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Notes on Consumption Theory

Consider a consumer (agent) who receives income y_1 in the first period and y_2 in the second period, where income is measured in consumption goods. Incomes y_1 and y_2 do not depend on what the consumer does and, hence, y_1 and y_2 are referred to as the consumer's endowment. Assume these values are known with certainty. The objective of the consumer is to choose how much to consume in each period to maximize his total utility:

$$U(c_1, c_2) = u(c_1) + \beta u(c_2)$$

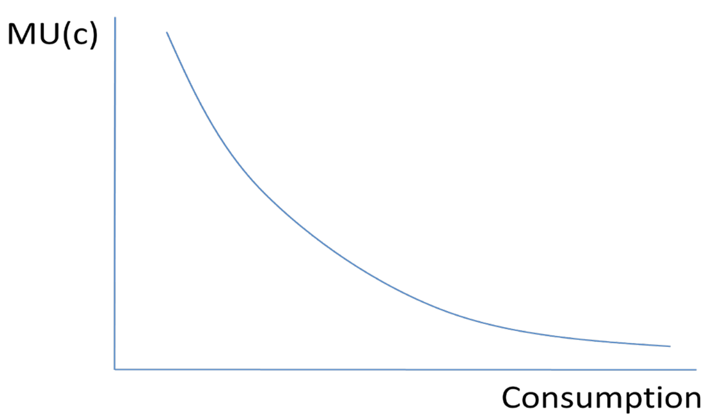
where c_i is his consumption in period i (either 1 or 2), $u(c_i)$ is the utility achieved from consumption in period i, and $U(c_1, c_2)$ is his lifetime utility. The parameter β is a constant term which determines how the consumer weighs utility in period 1 relative to period 2. For example, if $\beta = 1$, then utility in period 1 is weighted the same as utility in period 2. The agent is allowed to save or borrow in the first period, at interest rate *1+r*, but is not allowed to have any debt remaining at the end of the second period.

The utility function

We will make two assumptions about the agent's utility function:

1- More consumption is always preferred to less consumption
2- The agent has diminishing marginal utility

The first assumption simply implies that our consumer will always use up all of his income, since any unused income could have been used for consumption, which would have increased his utility. The second assumption implies that the agent receives less additional satisfaction with each additional unit of consumption. ***Marginal utility*** (MU) is the extra utility the agent gets from consuming one more unit. These two assumptions can be summarized as MU(1)>MU(2)>MU(3)>…>0 where the number in parentheses indicates the amount consumed. The first good brings the most extra utility, the second less, the third less than that, and so on, but each additional unit continues to make you at least somewhat better off than before. Graphically, this could be depicted as follows



The key question for the consumer is how to allocate his consumption over time to maximize utility. We can break the problem into 2 components. First, how does the consumer want his consumption to change across the two periods (the *tangency* condition)? Second, what is the maximum level of consumption available across the two periods (the *intertemporal budget constraint*)? Together, these two conditions will deliver the consumption and saving functions, which will tell us how much our agent will consume and save today given his total income, his utility *u* and time preference, and the interest rate.

1- The tangency condition

To determine the optimal timing of consumption, we will use a logical argument known as the perturbation method. The idea is as follows. Suppose we give our consumer one more unit of income in the first period. Our agent has two options.

a) Consume it today: if our agent consumes the extra income today, this will raise his total utility by the marginal utility of consumption in the first period: $MU(c_1)$.
b) Save it and consume in the second period: if our agent saves the extra income, next period he will have $1+r$ units to consume. If he then consumes the 1+r units, this will give him $(1+r)MU(c_2)$ extra utility next period. Finally, we want to know how he values this today, which means we have to discount it by the discount factor β. Thus, the perceived increase in utility today from saving the unit and consuming it next period is $\beta(1+r)MU(c_2)$.

Now consider what has to be true about a consumer who is maximizing his utility: the two options have to yield identical benefits. I.e., *a utility-maximizing consumer should be indifferent between the two options.* Mathematically, this can be expressed as:

$$MU(c_1) = \beta(1+r)MU(c_2)$$

which is known as the ***tangency condition***.

Why does this have to be true? Let's suppose it isn't. Specifically, suppose that $MU(c_1)>\beta(1+r)MU(c_2)$. This means the consumer derives more utility from consuming an extra unit today than from saving it and consuming it next period. But this means our agent could not have been maximizing his utility in the first place. This is because he could reduce his saving by one unit (thereby losing $\beta(1+r)MU(c_2)$ in utility) and consume that unit (thereby earning $MU(c_1)$ in extra utility). Because the return to the latter exceeds the return to the former, this would make him strictly better off. Thus, he could not have been maximizing his utility in the first place. So what should he do? He should keep reducing his saving and raising his current consumption. As he reduces his saving (and hence his consumption next period) the marginal utility of consumption in the second period would rise. As he increases his consumption today, his marginal utility of consumption in the first period will fall. Our agent should keep reallocating consumption to the first period until the tangency condition is restored, i.e. until the consumer is indifferent between consuming an extra unit today and saving an extra unit today.

Thus, the first key condition describing a utility-maximizing consumer who is free to borrow or save at an interest rate $1+r$ is the tangency condition. Looking at the tangency condition, you'll notice that the consumer's income is not in the equation. Instead, the tangency condition describes how the consumer wants his consumption to change across the two periods given the interest rate and the discount factor. It does not pin down the level of consumption, but rather the desired change. To see this more precisely, let's suppose that the marginal utility function is given by MU(c)=1/c, which corresponds to the logarithmic utility function. In other words, $u(c_1) = \log(c_1)$. In this case, the tangency condition can be written as:

$$\frac{1}{c_1} = \beta(1+r)\left(\frac{1}{c_2}\right)$$

$$\Leftrightarrow \frac{c_2}{c_1} = \beta(1+r)$$

If $\beta(1+r)$ is greater than one, then $c_2>c_1$ and our consumer wants consumption to be higher next period than it is today. If $\beta(1+r)$ is less than one, then $c_2<c_1$ and our consumer wants consumption to be lower next period than it is today. Finally, if $\beta(1+r)$ is exactly equal to one, then our consumer wants equal consumption in the two periods. Thus, we can see how the tangency condition is a description of the desired change in consumption across the two periods. However, it does not tell us exactly what the level of consumption should be. This requires a second condition, which links consumption with the consumer's income.

2- The intertemporal budget constraint

Now let's turn to the maximum level of consumption that our consumer could afford. We'll describe these using budget constraints. In the first period, our consumer receives income of y_1. With this income, he can do one of two things: consume (c_1) or save (s_1). The budget constraint for the consumer in the first period is simply

$$c_1+s_1=y_1$$

This just says that given income y_1 in the first period, the consumer will either consume or save all the goods (technically, the consumer could also leave some goods unused but we'll leave aside this scenario which cannot be optimal). Note that we could rewrite this as $s_1=y_1-c_1$, so that saving is the difference between income and consumption. If $s_1>0$, then income exceeds consumption and the consumer is saving. If $s_1<0$, then consumption exceeds income, and the consumer is borrowing.

In the second period, the consumer receives his endowment (y_2) plus interest income (or payments) from any saving (or borrowing) in the first period ($s_1(1+r)$) and consumes all this income. There can be no additional saving in the second period since the consumer is not alive for a third period and would be leaving resources unused. Therefore, the budget constraint in the second period is

$$c_2=y_2+s_1(1+r)$$

Note that this can be rearranged as $s_1=(c_2-y_2)/(1+r)$. If we plug this into the first period's budget constraint, we get $c_1+(c_2-y_2)/(1+r)=y_1$ which can be rewritten as

$$\boldsymbol{c_1+\frac{c_2}{1+r}=y_1+\frac{y_2}{1+r}}$$

This is known as the ***intertemporal budget constraint***. It equates the present discounted sum of consumption (the left-hand side) to the present discounted sum of income (the right hand side). It simply says that total consumption cannot exceed total income over the two periods.

The Consumption Function and the Savings Function

We now have two conditions that must be satisfied for an optimal consumption allocation: the tangency condition and the intertemporal budget constraint. Let's simplify the problem and assume a specific form for the marginal utility of consumption: $MU(c)=1/c$. Note that since $c>0$, $1/c>0$ as well. Also, as c rises, $1/c$ falls so the marginal utility of consumption is declining in the amount consumed. This formulation of marginal utility satisfies our assumptions.

Let's express the tangency condition given this expression for the marginal utility of consumption:

$$MU(c_1) = \beta(1+r)MU(c_2)$$

$$\Rightarrow \frac{1}{c_1} = \beta(1+r)\left(\frac{1}{c_2}\right)$$

$$\Leftrightarrow c_2 = \beta(1+r)c_1$$

Now let's plug this expression into the intertemporal budget constraint

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$

$$c_1 + \frac{\beta(1+r)c_1}{1+r} = y_1 + \frac{y_2}{1+r}$$

$$\boldsymbol{c_1 = \frac{1}{(1+\beta)}\left(y_1 + \frac{y_2}{1+r}\right)}$$

This last expression is the ***consumption function***. It tells us how much the consumer will spend in the first period given his endowments, the interest rate, and the discount factor β. Crucial things to note from the consumption function are that:

a) More income today (y_1) raises consumption today.
b) More income in the future (y_2) raises consumption today.
c) A higher interest rate ($1+r$) lowers consumption today.
d) A higher discount factor (β) lowers consumption today.

Suppose β=1, so that the consumer weighs current and future utility equally, then the consumption function is just

$$\boldsymbol{c_1 = \frac{1}{2}\left(y_1 + \frac{y_2}{1+r}\right)}$$

i.e. a simple average of current income and discounted future income.

From the consumption function, we can easily back-out the ***savings function***:

$$\boldsymbol{s_1 = y_1 - c_1 = \frac{\beta}{(1+\beta)}y_1 - \frac{1}{(1+\beta)}\left(\frac{y_2}{1+r}\right)}$$

<u>Consumption decisions with calculus</u>

Here we derive the same optimality conditions using calculus which simplifies the solution and makes it more general. Note that we can write the problem in the following form

$$\max\{\log(c_1) + \beta\log(c_2)\}$$

$$\text{subject to } \mathrm{c}_1 + \frac{\mathrm{c}_2}{1+\mathrm{r}} = \mathrm{y}_1 + \frac{\mathrm{y}_2}{1+\mathrm{r}}$$

Now express c_1 as a function of c_2, y_1 and y_2 and substitute into the objective function:

$$\max\left\{\log\left(y_1 + \frac{y_2}{1+r} - \frac{c_2}{1+r}\right) + \beta\log(c_2)\right\}$$

From calculus, we know that the necessary condition for a function to achieve maximum in a given point is that the derivate of this function is equal to zero at this point (this is called the first order condition). In our case, this condition is

$$\frac{1}{y_1 + \frac{y_2}{1+r} - \frac{c_2}{1+r}} \times \left(\frac{-1}{1+r}\right) + \frac{\beta}{c_2} = 0 \Rightarrow$$

$$c_2 = \beta(1+r)\left(y_1 + \frac{y_2}{1+r} - \frac{c_2}{1+r}\right) \Rightarrow$$

$$c_2 = \frac{\beta(1+r)}{1+\beta}\left(y_1 + \frac{y_2}{1+r}\right)$$

Using the budget constraint

$$c_1 = y_1 + \frac{y_2}{1+r} - \frac{c_2}{1+r} = \frac{1}{1+\beta}\left(y_1 + \frac{y_2}{1+r}\right)$$

Note that if r = 0 and $\beta = 1$, then

$$c_2 = c_1 = \frac{1}{2}(y_1 + y_2)$$

Theoretical Implications of the Consumption Function

A- Consumption Smoothing: Suppose the agent has the following endowment pattern: $y_1=0$ and $y_2=1$ and the 1+r=1 (for simplicity). Optimal consumption in each period is ½, i.e. the consumer seeks to smooth his consumption. If the consumer has the reverse endowment pattern: $y_1=1$ and $y_2=0$, *the optimal consumption path is identical*. This feature, which reflects diminishing marginal utility, is known as ***consumption smoothing***.

B- Permanent Income Hypothesis: Suppose income in each period has a permanent component and a transitory component: $y_t=y^p+y^T_t$ where y^p is the permanent component and y^T_t is the transitory component which is different every period. Now suppose we give the consumer an extra unit of the good in the first period, as a transitory boost to his income. Consumption smoothing says that the consumer should consume a little piece of that extra good each period, i.e. his consumption will go up by only a fraction of the change in income that period. Suppose instead the consumer gets a one unit increase in his permanent income that the consumer should consume a little piece of that extra good each period, i.e. his consumption will go up by only a fraction of the change in income that period. Suppose instead the consumer gets a one unit increase in his permanent income y^p which will last for all periods. Then his consumption should immediately and permanently rise by one unit each period. In other words, consumption should respond strongly to changes in permanent income, but very little to transitory changes in income: this is the ***Permanent Income Hypothesis*** (PIH) proposed by Milton Friedman.

C- Random Walk Hypothesis: The theory predicts that current consumption depends on current and expected future income. Suppose that you know today that your income will rise next year, then the PIH predicts that your current consumption embodies that information, i.e. there should be no change in your consumption next year when you actually get the increase in income. What then causes changes in consumption? News about current or future income prospects can be the only thing that change consumption. In other words, *changes in consumption should be unpredictable*. Suppose we add uncertainty to our tangency condition

$$MU(c_1) = (1+r)\beta E[MU(c_2)]$$

and suppose that $\beta(1+r)=1$. Then the tangency condition is given by

$$1/c_1 = E[1/c_2]$$

which is approximately equal to

$$E[c_2 - c_1] = 0$$

Consumption should follow a ***random walk*** (i.e., the best forecast for next period consumption is consumption in the current period: $E[c_2 - c_1] = 0 \Longrightarrow E(c_2) = c_1$), a hypothesis originally proposed by Bob Hall.

Aggregation of the Consumption and Savings Functions

Suppose we have N_A agents of type A, where type A refers to an endowment pattern (y_1^A, y_2^A) and N_B agents of type B with endowments (y_1^B, y_2^B). The *aggregate consumption function* is just the sum across all consumption functions

$$C_1 = N_A c_1^A + N_B c_1^B$$

$$= \left(\frac{1}{1+\beta}\right)\left[(N_A y_1^A + N_B y_1^B) + \frac{(N_A y_2^A + N_B y_2^B)}{(1+r)}\right]$$

$$= \left(\frac{1}{1+\beta}\right)\left[Y_1 + \frac{Y_2}{(1+r)}\right]$$

where capital letters indicate aggregate variables (i.e. $Y_1 = N_A y_1^A + N_B y_1^B$). Aggregate consumption depends all future income (W) and the interest rate: $\boldsymbol{C=C(W,r)}$. When income stream (W) rises, aggregate consumption rises. When interest rates rise, aggregate consumption falls. Note that in general, permanent income is based on many periods, not just 2 as assumed here, and W will be largely independent of current income Y. The key result is that the aggregate consumption function has the same properties has the individual's consumption function, but depends on aggregate income (both current and future) rather than individual income.

The *aggregate savings function* is defined as $S=Y-C(W,r)=S(Y,W,r)$. Hence, unlike aggregate consumption, the aggregate savings function depends directly on current income Y. Intuitively, if income today rises and we see little increase in consumption because of the PIH, then that income must be saved, causing aggregate savings to rise.

Empirical Tests (or Does This Theory Actually Work?)

A key implication of the PIH and random walk hypothesis is that predictable changes in income should have no effect on consumption, since they should already have been taken into account in consumption decisions. This is what allowed us to state that the aggregate consumption function was $C(W,r)$ which is largely independent of current Y.

Three tests:

1) Campbell and Mankiw (1989): First, they statistically pull out the predictable component of income changes. Second, they see whether these predictable changes in income are correlated with changes in consumption. Theory predicts no correlation, but they get positive correlation This is known as the *excess sensitivity of consumption puzzle*.
2) Shea (1995) looks at wage-earners who have long-term union contracts. These contracts contain predictable increases in income over time. Theory predicts consumption should not rise when their wages rise. Shea finds that, in fact, consumption does rise strongly with predictable increases in wages.
3) Johnson, Parker and Souleles (2004): The 2001 tax cut gave an immediate $300 rebate to individuals at different times, depending on social security numbers. They looked at whether consumption rose significantly for those individuals when they received their checks. They found that nearly all the amount was spent immediately, contrary to the predictions of the theory.

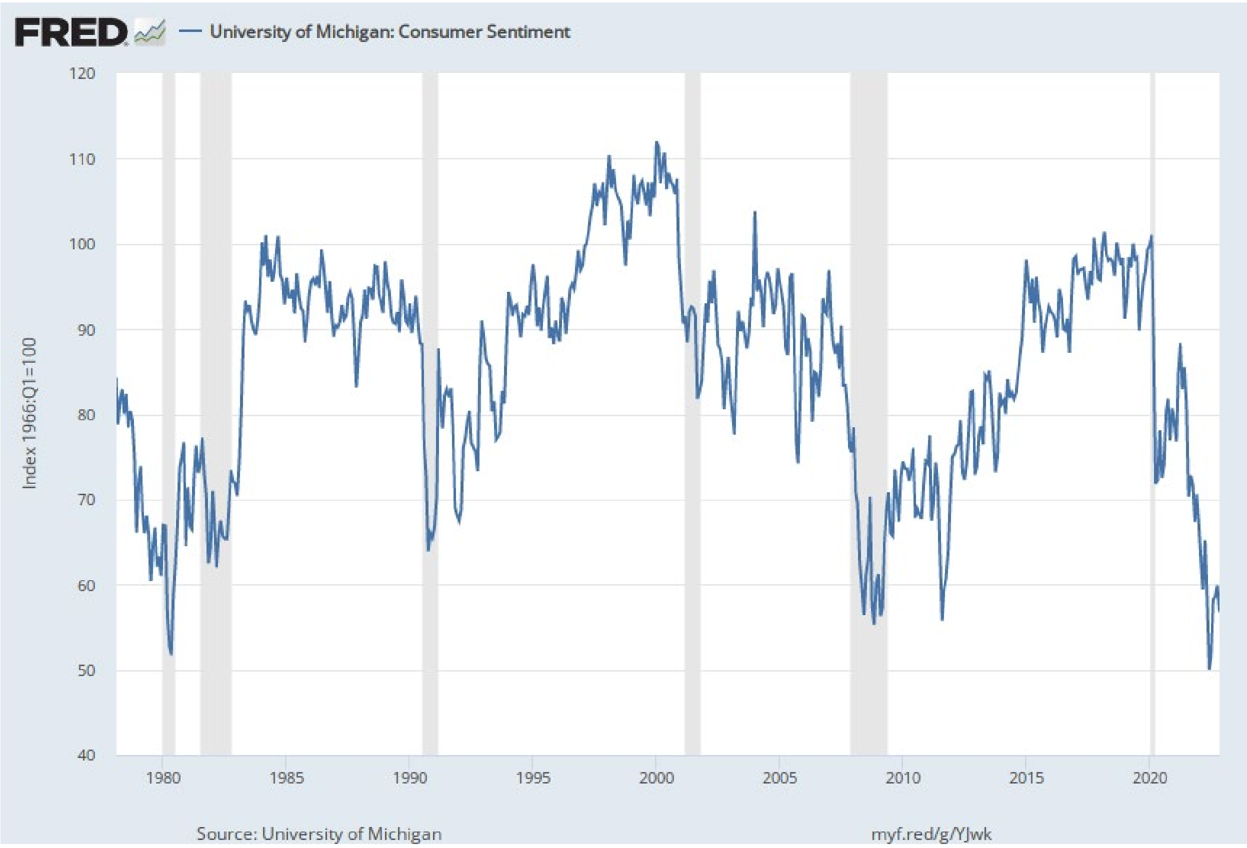
Potential explanations:

1) Some agents live "hand to mouth", i.e. are irrational and just spend their income.
2) Some agents are liquidity constrained, i.e. may be unable to borrow as much as they want against their future income.
3) Some agents simply may not have access to credit markets.

Thus, in general, for many consumers, the correct consumption function is $c_j=c_j(w_j,y_j,r)$. This means a more general aggregate consumption function should look like $\boldsymbol{C=C(Y,W,r)}$. When current income goes up by one dollar, the corresponding increase in consumption is referred to as the marginal propensity to consume (*mpc*). We will assume that $0<mpc<1$. Since the savings function comes from $S=Y-C(Y,W,r)$, then the general savings function is $\boldsymbol{S=S(Y,W,r)}$ which is increasing in Y (by $0<mpc<1$), decreasing in W, and increasing in r.

Expectations of Permanent Income and Consumer Sentiment

Note that permanent income W is not something that is readily observed, but rather is quite subjective. In our model, future income was known by assumption. In the real world, of course, there is always uncertainty about future income. Thus, what really matters for consumption is perceived or expected permanent income. Of course, this is difficult for economists to track. However, some measures of consumer confidence exist. For example, the University of Michigan has a monthly survey of consumers that asks them about what they think will happen to the economy in the future and to their incomes. This index allows us to track somewhat people's expectations of future income. We will henceforth refer to this variable as ***Consumer Sentiment*** (CS), reflecting the fact that W is an unknown and is affected by people's subjective expectations. The figure below illustrates how the Michigan index of consumer sentiment has varied over time.



To summarize, the aggregate consumption function that we will use in our model is:

$$C = C(Y - T, CS, r)$$

a) CS is our measure of consumer sentiment, or people's perceptions of their permanent income. We will treat this as an exogenous variable. An increase in CS is interpreted as an increase in perceived permanent income, leading to an increase in consumption, holding everything else constant.

b) Y-T is current after-tax disposable income. This enters the consumption function to be consistent with departures from the PIH apparent in the data. Thus, when current after-tax income rises, aggregate consumption will rise, holding everything else constant. Note that disposable income can rise if current income rises or if taxes decrease.

c) The interest rate r has a negative effect on aggregate consumption. Higher interest rates raise the incentive to save and hence tend to decrease consumption, holding everything else constant.

Extension 1: An Endowment Economy

We can use our description of the consumer's problem to consider a very stylized representation of the economy known as an endowment economy. Specifically, the economy consists only of consumers, who borrow from and lend to each other. Their income is determined exogenously, so we treat it as given. There is no investment. The idea is to solve for each agent's optimal consumption and saving functions, and use those to solve for the equilibrium interest rate. With this interest rate, we can study the actual consumption and saving outcomes.

Consider an economy with two types of agents: A and B. There are N agents of each type. A agents have an endowment pattern of $(y_{1A}, y_{2A}) = (0,1)$ while B agents have endowments of $(y_{1B}, y_{2B}) = (1,0)$. There is no investment or government spending. Let's solve for the consumption and savings functions for each type of agent. We'll then solve for the equilibrium interest rate, then finally solve for actual consumption allocations.

Part 1: Consumption and Savings functions

A agents have endowments of (0,1), so their consumption function is given by:

$$c_1^A = \frac{1}{2}\left(y_1^A + \frac{y_2^A}{1+r}\right) = \frac{1}{2}\left(0 + \frac{1}{1+r}\right) = \frac{1}{2}\left(\frac{1}{1+r}\right)$$

while their savings function is

$$s_1^A = y_1^A - c_1^A = 0 - \frac{1}{2}\left(\frac{1}{1+r}\right) = -\frac{1}{2}\left(\frac{1}{1+r}\right)$$

For the B agents, we can easily find that their consumption function is given by

$$c_1^B = \frac{1}{2}\left(y_1^B + \frac{y_2^B}{1+r}\right) = \frac{1}{2}\left(1 + \frac{0}{1+r}\right) = \frac{1}{2}$$

And their savings function is then just $s_1^B=y_1^B-c_1^B=1-1/2=1/2$.

Part 2: Solving for the equilibrium interest rate

Now recall the goods-market clearing condition: $Y=C+I+G+NX$. This economy is closed, so $NX=0$. There is no government, so $G=0$. We've also assumed there is no investment, so $I=0$. Thus, the goods market clearing condition is $Y=C$. So let's plug in for aggregate variables:

$$Y = C$$

$$Ny_1^A + Ny_1^B = Nc_1^A + Nc_1^B$$

$$0 + N = \frac{N}{2}\left(\frac{1}{1+r}\right) + \frac{N}{2}$$

$$\frac{1}{2} = \frac{1}{2}\frac{1}{1+r}$$

$$\mathbf{1} + \boldsymbol{r} = \mathbf{1}$$

Alternatively, we could use the savings market condition: $S=I$, combined with the fact that $I=0$ and each agent's savings function to get the same answer for the equilibrium interest rate. This is a good exercise.

Part 3: Solving for Consumption Allocations

We can now plug in the interest rate into the A agents' consumption function to get that $c_1^A=1/2$, the same as B's consumption in the first period.

General procedure to solve 2-period endowment economy models:

1) Solve for the consumption or savings functions of each agent, plugging in their endowments.
2) Use either the goods-market clearing condition (Y=C) or the savings market condition (S=0) to solve for the equilibrium interest rate.
3) Plug the equilibrium interest rate back into consumption and/or savings functions to solve for actual consumption allocations.

Extension 2: The Effects of Government Spending in the Endowment Economy

Let's now assume that there is a government that finances its spending with lump-sum taxes each period (T_1,T_2). Government spending each period is given by (G_1,G_2). For simplicity, let's assume that there is only one type of agent, and there are N of these agents. Assume also for simplicity that β=1. Government spending uses up resources and must be taken into account in the goods market clearing condition: C+G=Y.

Case 1: No government spending

Recall that the tangency condition for the consumer was, given that β=1 and MU(c)=1/c,

$$\frac{c_2}{c_1} = 1 + r$$

And the consumer's intertemporal budget constraint was given by

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$

Plugging in for c_2 yields the consumption function:

$$c_1 = \left(\frac{1}{2}\right)\left[y_1 + \frac{y_2}{1+r}\right]$$

The goods-market clearing condition in the first period is

$$Nc_1 = Ny_1$$

Or just c_1=y_1. Plugging this back into the consumption function and solving for the rate of return yields

$$1 + r = \frac{y_2}{y1}$$

Note that the higher is income in the second period relative to the first period, the higher is the equilibrium interest rate. This is because with high income in the future, consumers want to borrow against their future income to smooth their consumption. However, because there are no other agents willing to lend them resources, the interest rate must be high enough to dissuade consumers from borrowing as much as they would want to. In effect, the high interest rate exactly offsets the high future income, leaving the consumers satisfied with just consuming their income each period. Of course, the opposite would hold when current income exceeds future income: the interest rate would have to be very low to make consumers not want to save their current income.

Part 2: Government Spending

Now suppose we let the government spend some resources G_1 in the first period and G_2 in the second period. What the government spends resources on here is irrelevant. The important thing is that resources are being subtracted from those available for consumption.

The goods market clearing condition in the first period is now given by

$$Nc_1 + G_1 = Ny_1$$

And in the second period is

$$Nc_2 + G_2 = Ny_2$$

Dividing through by N and defining $g_t=G_t/N$ as government spending per capita, we get

$$c_1 = y_1 - g_1$$

$$c_2 = y_2 - g_2$$

Let's take the ratio of the two expressions:

$$\frac{c_2}{c_1} = \frac{y_2 - g_2}{y_1 - g_1}$$

Now let's use the tangency condition, which says that $c_2/c_1=1+r$ to get

$$1 + r = \frac{y_2 - g_2}{y_1 - g_1}$$

Note that if government spending in the future is less than current government spending ($g_2<g_1$), then the interest rate tends to be high. This reflects the fact that there are fewer goods available today, so consumers want to borrow against next period when more goods are available for consumption. To offset this, interest rates must be high, which makes consumers not want to borrow. The two effects must exactly cancel each other out, leaving consumers satisfied with their current consumption.

Part 3: Government Financing

Let's take into account the fact that the government needs to finance its spending with taxes T_1 and T_2. Let's derive the budget constraint. In the first period, the government can spend its tax revenue and borrow/lend the difference between the two:

$$G_1 + S_1^G = T_1$$

In the second period, the government cannot end with any debt, so its budget constraint is

$$G_2 = T_2 + S_1^G(1 + r)$$

Solving for government savings in the second period and plugging it into the budget constraint for the first period yields the government's intertemporal budget constraint:

$$G_1 + \frac{G_2}{1 + r} = T_1 + \frac{T_2}{1 + r}$$

which states that, like for consumers, the present discounted sum of government spending must equal the present discounted sum of government revenues.

Taxes also affect the consumer's budget constraint. Suppose that the taxes are "lump-sum", i.e. a fixed amount to be paid. Let's denote per-capita taxes by $\tau_t = T_t/N$. Then, the budget constraint for the consumer in each period is

$$c_1 + \tau_1 + s_1 = y_1$$

$$c_2 + \tau_2 = y_2 + s_1(1+r)$$

So solving for s_1 in the second expression, plugging it into the first and rearranging yields

$$c_1 + \frac{c_2}{1+r} = (y_1 - \tau_1) + \frac{(y_2 - \tau_2)}{1+r}$$

So that the present discounted sum of consumption must now equal the present discounted sum of *after-tax* income.

Budget Deficits and the Interest Rate

Common wisdom has it that budget deficits increase interest rates. This is based on the following idea: total saving consists of private saving and public (government) saving. Government saving is the difference between tax revenues and government spending. If the deficit increases, then government saving should fall, causing total saving to fall (holding private saving constant). Thus, the supply of savings falls (shifts back), which should cause the price of saving to rise. The price of saving is the interest rate.

In our model, this is not the case conditional on government spending. Let's hold government spending each period fixed at G_1 and G_2. We know the government's ITBC has to hold, but we can consider various tax regimes: all taxes in the first period, all taxes in the last period, or some tax in each period. If most taxes are in the second period, the government runs a deficit in the first period. How does this affect interest rates? It doesn't. Recall that the interest rate is given by

$$1 + r = \frac{y_2 - g_2}{y_1 - g_1}$$

which only depends on incomes and government spending. The timing of taxes here is irrelevant! This result is known as ***Ricardian Equivalence***. More formally, we say that under Ricardian Equivalence, *the equilibrium consumption levels and real rate of return are independent of the choice of government financing.* How can it be that a deficit today does not push the interest rate up? The answer is that consumer's savings decisions offset changes in public saving. Suppose the government cuts taxes today, causing a deficit. Then consumers know that taxes will have to rise later, so they save the extra income today to pay for their future tax burdens.

More formally, recall the consumer's ITBC

$$c_1 + \frac{c_2}{1+r} = (y_1 - \tau_1) + \frac{(y_2 - \tau_2)}{1+r}$$

which can be rewritten as

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} - \left(\tau_1 + \frac{\tau_2}{1+r}\right)$$

And recall the government's ITBC

$$G_1 + \frac{G_2}{1+r} = T_1 + \frac{T_2}{1+r}$$

which, divided through by N, can be written as

$$g_1 + \frac{g_2}{1+r} = \tau_1 + \frac{\tau_2}{1+r}$$

Plugging this into the consumer's ITBC yields

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} - \left(g_1 + \frac{g_2}{1+r}\right)$$

In other words, consumers should care only about the present discounted sum of government spending. As long as they can borrow/save freely in credit markets, the timing of taxes should be irrelevant to their decisions and also to the equilibrium interest rate.

Does Ricardian Equivalence Hold in Practice?

No. There are many reasons why this strong result does not actually hold. For example, people die. Taxes outside the span of their lifetimes will not have to be repaid by them, hence they will not take these into account. Secondly, most taxes are not lump-sum taxes but rather are distortionary: think of income taxes, capital gains taxes… Third, credit markets do not operate as perfectly as assumed here. First, people tend to face higher interest rates when they borrow than when they lend. Also, people face borrowing limits and some have no access to credit markets at all. Clearly, the strong assumptions of the model will tend to fail in the real world, and so will Ricardian Equivalence. However, the spirit of it may still hold: agents are forward-looking and will take into account the government's budget constraint in their decisions. A cut in taxes with no cut in government spending will need to be repaid at some point, and consumers will tend to save some of the cut in taxes to repay future taxes.