PROBLEM SET 3

1. Consider a random variable vector (X, Y) with joint pdf

$$f(x,y) = \begin{cases} e^{-y}, & 0 < x < y < \infty \\ 0, & otherwise \end{cases}.$$

(a) Compute $P(X + Y \geq 1)$.

(b) Find the marginal pdfs f_X and f_Y.

2. Let X_i, $i = 1, 2, ...$ be independent exponential random variables with rate η_i. Let $Z = min\{X_1, X_2, ..., X_n\}$ and $Y = max\{X_1, X_2, ..., X_n\}$. Find the distributions of Z and Y.

3. Prove the following statements:

(a) $Cov(X, Y) = Cov(Y, X)$

(b) $Cov(X, X) = Var(X)$

(c) $Cov(aX, Y) = aCov(X, Y)$

(d) $Cov(\sum_{i=1}^{n} X_i, \sum_{j=1}^{m} Y_i) = \sum_{i=1}^{n} \sum_{j=1}^{m} Cov(X_i, Y_j)$

4. Suppose that X and Y are independent continuous random variables. Find the distribution of $X + Y$.