increase in GDP at the time of IPO results in a 30 percent greater amount of capital injection. Column (3) additionally considers the cyclicality of stock market performance represented by Shiller price-to-earnings (PE) ratio to confirm that the relationship with economic output cyclicality is not simply due to stock market fluctuations. This confirms that the amount of capital injection is indeed related to the economic state at the time of the IPO, consistent with existing literature (Tran and Jeon, 2011).

B Numerical Appendix

B.1 Interest rate

The interest rate in the model is defined as the inverse of the expected stochastic discount factor. By using the law of motion of aggregate productivity shock in (3), the one-period interest rate is expressed as

$$\begin{aligned}
\frac{1}{R_t} &= E[M(z_t, z_{t+1})] = E[\beta \exp(\phi_0 z_t + \phi_1 z_{t+1})] \\
&= \beta E[\exp(\phi_0 z_t + \phi_1 z_{t+1})] \\
&= \beta E[\exp(\phi_0 z_t + \phi_1 \rho_z z_t + \phi_1 \epsilon_{z,t})] \\
&= \beta \exp(\phi_0 z_t + \phi_1 \rho_z z_t) E[\exp(\phi_1 \epsilon_{z,t})] \\
&= \beta \exp(\phi_0 z_t + \phi_1 \rho_z z_t) \exp\left(0 + \frac{\phi_1^2 \sigma_z^2}{2}\right) \\
&= \beta \exp\left((\phi_0 + \phi_1 \rho_z) z_t + \frac{\phi_1^2 \sigma_z^2}{2}\right) \\
R_t &= \frac{1}{\beta} \exp\left(-(\phi_0 + \phi_1 \rho_z) z_t - \frac{\phi_1^2 \sigma_z^2}{2}\right) .
\end{aligned}$$

B.2 Labor decision

Optimal level of labor is determined by maximizing the profit function, expressed as

$$l = \text{argmax}_l sz(k^\alpha l^{1-\alpha})^\eta - wl$$

First order condition is

$$\begin{aligned}
&(sz)\eta(k^\alpha l^{1-\alpha})^{\eta-1}(1-\alpha)k^\alpha l^{-\alpha} - w = 0 \\
&(sz)(1-\alpha)\eta k^{\alpha\eta} l^{(1-\alpha)\eta-1} = w \\
&l^{(1-\alpha)\eta-1} = \frac{w}{(1-\alpha)\eta(sz)} k^{-\alpha\eta}.
\end{aligned}$$

Therefore,

$$l^* = \left(\frac{w}{(1-\alpha)\eta(sz)} k^{-\alpha\eta}\right)^{\frac{1}{(1-\alpha)\eta-1}} = \left(\frac{(1-\alpha)\eta}{w(z)}\right)^{\frac{1}{1-(1-\alpha)\eta}} (sz)^{\frac{1}{1-(1-\alpha)\eta}} k^{\frac{\alpha\eta}{1-(1-\alpha)\eta}}.$$

Substituting the optimal labor into the profit function yields

$$
\begin{aligned}
\pi =& szk^{\alpha\eta}l^{(1-\alpha)\eta} - wl - (r+\delta)k + (1+r)a \\
=& \left(\frac{(1-\alpha)\eta}{w}\right)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}} (sz)^{\frac{1}{1-(1-\alpha)\eta}} k^{\frac{\alpha\eta}{1-(1-\alpha)\eta}} \\
& - w\left(\frac{(1-\alpha)\eta}{w}\right)^{\frac{1}{1-(1-\alpha)\eta}} (sz)^{\frac{1}{1-(1-\alpha)\eta}} k^{\frac{\alpha\eta}{1-(1-\alpha)\eta}} - (r+\delta)k + (1+r)a \\
=& \left(((1-\alpha)\eta)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}} - ((1-\alpha)\eta)^{\frac{1}{1-(1-\alpha)\eta}}\right) w^{\frac{-(1-\alpha)\eta}{1-(1-\alpha)\eta}} (sz)^{\frac{1}{1-(1-\alpha)\eta}} k^{\frac{\alpha\eta}{1-(1-\alpha)\eta}} \\
& - (r+\delta)k + (1+r)a \\
=& \left(1-(1-\alpha)\eta\right)\left((1-\alpha)\eta\right)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}} w^{-\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}} (sz)^{\frac{1}{1-(1-\alpha)\eta}} k^{\frac{\alpha\eta}{1-(1-\alpha)\eta}} \\
& - (r+\delta)k + (1+r)a \\
=& \left(1-(1-\alpha)\eta\right)\left(\frac{(1-\alpha)\eta}{w}\right)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}} (sz)^{\frac{1}{1-(1-\alpha)\eta}} k^{\frac{\alpha\eta}{1-(1-\alpha)\eta}} - (r+\delta)k + (1+r)a.
\end{aligned}
$$

B.3 Capital allocation

Optimal level of capital investment is determined by maximizing the profit function of the next period, expressed as

$$
k' = \operatorname{argmax} \mathbb{E}_{s',z'}\left[\, \pi(a', s', z')\,\right] \quad \text{s.t.} \quad k' \leq \frac{1}{1-\theta}a'
$$

The first order condition is

$$
\begin{aligned}
& \frac{\partial}{\partial k'}\mathbb{E}_{s',z'}\left[(1-(1-\alpha)\eta)\left(\frac{(1-\alpha)\eta}{w(z')}\right)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}} (s'z')^{\frac{1}{1-(1-\alpha)\eta}} k'^{\frac{\alpha\eta}{1-(1-\alpha)\eta}} - (r(z')+\delta)k' + (1+r)a'\right] \\
=& \mathbb{E}_{s',z'}\left[\frac{\partial}{\partial k'}\left((1-(1-\alpha)\eta)\left(\frac{(1-\alpha)\eta}{w(z')}\right)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}} (s'z')^{\frac{1}{1-(1-\alpha)\eta}} k'^{\frac{\alpha\eta}{1-(1-\alpha)\eta}} - (r(z')+\delta)k' + (1+r)a'\right)\right] \\
=& \mathbb{E}_{s',z'}\left[\alpha\eta\left(\frac{(1-\alpha)\eta}{w(z')}\right)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}} (s'z')^{\frac{1}{1-(1-\alpha)\eta}} k'^{\frac{\eta-1}{1-(1-\alpha)\eta}} - (r(z')+\delta)\right] = 0.
\end{aligned}
$$

Rearranging them,

$$\alpha\eta((1-\alpha)\eta)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}} k'^{\frac{\eta-1}{1-(1-\alpha)\eta}} \mathbb{E}_{s',z'}\left[\left(\frac{s'z'}{w(z')^{(1-\alpha)\eta}}\right)^{\frac{1}{1-(1-\alpha)\eta}}\right] = \mathbb{E}_{z'}[r(z')+\delta]$$

$$k'^{\frac{\eta-1}{1-(1-\alpha)\eta}} = \frac{\mathbb{E}_{z'}[r(z')+\delta]}{\alpha\eta((1-\alpha)\eta)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}} \mathbb{E}_{s',z'}\left[\left(\frac{s'z'}{w(z')^{(1-\alpha)\eta}}\right)^{\frac{1}{1-(1-\alpha)\eta}}\right]}$$

$$k' = \left(\frac{\alpha\eta((1-\alpha)\eta)^{\frac{(1-\alpha)\eta}{1-(1-\alpha)\eta}} \mathbb{E}_{s',z'}\left[\left(\frac{s'z'}{w(z')^{(1-\alpha)\eta}}\right)^{\frac{1}{1-(1-\alpha)\eta}}\right]}{\mathbb{E}_{z'}[r(z')+\delta]}\right)^{\frac{1-(1-\alpha)\eta}{1-\eta}}.$$

Therefore, the first-best capital investment is

$$k' = (\alpha\eta)^{\frac{1-(1-\alpha)\eta}{1-\eta}}((1-\alpha)\eta)^{\frac{(1-\alpha)\eta}{1-\eta}}\left(\frac{\int\int\left(\frac{s'z'}{w(z')^{(1-\alpha)\eta}}\right)^{\frac{1}{1-(1-\alpha)\eta}} dH(s'|s)dG(z'|z)}{\int(r(z')+\delta)dG(z'|z)}\right)^{\frac{1-(1-\alpha)\eta}{1-\eta}},$$

which implies the optimal capital choice given a' as

$$k'^{*} = \max\left(\frac{a'}{1-\theta}, (\alpha\eta)^{\frac{1-(1-\alpha)\eta}{1-\eta}}((1-\alpha)\eta)^{\frac{(1-\alpha)\eta}{1-\eta}}\left(\frac{\int\int\left(\frac{s'z'}{w(z')^{(1-\alpha)\eta}}\right)^{\frac{1}{1-(1-\alpha)\eta}} dH(s'|s)dG(z'|z)}{\int(r(z')+\delta)dG(z'|z)}\right)^{\frac{1-(1-\alpha)\eta}{1-\eta}}\right).$$

B.4 Price of Share

The price of share described in (11) can be reformulated as

$$\chi p(a_t,k_t,s_t,z_t) = \mathbb{E}_t\Big[M(z_t,z_{t+1})\max\{(\chi d_{t+1} + \chi p(a_{t+1},k_{t+1},s_{t+1},z_{t+1})), \chi d_{t+1}^{x,pb}\}\Big],$$

where $d_{t+1} = \pi_{t+1}(a_{t+1},k_{t+1},s_{t+1},z_{t+1}) - f - a_{t+2}(a_{t+1},k_{t+1},s_{t+1},z_{t+1})$ and π_{t+1} is as described in (14). By cancelling out χ, it is reduced to

$$p(a_t,k_t,s_t,z_t) = \mathbb{E}_t\Big[M(z_t,z_{t+1})\max\{(d_{t+1} + p(a_{t+1},k_{t+1},s_{t+1},z_{t+1})), d_{t+1}^{x,pb}\}\Big]. \quad (25)$$

In the meantime,

$$
\begin{aligned}
&\mathbb{E}_t\Big[M(z_t,z_{t+1})V^{pb}(a_{t+1},k_{t+1},s_{t+1},z_{t+1})\Big]\\
&\quad=\mathbb{E}_t\Big[M(z_t,z_{t+1})\max\{\tilde{V}^{pb}(a_{t+1},k_{t+1},s_{t+1},z_{t+1}),V^{x}(a_{t+1},k_{t+1},s_{t+1},z_{t+1})\}\Big]\\
&\quad=\mathbb{E}_t\Big[M(z_t,z_{t+1})\max\{(1-\chi)d_{t+1}+\mathbb{E}_{t+1}\Big[M(z_{t+1},z_{t+2})V^{pb}(a_{t+2},k_{t+2},s_{t+2},z_{t+2})\Big],(1-\chi)d^{x,pb}_{t+1}\}\Big]\\
&\quad=(1-\chi)\mathbb{E}_t\Big[M(z_t,z_{t+1})\max\{d_{t+1}+\frac{1}{1-\chi}\mathbb{E}_{t+1}\Big[M(z_{t+1},z_{t+2})V^{pb}(a_{t+2},k_{t+2},s_{t+2},z_{t+2})\Big],d^{x,pb}_{t+1}\}\Big]
\end{aligned}
$$

which can be reformulated as

$$
\begin{aligned}
&\frac{1}{1-\chi}\mathbb{E}_t\Big[M(z_t,z_{t+1})V^{pb}(a_{t+1},k_{t+1},s_{t+1},z_{t+1})\Big]\\
&\quad=\mathbb{E}_t\Big[M(z_t,z_{t+1})\max\{d_{t+1}+\frac{1}{1-\chi}\mathbb{E}_{t+1}\Big[M(z_{t+1},z_{t+2})V^{pb}(a_{t+2},k_{t+2},s_{t+2},z_{t+2})\Big],d^{x,pb}_{t+1}\}\Big].
\end{aligned}
\tag{26}
$$

The condition $p(a_t,k_t,s_t,z_t)=\frac{1}{1-\chi}\mathbb{E}_t\Big[M(z_t,z_{t+1})V^{pb}(a_{t+1},k_{t+1},s_{t+1},z_{t+1})\Big]$ makes the Equation (25) and (26) equivalent. Therefore,

$$
\chi p(a,s,z)=\frac{\chi}{1-\chi}\mathbb{E}_t\Big[M(z,z')V^{pb}(a',s',z')\Big].
$$