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The IS-LM Model and Aggregate Demand

These notes will construct our model of aggregate demand. This will be the standard "IS-LM" approach to aggregate demand, based on Keynes and Hicks. Essentially, we will describe two important macroeconomic markets: the market for savings and the market for money holdings. We will consider each market independently, taking the level of output as exogenous in each market but solving for the equilibrium interest rate in each market. We will then combine the two markets, using the IS-LM approach, to determine the equilibrium level of output and interest rates that ensure that both markets clear.

Part 1: The Savings (Loanable Funds) Market

This is the market for savings and investment. We will examine what the supply and demand for savings are, as well as to characterize how the price of savings (the real interest rate) is determined.

The supply of savings

Total saving in the economy can be divided into two components: public and private saving. ***Private saving*** S_p is the saving done by consumers and is equal to after-tax income minus consumption:

$$S_p=Y-T-C.$$

Public saving S_g is the difference between tax revenues and government expenditures

$$S_g= T-G.$$

Total saving S is the sum of public and private saving, and so

$$S=S_p+S_g=(Y-T-C)+(T-G)=Y-C-G.$$

Note that it appears that total savings does not directly depend on taxes. However, to the extent that changes in taxes can affect consumption or output, total saving will in general change with exogenous changes in taxes.

Recall from our work on consumption that aggregate consumption could be described by a function of current after-tax income, consumer sentiment (as a proxy for permanent income), and the interest rate:

$$C=C(Y-T,CS,r).$$

Hence, plugging this into our expression for total saving, we get

$$S=Y-C(Y-T,CS,r)-G.$$

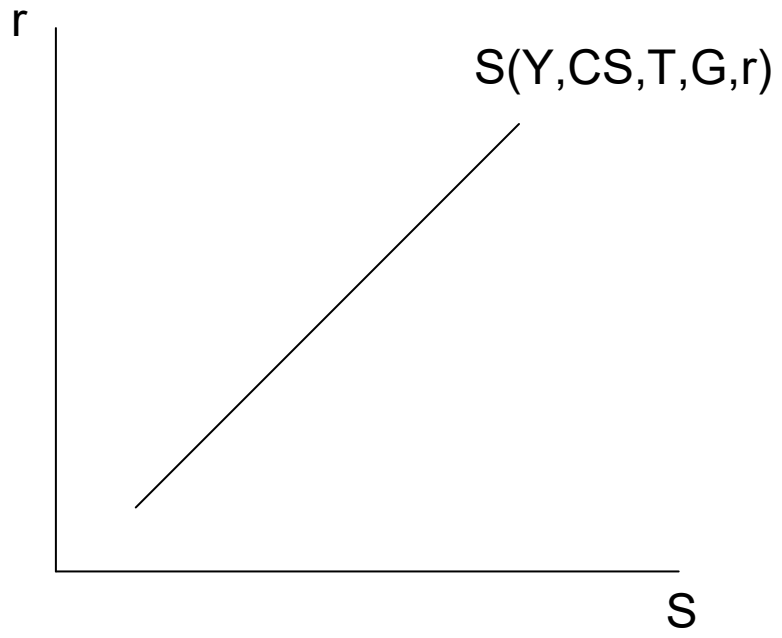
Recall that consumption is increasing in after-tax income *Y-T* and consumer sentiment *CS* but decreasing in the real interest rate *r*. Hence, aggregate savings must be increasing in *r* but decreasing in *CS*. How does an increase in *Y* affect savings? An increase in *Y* has a direct positive effect on saving (since it is on the RHS of the expression above) but it also increases consumption which tends to decrease savings.

We will refer to the increase in consumption from a $1 change in after-tax income the ***marginal propensity to consume*** (*mpc*). Suppose the *mpc* is 0.8, then if income goes up by a dollar, consumers will tend to raise their consumption by $0.80. From the permanent income hypothesis, we know that transitory changes in income have small (less than one-for-one) effects on consumption, which tells us that as long as the change in Y is not permanent, then it must be that *0<mpc<1*. Consumption rises when income rises but by less than one-for one. Therefore, when Y goes up by a dollar, consumption goes up by less than a dollar, which means that total savings must rise. The amount by which savings rises is known as the ***marginal propensity to save*** (*mps*). Since income is either consumed or saved, it must be that *mps=1-mpc*.

We can then summarize all these effects via a general ***savings function:***

$$S = S\left(\overset{+}{Y}, \overset{+}{T}, \overset{-}{CS}, \overset{-}{G}, \overset{+}{r}\right)$$

where + or – indicates whether an increase in that variable raises or lowers total savings. This implies that we can plot the supply of savings as a function of the price of savings (r) as follows:



Note that the supply curve for savings is upward sloping. This reflects the fact that as the interest rate rises, the incentive for consumers to save rises. Note also that increases in *Y* and *T* and decreases in *CS* and *G* all *shift* the supply of savings curve to the right.

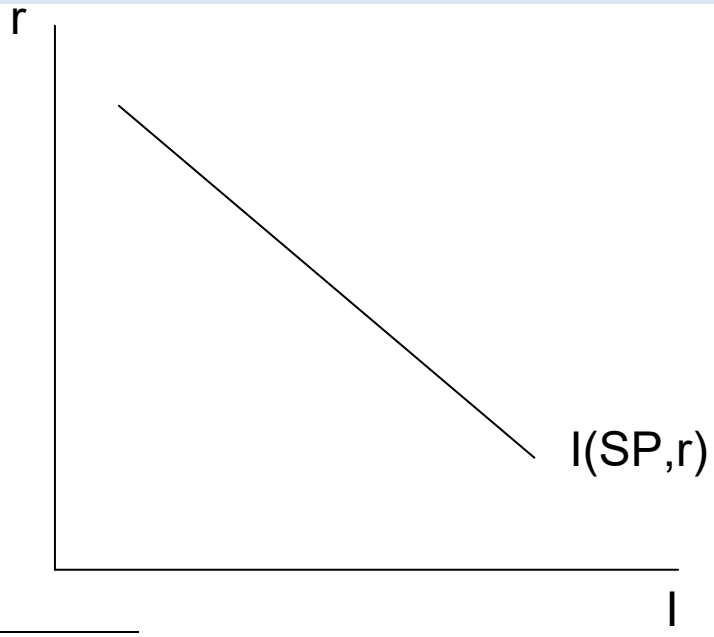
The demand for savings: Investment

When a consumer saves, he/she places money in the bank in exchange for the promise of getting their money back, with interest, in the future. However, the bank does not hold on to this money but instead immediately lends it out.[1] The people who borrow this money represent the demand for savings. In general, these loans are taken out for two reasons: purchasing houses and business investment (i.e., factories, machines, trucks, etc.). Both of these qualify as investment according to GDP definitions. Thus, we can think of investment as the demand for savings.

Recall that in the notes on investment, we derived the following ***investment function*:**

$$I = I\left(\overset{+}{\widetilde{SP}}, \overset{-}{\widetilde{r}}\right)$$

The variable SP captured "animal spirits", or the optimism of firms about future profits from investment. Higher levels of SP tend to lead to greater investment, ceteris paribus. An increase in the interest rate raises the opportunity cost of investment and so, holding all else constant, tends to lower investment. This inverse relationship between investment and the real interest rate gives us the demand for savings:



[1] We will talk more about the banking system and how deposits get lent out in the section on the money market.

Equilibrium in the savings market

To understand how the loanable funds market clears, we can use the national income accounting identity:

$$Y = C + I + G + NX$$

which can be rearranged as

$$Y - C - G = I + NX.$$

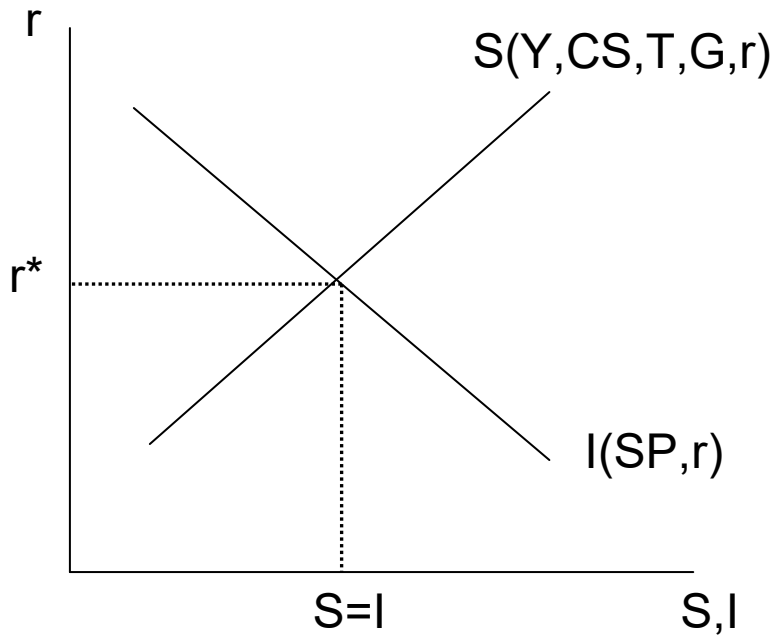
Note that the left-hand side is equal to total supply of savings:

$$S = I + NX$$

which tells us that, mechanically, savings must equal investment plus net exports. For now, we will focus on a closed economy in which net exports are always zero. Therefore, we must have that

$$\boldsymbol{S = I}.$$

Savings must be equal to investment. For this to be true in our model, the interest rate must adjust to the point where the savings and investment curves intersect, as shown below. We will refer to this as *r**.



Part 2: The Money Market

The money market consists of the supply and demand for real money balances. Money balances are the amount of liquid money in the economy. Money here can be defined in different ways, but for our purposes, we can think of it as the amount of cash floating around the economy plus deposits in banks. We will denote the stock of money with *M*. However, for the purposes of this market, we will focus on the real money stock: *M/P*. The real money stock is important because it tells us about the amount of money available relative to the price of goods: what money is used to buy.

Demand for Real Money Balances

When thinking about the demand for money, what we are talking about is how much of one's assets one wishes to hold in the form of cash or bank deposits. People need to hold cash to be able to purchase goods. When goods are expensive, you need more money to buy the same good. By focusing on real money balances, *M/P*, we are focusing on people's desire to hold cash after controlling for the price of goods. We will refer to the demand for real money balances as $(M/P)^d$.

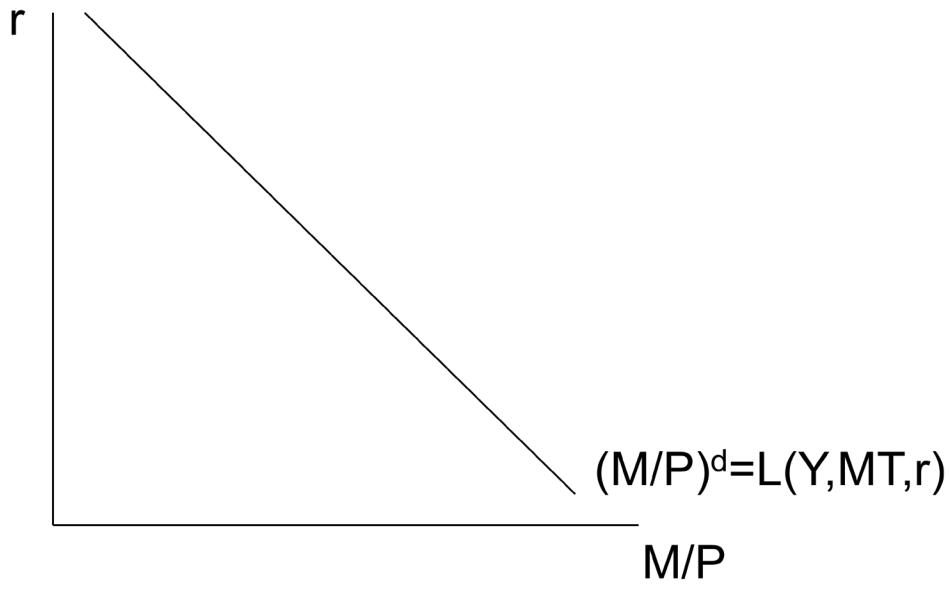
The three key factors that affect the demand for real money balances are income and the interest rate.

- *Y*: Income matters because the richer people are, the more they will be able to buy. But buying more goods means engaging in more transactions, which requires more money. Hence, *as income goes up, people will want to hold more cash*.
- *r*: The interest rate is also important because it represents the opportunity cost of holding money. When you keep money in your wallet, you are giving up the interest that you would have earned had you stored that wealth in an interest-bearing account. *The higher the interest rate, the greater the loss from holding cash, the less cash people will want to hold*.
- *MT*: Technologies related to money demand. People's desire to hold money, given the interest rate and income, also depends on how easy it is to access money holdings. For example, the presence of ATM machines means that people can walk around with less cash in their pockets since it will be easier for them to get cash when they need it. This tends to make people want to keep less of their assets in the form of cash. We'll assume that *improvements in "money technologies" (i.e. MT goes up) reduces the demand for money, holding all else constant*.

Hence, we can summarize the ***demand for real money balances*** using the following function:

$$(\boldsymbol{M/P})^{\boldsymbol{d}} = \boldsymbol{L}\left(\overset{+}{\boldsymbol{Y}}, \overset{-}{\boldsymbol{MT}}, \overset{-}{\boldsymbol{r}}\right)$$

Graphically, this can be depicted as a demand curve where the price of holding money (the opportunity cost) is the interest rate.



Note that the demand curve for real money balances is downward-sloping because higher interest rates imply that people are giving up greater rates of return by holding cash (rather than interest-bearing assets) and therefore people want to hold less cash when interest rates are high. Note also that an increase in income (*Y*) will shift money demand to the right, while an increase in money technology (*MT*) will shift the money demand curve to the left.

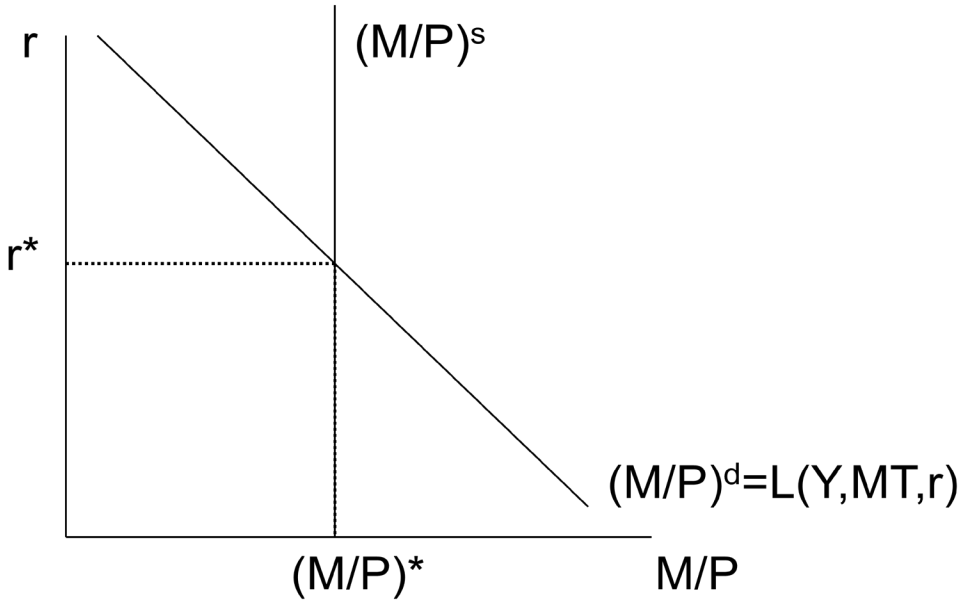
Supply of Real Money Balances

The central bank of the economy (the Federal Reserve Bank for the U.S.) controls the amount of money in the economy. This is done via *open-market operations*. We will study this more closely later in the class. Thus, like government spending, we can treat the amount of money supplied as an exogenous variable. The supply of real money balances $(M/P)^s$, is therefore independent of the interest rate. Graphically, we can be represented as a vertical line in the graph below. Note that when the price level goes up, the real money supply (*M/P*) goes down unless the central bank increases the nominal money

supply (M) by a proportional amount. Hence, increases in the price level, holding all else constant, shift the real money supply curve to the left.

Equilibrium in the Money Market

For real money supply to equal real money demand, the interest rate must adjust to r^*.



Summary

At this point, we've presented two markets in isolation: the money market and the loanable funds market. In each case, we had the interest rate on the Y-axis and said that the interest rate would adjust to ensure an equilibrium outcome. But if the interest rate adjusts to clear one market, how can there be an equilibrium in the other market? The solution lies in the fact that, in each market, we've treated the level of output Y as exogenous. In fact, both output and interest rates will adjust to ensure that equilibrium in both markets occurs.

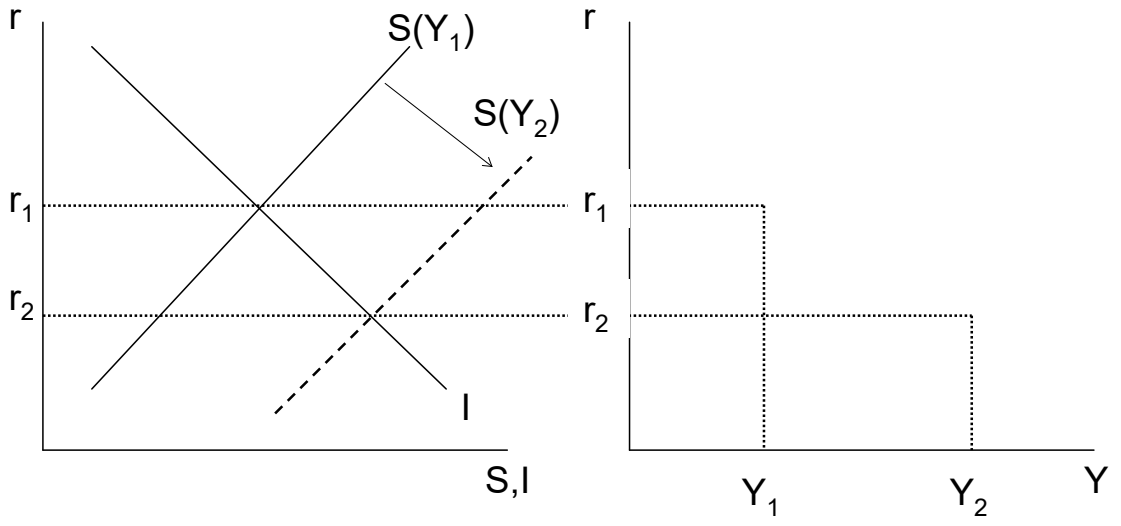
The IS-LM Model of Aggregate Demand

How can we have an equilibrium in both markets if the interest rate is the only variable adjusting to clear each market? The answer is that the interest rate is *not* the only endogenous variable adjusting to clear the market. Total income *Y*, which we treated as exogenous in each case, is also an endogenous variable in the economy. Importantly, total income affects both markets: the supply of savings in the loanable funds market and the demand for real money balances in the money market.

The IS-LM approach to aggregate demand consists of two curves: IS and LM. The IS curve is going to tell us: for a given level of *Y*, what is the interest rate that clears the saving market. The LM curve will tell us: for a given level of *Y*, what is the interest rate that clears the money market. The equilibrium outcome will be the level of *both* Y and r such that both markets are in equilibrium.

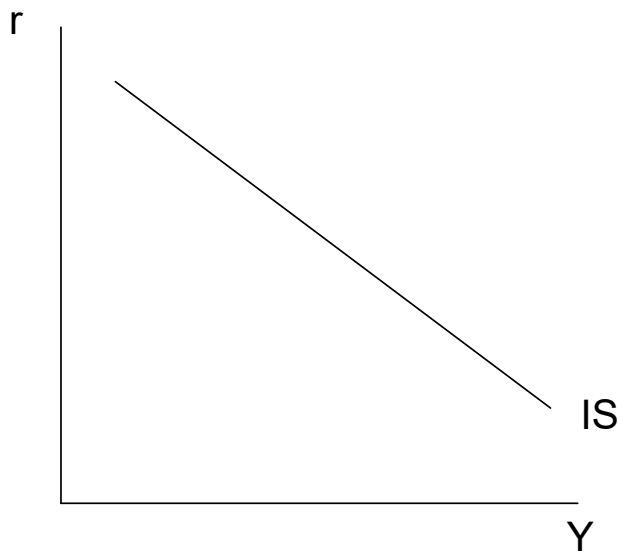
The IS Curve:

Consider first the loanable funds market. The graph for the loanable funds market is shown below, on the left, with another graph to the right which has the interest rate on the Y-axis and total income on the X-axis.



Suppose income is initially equal to Y_1. Then, total savings is given by $S(Y_1)$, as shown on the graph. The interest rate that clears the savings market given Y_1 is r_1. This combination (Y_1,r_1) is a single point on the right-hand side graph. Now consider what happens when income rises to Y_2. An increase in income must shift the supply of savings curve to the right, as shown in the graph. This increase in the supply of

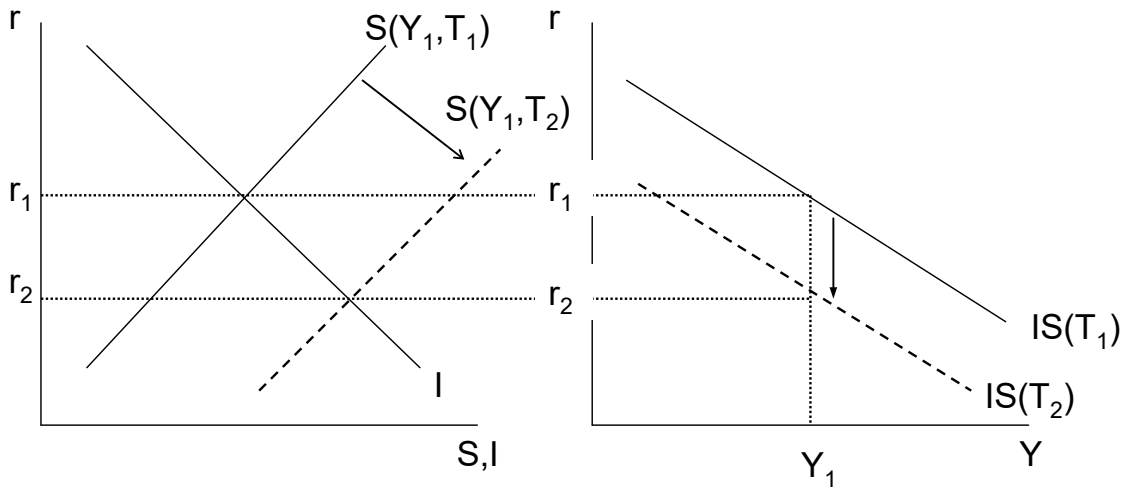
savings pushes the equilibrium interest rate down to r_2. This new interest rate-income combination (Y_2,r_2) is another point on the diagram on the RHS. Note that with a higher Y, we get a lower interest rate. In other words, with greater income comes higher saving and lower interest rates. We can generalize this as the following IS curve:



The IS Curve gives the interest rate that clears the loanable funds market for each possible level of income.

Shifts in the IS Curve:

Having now derived the IS curve, we can now ask how changes in exogenous variables will shift the IS curve around. Consider, for example, the case of an increase in taxes, illustrated in the figure below. The graph to the left shows the loanable funds market and the one to the right is the IS curve. Suppose we start off with income Y_1, then r_1 is the equilibrium interest rate when taxes are T. Now suppose taxes go up to T_2. We know that this tends to increase savings by shifting the supply of savings curve to the right. The new equilibrium interest rate is r_2, lower than the original. In terms of the IS curve, this means that for the same level of income, Y_1, it now takes a lower interest rate (r_2) to clear the savings market. Hence, the IS curve has shifted down. Note that the same results would hold if G or CS went down.



Another exogenous change we could consider is that of animal spirits. If animal spirits go up (*SP*), then the IS would shift up. The reason is that the increase in animal spirits increases the demand for investment, thereby shifting the investment demand curve up. This would lead to a higher equilibrium interest rate. Thus, for the same level of income, it would now take a higher interest rate to clear the market, hence the IS curve would shift up.

We can summarize the effects of changes in exogenous variables on the IS curve using the following function:

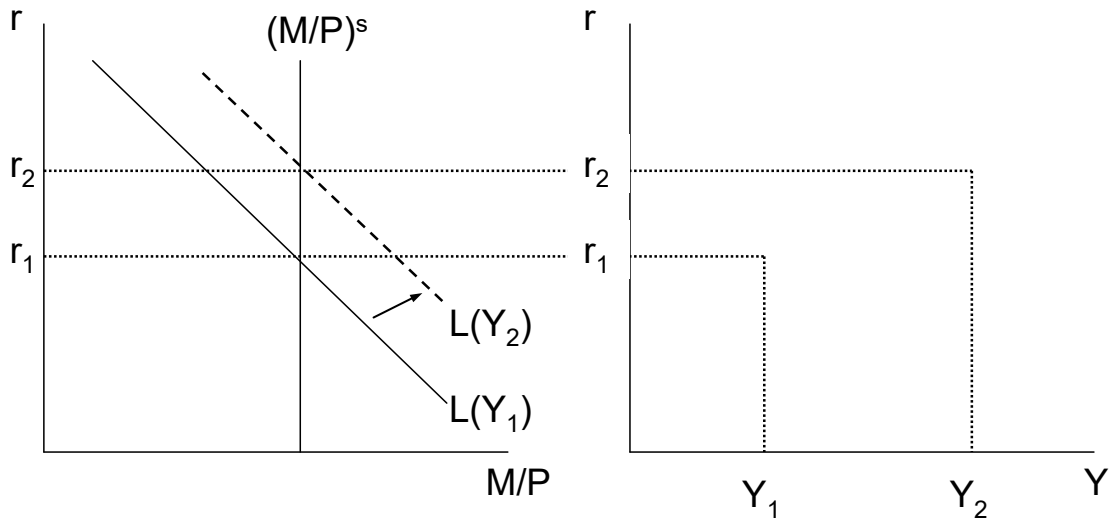
$$\boldsymbol{IS} = \boldsymbol{IS}(\overset{+}{\widetilde{\boldsymbol{CS}}}, \overset{-}{\widetilde{\boldsymbol{T}}}, \overset{+}{\widetilde{\boldsymbol{G}}}, \overset{+}{\widetilde{\boldsymbol{SP}}}, \overset{-}{\widetilde{\boldsymbol{r}}})$$

where the + or – indicates whether an increase in the variable shifts the IS curve to the right (+) or to the left (-).

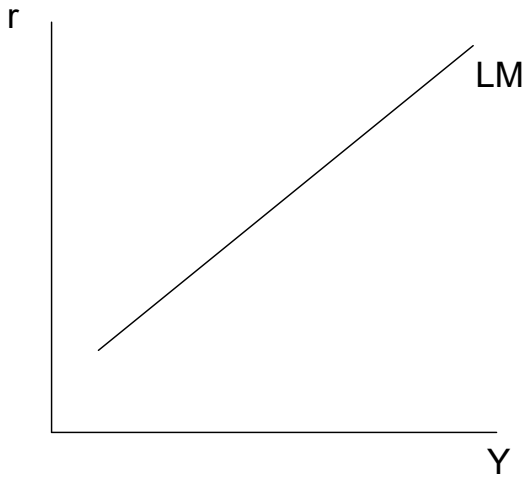
Important caveat: *changes in r do not shift the IS curve, they are movements along the IS curve!*

The LM Curve:

We can now go through exactly the same process with the money market. The graph for the money market is shown below, on the left, with another graph to the right which has the interest rate on the Y-axis and total income on the X-axis.



Suppose income is initially equal to Y_1. Then, total money demand is given by $(M/D)^d=L(Y_1)$, as shown on the graph. The interest rate that clears the money market given Y_1 is r_1. This combination (Y_1,r_1) is a single point on the right-hand side graph. Now consider what happens when income rises to Y_2. An increase in income increases the demand for real money balances, shifting the money demand curve right, as shown in the graph. This increase in the demand for money balances pushes the equilibrium interest rate up to r_2. This new interest rate-income combination (Y_2,r_2) is another point on the diagram on the RHS. Note that with a higher Y, we get a higher interest rate. In other words, with greater income comes a greater demand for cash to purchase more goods. Because the money supply is fixed, the interest rate rises and makes people want to hold less money. We can generalize this as the following LM curve:

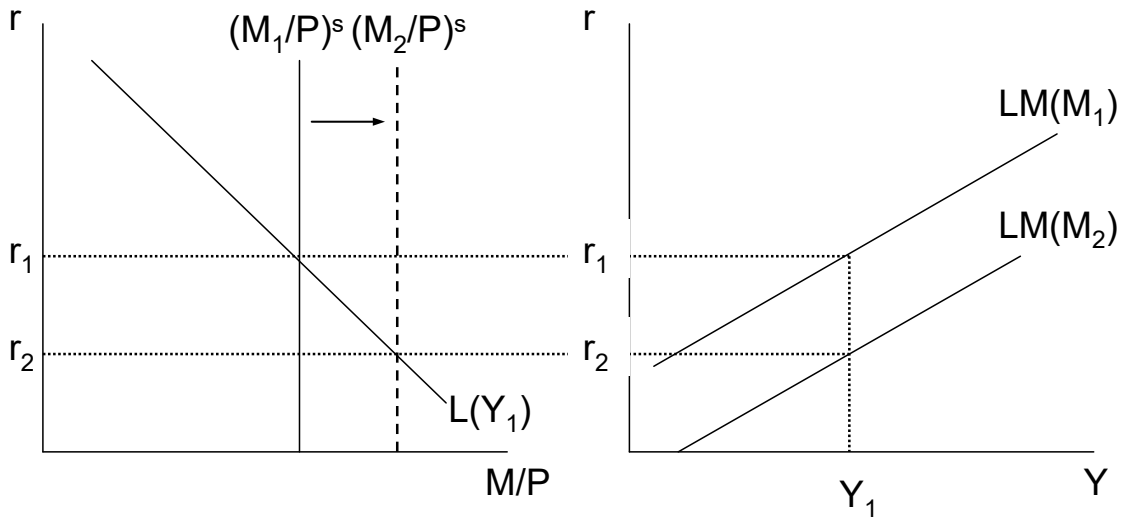


The LM Curve gives the interest rate that clears the money market for each possible level of income.

Shifts in the LM Curve:

Now that we've derived the LM curve, we want to know how it depends on exogenous variables. To figure this out, we can run through the same exercise as before: for a given level of income Y_1, we need to figure out whether a change in the exogenous variable raises or lowers the equilibrium interest rate. This determines whether the LM curve shifts up or down.

Suppose we are initially given income Y_1, and the money supply is M_1. The equilibrium interest rate that makes money supply equal to money demand is r_1. Now suppose the money supply goes up to M_2. Then the real money supply curve shifts to the right. This pushes down the equilibrium real interest rate to r_2. This means that for the same income level Y_1, it now takes a lower interest rate to clear the money market. This means the LM curve must now be directly under the original one.



Note that exactly the same pattern would be true if instead of looking at an increase in the money supply M, we considered a decrease in the price level P. In each case, real money supply M/P goes up, shifting the real money supply curve to the right, pushing the interest rate down, and therefore shifting the LM curve to the right.

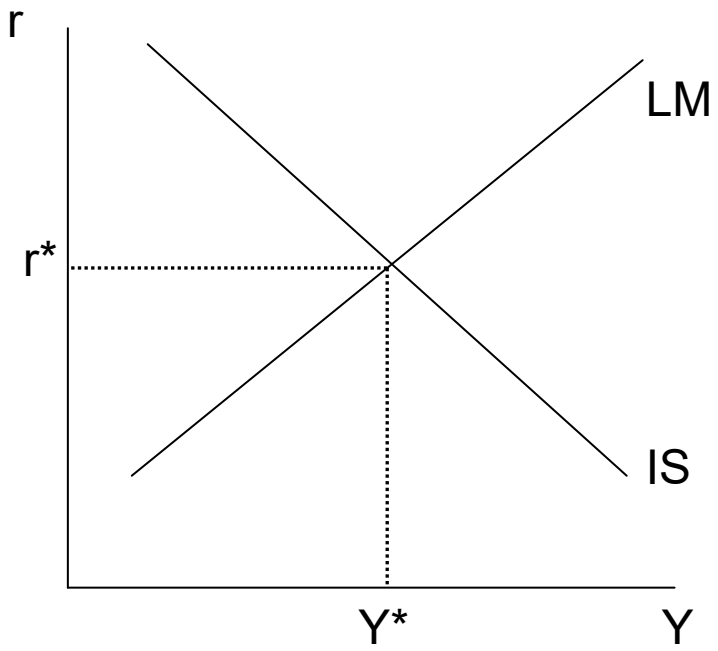
Thus, we can summarize changes in the LM curve using the following function:

$$\boldsymbol{LM = LM}\left(\overset{+}{\tilde{\boldsymbol{M}}}, \overset{-}{\tilde{\boldsymbol{P}}}, \overset{+}{\widetilde{\boldsymbol{MT}}}, \overset{+}{\tilde{\boldsymbol{r}}}\right)$$

where a + indicates that an increase in the variable shifts the LM curve to the right, *except for the real interest rate which cause movements along the LM curve.*

Combining the IS and LM Curves:

Now that we've summarized equilibrium combinations of income and interest rates for each market, we can put both curves on the same graph. This is done below:

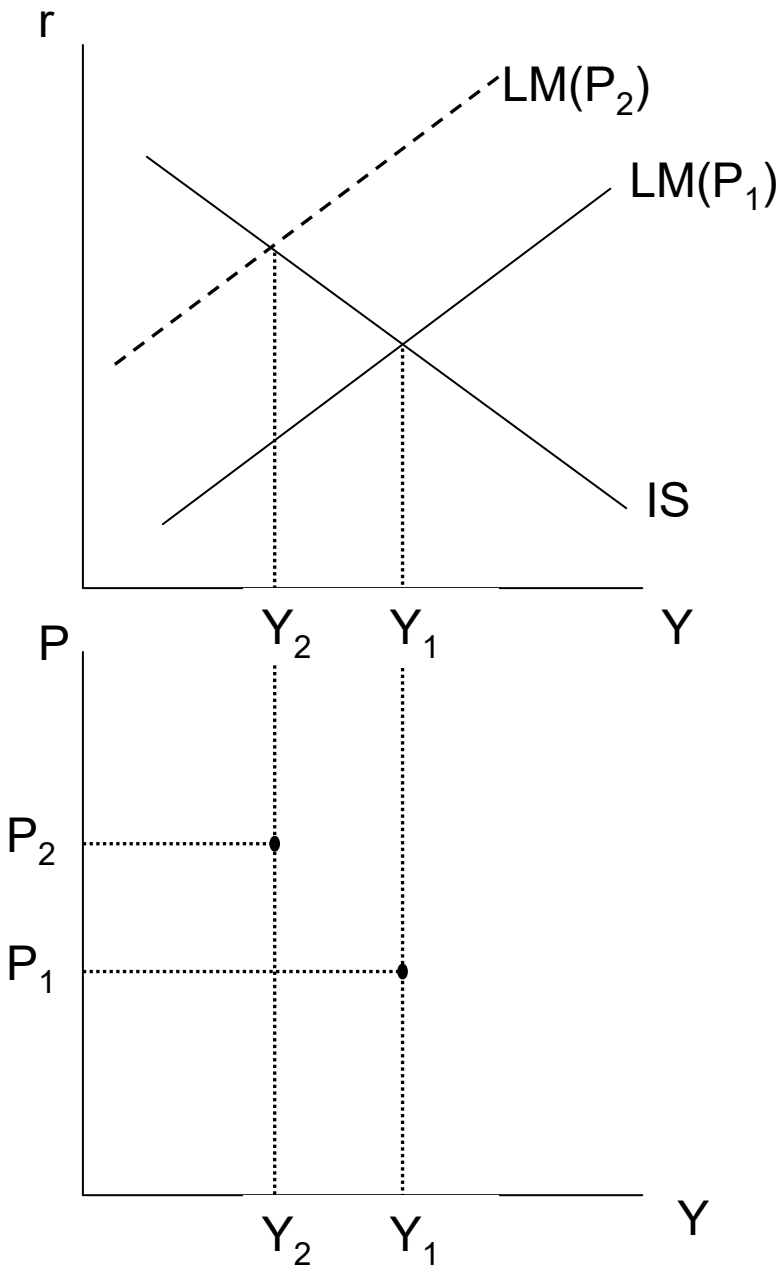


Any point on the IS curve gives the interest rate that clears the loanable funds market given a specific level of output. Any point on the LM curve gives the interest rate that clears the money market given a specific level of output. Thus, having an equilibrium in both markets requires being on both curves at the same time. This only occurs when the two curves intersect. As illustrated in the graph above, *the combination (Y*,r*) is the unique interest rate and income level that clears both markets given the exogenous variables and the price level. At this point, the IS-LM model jointly determines the equilibrium levels of output and interest rates in the economy that clear two markets: the savings market and the money market. This is done conditional on a set of exogenous variables (CS, T, G, SP,MT) and the price level.*

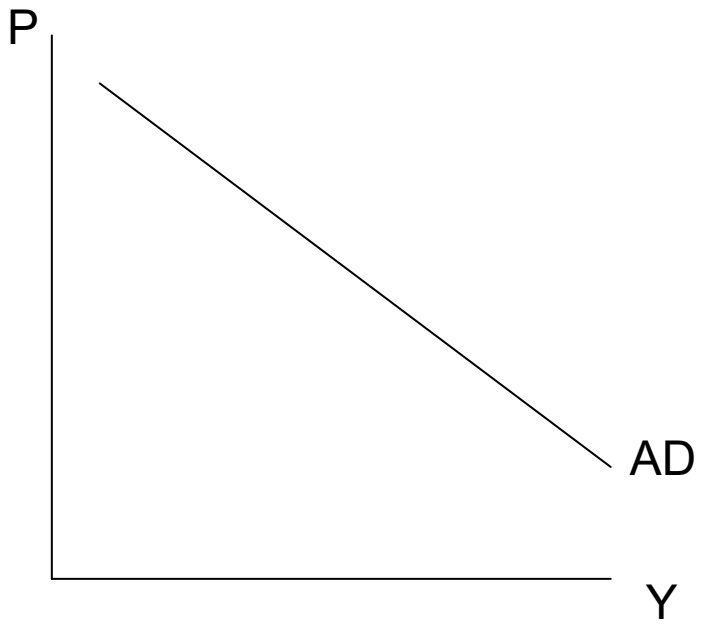
However, our model is not complete until we can also pin down prices. This requires linking the IS-LM model to the supply side of the economy. The IS-LM approach gives us equilibrium conditions that clear two other markets, but does so *given the price level*. We are now ready to convert the IS-LM approach into an aggregate demand curve which, combined with aggregate supply, will determine both prices and output in the economy.

IS-LM and Aggregate Demand

The goal now is to use our IS-LM model to derive an equilibrium relationship between prices and total output. To do so, we use both the IS-LM diagram, which holds for a given price level, and the P-Y diagram which will tell us how output changes with prices. Suppose we start off with a given price level P_1. Given this price level, the IS-LM diagram tells us the interest rate and the level of output that must hold to clear both the saving and money markets. Let's call this level of output Y_1. This combination of P_1 and Y_1 is a single point in P-Y diagram. Now suppose prices go up to P_2. What happens in the IS-LM diagram? Remember that an increase in the price level reduces the real money supply (M/P). This shifts the LM curve to the left, as illustrated in the diagram. The new intersection of the IS-LM curves implies a higher interest rate and a lower level of output, Y_2. This gives us our second point in P-Y space.



Thus, what we can see is that as the price level goes up, the real money supply decreases and pushes real interest rates up and output down. There is a negative relationship between the price level and output implied by the IS-LM model. This can thus be drawn as an ***aggregate demand*** curve:

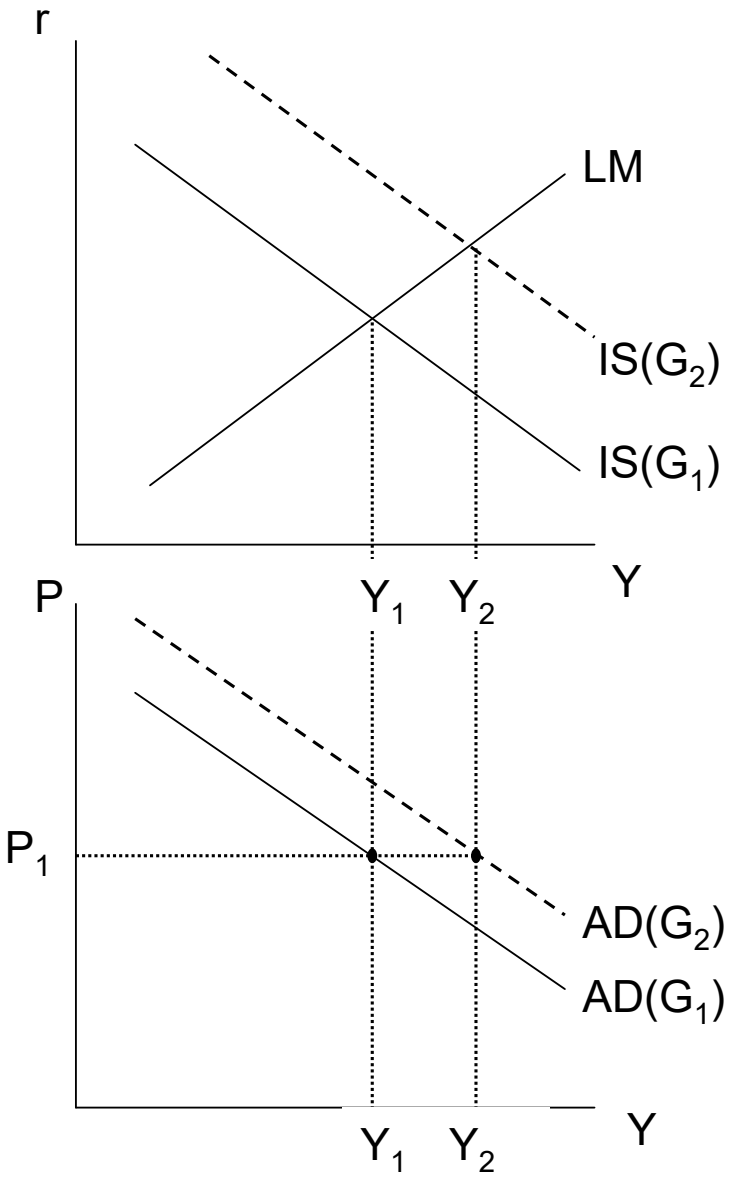


What shifts the AD curve?

A shift to the right of the AD curve implies that for the same price level, output must now be higher to clear both the savings and money markets. In terms of the IS-LM diagram, this will occur whenever a change in an exogenous variable causes the equilibrium level of output in IS-LM space to increase.

A shift in the IS Curve:

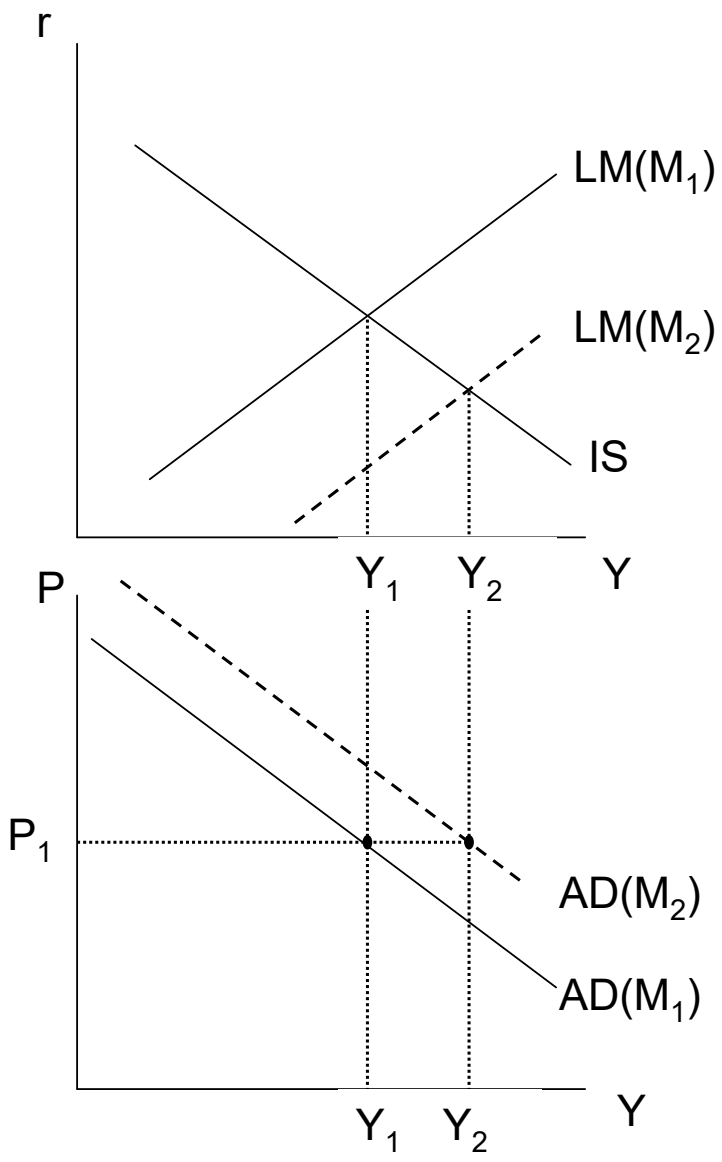
Suppose that prices and output are originally P_1 and Y_1 respectively. Let's assume that government spending is initially G_1. Now suppose that government spending rises to G_2. The increase in government saving reduces total savings and shifts the IS curve to the right. The equilibrium level of output that clears the IS-LM market is now Y_2. This means that, for the same price level P_1, we now need a higher level of output to clear the savings and money markets: i.e. the AD curve must shift to the right, as illustrated in the graph below.



The same pattern will hold true for any other change in an exogenous variable that shifts the IS curve to the right: i.e. if *CS* or *SP* go up, or if taxes *T* go down.

A shift in the LM Curve:

Suppose that prices and output are originally P_1 and Y_1 respectively. Let's assume that the money supply is initially M_1. Now suppose that the money supply rises to M_2. The increase in M increases the real money supply and shifts the LM curve to the right. The equilibrium level of output that clears the IS-LM market is now Y_2. This means that, for the same price level P_1, we now need a higher level of output to clear the savings and money markets: i.e. the AD curve must shift to the right, as illustrated in the graph below.



Summary:

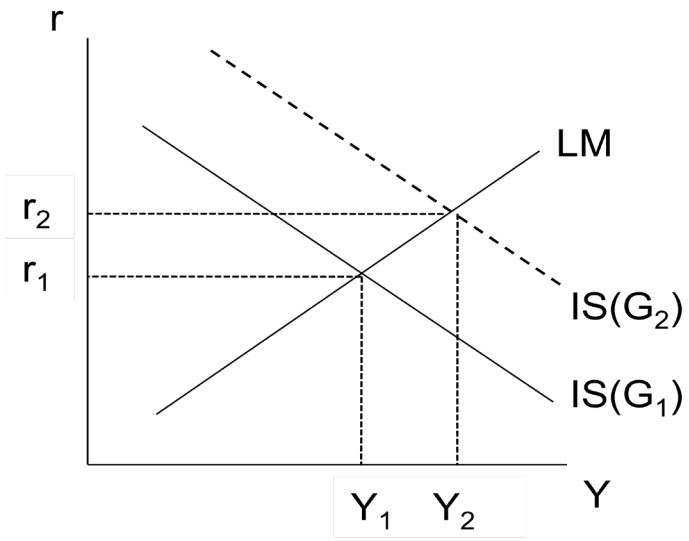
We can thus summarize how the aggregate demand curve will shift with exogenous variables as follows:

$$AD = AD\left(\overset{+}{\overbrace{CS}}, \overset{-}{\overbrace{T}}, \overset{+}{\overbrace{G}}, \overset{+}{\overbrace{SP}}, \overset{+}{\overbrace{M}}, \overset{+}{\overbrace{MT}}; \overset{-}{\overbrace{P}}\right)$$

where a + indicates that an increase in the exogenous variable shifts the curve to the right, with the exception of changes in the price level which are movements along the aggregate demand curve.

Comparative Statics of IS-LM model

Suppose the government increases its spending *G*. Based on the IS-LM model, what are the implications for interest rates, output, consumption, investment, and savings? First, let's start with the IS-LM diagram. Recall that *G* going up shifts IS curve right.



The new equilibrium (Y_2, r_2) requires a higher interest rate and higher output. Once we know what happens to the interest rate and output, we can figure out what happens to the other variables.

1- Investment: recall that $I = I\left(\overset{+}{\widetilde{SP}}, \overset{-}{\tilde{r}}\right)$. Since *SP* is unchanged, and *r* goes up, investment must fall.
2- Saving: Since saving is equal to investment and investment falls, saving must fall.
3- Consumption: Theoretically ambiguous. Remember that the consumption function depends on income and interest rates. Higher income implies more consumption but higher interest rates imply less consumption. We cannot determine whether consumption would increase or decrease in this case.

One could go through and do this same exercise for changes in all exogenous variables: *CS, T, G, M, SP, P, and MT*. I summarize the results below and leave these as an exercise.