MAT2013: Probability and Statistics **Spring 2017**

Problem Set 1

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1. Prove that if $P(\cdot)$ is a legitimate probability function and B is a set with $P(B) > 0$, then $P(\cdot|B)$ also satisfies Kolmogorov's Axioms. **Sketch:**

$$P(A|B) \underbrace{=}_{Def.} \frac{P(A \cap B)}{P(B)} \underbrace{\geq}_{P(A)\geq P(A\cap B)} \frac{P(A)}{P(B)} \underbrace{\geq}_{P(A)\geq 0,\, P(B)>0} 0$$

$$P(\Omega|B) \underbrace{=}_{Def.} \frac{P(\Omega \cap B)}{P(B)} \underbrace{=}_{P(\Omega\cap B)=P(B)} \frac{P(B)}{P(B)} = 1$$

$$P(\bigcup_{i=1}^{\infty} A_i|B) \underbrace{=}_{Def.} \frac{P((\bigcup_{i=1}^{\infty} A_i) \cap B)}{P(B)} \underbrace{=}_{Dist.law} \frac{P(\bigcup_{i=1}^{\infty}(A_i \cap B))}{P(B)} \underbrace{=}_{Axiom(3)} \sum_{i=1}^{\infty} \frac{P(A_i \cap B)}{P(B)} \underbrace{=}_{Def.} \sum_{i=1}^{\infty} P(A_i|B)$$

Proof:
Let $(\Omega, \mathcal{A}, P)$ be a probability space, then the probability measure P satisfies following axioms:

1. $P : \mathcal{A} \to [0, \infty)$ (i.e. $\forall A \in \mathcal{A}, \quad P(A) \geq 0$)
2. $P(\Omega) = 1$
3. If A_i is a sequence of disjoint sets that belongs to $\mathcal{A}$, then

$$P(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$$

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