PROBLEM SET 1

1. Prove that if $P(\cdot)$ is a legitimate probability function and B is a set with $P(B) > 0$, then $P(\cdot \mid \mathrm{B})$ also satisfies Kolmogorov's Axioms.

2. A fair die is cast until a 6 appears. What is the probability that it must be cast more than five times?

3. A pair of events A and B cannot be simultaneously mutually exclusive and independent. Prove that if $P(A) > 0$ and $P(B) > 0$, then:

(a) If A and B are mutually exclusive, they cannot be independent.

(b) If A and B are independent, they cannot be mutually exclusive.

4. Suppose that 5% of men and 0.25% of women are color-blinded. A person is chosen at random and that person is color-blinded. What is the probability that the person is male?

(Assume males and females to be in equal numbers.)

5. Police plan to enforce speed limits by using radar traps at four different locations within the city limits. The radar traps at each of the locations L_1, L_2, L_3, and L_4 will be operated 40%, 30%, 20%, and 30% of the time.

(a) If a person who is speeding on her way to work has probabilities of 0.2, 0.1, 0.5, and 0.2, respectively, of passing through these locations, what is the probability that she will receive a speeding ticket?

(b) If the person in (a) received a speeding ticket on her way to work, what is the probability that she passed through the radar trap located in L_2?