- The price of "consumption at time t" in terms of "consumption at time 1" is
  $1/(1+r)^{T-1}$
- In general, the value of a given path of consumption choices $\{c_1, c_2, \dots, c_T\}$ will be
  $$PV_c = p_1 c_1 + p_2 c_2 + \dots + p_T c_T$$
  $$= 1\, c_1 + \frac{1}{1+r} c_2 + \dots + \frac{1}{(1+r)^{T-1}} c_T$$
  $$= c_1 + \frac{1}{1+r} c_2 + \dots + \frac{1}{(1+r)^{T-1}} c_T$$
- If we generalize to N periods the value of our incomes in various periods (endowments): $\{w_1, w_2, \dots, w_T\}$ will be
  $$PV_w = p_1 w_1 + p_2 w_2 + \dots + p_T w_T$$
  $$= w_1 + \frac{1}{1+r} w_2 + \dots + \frac{1}{(1+r)^{T-1}} w_T$$
- If we have that consumption is the same across time, $(c_1 = c_2 = c_T = c)$, then
  $$PV_c = c + \frac{1}{1+r} c_2 + \dots + \frac{1}{(1+r)^{T-1}} c_T$$
  $$= c\left(1 + \frac{1}{1+r} + \frac{1}{(1+r)^{T-1}}\right)$$
- Given $PV_c = PV_w$:
  $$PV_w = PV_c = c\left(1 + \frac{1}{1+r} + \dots + \frac{1}{(1+r)^{T-1}}\right)$$
  $$c = \frac{w_1 + \frac{1}{1+r} w_2 + \dots + \frac{1}{(1+r)^{T-1}} w_T}{1 + \frac{1}{1+r} + \dots + \frac{1}{(1+r)^{T-1}}}$$
- If interest rate $= 0$ $(r=0)$, then:
  $$c = \frac{1}{T}(w_1 + w_2 \dots + w_T)$$

- Maximum is when derivative is equal to 0
  x^* is a max if $f'(x^*) = 0$
- ex) 1) Derivative of functions
  1) $f(x) = 2x^{4} \Rightarrow 8x^3$
  2) $f(x) = 12x + x^5 \Rightarrow 12 + 5x^4$
  3) $f(x) = (4x-3)(1-2x) \Rightarrow 4(1-2x) + (4x-3)(-2)$
  4) $f(x) = (2x-1)/(3x+5) \Rightarrow 2/(3x+5) - 3(2x-1)/(3x+5)^2$
  2) utility function $u(x,y)$, solve for marginal utility $MU(x) = du/dx$
  1) $u(x,y) = 2x + y \Rightarrow MU(x) = 2$
  2) $u(x,y) = 2y \Rightarrow MU(x) = 2y$
  3) $u(x,y) = 4xy^2 \Rightarrow MU(x) = 4y^2$
  4) $u(x,y) = (3x^2-1)^2(2y-5) \Rightarrow MU(x) = 2(3x^2-1)(6x)(2y-5)$
  3) Take total derivatives to decompose changes in Y into N, K, A
  1) $Y = AKN \Rightarrow dY = KN\,dA + AN\,dK + AK\,dN$
  2) $Y = AK^{\alpha} N^{1-\alpha} \Rightarrow dY = K^{\alpha} N^{\alpha} dA + \alpha A K^{\alpha-1} N^{\alpha} dK + (1-\alpha) A K^{\alpha} N^{-\alpha} dN$
  4) Find the value of N that maximizes profits
  1) P is exogenous, $Y = N^{\alpha}$, w is exogenous
  $$N = (w/\alpha P)^{1/(\alpha-1)}$$
  2) $P = Y^{-\gamma}$, $Y = N$, and w is exogenous
  $$N = (w/\alpha(1-\gamma)P)^{1/(\alpha-1)}$$

- Tangency condition:
  a) Consume it today:
  $$MU(c_1)$$
  b) Save it and consume second period:
  $$MU(c_1) = \beta(1+r)\,MU(c_2)$$
  ex) If $MU(c) = 1/c$, then tangency condition is:
  $$u(c_1) = \log(c_1)$$
  $$\frac{1}{c_1} = \beta(1+r)\left(\frac{1}{c_2}\right)$$
  $$\frac{c_2}{c_1} = \beta(1+r)$$

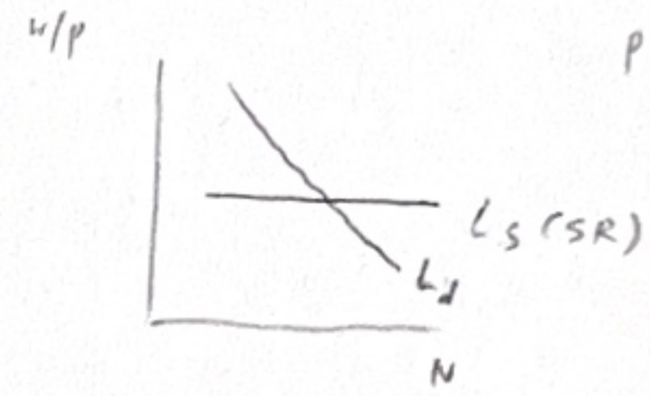
  → if $\beta(1+r) > 1$: $c_2 > c_1$ and our consumer wants consumption to be **higher** next period.
  if $\beta(1+r) < 1$: $c_2 < c_1$, our consumer wants consumption to be lower
  if $\beta(1+r) = 1$: **equal** consumption

- Intertemporal Budget Constraint
  a) First period:
  $$c_1 + s_1 = Y_1$$
  b) Second period:
  $$c_2 = Y_2 + s_1(1+r)$$
  c) IBC:
  $$c_1 + \frac{c_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$$
- Consumption Function
  1) Tangency condition
  ↓
  IBC → simplify
  ex) $MU(c) = 1/c$
  $$c_1 = \frac{1}{(1+\beta)}\left(Y_1 + \frac{Y_2}{1+r}\right)$$

Ricardian Equivalence:
Consumers are looking forward to and can internalize the government's budget constraint.
If taxes are low, save.

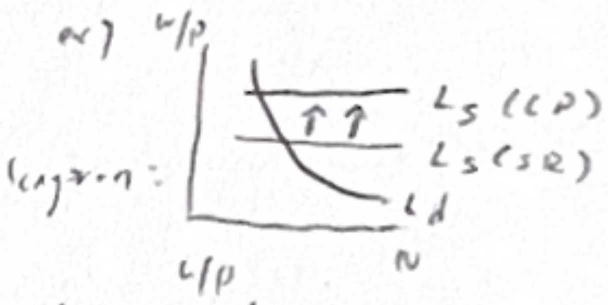
Agent i has consumption function:
$$c_1^i = \frac{1}{1+\beta}\left(Y_1^i + \frac{Y_2^i}{1+r}\right)$$

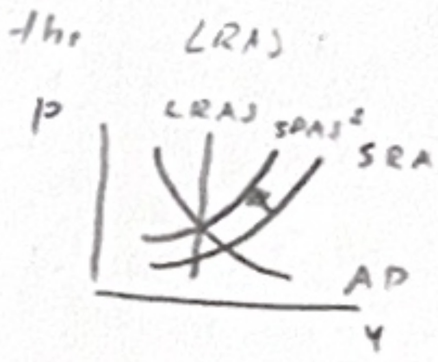
AS/AD



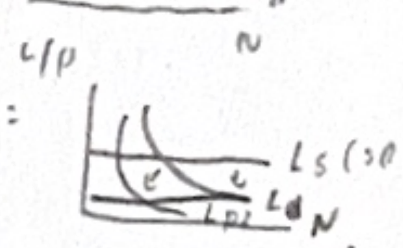
- always goes towards the LRAS:

ex) long run:





short run:



a) Increase of M: (money supply)

	P	Y	r	I	C	N	w/p
Short Run	+	+	−	+	+	+	−
Long Run	+	0	0	0	0	0	0

b) Increase of G: (government spending)

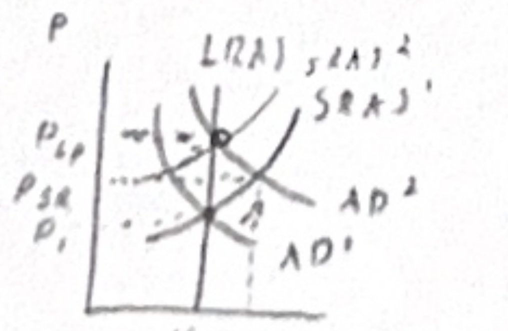
	P	Y	r	I	C	N	w/p
Short Run	+	+	+	−	?	+	−
Long Run	+	0	+	−	−	0	0

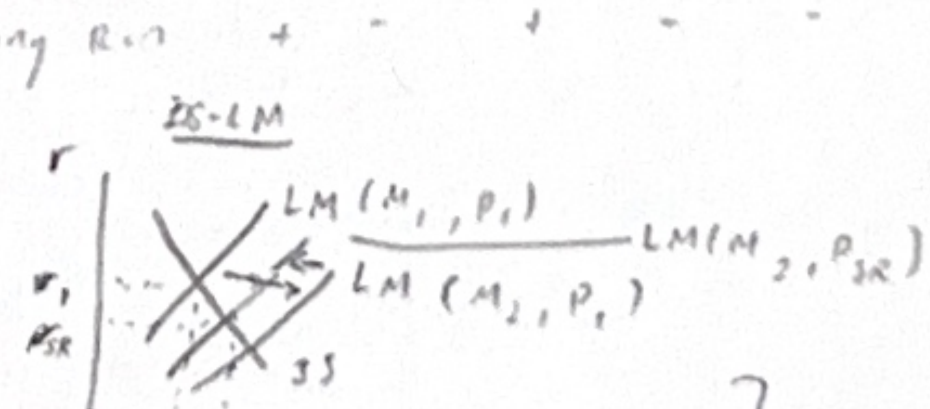
c) Increase of markups (μ):

	P	Y	r	I	C	N	w/p
Short Run	+	−	+	−	−	−	−
Long Run	+	−	+	−	−	−	0

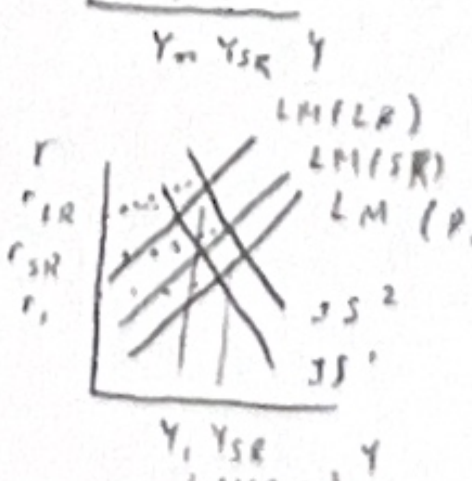
IS-LM

a) M ↑ ⇒

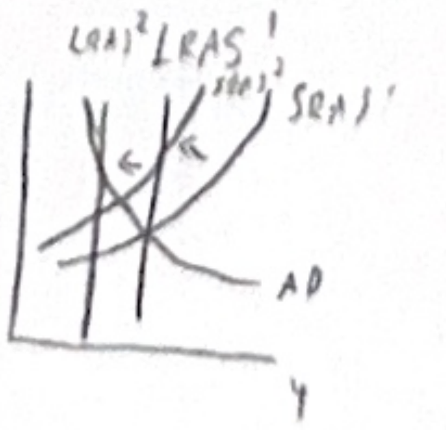


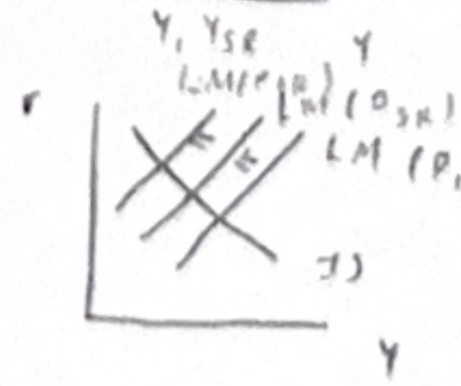


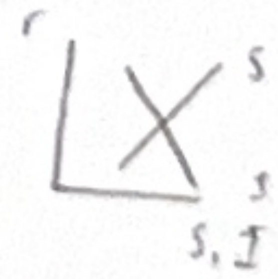
b) G ↑ ⇒
↓
'' ''



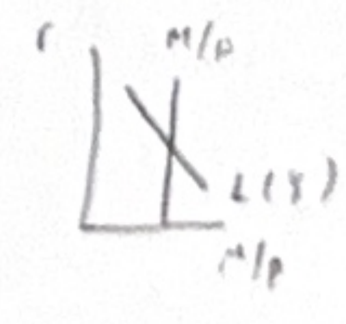
c) μ ↑ ⇒







moves in same direction
SCS ⇒ LM



moves in direction
LM ⇒ M/p line