PROBLEM SET 2

1. Let X be a (absolutely) continuous random variable with probability density f_X. What is the density of $Y = X^2$?

2. A certain river floods every year. Suppose that the low-water mark is set at 1 and the high-water mark Y has distribution function

$$F_Y(y) = 1 - \frac{1}{y^2}, \;\; 1 \leq y < \infty.$$

(a) Verify that $F_Y(y)$ is a probability distribution function.

(b) Find $f_Y(y)$, the pdf of Y.

(c) If the low-water mark is reset at 0 and we use a unit of measurement that is $\frac{1}{10}$ of that given previously, the high-water mark become $Z = 10(Y-1)$. Find $F_Z(z)$.

3. (Geometric Distribution) Consider a random experiment where we do Bernoulli trial independently with $P(success) = p$ until the first success occurs.

(a) Construct a proper probability space $(\Omega, \mathcal{A}, P)$ and define a proper random variable X so to induce a proper pmf for it.

(b) Compute the mean and variance of X.

(c) Prove that X satisfies the memoryless property: $P(X > s \mid X > t) = P(X > s - t), \;\; s > t$.

(d) Suppose that the probability is 0.001 that a light bulb will fail on any given day, then what is the probability that it will last at least 3 days.

4. Let X be a random variable having the geometric distribution with $P(success) = p$.

(a) Obtain the pdf for the random variable Y defined as

$$Y = \frac{1}{X}.$$

(b) Obtain $E[Y]$ and compare it to $E[X]$.

5. Given a random variable X with pdf $f_X = 1$ for $0 < x < 1$ and any two points a_1, a_2 in the interval $(0, 1)$, such that $a_1 < a_2$ and $a_1 + a_2 \leq 1$,

(a) show that

$$P[a_1 < X < (a_1 + a_2)] = a_2.$$

(b) In general, if $f(x)$ is uniform in the interval (a, b), and if $a \leq a_1$, $a_1 < a_2$, and $a_1 + a_2 \leq b$, show that

$$P[a_1 < X < (a_1 + a_2)] = \frac{a_2}{(b - a)}.$$

6. Compute the variance $Var(X)$ of a random variable X following *binomial* distribution.