<u>AE 678 – Aeroelasticity</u>

Static Aeroelastic Analysis of XB-47 Bomber with Swept wing

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Course Instructor - PPM sir

Problem statement:

The given data below pertains to the wing of the XB-47 bomber airplane with swept back wings. Show that the airplane is divergence free in the subsonic flight range. Compute the aero elastic response of the airplane when subjected to a symmetric maneuver in the pitch plane corresponding to a 3g pull-out at a Mach number of 0.8 at 6 km altitude. The geometry details of wing are given below in Figure 1. Use beam theory (with effective root concept) for structural analysis and corrected strip theory for aerodynamic calculations. The basic airplane data is given below:

Wing span: 35.35m

Aspect Ratio: 9.43
Gross weight: 66000 kg
Angle of Sweep: 34o to elastic axis and 35o to quarter-chord line.
The uncorrected 2-d lift curve slope is 6.28 per radian (use aspect ratio, sweep and Mach number corrections in your solution) Pitching Moment coefficient (wrt aerodynamic centre) of the wing sections in stream wise direction is -0.015 at all stations .
The line of CG as 0.35c

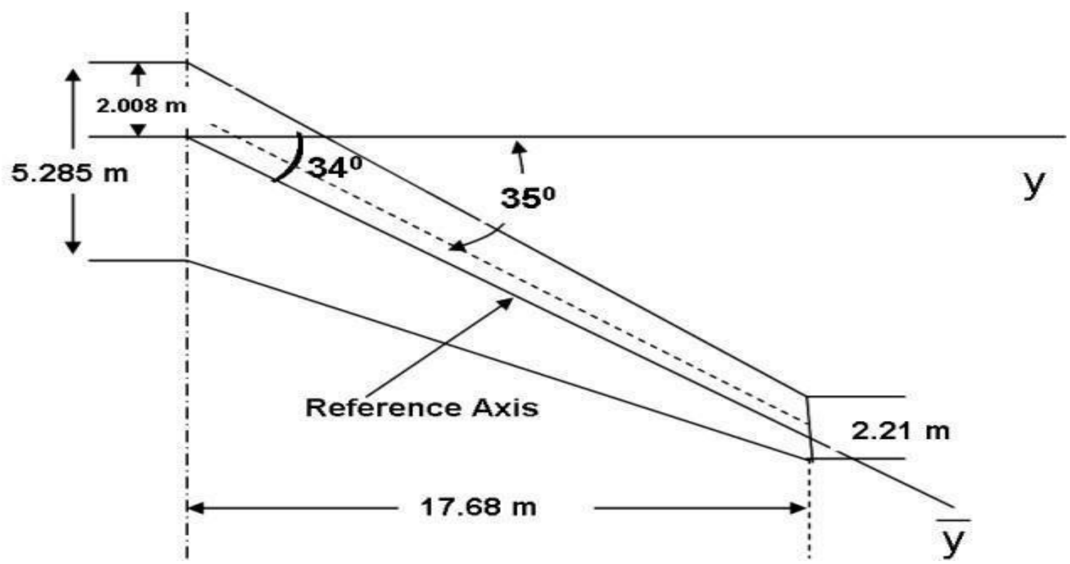


Fig1: Problem Statement Diagram

Structural Case II: Consider a modified version of the XB-47 wing where the sweep of elastic axis is same as that of the line of aerodynamic centres and is equal to 35o and the wing chord is uniform. Calculate the equivalent chord keeping the wing area and wing span to be the same as the

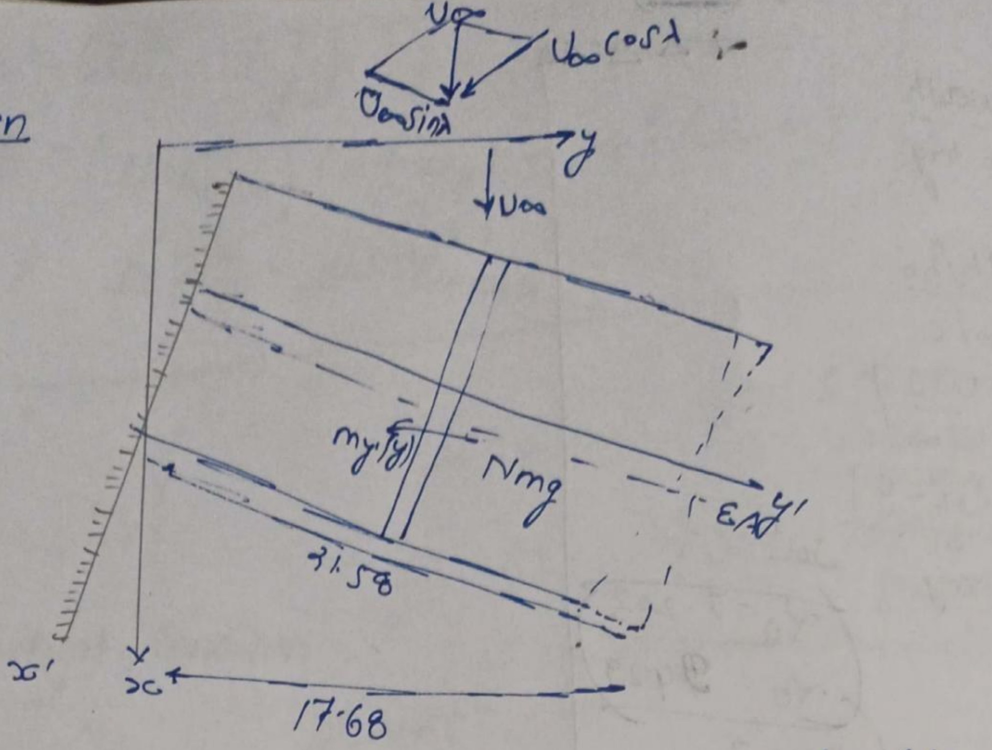
Structural Case II: Consider a modified version of the XB-47 wing where the sweep of elastic axis is same as that of the line of aerodynamic centres and is equal to 35o and the wing chord is uniform. Calculate the equivalent chord keeping the wing area and wing span to be the same as actual XB-47 wing. Take EIavg=2.0E+08; GJavg=1.75E+08 and mavg= 500 kg/m and assume them to be constant along the effective elastic axis system. In both cases take the line of CG as 0.35c

Formulation & Solution Methods:

(F3) Energy method with aerodynamic forces calculated on streamwise strips and transformed to the effective (beam) axes system.

S6 Ritz Method - Energy method with global trial functions over the

Given



$EI_{avg.} = 2 \times 10^{8}\ Nm^2$, $M = 0.8$ at 6 km

$GJ_{avg.} = 1.75 \times 10^{9}\ Nm^2$

Semispan $s = \frac{b}{2} = 17.68\ m$

Aspect Ratio $= 9.43$

$$\bar{l} = \frac{17.68}{\cos 35^\circ}$$

$$= 21.58\ m$$

Wing Area $\Rightarrow S = b^2/AR = \frac{(17.68 \times 2)^2}{9.43} = 132.59\ m^2$

half wing $\frac{S}{2} = \frac{132.59}{2} = 66.295\ m^2$

$$\bar{l} \times \bar{c} = 66.295$$

$$\bar{c} = \frac{66.295}{21.58} \Rightarrow \boxed{\bar{c} = 3.072\ m}$$

$$\bar{c} = c \cos 35^\circ \Rightarrow c = \frac{3.072}{\cos 35^\circ} = 3.75\ m$$

By Prandtl - Glauret correlation

$$C_{L\alpha\ \text{corrected}} = \frac{C_{L\alpha-2D} \cos\Lambda}{\sqrt{1 - (M\cos\Lambda)^2 + \left(\frac{C_{L\alpha 2D}\cos\Lambda}{\pi AR}\right)^2} + \left(\frac{C_{L\alpha-2D}\cos\Lambda}{\pi AR}\right)}$$

putting value $C_{L\alpha-20} = 6.28$

$$B = \frac{C_{L\alpha-20} \times \cos 35^\circ}{\pi \, A.R. \sqrt{1-(M\cos 35^\circ)^2}} = 0.22$$

$$C_{L\alpha\,corrected} = \frac{\pi \, AR \, B}{B + \sqrt{1+B^2}} =$$

$\boxed{C_{L\alpha\,corrected} = 5.42}$ /radian

~~= 5.845 /radian.~~

$V_\infty = ?$

$T_{6km} = T_{sea\,level} - 6.5 \times 6.$

$$\boxed{T_{6km} = 288.15 - 6.5 \times 6 = 249.15}$$

Speed of sound $a = \sqrt{\gamma R T} = \sqrt{1.4 \times 287 \times 249.15} = 316.4$

$V_\infty = M \times a$

$V_\infty = 0.8 \times 316.4$

$\boxed{V_\infty = 253.12 \text{ m/sec}}$

$\rho_{6km} = 0.66 \text{ kg/m}^3.$

(Dynamic pressure) $P_{dyn} = \frac{1}{2} \rho V_\infty^2$

$= \frac{1}{2} \times 0.66 \times 253.12^2$

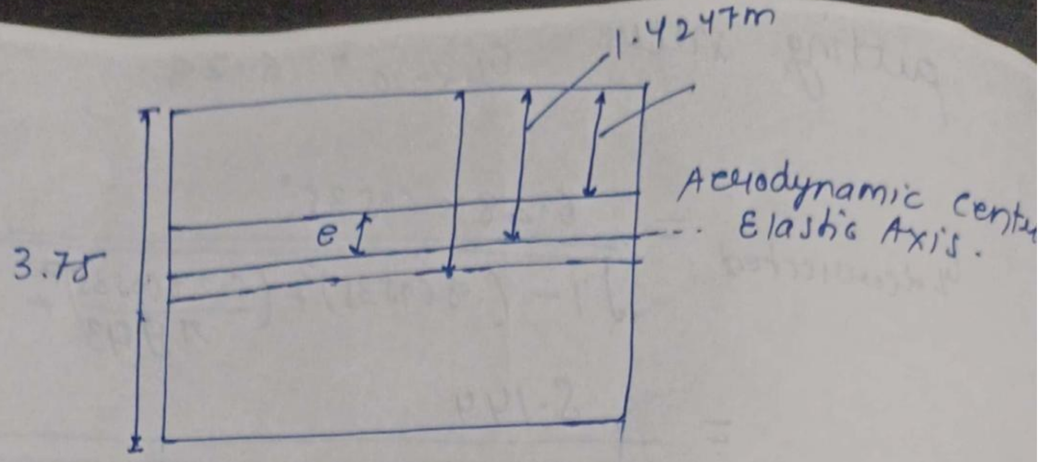
$= 21143.02 \text{ Pa}$

New Elastic Axis position

$$\frac{x_{EA}}{\bar{c}} = \frac{2.008}{5.285}$$ $\therefore c = 3.75$ from fig.

$$x_{EA} = \left(\frac{2.008}{5.285}\right) \times 3.75$$

$\boxed{x_{EA} = 1.4247}$



Where e is the fractional distance between elastic axis and aerodynamic centre.

$$e\bar{c} = 2.008 - 0.25\,(C_{root}) \text{ from leading edge}$$

$$= (2.008 - 0.25(5.285))/3.75$$

$$\boxed{e = 0.13}$$

According to case
I am neglecting the variation of e along y and will assume it to be constant value of 0.13.

x_{cg} - Distance b/w CG & EA

$$x_{cg} = \left(0.356 - \frac{2.008}{5.285} \times C\right)$$

$$\boxed{x_{cg} = -0.1123\ m}$$

For α_R calculation

$$W = L = \frac{1}{2}\rho V_\infty^2 S C_L$$

$$C_L = \frac{2W}{\rho V_\infty^2 S}$$

$$\boxed{C_L = 0.2307} \text{ (radian)}$$

$C_L = \partial(C_{L\alpha})\,\alpha_R$

$\alpha_R = 0.2307 \div 5.[illegible]4$

$\cancel{\alpha_R = 0.0395}$ (radian)

$\boxed{\cancel{\alpha_R = 2.263^\circ}}$ $\alpha_R = 2.44$

pitching Moment coeff.

$\overline{C}_{maco} = -0.015 \times \cos^2(35)$

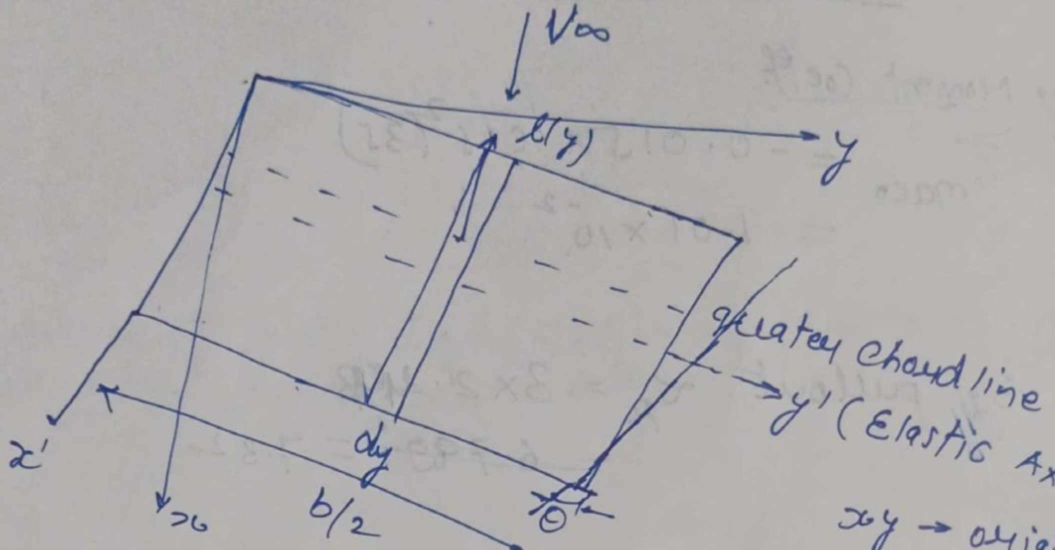
$= 1.01 \times 10^{-2}$

$\therefore$ 3g pullout $\alpha_R = 3 \times 2.44$

$\cancel{= 6.789}^\circ = 7.32$

Formulation Method – F3

Energy method with aerodynamic force calculated on streamwise strips and transformed to the effective (beam) axes system.



$xy \rightarrow$ original Axis
$x'y' \rightarrow$ Effective axes system.

~~Aerodynamic force~~

The running lift on the strip along y axis

$$L(y) = P_{dyn} c \, C_{L_\alpha} [\alpha + \theta(y)]$$

Total force on the strip

$$l(y) = L(y) - N.m.g.$$

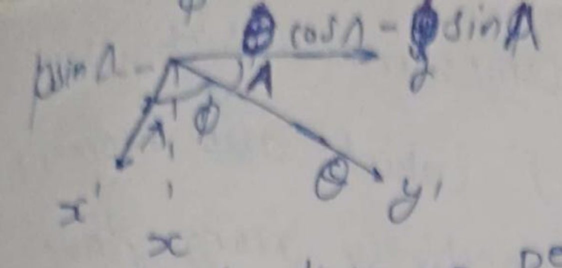
$$= P_{dyn} c \, C_{L_\alpha} [\alpha + \theta(y)] - Nmg \quad - (1)$$

N – load factor.

Moment of the strip about an axis parallel to y-axis and passes through the point of intersection of the elastic axis with stream-wise segment.

$$m(y) = P_{dyn} c \, C_{L_\alpha} [\alpha + \theta(y)] . ec + P_{dyn} c^2 C_{mac_0} - Nmg \, x_{cg} \quad (2)$$

here.
$c \rightarrow$ mean aerodynamic chord



$\phi = \frac{dw_B}{dy'}$ → Deflecting due to bending

$\theta\cos\Lambda$ is due to twisting moment acting against the stream-wise positive angle.

$\phi\sin\Lambda$ is the twist due to bending of the wing

$$\boxed{\alpha_{eff} = \alpha_R + \theta'(y) = \alpha_R + \theta\cos\Lambda - \phi.\sin\Lambda}$$

from eq^n (1) & (2), we get

$$\boxed{l(y) = P_{dyn}\, c\, C_{l\alpha}(\alpha_R + \theta\cos\Lambda - \phi\sin\Lambda) - N mg}$$

$$\boxed{m(y) = P_{dyn}\, e c^2 C_{l\alpha}(\alpha + \theta\cos\Lambda - \phi\sin\Lambda) + P_{dyn}\, c^2 C_{mac0} - N mg\, \bar{x}_{cg}}$$

convert axes system $l(y)$, $m(y)$ along y' axes

$l(y') = l(y)\cos\Lambda$

$m(y') = m_y(y)\cos\Lambda$

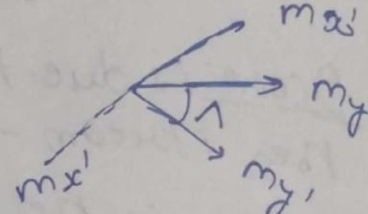
putting these value in above eq^n.

$$\boxed{l(y') = P_{dyn}\, c\, C_{l\alpha}(\alpha_R + \theta\cos\Lambda - \phi\sin\Lambda)\cos\Lambda - N mg\cos\Lambda}$$

$$m(y') = P_{dyn}\, e c^2 C_{l\alpha}(\alpha_R + \theta\cos\Lambda - \phi\sin\Lambda)\cos\Lambda + P_{dyn}\, c^2 C_{mac0}\cos\Lambda - N mg\, \bar{x}_{cg}\cos\Lambda$$

$m_y(y')$ into two components which are $m_{x'}(y')$ and $m_{y'}(y')$ which are moment about x' and y' axis respectively along y' axis.

$$\boxed{\begin{aligned} m_{x'}(y') &= m_y(y')\sin\Lambda \\ m_{y'}(y') &= m_y(y')\cos\Lambda \end{aligned}}$$



putting these value from $m_y(y')$, m

$$\boxed{\begin{aligned} m_{x'}(y') = {} & P_{dyn}\, e c^2 C_{l\alpha}(\alpha_R + \theta\cos\Lambda - \phi\sin\Lambda)\cos\Lambda\sin\Lambda \\ & + P_{dyn}\, c^2 C_{mac}\cos\Lambda\sin\Lambda - N m g \bar{x}_{cg}\cos\Lambda\sin\Lambda \end{aligned}}$$

$$\boxed{\begin{aligned} m_{y'}(y') = {} & P_{dyn}\, e c^2 C_{l\alpha}(\alpha_R + \theta\cos\Lambda - \phi\sin\Lambda)\cos^2\Lambda + \\ & + P_{dyn}\, c^2 C_{mac_0}\cos^2\Lambda - N m g \bar{x}_{cg}\cos^2\Lambda \end{aligned}}$$

In My problem, the strain Energy is due to bending and twisting, the External work done is also due to bending and twisting

Strain Energy, due to bending $U_{bending} = \dfrac{1}{2}\displaystyle\int_0^{b/2} \bar{EI}(\bar{y}')\left[\frac{d^2 w}{d\bar{y}'}(\bar{y}')\right]^2 d\bar{y}'$

External work done in bending

$$W_{bending} = \int_0^{b/2} l_2(\bar{y}')\, w(\bar{y}')\, d\bar{y}'$$

potential Energy functional due to bending

$$\pi_{bending} = U_{bending} - W_{bending}$$

$$\boxed{\pi_{bending} = \frac{1}{2}\int_0^{b/2} EI_{avg}(\bar{y}')\left[\frac{d^2w}{d\bar{y}'}(\bar{y}')\right]^2 d\bar{y}' - \int_0^{b/2} l_2(\bar{y}')\,w(\bar{y}')d\bar{y}'}$$ —(3)

Similarily for twisting.

$$U_{twisting} = \frac{1}{2}\int_0^{b/2} GJ(\bar{y}')\left[\frac{d\theta(\bar{y}')}{d\bar{y}'}\right]^2 d\bar{y}'$$

External workdone in twisting is given by

$$W_{twisting} = \int_0^{b/2} m_y(\bar{y}')\,\theta(\bar{y}')d\bar{y}'$$

potential Energy functional due to twisting

$$\boxed{\pi_{twisting} = \frac{1}{2}\int_0^{b/2} GJ(\bar{y}')\left[\frac{d\theta(\bar{y}')}{d\bar{y}'}\right]^2 d\bar{y}' - \int_0^{b/2} m_y(\bar{y}')\,\theta(\bar{y}')d\bar{y}'}$$ —(4)

for my case $\bar{y}' = y'$

Total potential Energy function is given by

$$\pi_{Total} = \pi_{twisting} + \pi_{bending}$$

$$\boxed{\begin{aligned}\pi_{total} = {} & \frac{1}{2}\int_0^{b/2} EI(y')\left[\frac{d^2w}{dy'}(y')\right]^2 dy' + \frac{1}{2}\int_0^{b/2} GJ(y')\left[\frac{d\theta(y')}{dy'}\right]^2 dy' \\ & - \int_0^{b/2} l_2(y')w(y')dy' - \int_0^{b/2} m_y(y')\left[\frac{d\theta(y')}{dy'}\right]^2 dy'\end{aligned}}$$

This eqn used in solution Method.

Solution Method - (S-6)

(Ritz Method - Energy Method with global trail function over the beam span)

Explanation :-

The Ritz method is based on the principle of minimum potential Energy for conservative system. It is an approximate function method use to find the displacements function. It assumes a shape function for the unknown displacement and twist. They may be polymomials, trigonometric function but usedy polynomial is easy to construct can be differentiated and integrated easily.

The potential energy function of the system is written in terms of these parameters and the value of these parameters that would minimize the potential Energy function of the system are calculated.

Required of Admissible function

1. It must satisfy Essential Boundary condition
2. It must be continuous as required by VES (variational Energy statement.)
3. It must be linearly independent and complete.

Approximation function for ($w(y')$) and $\theta(y')$

$$w(y') = a_1 + a_2 y' + \dots\dots\dots a_{n+1} y'^{n}$$

$$\theta(y') = b_1 + b_2 y' + \dots\dots\dots a_{n+1} y'^{n}$$

Boundary Condition (Flexible wing)

Displacement at fixed end — $w(y') = 0$. $\therefore y' = 0$.

Twist at fixed end → $\theta(y') = 0$ $\therefore y' = 0$.

Slope at root — $\frac{dw(0)}{dy'} = 0$.

Moment at tip → $EI \frac{d^2 w(b/2)}{dy'^2} = 0$.

Shear force at tip $\frac{d}{dy'}\left(EI \frac{d^2 w(b/2)}{dy'^2}\right) = 0$.

Torque at tip → $\frac{GJ\, d\theta(b/2)}{dy'^2} = 0$

Apply Boundary Condition for (Rigid wing)

Deflection at root and tip $w(0) = 0$ $w(b/2) = 0$

Slope at root & tip $\frac{\partial w(0)}{dy'} = 0$

Moment at root & tip $EI \frac{d^2 w(0)}{dy'^2} = 0$ & $EI \frac{d^2 w(b/2)}{dy'^2} = 0$

Shear force at root & tip $\frac{d}{dy'}\left[EI \frac{d^2 w(0)}{dy'^2}\right) = 0$

$$\frac{d}{dy'}\left[EI \frac{d^2 w(b/2)}{dy'^2}\right] = 0$$

Twist at root and tip $\theta(0) = 0$ $\theta(b/2) = 0$

Torque at root & tip $GJ \frac{d\theta(0)}{dy'^2} = 0$

$$GJ \frac{d\theta(S')}{dy'^2} = 0$$

Scanned with OKEN Scanner

Appling Boundary Condition

$$w(y') = a_1 + a_2 y' + a_3 y'^2 \cdots a_{n+1} y'^n$$

$$w(0) = 0 \Rightarrow a_1 = 0$$

$$\frac{d\,w(0)}{dy'} = 0 \qquad a_2 = 0$$

$$\boxed{w(y') = a_3 y'^2 + a_4 y'^3 \cdots\cdots a_{n+1} y'^n}$$

Similarly

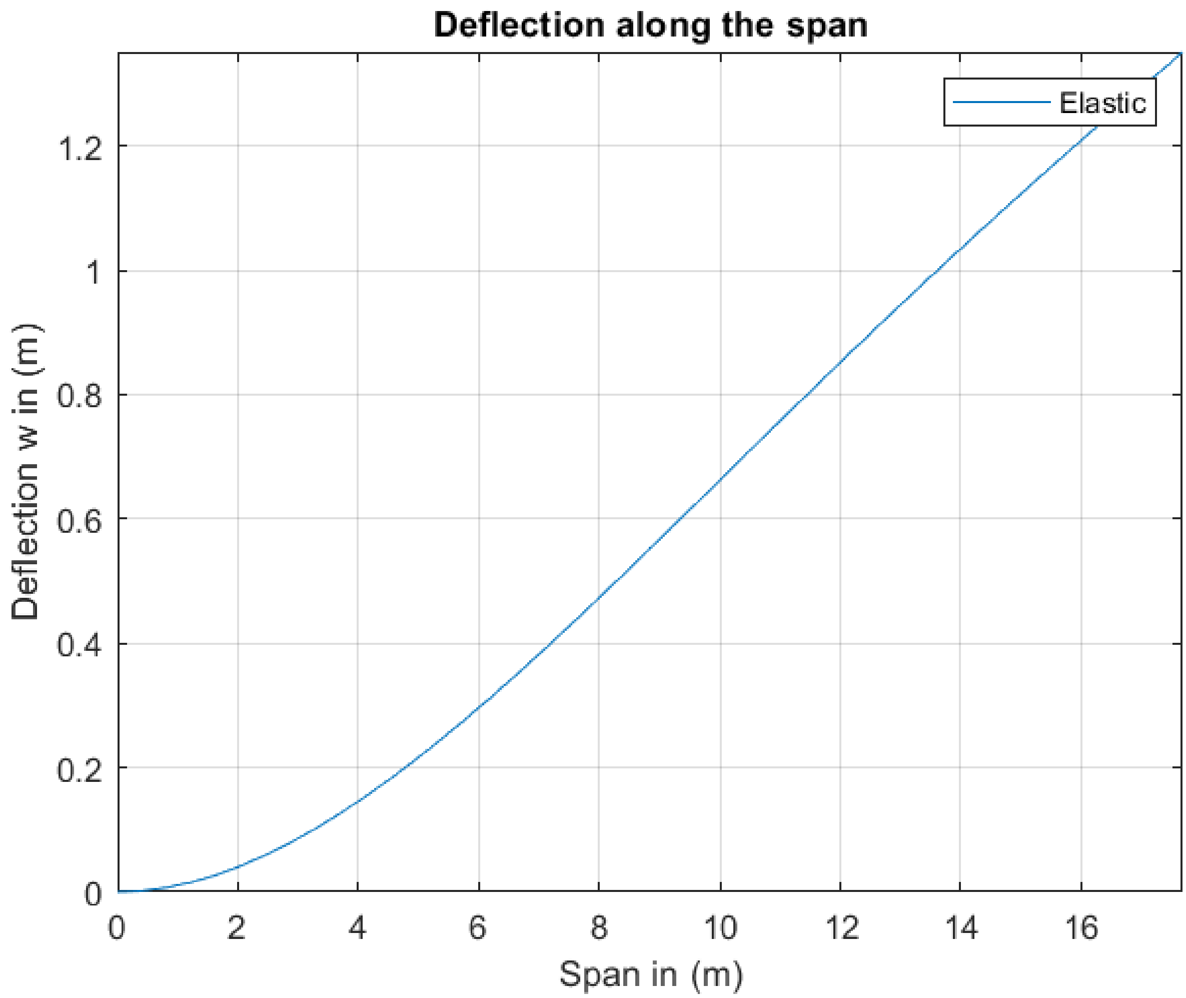
$$\boxed{\theta(y') = b_2 y' + b_3 y'^2 \cdots b_{n+1} y'^n}$$

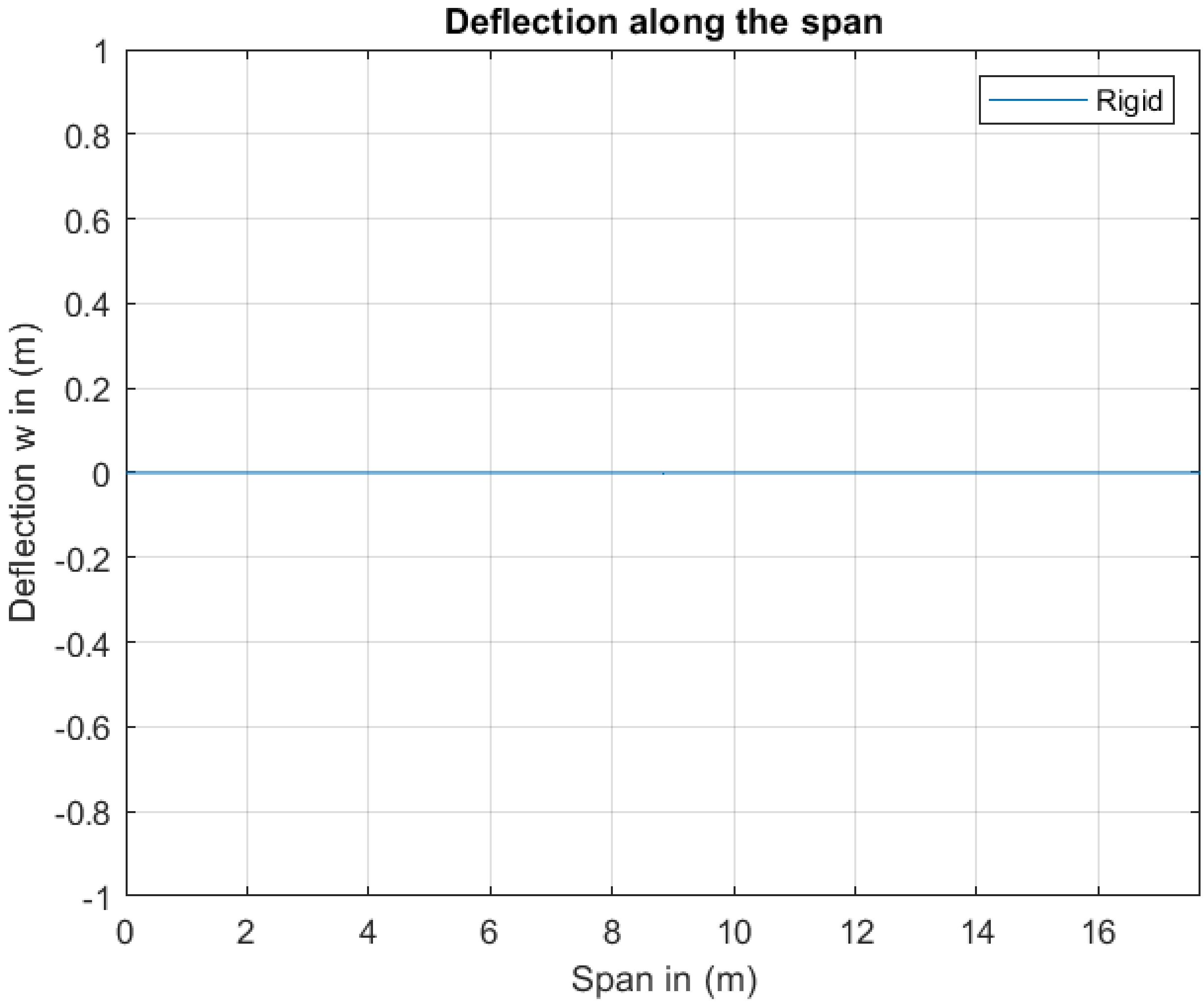
Matlab Code

Aeroelastic Response

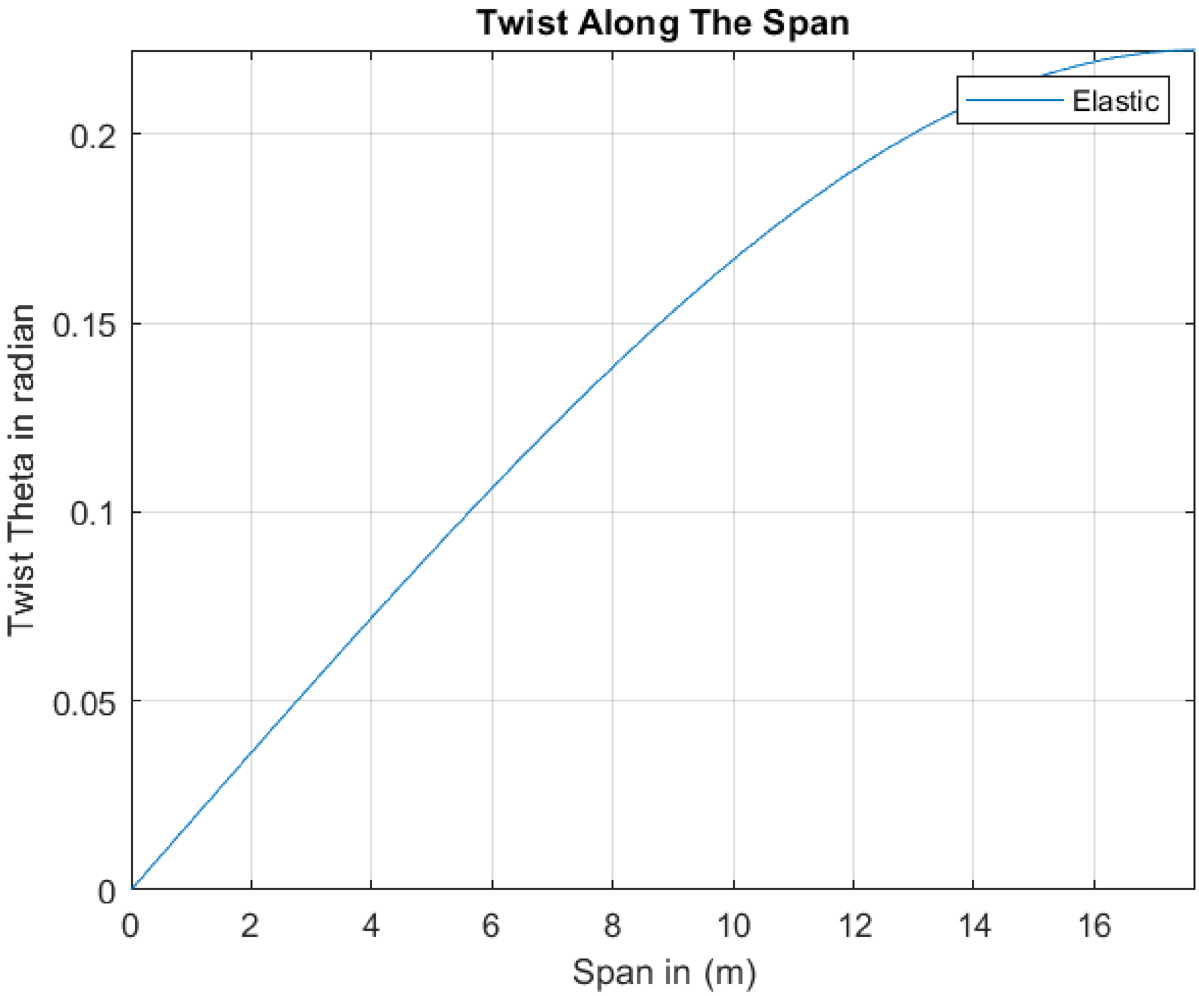
The rigid angle of attack associated with the wing for the given manevour is alphaR = 4.978672 in deg

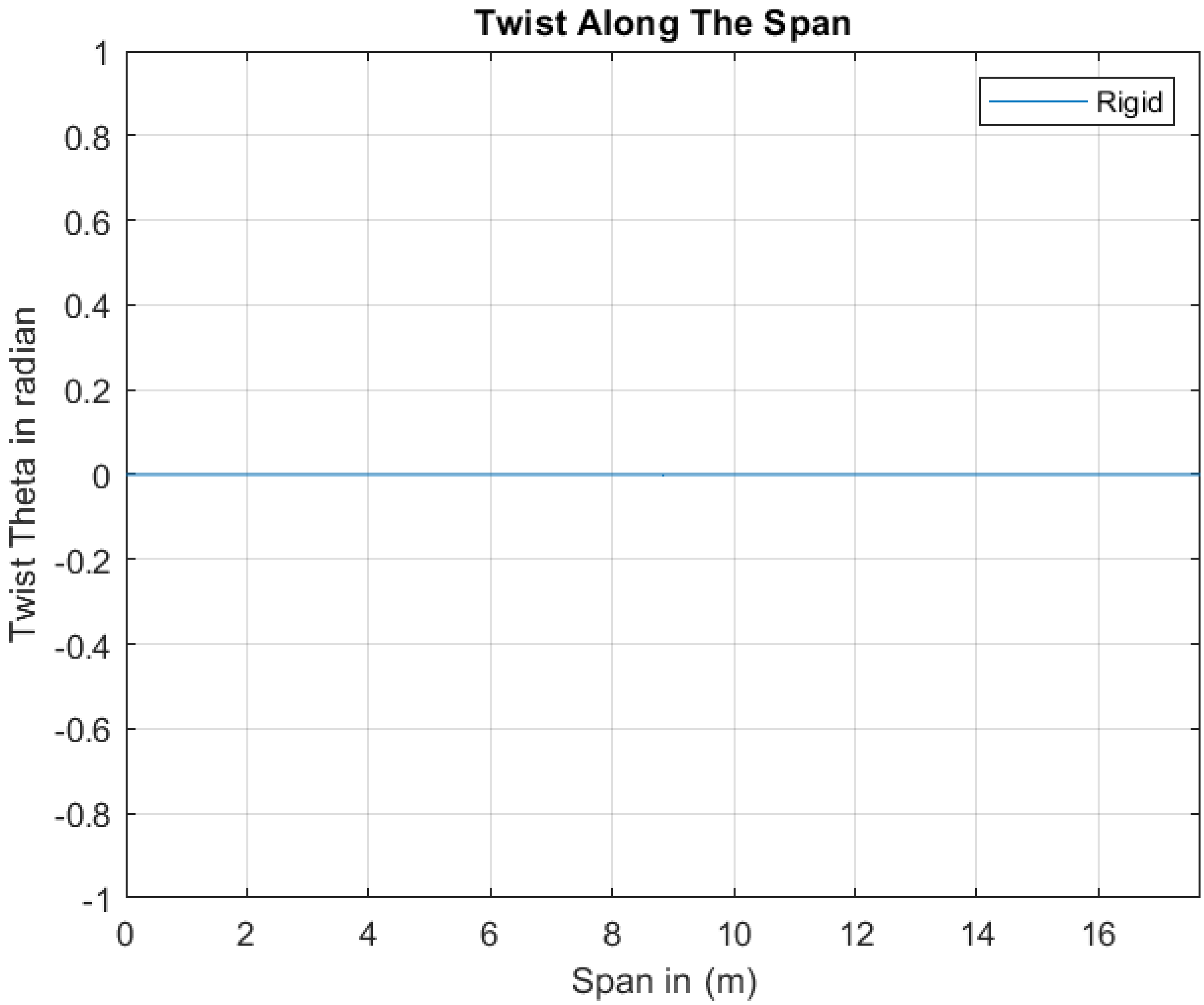
Tip deflection – 1.35 mtip
Twist – 12.74
Total Lift = 1942380.000000 in N



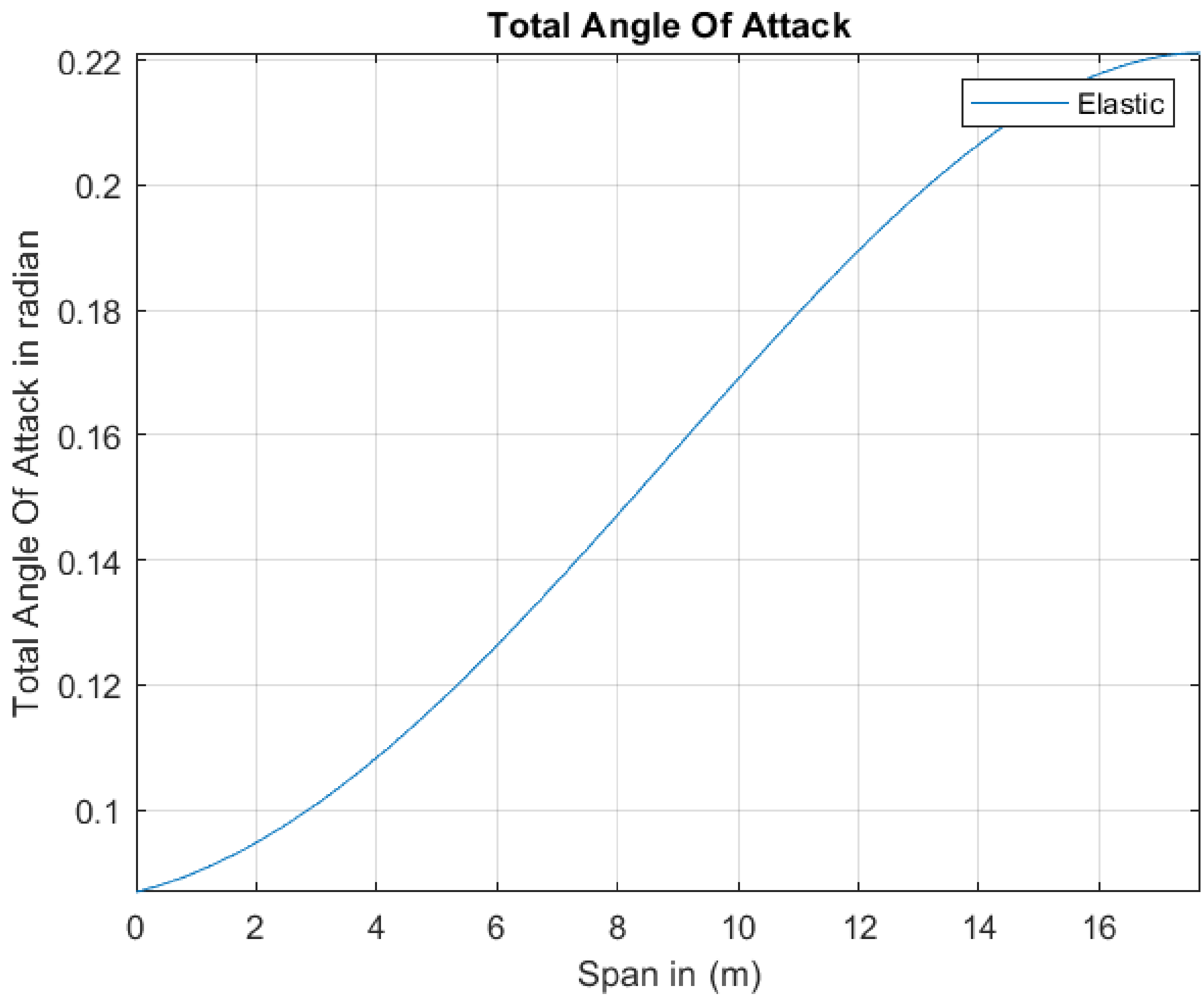


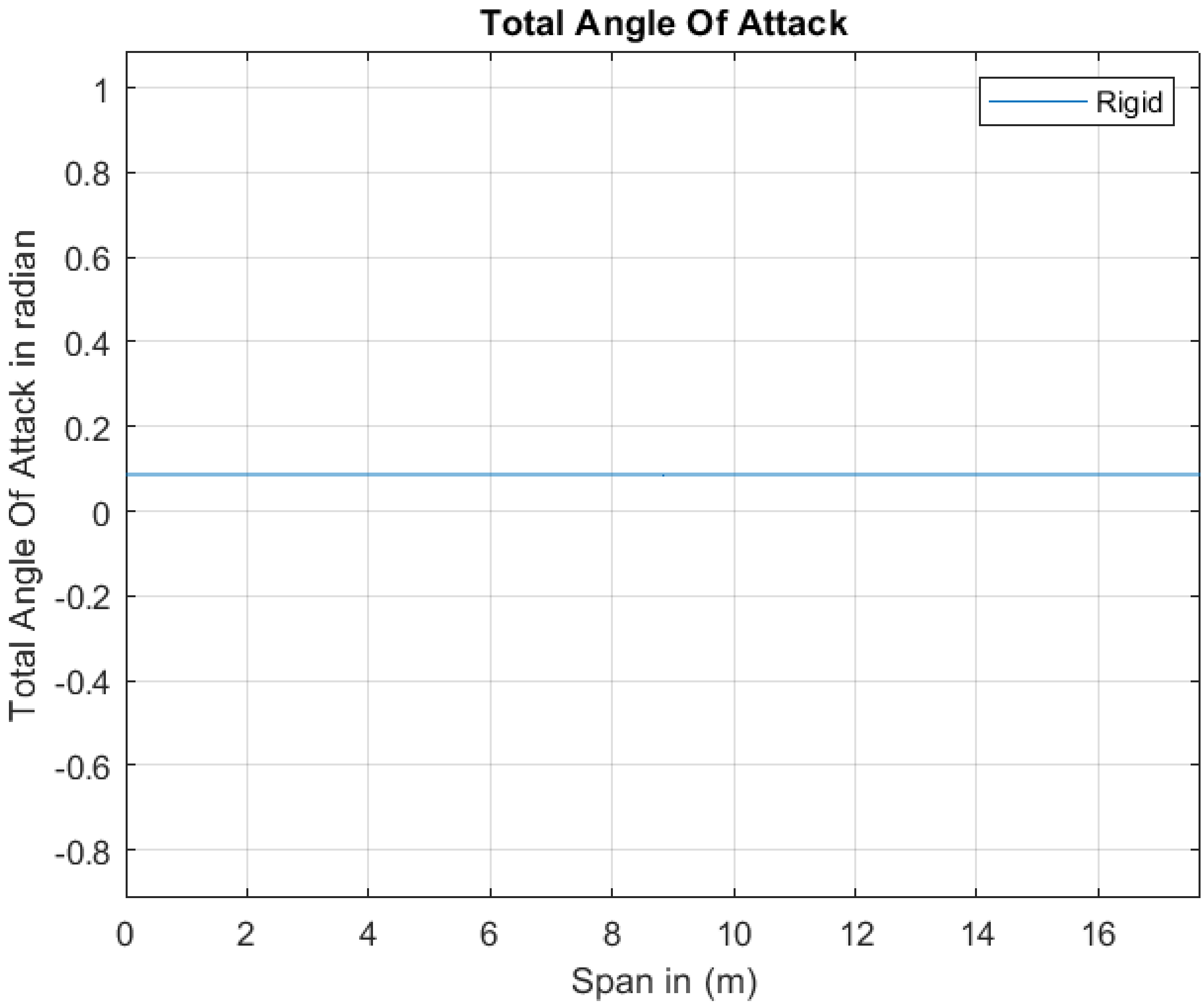
deflection is zero at the fixed end in both case and 1.35 m at the free end for elastic case, whereas deflection is zero in rigid case.


Twist Along The Span
Elastic
Twist Theta in radian
0
0.05
0.1
0.15
0.2
0
2
4
6
8
10
12
14
16
Span in (m)

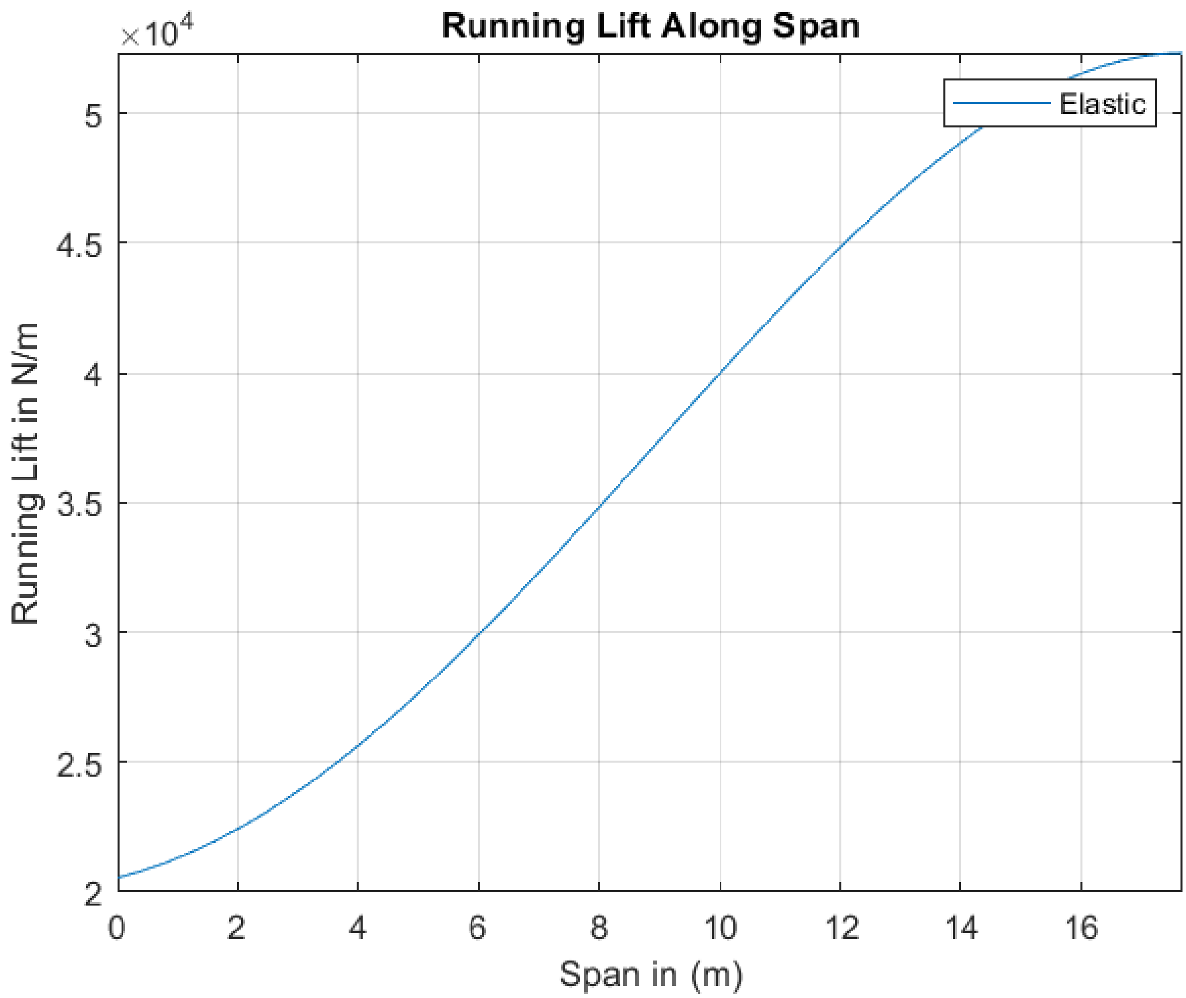


the twist is zero at the fixed end in both case and 0.226 Radian at the free end for elastic case , whereas twist is zero for the rigid case.

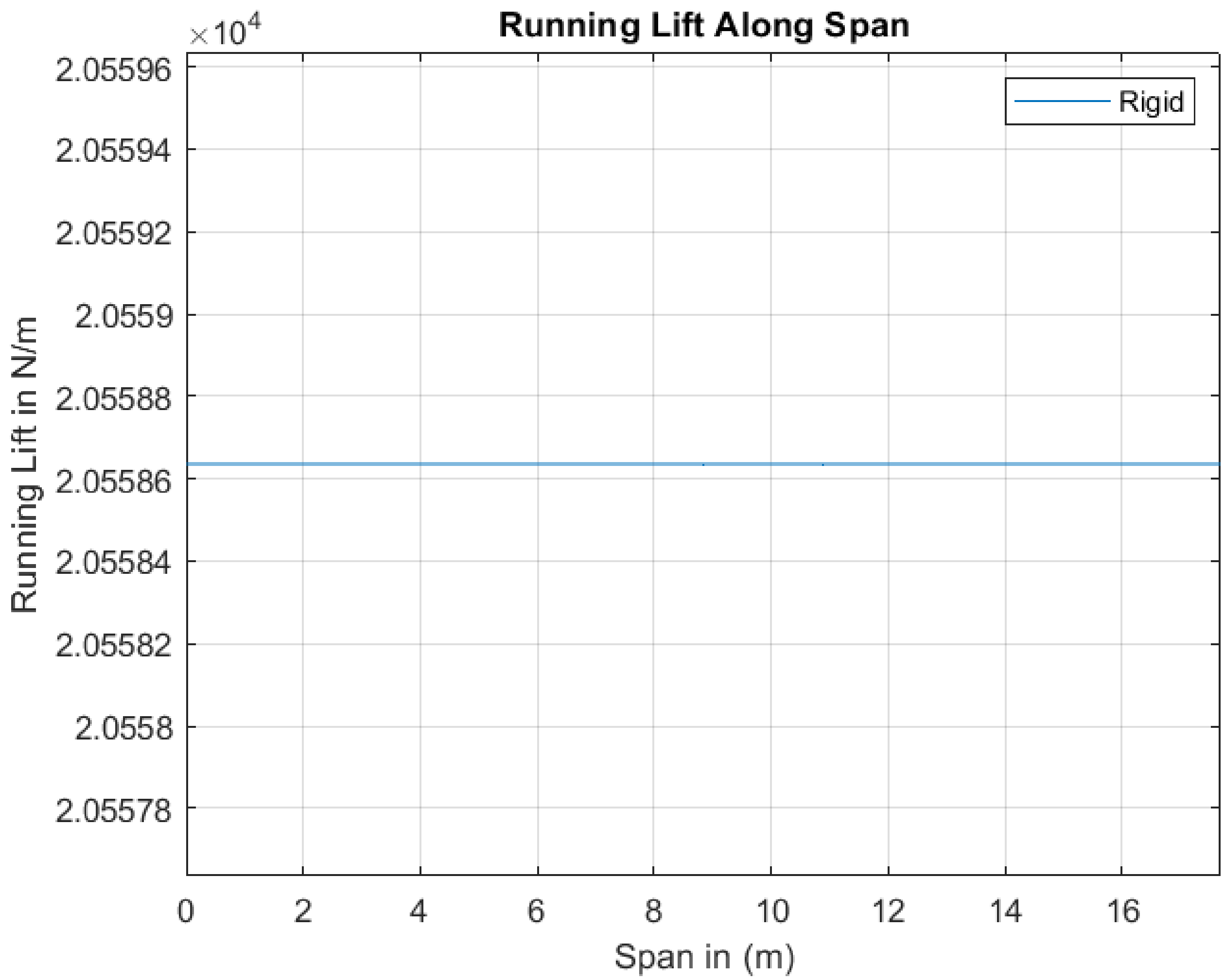

Total Angle Of Attack
Elastic
Total Angle Of Attack in radian
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0
2
4
6
8
10
12
14
16
Span in (m)



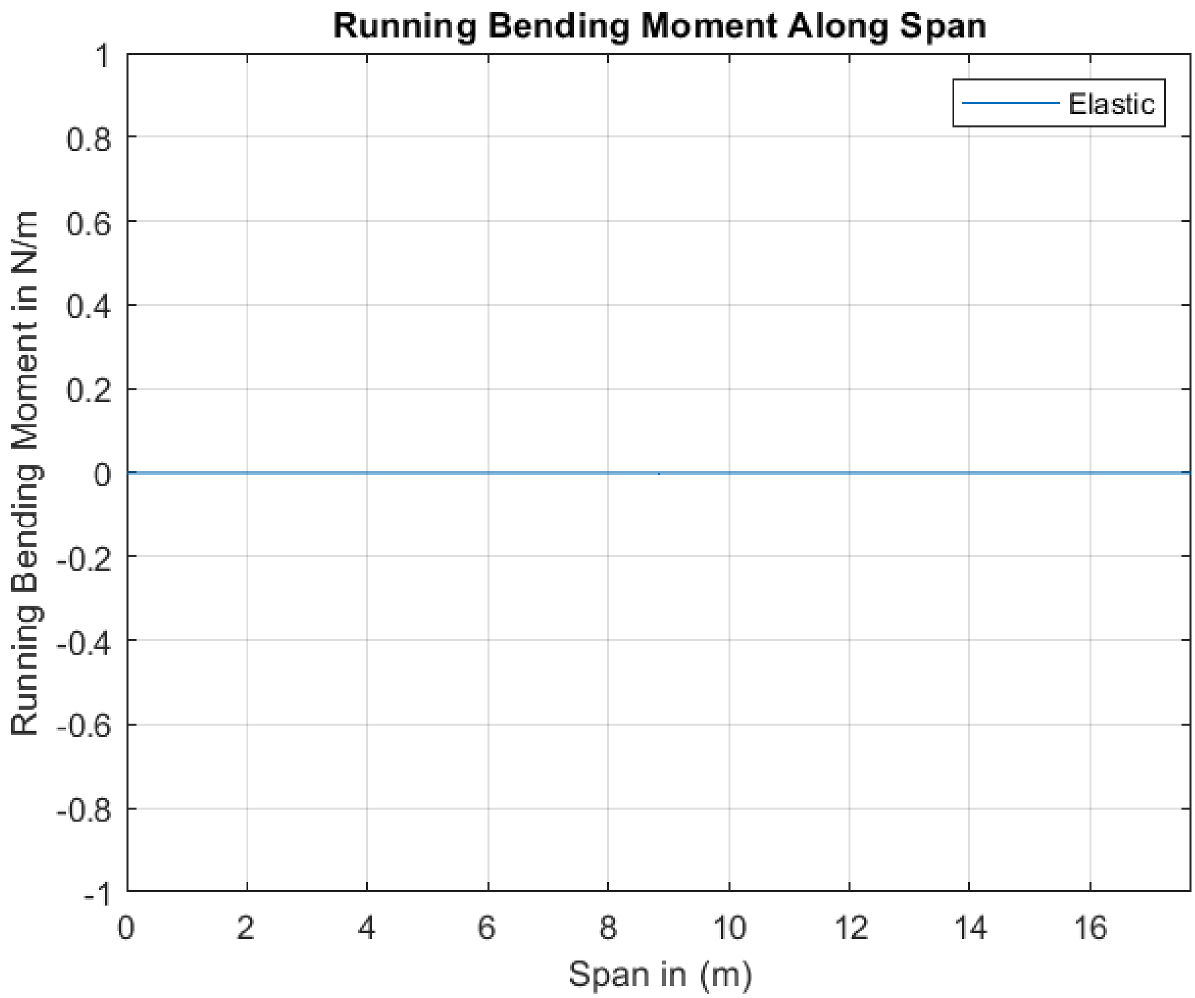
At the fixed end the total angle of attack for both the elastic and rigid case is the same equal to 0.087 radian at the fixed end.for elastic case increases along the span

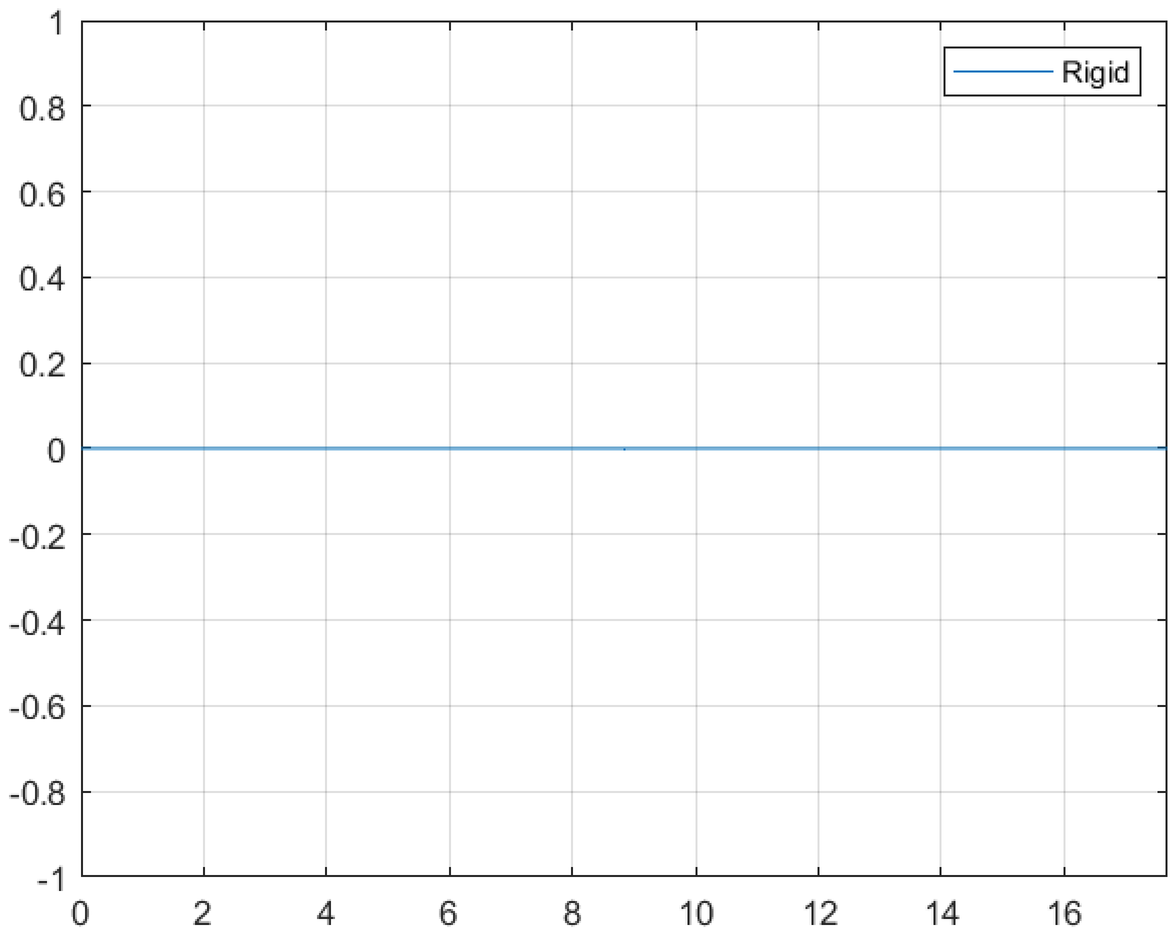


Running Lift Along Span
×10^4
Elastic
5
4.5
4
3.5
3
2.5
2
0
2
4
6
8
10
12
14
16
Running Lift in N/m
Span in (m)

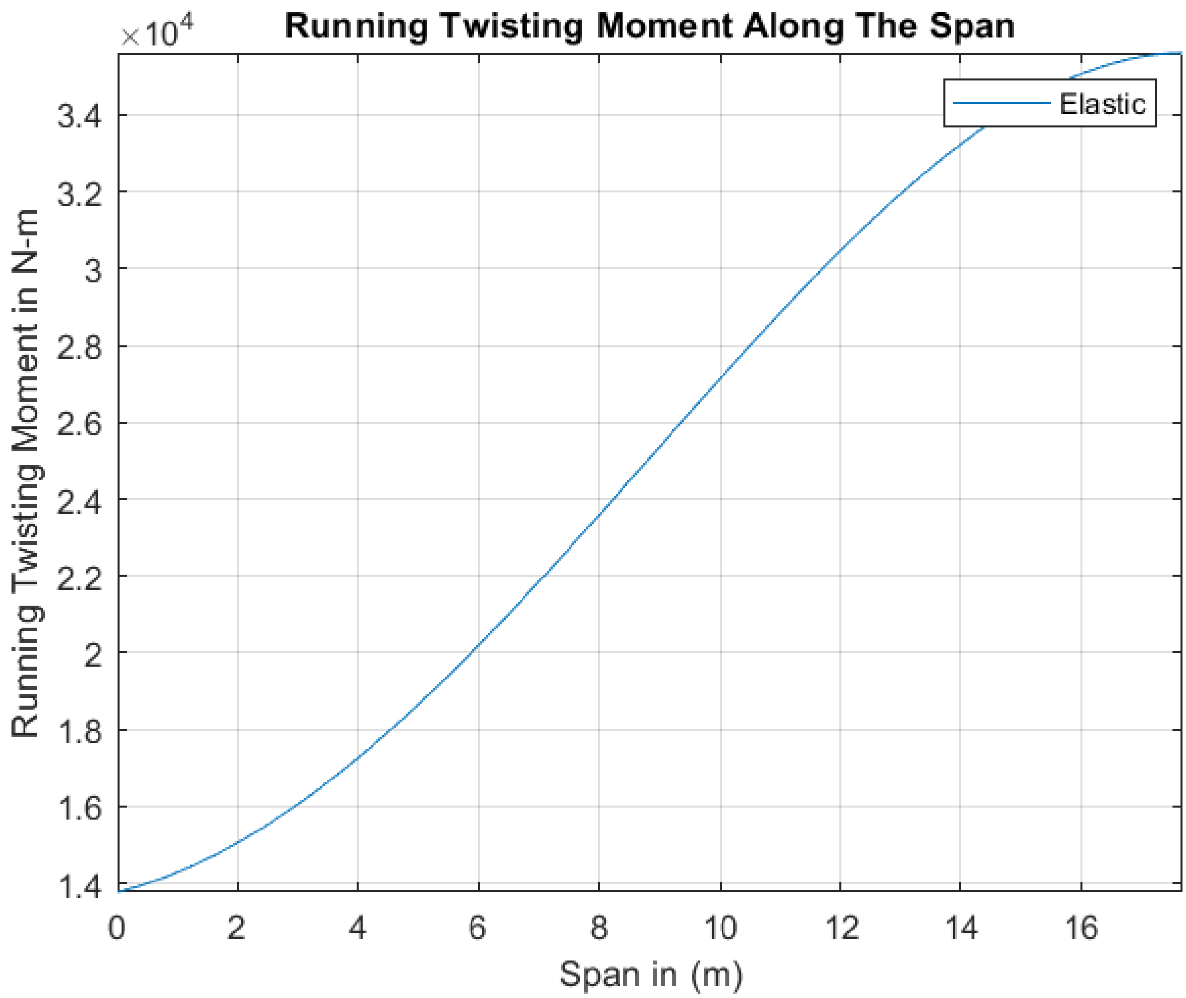


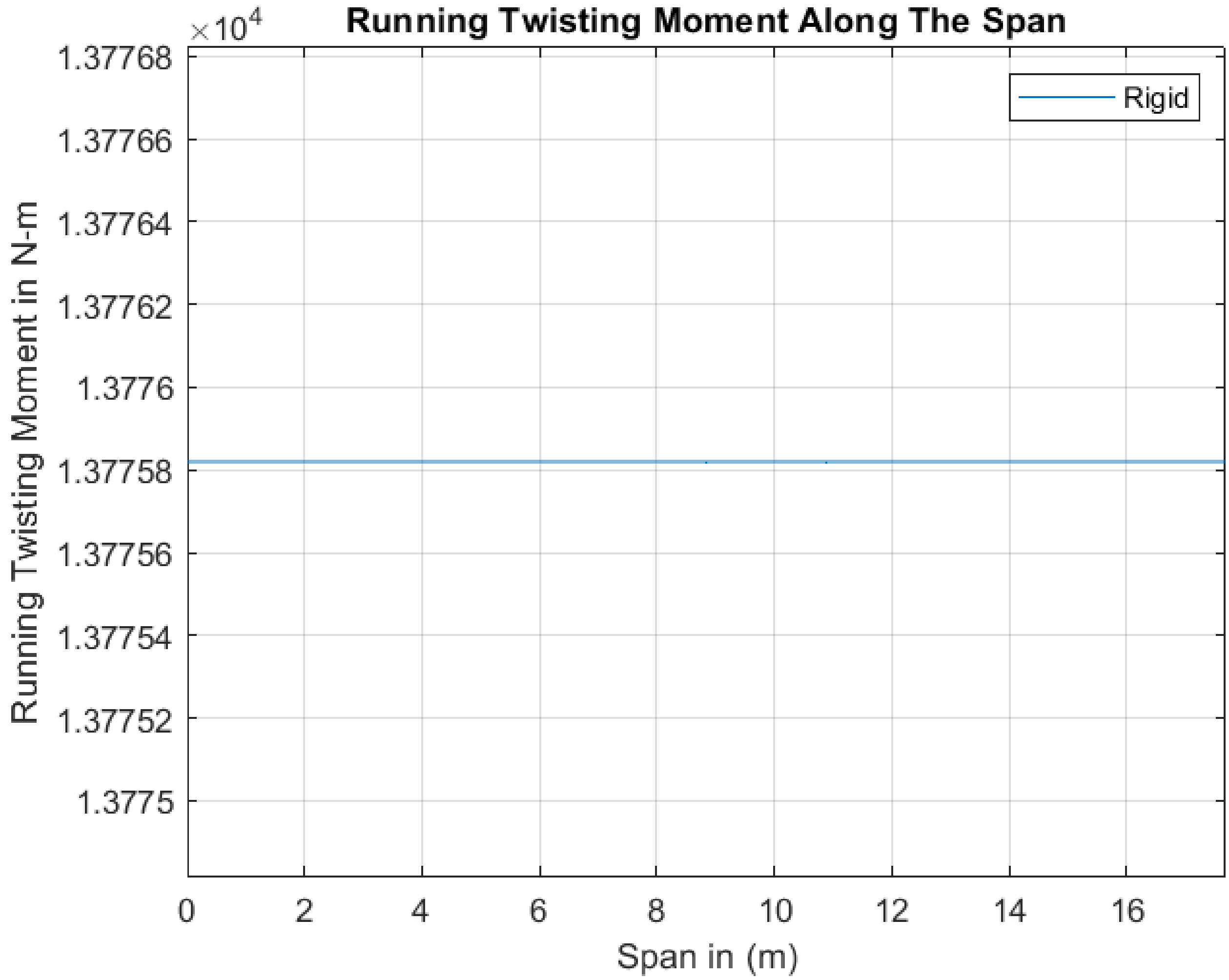
In rigid wing running lift is constant because the wing chord is uniform and elastic case the running lift increases as we move along the length of the span. The tip lift is non zero in the strip theory formulation.


Running Bending Moment Along Span
Elastic
Running Bending Moment in N/m
Span in (m)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
2
4
6
8
10
12
14
16



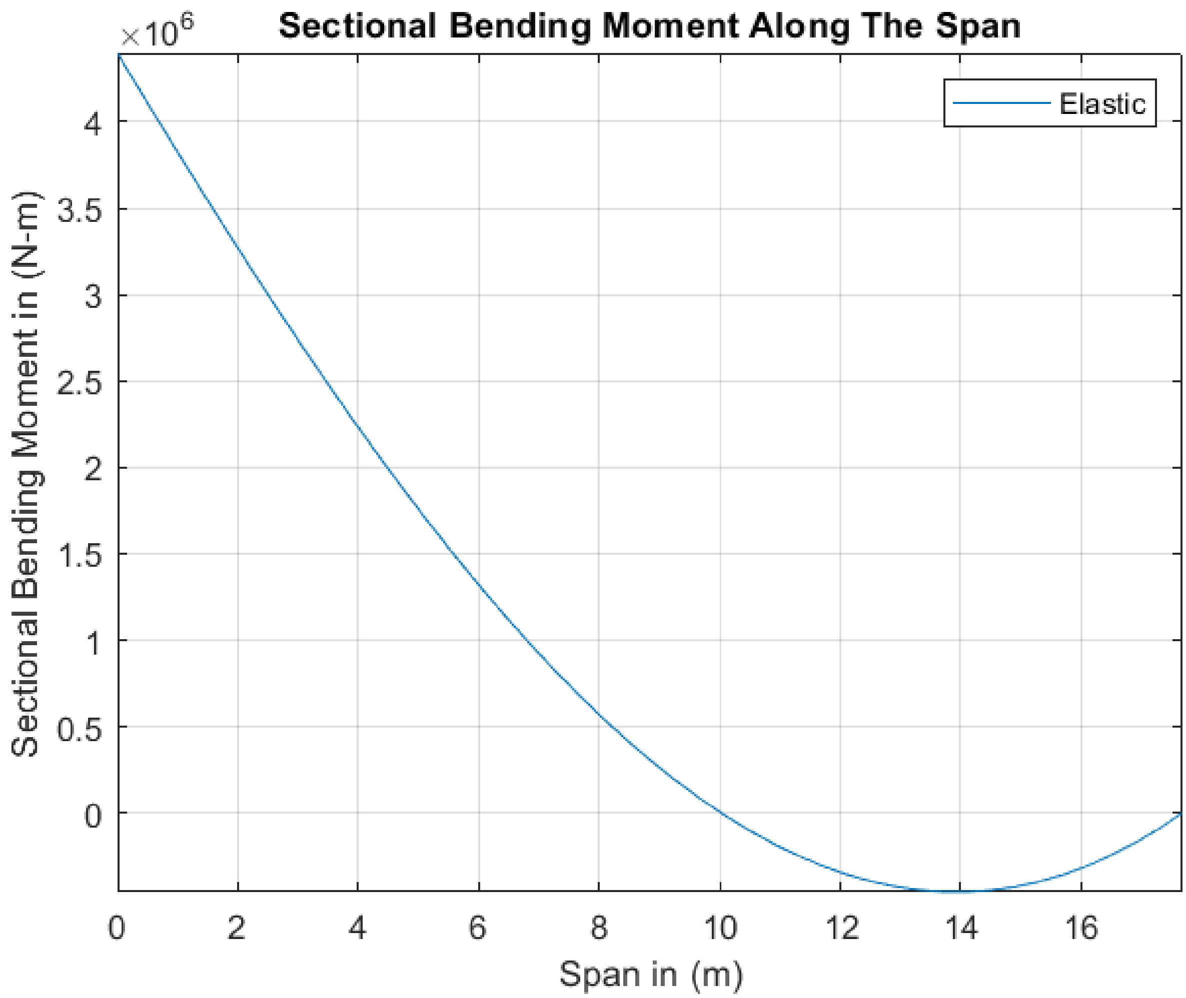
The running bending moment is zero both the rigid and elastic case. It is wrong in elastic case.


Running Twisting Moment Along The Span
×10^4
Elastic
Running Twisting Moment in N-m
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3
3.2
3.4
0
2
4
6
8
10
12
14
16
Span in (m)

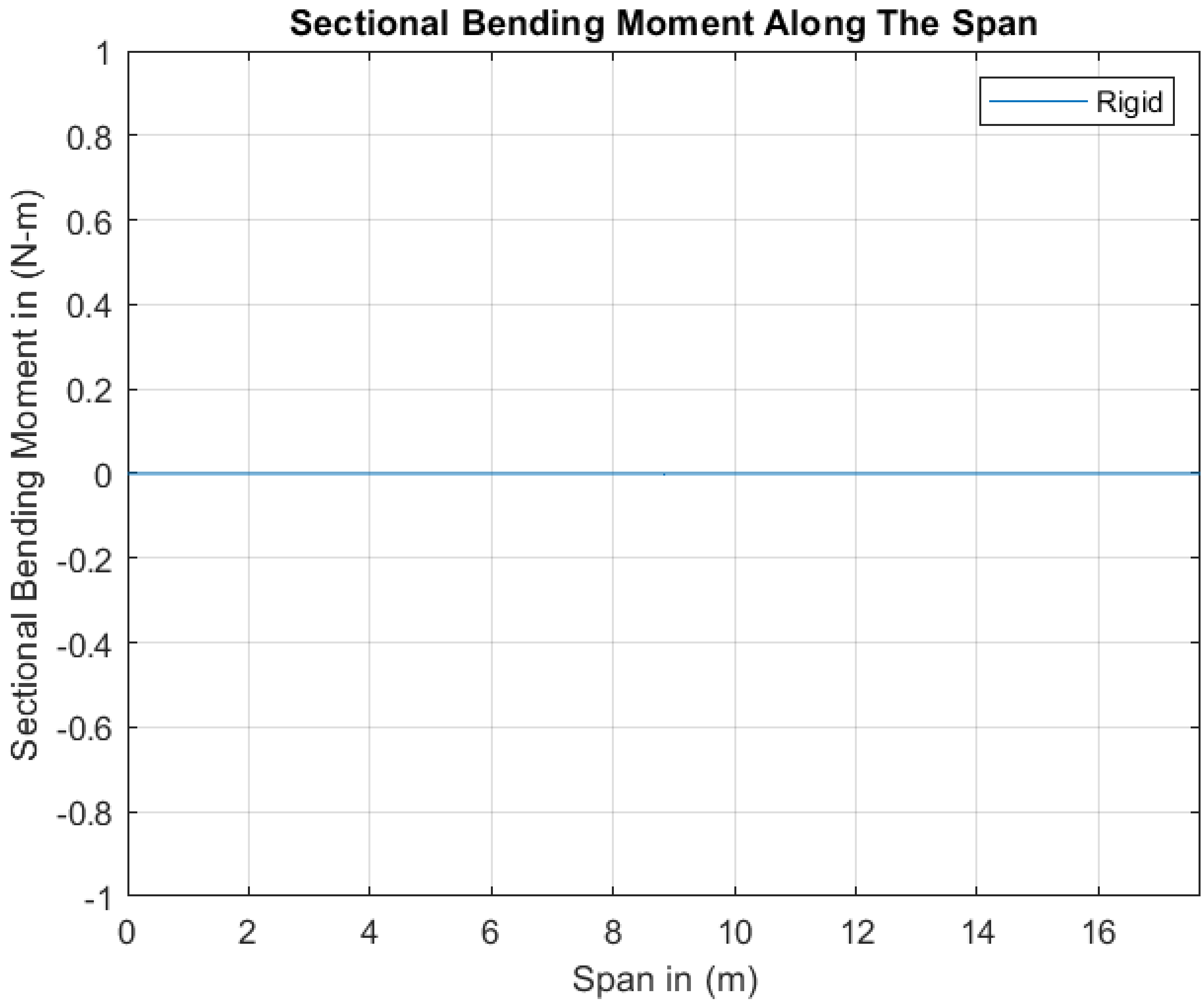


Results

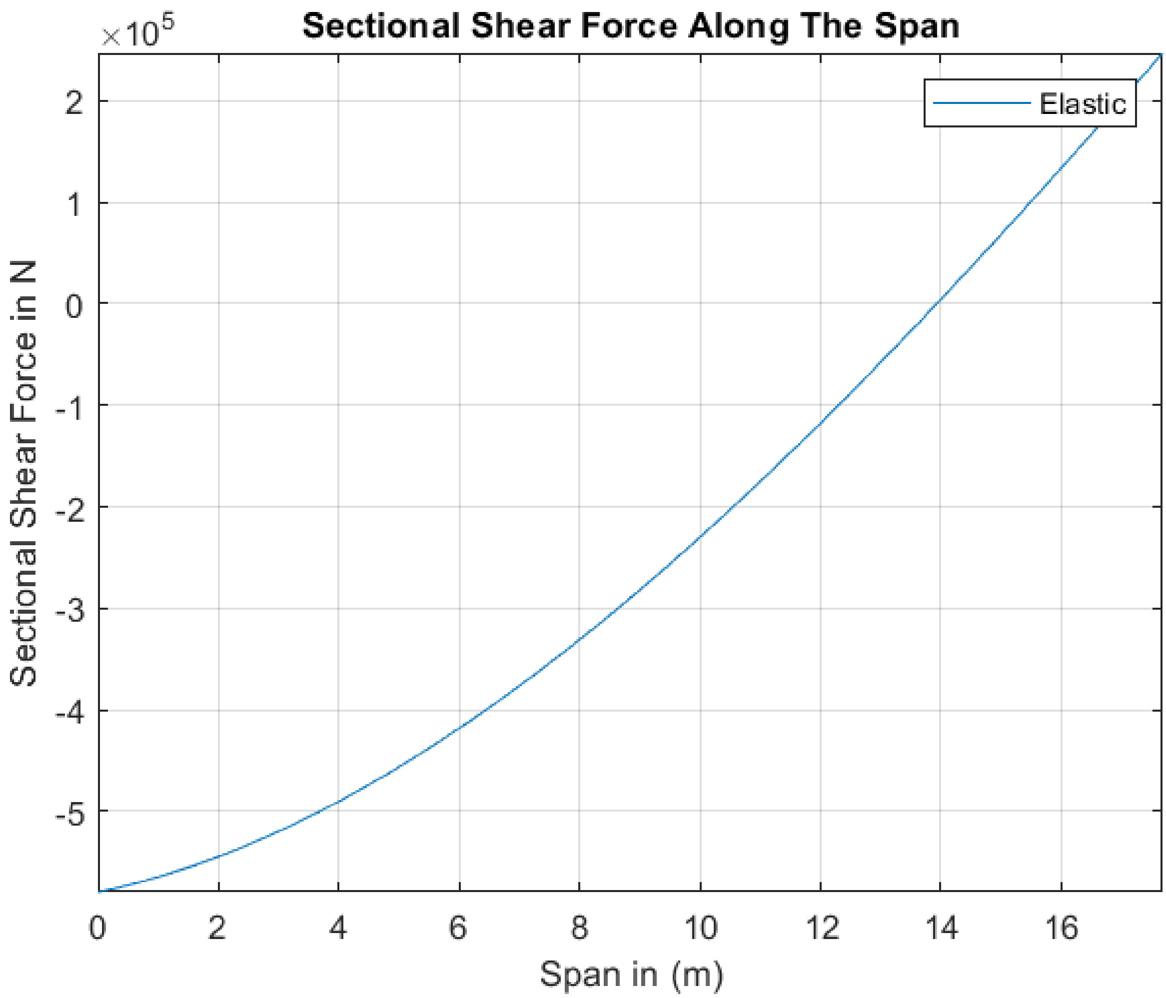
The running twisting moment is the same for both rigid and elastic case at the fixed end . The running twist for elastic case increases as we move along the length of span and rigid case as we move along the length of span is constant.



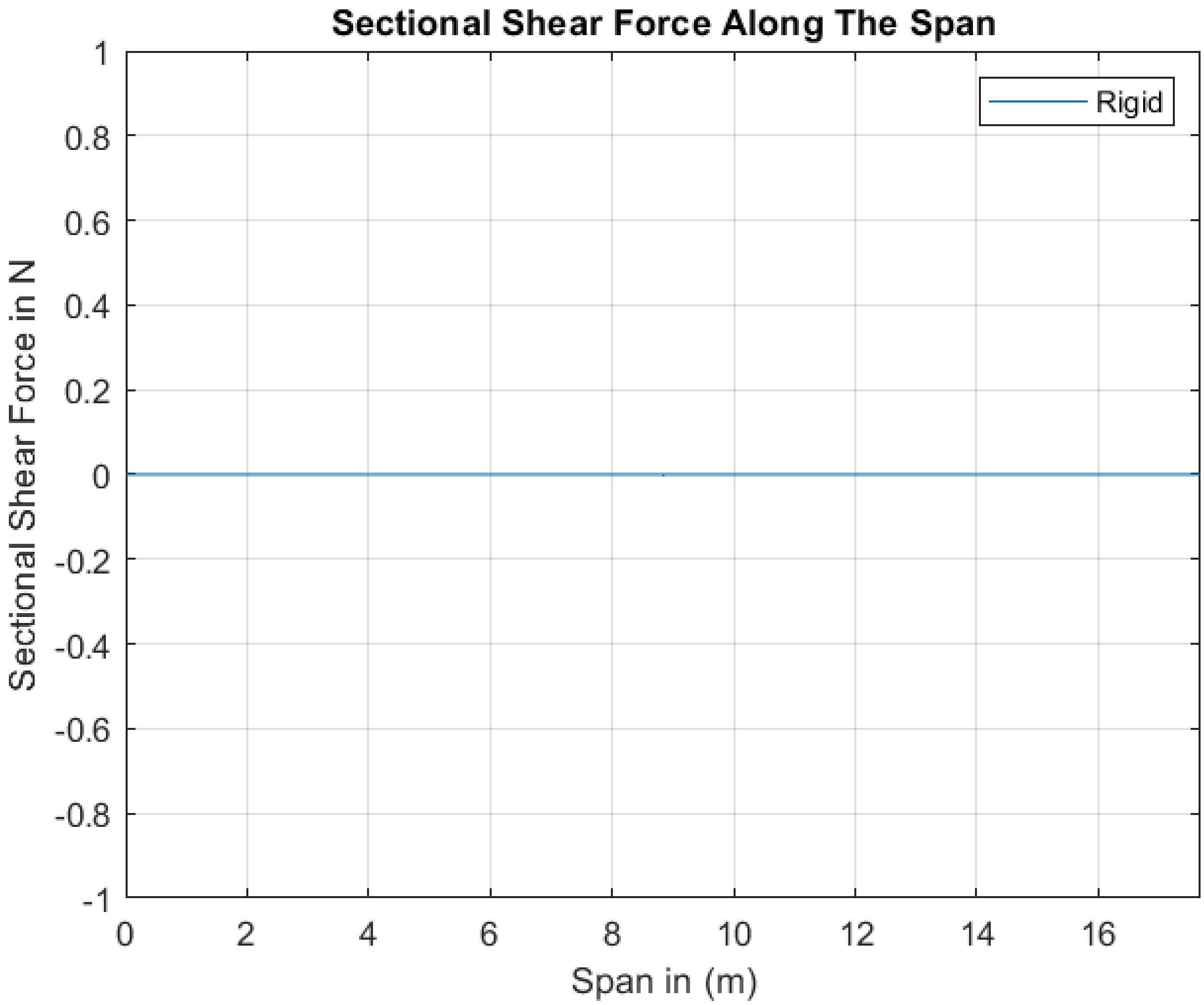
Sectional Bending Moment Along The Span
×10^6
Elastic
Sectional Bending Moment in (N-m)
4
3.5
3
2.5
2
1.5
1
0.5
0
0
2
4
6
8
10
12
14
16
Span in (m)



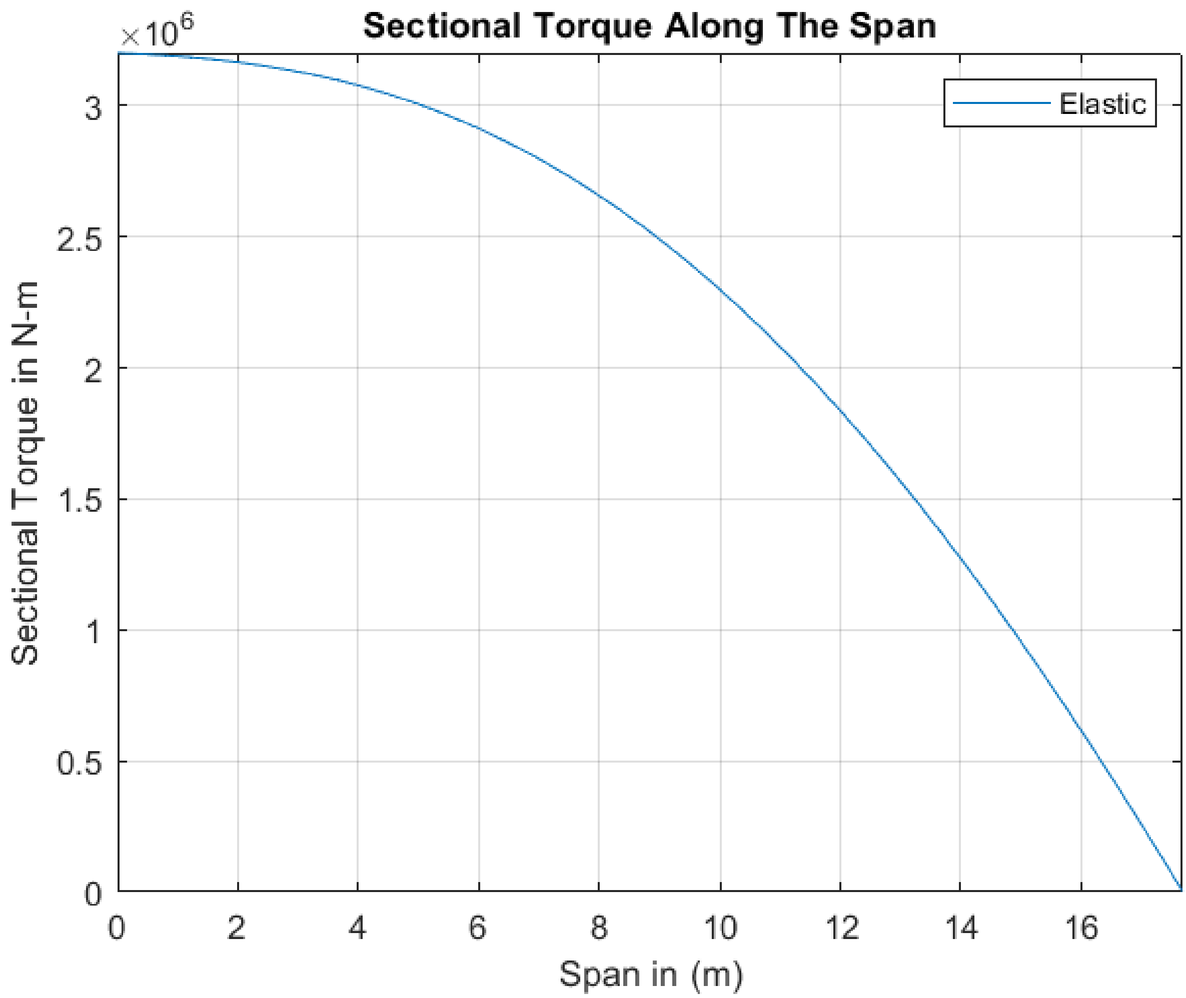
The sectional bending moment for rigid case is zero, whereas the sectional bending moment for the elastic case is maximum at the root of wing and decreases as we move along the span of wing, and becomes almost zero at the free end .



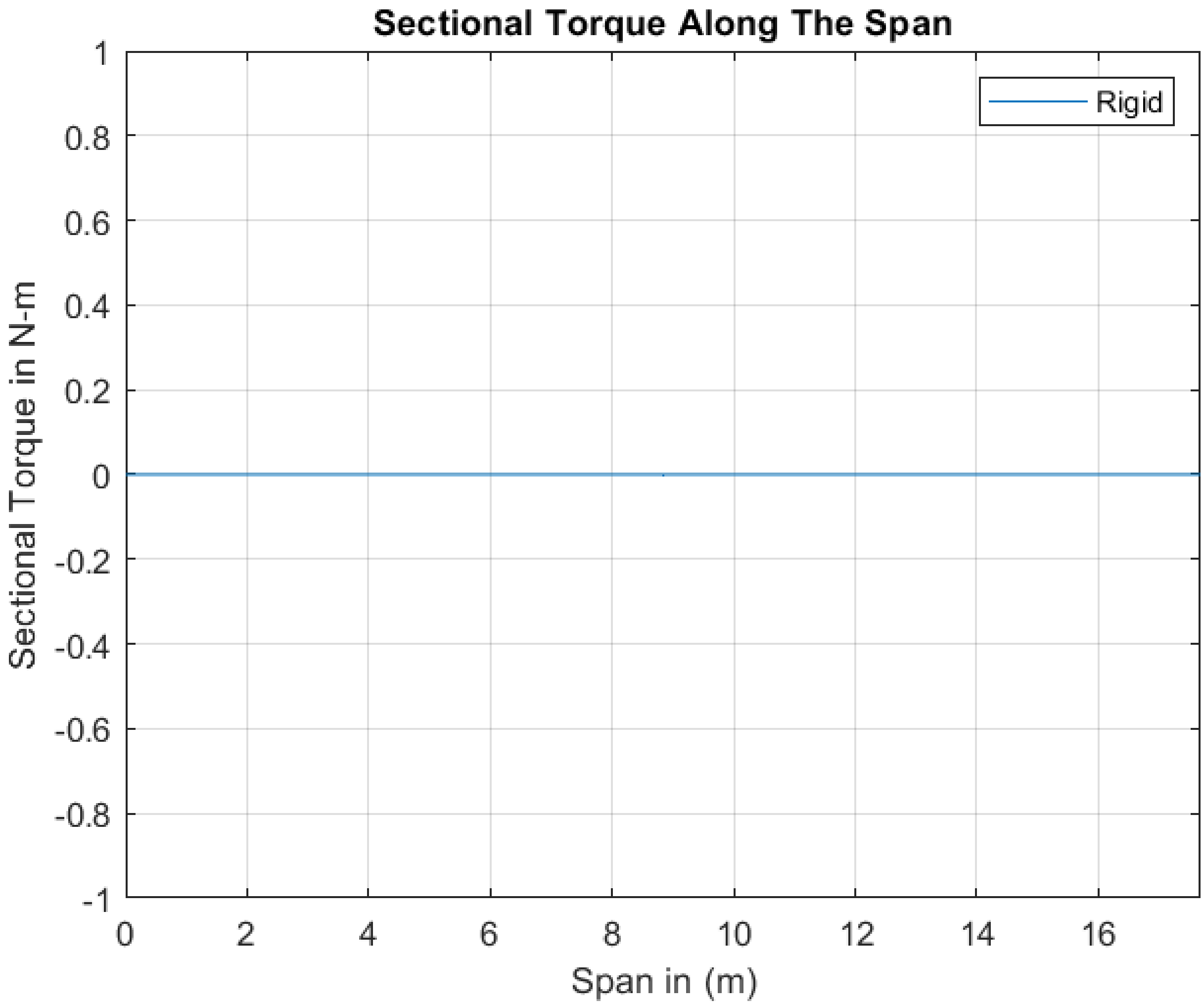
Sectional Shear Force Along The Span
×10^5
2
1
0
-1
-2
-3
-4
-5
Sectional Shear Force in N
Elastic
0
2
4
6
8
10
12
14
16
Span in (m)



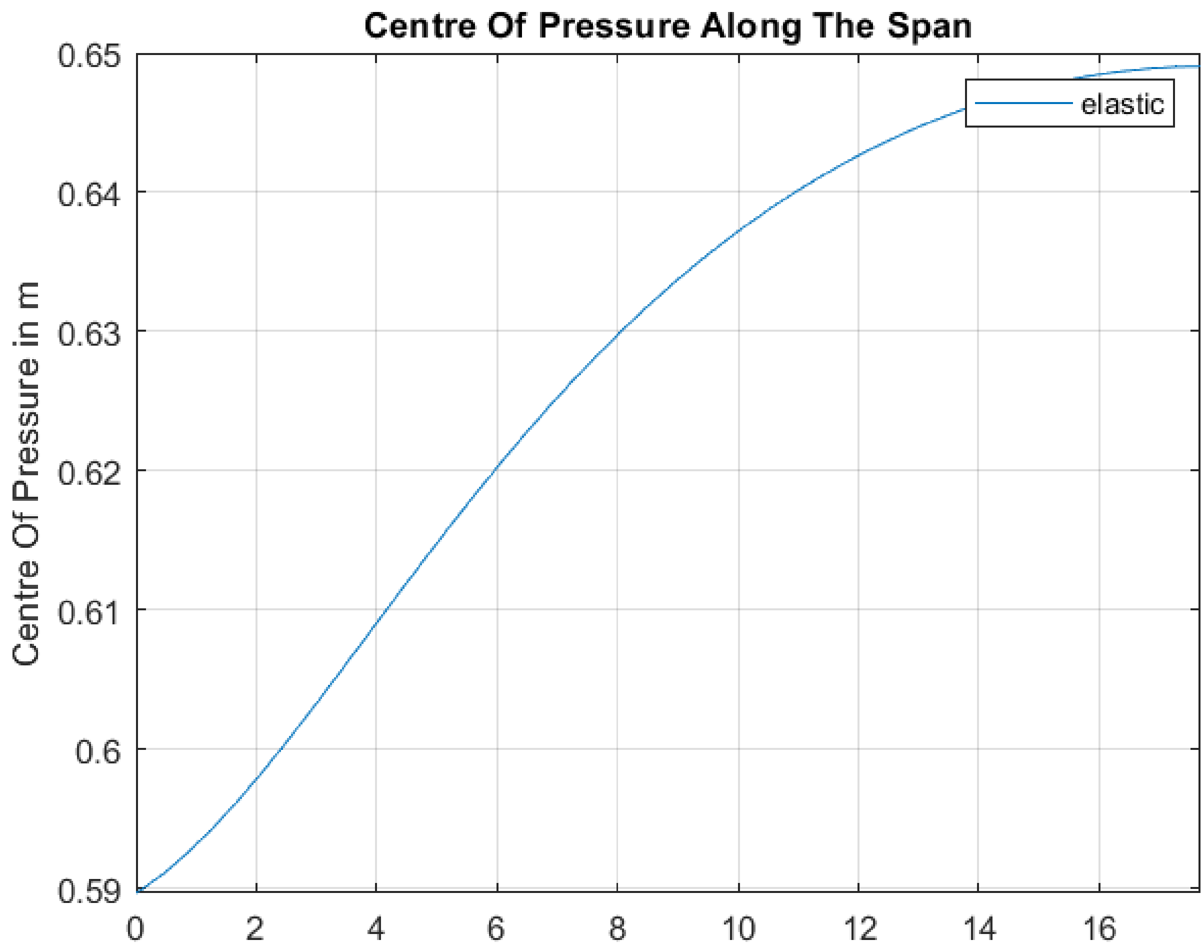
The sectional shear force is zero for rigid case. And elastic case it is negative at fixed end and continuously increases as we move along the span. It is wrong or error because its not zero at tip.

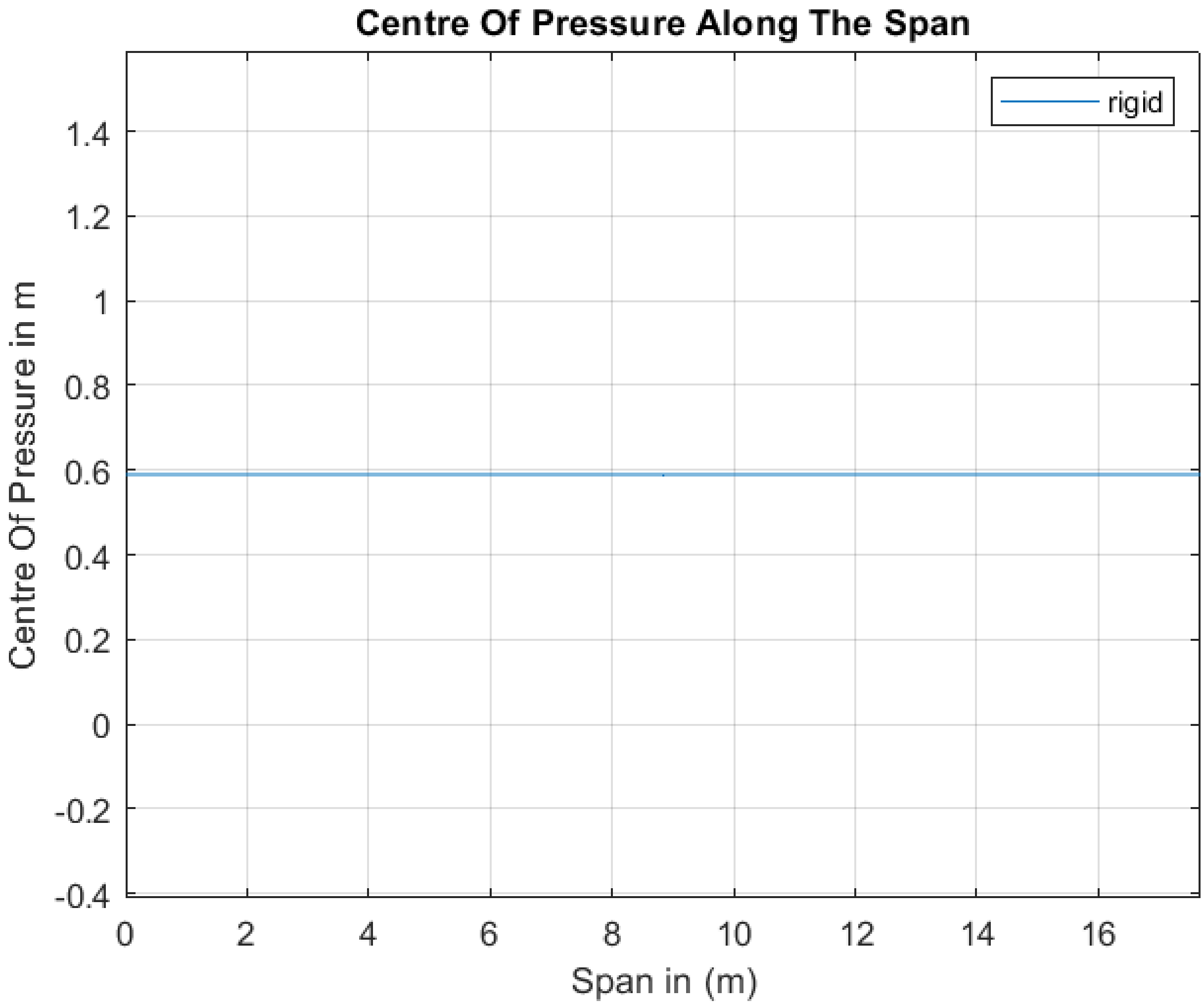


Sectional Torque Along The Span
×10^6
3
2.5
2
1.5
1
0.5
0
0
2
4
6
8
10
12
14
16
Span in (m)
Sectional Torque in N-m
Elastic



The sectional torque is zero for rigid case. For elastic case sectional torque is maximum at the fixed end and continuously decreases to zero at tip .


Centre Of Pressure Along The Span
Centre Of Pressure in m
elastic
0.65
0.64
0.63
0.62
0.61
0.6
0.59
0
2
4
6
8
10
12
14
16



the centre of pressure for rigid is a straight line, whereas the centre of pressure for the elastic case is a non-linear line.

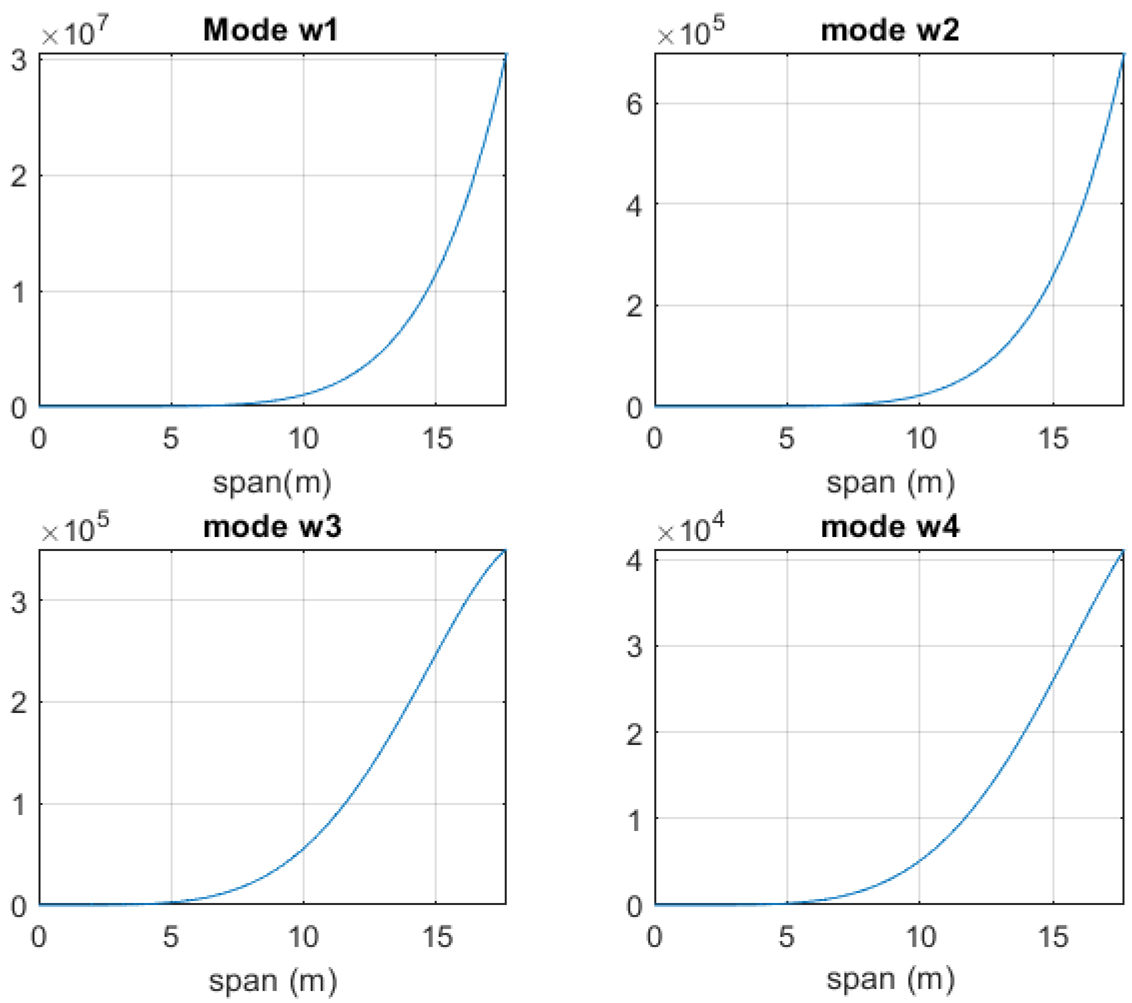
Divergence condition:

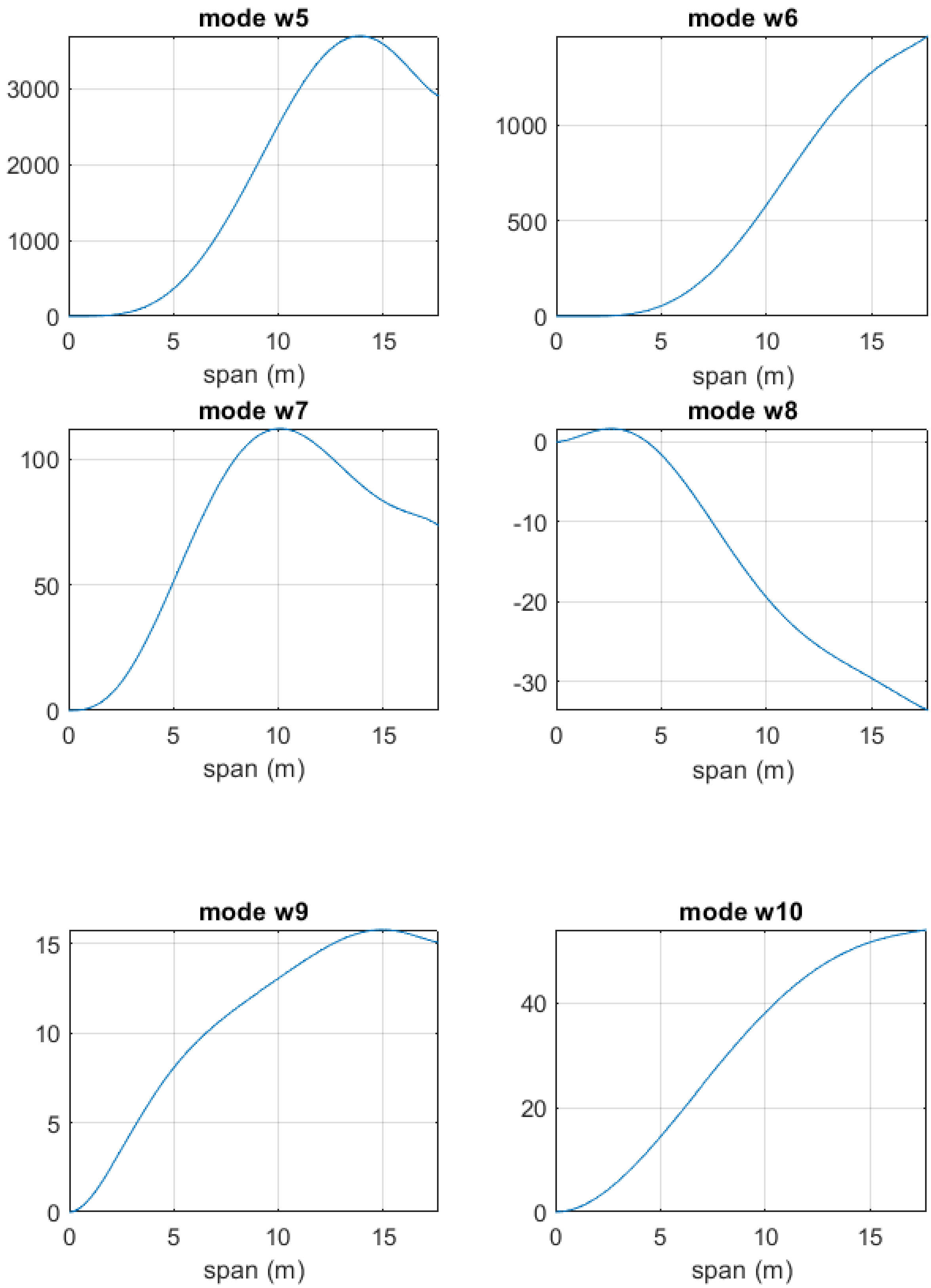
Divergence Dynamic Pressure, PdynD = 108917.393901 Pa

Divergence Velocity V(Div) = 574.458622 m/s

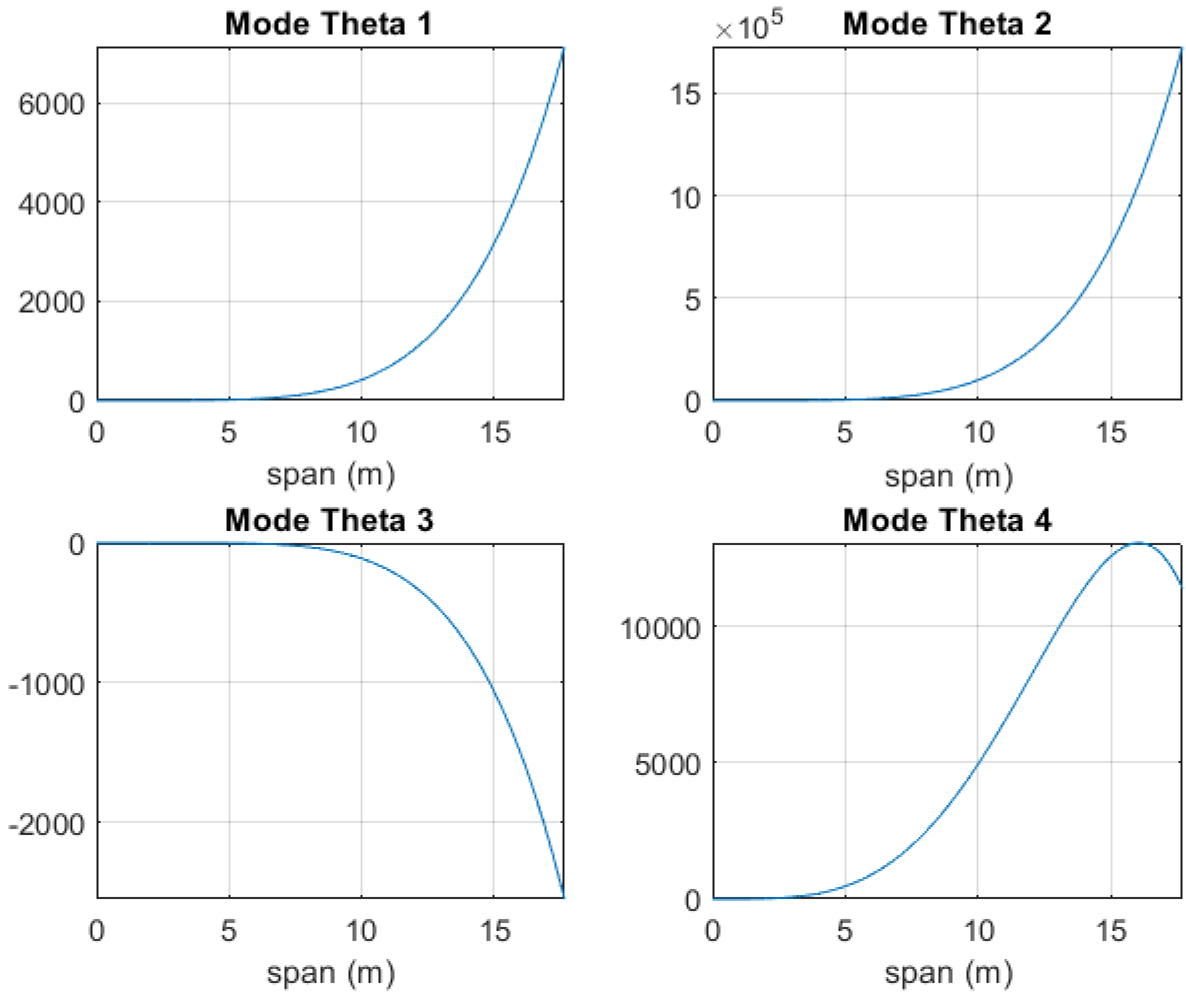
Divergence Mach Number M(Div) = 1.817907 m/s

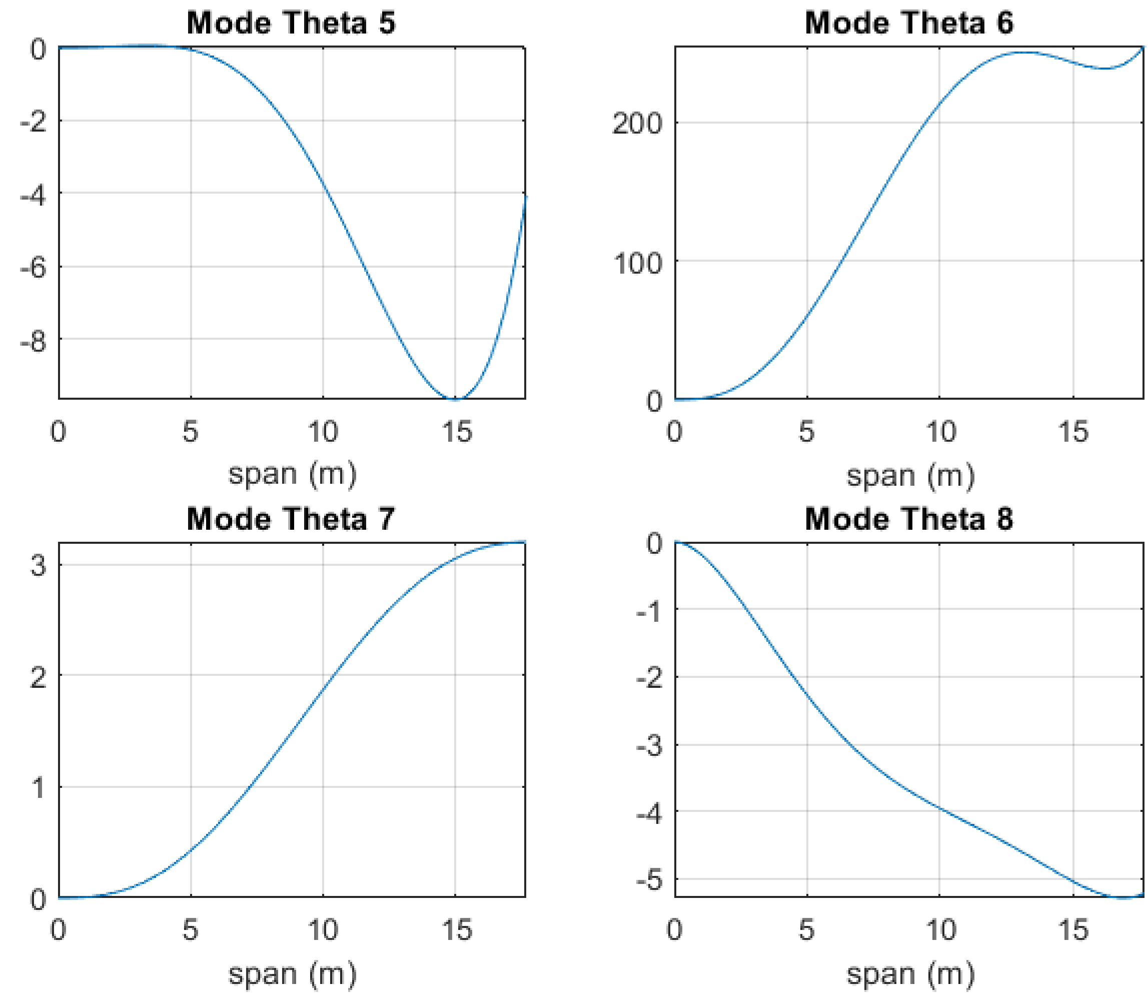
Mode shape for deflections

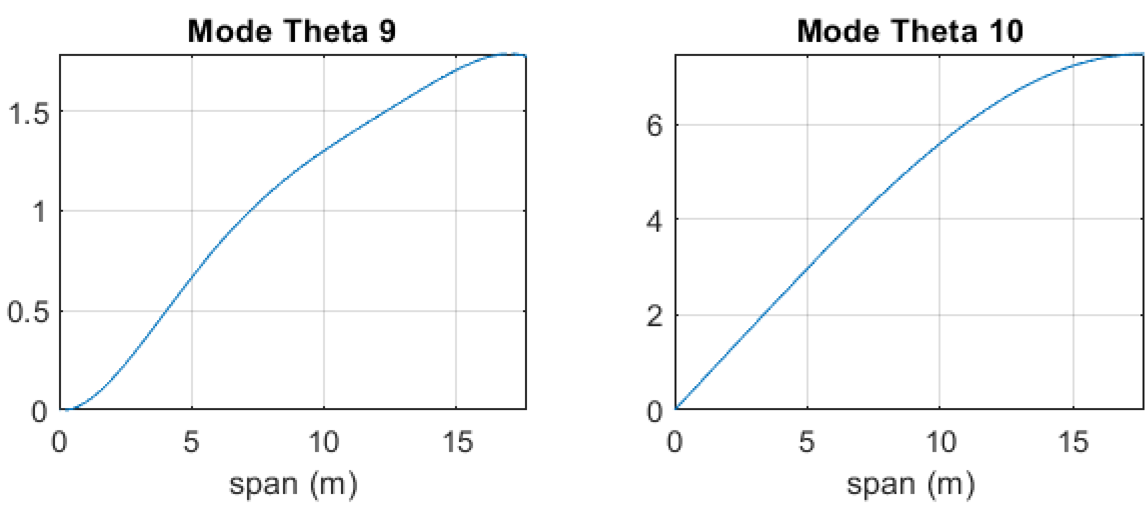



mode w5
0
1000
2000
3000
0
5
10
15
span (m)
mode w6
0
500
1000
0
5
10
15
span (m)
mode w7
0
50
100
0
5
10
15
span (m)
mode w8
0
-10
-20
-30
0
5
10
15
span (m)
mode w9
0
5
10
15
0
5
10
15
span (m)
mode w10
0
20
40
0
5
10
15
span (m)

Mode shape for Twist




Mode Theta 5
0
-2
-4
-6
-8
0
5
10
15
span (m)
Mode Theta 6
200
100
0
0
5
10
15
span (m)
Mode Theta 7
3
2
1
0
0
5
10
15
span (m)
Mode Theta 8
0
-1
-2
-3
-4
-5
0
5
10
15
span (m)


Mode Theta 9
1.5
1
0.5
0
0
5
10
15
span (m)
Mode Theta 10
6
4
2
0
0
5
10
15
span (m)

Matlab code for elastic and rigid wing:

```
% Defining the variables
clc;
syms Pdyn y w theta alphaR a b g N a1 a2 a3 a4 a5 b1 b2 b3 b4 b5
% Given Data
[Clalphac,lambda,AR,M,g,N,mass,Vs,EI,GJ,m,c,s,ec,xcg,cmac0]=
deal(5.422,0.61,9.43,0.8,9.81,3,66000,316.4,2E8 ,1.75E8
,500,3.072,17.68,0.6875,-0.1123,-0.015);

% Clalphac= Cl alpha corrected (per radian)
% lambda =  Sweep angle wrt to EA axis (radian)
% AR =  Aspect Ratio
% M =  Mach Number
% g =  Acceleration Due To Gravity
% N = Load Factor
% mass =  Mass in Kg
% Vs = Velocity Of Sound (m/sec) at 6 Km
Pdyn = (0.5*0.66*((M*Vs)^2)); % Dynamic Pressure at 6 km
% % Properties Of Wing
% EI = Variation Of Product Of E and I with y
% GJ =  Variation Of Product Of G and J with y
% m = Variation Of Mass Of Wing With y
% c =  Variation Of Chord Length With y
% s = Span(m) in y' direction i.e. Elastic Axis
% ec = Distance from EA to AC
% xcg =  Distance from EA to AC
% cmac0 = Aerodynamic Moment Co-Efficent
% % Assuming Approximate Function For 'w' And 'theta'
ai =5 %Number Of constants In The Approximate Function For w
bi = ai; % Number Of constants In The Approximate Function For theta
a = sym('a',[1 ai]); % Creating an array for a's parameters
a = a.'; % Converting it into a Column Vector
b = sym('b',[1 bi]); % Creating an array for b's parameters
```

```
b = b.'; % Converting it into a Column Vector
w = 0; % Initializing The Approximate Function For Deflection 'w'
theta =0; % Initialing The Approximate Function For Theta
for i = 1:ai
 w = a(i,1)*y^(i+1) + w;
end
for i = 1:bi
 theta = b(i,1)*y^(i) + theta;
end
% Effective Total Angle Of Attack
phi = diff(w,y);
alphaT = alphaR + theta*cos(lambda) - phi*sin(lambda);
% Forces And Moment Relations
l = Pdyn*c*Clalphac*alphaT; % Running Lift / Unit Length
l_ = (l - N*m*g)*cos(lambda); % Net Force Per Unit Length Of Span
my = ((l*ec) - (N*m*g*xcg))*(cos(lambda)^2) +
(Pdyn*(cos(lambda)^2)*(c^2)*cmac0);
% Twisting Moment Per Unit Length Of Span
% Differentitation Terms
dw_dy = diff(w, y);
dw2_dy2 = diff((dw_dy), y);
dtheta_dy = diff(theta, y);
% Strain Energy, Work Done And Potential Energy Functional
UB = 0.5*int((EI*((dw2_dy2)^2)),y,[0 s]); % Bending Strain Energy
UT = 0.5*int((GJ*((dtheta_dy)^2)),y,[0 s]);
% Torsional Strain Energy
WeB = int((l_*w), y,[0 s]); % Workdone In Bending
WeT = int((my*theta), y,[0 s]); % Workdone In Twisting
PEB = vpa((UB-WeB), 4); % Bending Energy Functional
PET = vpa((UT-WeT), 4); % Twisting Energy Functional
PEF = vpa((PEB+PET), 4); % Potential Energy Functional
% Rayleigh Ritz Method Minimizing The Potential Energy Functional
eqn = cell(ai+bi, 1); % Cell Defined For Equation
Coeff = cell(ai+bi, 1); % Cell Defined For Co-efficients
for i = 1:ai
```

```
 eqn{i} = diff(PEF, a(i)); % Differentiation wrt to ai
end
for i = 1:bi
 eqn{i+ai} = diff(PEF, b(i)); % Differentiation wrt to bi
end
for i = 1:ai+bi
 Coeff{i} = fliplr(coeffs(eqn{i}, [a b]));
end

S = solve(eqn{:});
[a1,a2,a3,a4,a5,b1,b2,b3,b4,b5]=
deal(S.a1,S.a2,S.a3,S.a4,S.a5,S.b1,S.b2,S.b3,S.b4,S.b5)

% Rigid Angle Of Attact For The Given Manoeuvre
Lift = 2*int(l, [0 s]); % Lift Of Both The Wings
Lift = vpa(Lift,5);
alphaR_eqn = subs(Lift-(mass*N*g)==0);
alphaR = solve(alphaR_eqn);
alphaR = vpa(alphaR, 5);
alphaR_deg = (alphaR*180)/3.1416; % In terms of degrees
fprintf('The rigid angle of attack associated with the wing for the given manevour
is alphaR = %f in deg \n', alphaR_deg);
% Total Lift For The Given Manoeuver
l = subs(l);
Total_Lift = 2*int(l,[0 s]);
Total_Lift = subs(Total_Lift,alphaR);
fprintf('Total Lift = %f in N \n', Total_Lift);
lr = Pdyn*cos(lambda)^2*c*Clalphac*alphaR;
Pzr = lr - (N*mass*g);
% Span wise distribution of w deflection
w = vpa(subs(subs(w)),4);
figure(1);
k = 0;
fplot(w,[0 s]);
grid on;
```

```
hold on;
fplot(k,[0 s]);
grid on;
hold  off;
legend('Elastic','Rigid');
ylabel('Deflection w in (m)');
title('Deflection along the span');
xlabel('Span in (m)');
fprintf('Deflection function w = %s \n',w);
% Span Wise Distibution Of Twist Theta
theta = vpa(subs(subs(theta)), 4);
figure(2);
fplot(theta, [0 s]);
grid on;
hold on;
k = 0;
fplot(k, [0 s]);
grid on;
hold off;
legend('Elastic', 'Rigid');
ylabel('Twist Theta in radian');
title('Twist Along The Span');
xlabel('Span in (m)');
fprintf('Twist Function Theta = %s\n', theta);
% Spanwise Distribution Of Total Angle Of Attack
alphaT = subs(alphaR+theta*cos(lambda)-(diff(w,y)*sin(lambda)));
figure(3);
fplot(alphaT, [0 s]);
grid on;
hold on;
alphaTR = subs(alphaR);
fplot(alphaTR, [0 s]);
grid on;
hold off;
legend('Elastic', 'Rigid');
```

```
ylabel('Total Angle Of Attack in radian');
xlabel('Span in (m)');
title('Total Angle Of Attack');
% Spanwise Distribution Of Running Lift
l = (Pdyn*(cos(lambda)^2)*c*(Clalphac)*alphaT);
figure(4);
fplot(l, [0 s]);
grid on;
hold on;
lR = (Pdyn*(cos(lambda)^2)*c*(Clalphac)*alphaTR);
fplot(lR, [0 s]);
grid on;
hold off;
legend('Elastic', 'Rigid');
ylabel('Running Lift in N/m');
xlabel('Span in (m)');
title('Running Lift Along Span');
% SpanWise Distribution Of Running Bending Moment
rBM = 0;
figure(5);
fplot(rBM, [0 s]);
grid on;
hold on;
fplot(rBM, [0 s]);
grid on;
hold off;
legend('Elastic', 'Rigid');
ylabel('Running Bending Moment in N/m');
xlabel('Span in (m)');
title('Running Bending Moment Along Span');

% Spanwise Distribution Of Running Twisting Moment
my = subs((l*ec) - (N*m*g*xcg) + (Pdyn*(cos(lambda)^2)*(c^2)*cmac0));
figure(6);
fplot(my, [0 s]);
```

```
grid on;
hold on;
myR = subs((lR*ec) - (N*m*g*xcg) + (Pdyn*(cos(lambda)^2)*(c^2)*cmac0));
fplot(myR, [0 s]);
grid on;
hold off;
legend('Elastic', 'Rigid');
ylabel('Running Twisting Moment in N-m');
xlabel('Span in (m)');
title('Running Twisting Moment Along The Span');
% SpanWise Distribution Of Sectional Bending Moment
dw_dy = diff(w, y);
dw2_dy2 = diff((dw_dy), y);
SBM = (EI*(dw2_dy2));
figure(7);
fplot(SBM, [0 s]);
grid on;
hold on;
k = 0;
fplot(k, [0 s]);
grid on;
hold off;
legend('Elastic', 'Rigid');
ylabel('Sectional Bending Moment in (N-m)');
title('Sectional Bending Moment Along The Span');
xlabel('Span in (m)');
% SpanWise Distribution Of Sectional Shear Force
dw3_dy3 = diff(dw2_dy2, y);
SSF = (EI*(dw3_dy3));
figure(8);
fplot(SSF, [0 s]);
grid on;
hold on;
k = 0;
fplot(k, [0 s]);
```

```
grid on;
hold off;
legend('Elastic', 'Rigid');
ylabel('Sectional Shear Force in N');
title('Sectional Shear Force Along The Span');
xlabel('Span in (m)');
% SpanWise Distribution Of Sectional Torque
dtheta_dy = diff(theta, y);
ST = (GJ*(dtheta_dy));
figure(9);
fplot(ST, [0 s]);
grid on;
hold on;
k = 0;
fplot(k, [0 s]);
grid on;
hold off;
legend('Elastic', 'Rigid');
ylabel('Sectional Torque in N-m');
title('Sectional Torque Along The Span');
xlabel('Span in (m)');
% SpanWise Centre Of Pressure Distribution
COP = ((l*ec)+(Pdyn*(cos(lambda)^2)*(c^2)*cmac0))/l;
COPR = ((lR*ec)+(Pdyn*(cos(lambda)^2)*(c^2)*cmac0))/lR;
figure(10);
fplot(COP, [0 s]);
grid on;
hold on;
fplot(COPR, [0 s]);
grid on;
hold off;
legend('Elastic', 'Rigid');
ylabel('Centre Of Pressure in m');
title('Centre Of Pressure Along The Span');
xlabel('Span in (m)');
```

```
% Tip Deflection and Tip Twist
y = s;
tip_deflection = subs(w);
tip_deflection = vpa(tip_deflection, 4);
fprintf('Tip Deflection = %f m\n', tip_deflection);
tip_twist = subs(theta);
tip_twist = vpa(tip_twist, 4);
tip_twist_degrees = (tip_twist*180)/3.1416;
fprintf('Tip Twist = %f degrees\n',tip_twist_degrees);
% End Of Program
```

Matlab code for divergence analysis:

```
% Defining the variables
clc;
format compact
syms Pdyn y w theta alphaR a b g N a1 a2 a3 a4 a5 b1 b2 b3 b4 b5
% Given Data
[Clalphac,lambda,AR,M,g,N,mass,Vs,EI,GJ,m,c,s,ec,xcg,cmac0]=
deal(5.422,0.61,9.43,0.8,9.81,3,66000,316.4,2E8 ,1.75E8 ,500,3.072,17.68,0.6875,-
0.1123,-0.015);
% Clalphac= Cl alpha corrected (per radian)
% lambda =  Sweep angle wrt to EA axis (radian)
% AR =  Aspect Ratio
% M =  Mach Number
% g =  Acceleration Due To Gravity
% N = Load Factor
% mass =  Mass in Kg
% Vs = Velocity Of Sound (m/sec) at 6 Km
%Pdyn = (0.5*0.6601*((M*Vs)^2)); % Dynamic Pressure at 6 km
% % Properties Of Wing
% EI = Variation Of Product Of E and I with y
% GJ =  Variation Of Product Of G and J with y
% m = Variation Of Mass Of Wing With y
% c =  Variation Of Chord Length With y
% s = Span(m) in y' direction i.e. Elastic Axis
% ec = Distance from EA to AC
% xcg =  Distance from EA to AC
% cmac0 = Aerodynamic Moment Co-Efficent
% % Assuming Approximate Function For 'w' And 'theta'
ai =5; %Number Of constants In The Approximate Function For w
bi = ai; % Number Of constants In The Approximate Function For theta
a = sym('a',[1 ai]); % Creating an array for a's parameters
a = a.'; % Converting it into a Column Vector
b = sym('b',[1 bi]); % Creating an array for b's parameters
b = b.'; % Converting it into a Column Vector
w = 0; % Initializing The Approximate Function For Deflection 'w'
theta =0; % Initialing The Approximate Function For Theta
for i = 1:ai
 w = a(i,1)*y^(i+1) + w;
end
for i = 1:bi
 theta = b(i,1)*y^(i) + theta;
end
% Effective Total Angle Of Attack
phi = diff(w,y);
alphaT = (alphaR + theta - phi*tan(lambda));
% Forces And Moment Relations
l = Pdyn*cos(lambda)^2*c*Clalphac*alphaT; % Running Lift / Unit Length
Pz = l - (N*m*g); % Net Force Per Unit Length Of Span
my = (l*ec) - (N*m*g*xcg) + (Pdyn*(cos(lambda)^2)*(c^2)*cmac0);
% Twisting Moment Per Unit Length Of Span
% Differentitation Terms
dw_dy =  diff(w,  y);
dw2_dy2 = diff((dw_dy), y);
dtheta_dy = diff(theta, y);
% Strain Energy, Worrk Done And Potential Energy Functional
UB = 0.5*int((EI*((dw2_dy2)^2)),y,[0 s]); % Bending Strain Energy
```

```
UT = 0.5*int((GJ*((dtheta_dy)^2)),y,[0 s]);
% Torsional Strain Energy
WeB = int((Pz*w), y,[0 s]); % Workdone In Bending
WeT = int((my*theta), y,[0 s]); % Workdone In Twisting
PEB = vpa((UB-WeB), 4); % Bending Energy Functional
PET = vpa((UT-WeT), 4); % Twisting Energy Functional
PEF = PEB+PET;
% Rayleigh Ritz Method Minimizing The Potential Energy Functional
eqn = cell(ai+bi, 1); % Cell Defined For Equation
Coeff = cell(ai+bi, 1); % Cell Defined For Co-efficients
for i = 1:ai
 eqn{i} = diff(PEB, a(i)); % Differentiation wrt to ai
end
for i = 1:bi
 eqn{i+ai} = diff(PET, b(i)); % Differentiation wrt to bi
end
for i = 1:ai+bi
 Coeff{i} = fliplr(coeffs(eqn{i}, [a b]));
end
% Eigen Value Analysis For PdynD
Coeff_exp = vpa(cell2sym(Coeff), 4); % Expanding The Cell
Coeff_sq = Coeff_exp(1:ai+bi, 1:ai+bi); % Converting in to square matrix
Dm = det(Coeff_sq);
PdynD = solve(Dm==0, Pdyn); % Value Of PdynD
PdynD = vpa(subs(PdynD), 5);
PdynD = PdynD(imag(PdynD)==0); % Taking Only The Real Terms
PdynD = PdynD(PdynD>=0); % Taking Only Positive Terms
PdynD = vpa(min(PdynD), 4); % The Minimum Value Of Dynamic Pressure Is Divergence
% Divergence Velocity And Mach Number
VDiv = vpa((2*PdynD)/0.6601)^0.5; % rho of air at 6000 m
MDiv = VDiv/316; % Speed of sound 316 m/sec at 6000 m
Pdyn = PdynD; % Replacing Pdyn with PdynD for perturbation Solution
Coeff_sq = subs((Coeff_sq), Pdyn);
[Eve, Eval] = eig(Coeff_sq); % Eigen Value And Eigen Vector For Mode Shapes
% Displaying the input approximate functions and the desired output
% disp('Approximate Functions');
% fprintf('\n w = %s \n', w);
fprintf('\n theta = %s \n', theta);
fprintf('Divergence Dynamic Pressure, PdynD = %f Pa\n', PdynD);
fprintf('Divergence Velocity V(Div) = %f m/s\n', VDiv);
fprintf('Divergence Mach Number M(Div) = %f m/s \n', MDiv);
% % Mode Shapes For Deflection
[a1,a2,a3,a4,a5]=deal(Eve(1,1),Eve(2,1),Eve(3,1),Eve(4,1),Eve(5,1))
def_w  =  subs(w);
figure(1);
title('Displacement mode');
subplot(2,2,1);
fplot(def_w,[0 s]);
grid on;
hold on;
title('Mode w1');
xlabel('span(m)');
[a1,a2,a3,a4,a5]=deal(Eve(1,2),Eve(2,2),Eve(3,2),Eve(4,2),Eve(5,2))
def_w = subs(w);
subplot(2,2,2);
fplot(def_w, [0 s]);
grid on;
hold on;
```

```
title('mode w2');
xlabel('span (m)');
[a1,a2,a3,a4,a5]=deal(Eve(1,3),Eve(2,3),Eve(3,3),Eve(4,3),Eve(5,3))
def_w = subs(w);
subplot(2,2,3);
fplot(def_w, [0 s]);
grid on;
hold on;
title('mode w3');
xlabel('span (m)');
[a1,a2,a3,a4,a5]=deal(Eve(1,4),Eve(2,4),Eve(3,4),Eve(4,4),Eve(5,4))
def_w = subs(w);
subplot(2,2,4);
fplot(def_w, [0 s]);
grid on;
hold on;
title('mode w4');
xlabel('span (m)');
figure(2);
[a1,a2,a3,a4,a5]=deal(Eve(1,5),Eve(2,5),Eve(3,5),Eve(4,5),Eve(5,5))
def_w = subs(w);
subplot(2,2,1);
fplot(def_w, [0 s]);
grid on;
hold on;
title('mode w5');
xlabel('span (m)');
[a1,a2,a3,a4,a5]=deal(Eve(1,6),Eve(2,6),Eve(3,6),Eve(4,6),Eve(5,6))
def_w = subs(w);
subplot(2,2,2);
fplot(def_w, [0 s]);
grid on;
hold on;
title('mode w6');
xlabel('span (m)');
[a1,a2,a3,a4,a5]=deal(Eve(1,7),Eve(2,7),Eve(3,7),Eve(4,7),Eve(5,7))
def_w = subs(w);
subplot(2,2,3);
fplot(def_w, [0 s]);
grid on;
hold on;
title('mode w7');
xlabel('span (m)');
[a1,a2,a3,a4,a5]=deal(Eve(1,8),Eve(2,8),Eve(3,8),Eve(4,8),Eve(5,8))
def_w = subs(w);
subplot(2,2,4);
fplot(def_w, [0 s]);
grid on;
hold on;
title('mode w8');
xlabel('span (m)');
figure(3);
[a1,a2,a3,a4,a5]=deal(Eve(1,9),Eve(2,9),Eve(3,9),Eve(4,9),Eve(5,9))


def_w = subs(w);
subplot(2,2,1);
fplot(def_w, [0 s]);
```

```
grid on;
hold on;
title('mode w9');
xlabel('span (m)');
[a1,a2,a3,a4,a5]=deal(Eve(1,10),Eve(2,10),Eve(3,10),Eve(4,10),Eve(5,10))
def_w = subs(w);
subplot(2,2,2);
fplot(def_w, [0 s]);
grid on;
hold on;
title('mode w10');
xlabel('span (m)');
%mode Shapes For Theta
figure(4);
title('Twisting Modes');
[b1,b2,b3,b4,b5]=deal(Eve(6,1),Eve(7,1),Eve(8,1),Eve(9,1),Eve(10,1))
def_theta = subs(theta);
subplot(2,2,1);
fplot(def_theta, [0 s]);
grid on;
hold on;
title('Mode Theta 1');
xlabel('span (m)');
[b1,b2,b3,b4,b5]=deal(Eve(6,2),Eve(7,2),Eve(8,2),Eve(9,2),Eve(10,2))
def_theta = subs(theta);
subplot(2,2,2);
fplot(def_theta, [0 s]);
grid on;
hold on;
title('Mode Theta 2');
xlabel('span (m)');
[b1,b2,b3,b4,b5]=deal(Eve(6,3),Eve(7,3),Eve(8,3),Eve(9,3),Eve(10,3))
def_theta = subs(theta);
subplot(2,2,3);
fplot(def_theta, [0 s]);
grid on;
hold on;
title('Mode Theta 3');
xlabel('span (m)');
[b1,b2,b3,b4,b5]=deal(Eve(6,4),Eve(7,4),Eve(8,4),Eve(9,4),Eve(10,4))
def_theta = subs(theta);
subplot(2,2,4);
fplot(def_theta, [0 s]);
grid on;
hold on;
title('Mode Theta 4');
xlabel('span (m)');
figure(5);
[b1,b2,b3,b4,b5]=deal(Eve(6,5),Eve(7,5),Eve(8,5),Eve(9,5),Eve(10,5))
def_theta = subs(theta);
subplot(2,2,1);
fplot(def_theta, [0 s]);
grid on;
hold on;
title('Mode Theta 5');
xlabel('span (m)');
[b1,b2,b3,b4,b5]=deal(Eve(6,6),Eve(7,6),Eve(8,6),Eve(9,6),Eve(10,6))
def_theta = subs(theta);
```

```
subplot(2,2,2);
fplot(def_theta, [0 s]);
grid on;
hold on;
title('Mode Theta 6');
xlabel('span (m)');
[b1,b2,b3,b4,b5]=deal(Eve(6,7),Eve(7,7),Eve(8,7),Eve(9,7),Eve(10,7))
def_theta = subs(theta);
subplot(2,2,3);
fplot(def_theta, [0 s]);
grid on;
hold on;
title('Mode Theta 7');
xlabel('span (m)');
[b1,b2,b3,b4,b5]=deal(Eve(6,8),Eve(7,8),Eve(8,8),Eve(9,8),Eve(10,8))
def_theta = subs(theta);
subplot(2,2,4);
fplot(def_theta, [0 s]);
grid on;
hold on;
title('Mode Theta 8');
xlabel('span (m)');
figure(6);
[b1,b2,b3,b4,b5]=deal(Eve(6,9),Eve(7,9),Eve(8,9),Eve(9,9),Eve(10,9))
def_theta = subs(theta);
subplot(2,2,1);
fplot(def_theta, [0 s]);
grid on;
hold on;
title('Mode Theta 9');
xlabel('span (m)');
[b1,b2,b3,b4,b5]=deal(Eve(6,10),Eve(7,10),Eve(8,10),Eve(9,10),Eve(10,10))
def_theta = subs(theta);
subplot(2,2,2);
fplot(def_theta, [0 s]);
grid on;
hold on;
title('Mode Theta 10');
xlabel('span (m)');
% % End Of Program
```

Reference

1. Introduction to Finite Element Method by J.N. Reddy
2. Aeroelasticity class lectures and notes.
3. NPTEL lectures on Finite Element Method by Prof. Nachiketa Tiwari (IIT Kanpur, Mechanical Engineering Department).
4. Code help from Google/ chat gpt.