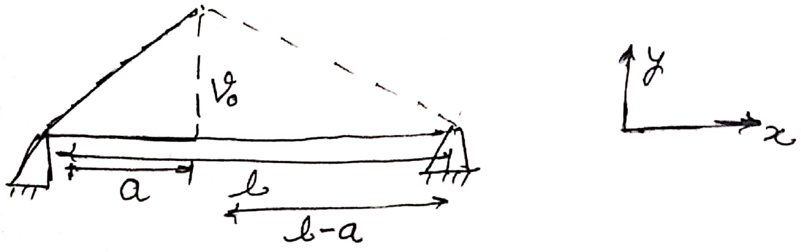
Structural Dynamics

Roll no. - 22M0040



Soln (I)

Governing Differential Eqn

$$\boxed{-m\frac{d^2 v}{dt^2} + P_y + \frac{d}{dx}\left[T\frac{dv}{dx}\right] = 0}$$

For free vibration $\therefore P_y = 0$.

$$\boxed{T\frac{d^2 v}{dx^2} - m\frac{d^2 v}{dt^2} = 0} \quad -(1)$$

Boundary Conditions.

$v(x, t) = 0$ at $x = 0, L$

Initial condition.

$v(x, 0)$, $\dot{v}(x, 0)$ are specified.

If T is constant

$$T\frac{d^2 v}{dx^2} - m\frac{d^2 v}{dt^2} = 0 .$$

ST. Boundary conditions & Initial condition

$v(x,t) = V(x)\,g(t)$ - [Synchronous motion under go normal normal of vibration.

putting in eqn (1)

$$T\frac{d^2V(x)}{dx^2}\cdot g - mV(x)\frac{d^2g}{dt^2} = 0.$$

$\because {}'' = \frac{d}{dx}$

$\because \dot{} = \frac{d}{dt}$

$$\frac{TV''}{V} - \frac{m\ddot{g}}{g} = 0.$$

$$\frac{TV''}{mV} = \frac{\ddot{g}}{g} = \text{constant} = -\omega_n^2.$$

So, $TV''(x) + \omega_n^2 m V(x)$

$$\frac{d^2V(x)}{d\bar{x}} + \frac{m\omega_n^2L^2}{T}V = 0 \qquad \bar{x} = \frac{x}{L}$$

λ^2.

S.T. $v(0,t) = v(\frac{L}{L},t) = 0.$

$$\frac{d^2V(\bar{x})}{d\bar{x}} + \lambda^2 V(\bar{x}) = 0.$$

$$V(\bar{x}) = A\sin\lambda\bar{x} + B\cos\lambda\bar{x}.$$

BCs.

$$0 = A\sin\lambda(0) + B\cos\lambda(0)$$

$$\boxed{0 = B}$$

$v(1,t) = 0.$

$0 \Rightarrow A\sin\lambda + B$

$A\sin\lambda = 0$

$A \neq 0$ so, $\sin\lambda = 0$

Scanned with OKEN Scanner

$\Rightarrow \lambda = n\pi \qquad n = 1, 2, 3 \cdots N$

So, $\lambda^2 = n^2\pi^2$

$$\frac{m\omega_n^2 L^2}{T} = n^2\pi^2$$

$$\boxed{\begin{aligned} \omega_{n_i}^2 &= \frac{n^2\pi^2 T}{mL^2} \\ \phi_i(x) &= A \sin \pi n \bar{x} \end{aligned}}$$

Modal Sol^n:

$$\boxed{v(\bar{x}, t) = \sum_{i=1}^{\infty} \phi_i(\bar{x})\, \eta_i(t)} \quad -$$

$\cancel{v(\bar{x}, t)} =$

$$\int_0^1 \frac{T}{L^2} \sum_{i=1}^{N} \phi_i'' \eta_i \, d\bar{x} - \int_0^1 \phi_i\, m \sum_{i=1}^{N} \ddot{\eta}_i \, d\bar{x} = 0 \quad (2)$$

multiply by ϕ_j.

$$\int_0^1 \phi_j \frac{T}{L^2} \sum_{i=1}^{N} \phi_i'' \eta_i \, d\bar{x} = \int_0^1 \phi_j\, m \phi_i \sum_{i=1}^{N} \ddot{\eta}_i \, d\bar{x} = 0$$

$$= \int_0^1 \phi_j \frac{T}{L^2} \phi_i'' \, d\bar{x} = -\omega_{n_i}^2 \int_0^1 \phi_j\, m\, \phi_i \, d\bar{x}$$

Integrating by parts.

$$\cancel{\phi_j \frac{T}{L^2} \phi' \Big|_0^1}^{=0} - \int \frac{d\phi_j}{d\bar{x}} \frac{T}{L^2} \phi' \, d\bar{x} = -\omega_{n_i}^2 \int_0^1 \phi_j\, m\, \phi_i \, d\bar{x}$$

$$-\int_0^1 \frac{d\phi_j}{d\bar{x}} \frac{T}{L^2} \phi_i' \, d\bar{x} = -\omega_{n_i}^2 \int_0^1 \phi_j\, m\, \phi_i \, d\bar{x} \quad \cdots (3)$$

from eqn ② multiply ϕ_i and Integrating by parts. we get.

$$-\int_0^1 \frac{d\phi_i}{d\bar{x}} \frac{T}{L} \frac{d\phi_j}{d\bar{x}} d\bar{x} = -\omega_{nj}^2 \int_0^1 \phi_i m \phi_j d\bar{x} \quad (4)$$

from ③ and ④

$$\boxed{(\omega_{ni}^2 - \omega_{nj}^2) \int_0^1 \phi_i m \phi_j d\bar{x} = 0}$$

$\boxed{\omega_{ni} \neq \omega_{nj}}$ $\quad i \neq j$

∴ orthogonality of mode shape for a shape

$$\boxed{\int_0^1 \phi_j m \phi_j d\bar{x} = 0.}$$

$i \neq j$

alternate

$$\boxed{\int_0^1 \frac{d\phi_j}{d\bar{x}} \frac{T}{L^2} \frac{d\phi_i}{d\bar{x}} d\bar{x} = 0} \text{ — orthogonality.}$$

$$m_{nj} \ddot{\eta}_j + k_{nj} \eta_j = 0 \qquad m_{nj} = \int_0^1 m \phi_j^2 d\bar{x} \quad i = j$$

$$\ddot{\eta}_j + \frac{k_{nj}}{m_{nj}} \eta_j = 0 \qquad k_{nj} = \int_0^1 \frac{T}{L^2} \phi_j'^2 d\bar{x}$$

$$\boxed{\ddot{\eta}_j + \omega_{nj}^2 \eta_j = 0}$$

Initial Condition

$\eta_j(0)$, $\dot{\eta}_j(0)$ are specified.

$$\boxed{\eta_j(t) = A_j \cos(\omega_{nj} t) + B_j \cos \omega_{nj} t}$$

$$\eta_j(0) = \frac{\int_0^1 m\phi_j\, v_j(\bar{x})\, d\bar{x}}{m_{\eta_j}}$$

$$m_{\eta_j} = \int_0^1 m\, \phi_j^2\, d\bar{x}$$

$$= \int_0^1 m \sin^2(n_i \pi \bar{x})\, d\bar{x}$$

$$= \frac{m}{2} \int_0^1 1 - \cos(2 n_i \pi \bar{x})\, d\bar{x}$$

$$= \frac{m}{2} \left[\bar{x} + \frac{\sin(2 n \pi \bar{x})}{2 n_i \pi} \right]_0^1$$

$$= \frac{m}{2}$$

Similarity

$$k_{\eta_j} = \frac{T}{L^2} \int_0^1 \phi_j'^2\, d\bar{x}$$

$$= \frac{T}{L^2} \int_0^1 (n_i \pi)^2 \cos^2(n_i \pi \bar{x})\, d\bar{x}$$

$$= \frac{T}{2L^2} (n_i \pi)^2 \left[\bar{x} - \frac{\sin(2 n_i \pi \bar{x})}{2 n_i \pi} \right]_0^1$$

$$k_{\eta_j} = \frac{T}{2L^2} (n_i \pi)^2$$

from

$$\omega_{\eta_j} = \sqrt{\frac{k_{\eta_j}}{m_{\eta_j}}} = \sqrt{\frac{\frac{T}{2L^2}(n_i \pi)^2}{m/2}}$$

$$\boxed{\omega_{\eta_j} = \frac{n_i \pi}{L} \sqrt{\frac{T}{m}}}$$

Scanned with OKEN Scanner

$$\eta_j(0) = 2\int_0^{a} \sin(n_i\pi\bar{x})\left(\frac{v_0}{a}\right)\bar{x}\,d\bar{x} + 2\int_a^1 \sin(n_i\pi\bar{x})\left[\frac{v_0}{1-a} - \frac{4v_0\bar{x}}{1-a}\right]d\bar{x}$$

$$= 2\left[\frac{v_0}{a}\bar{x}\left\{-\frac{\cos(n_i\pi\bar{x})}{n_i\pi}\right\}\right]_0^a - 2\int_0^a\left(\frac{v_0}{a}\right)\left(-\frac{\cos(n_i\pi\bar{x})}{n_i\pi}\right)d\bar{x}$$

$$2\left[\left(\frac{v_0}{1-a} - \frac{v_0\bar{x}}{1-a}\right)\left(-\frac{\cos(n_i\pi\bar{x})}{n_i\pi}\right)\right]_a^1 - \int_a^1\left(\frac{-v_0}{1-a}\right)\left(-\frac{\cos(n_i\pi\bar{x})}{i\pi}\right)d\bar{x}$$

$$= \cancel{\frac{-2v_0\cos(n_i\pi a)}{n_i\pi}} + 2\left[\frac{v_0}{a}\frac{\sin(n_i\pi\bar{x})}{(i\pi)^2}\right]_0^a + 2\left[\cancel{\frac{v_0}{i\pi}\cos(n_i\pi a)}\right]$$

$$- \frac{2v_0}{1-a}\left[\frac{\sin n_i\pi\bar{x}}{(n_i\pi)^2}\right]_a^1$$

$$= \cancel{\frac{2v_0}{n_i\pi}} \frac{2v_0}{(n_i\pi)^2}\left[\frac{1}{a} + \frac{1}{1-a}\right]\sin(n_i\pi\bar{x}$$

$$\boxed{\eta_j(0) = \frac{2v_0}{a(1-a)}\frac{\sin(n_i\pi\bar{x})}{(n_i\pi)^2}}$$

Similarily

$$\dot{\eta}_j(0) = 0.$$

$$\frac{\int_0^1 m\phi_j v_j'\,d\bar{x}}{m\eta_j} = 0.$$

$$\boxed{\eta_j(t) = A_j\cos(\omega_{nj}t)}.$$

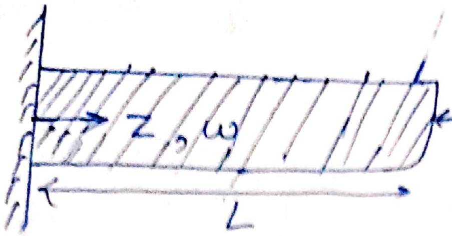
$$\boxed{\eta_j(t) = \frac{2v_0}{a(1-a)}\frac{\sin(n_i\pi a)}{(n_i\pi)^2}\cos\left[\frac{n_i\pi}{L}\sqrt{\frac{T}{m}}\,t\right]}$$

$$u(\bar{x},t) = \sum_{i=n}^{\infty}\phi(\bar{x})\eta_j(t)$$

$$= \sum_{i=n}^{\infty}\sin[n_i\pi\bar{x}]\left[\frac{2v_0}{a(1-a)}\frac{\sin(n_i\pi a)}{(n_i\pi)^2}\cos\left[\frac{n_i\pi}{L}\sqrt{\frac{T}{m}}\,t\right]\right.$$

Scanned with OKEN Scanner

Solⁿ ②



Governing Differential Eqⁿ

$$\frac{d}{dz} EA(z)\frac{dw}{dz} + P_z(z) = 0 \quad \text{[Static Equill. Eq.]}$$

$$P_z(z,t) = -\rho A \frac{d^2 w}{dt^2} \quad \text{| Free Vibration.}$$

$$\frac{d}{dz} EA(z)\frac{dw}{dz}(z,t) - \rho A \frac{d^2 w}{dt^2} = -P_{Ext.}(z,t) \quad \text{[Forced Vibration]}$$

$$EA(z)\frac{d^2 w}{dz^2}(z,t) - \rho A\frac{d^2 w}{dt^2} = -P_0 \sin(\omega_e t)$$

$\therefore \bar{z} = \frac{z}{L}$

$$\boxed{\frac{EA}{L^2}\frac{d^2 w}{d\bar{z}^2}(\bar{z},t) - \rho A\frac{d^2 w}{dt^2} = -P_0 \sin(\omega_e t)} \quad -①$$

$$w(\bar{z},t) = \sum_{i=1}^{N} \phi_i(\bar{z})\, \eta_i(t)$$

putting eqⁿ ① $\because$ A bar with uniform E and A, m (constant)

$$\frac{EA}{L^2}\sum_{i=1}^{N} \phi_i''(\bar{z})\eta_i(t) - \rho A \sum_{i=1}^{N} \phi_i(\bar{z})\ddot{\eta}_i(t) = -P_0 \sin(\omega_e t)$$

multiply by ϕ_j and Integrating both sides

For $j \neq i$

$$\int_0^1 \phi_j \frac{EA}{L^2}\sum_{i=1}^{N} \phi_i''(\bar{z})\eta_i(t)\,d\bar{z} - \int_0^1 \phi_j \rho A \sum_{i=1}^{N} \phi_i(\bar{z})\ddot{\eta}_i(t) = -\int_0^1 \phi_j P_0 \sin \omega_e t \, d\bar{z}$$

$$\int_0^1 \phi_i \frac{EA}{L^2}\phi_i''(\bar{z})\eta_i(t)\,d\bar{z} - \int_0^1 \phi_i \rho A \phi_i(z)\ddot{\eta}_i(t)\,d\bar{z} = -\int_0^1 P_0 \phi_j \sin \omega_e t \, d\bar{z}$$

$$\cancel{\frac{EA}{L^2}\phi_j \phi_i'}\Big|_0^1 - \int_0^1 \underbrace{\left[\phi_j \frac{EA}{L^2} \phi_i\, d\bar{z}\right]}_{k_{nj}} \eta_i(t) - \int_0^1 \underbrace{\phi_j \rho A \phi_i\, d\bar{z}}_{m_{nj}}\, \ddot{\eta}(t) = -\int_0^1 P_0 \phi_j \sin w_e t\, d\bar{z}$$

$$\Rightarrow \boxed{m_{nj}\, \ddot{\eta}(t) + k_{nj}\, \eta_j(t) = P_0 \sin w_e t}$$

$$\ddot{\eta}_j(t) + \frac{k_{nj}}{m_{nj}} \eta_j(t) = \frac{P_0 \sin w_e t}{m_{nj}}$$

$$\ddot{\eta}_j(t) + w_{nj}^2\, \eta_j(t) = \frac{P_0 \sin w_e t}{m_{nj}}$$

Soln:-

$\eta_j(t)$ = Complementary Function + Particular function.

$$C.F. = A_j \cos(w_{nj} t) + B_j \sin(w_{nj} t)$$

$$P.I. = \frac{P_0 \sin w_e t}{m_{nj}(w_e^2 - w_{nj}^2)}$$

$$\eta_j(t) = A_j \cos(w_{nj} t) + B_j \sin(w_{nj} t) + \frac{P_0 \sin w_e t}{m_{nj}(w_e^2 - w_{nj}^2)}$$

putting initial conditions

$\eta_j(0) = 0$. So, $\boxed{A_j = 0}$.

$$\eta_j(t) = B_j \sin(w_{nj} t) + \frac{P_0 \sin w_e t}{m_{nj}(w_e^2 - w_{nj}^2)}$$

second Initial Condition

$$\dot{\eta}_i(0) = 0 .$$

$$\dot{\eta}_i(t) = \omega_{n_i} B_i \cos(\omega_{n_i} t) + \frac{\omega_e P_0 \cos \omega_e t}{m_{n_i}(\omega_e^2 - \omega_{n_i}^2)}$$

$$0 = B_i \omega_{n_i} + \frac{\omega_e P_0}{m_{n_i}(\omega_e^2 - \omega_{n_i}^2)}$$

$$\Rightarrow B_i = \frac{-P_0 \omega_e}{\omega_{n_i} m_{n_i}(\omega_e^2 - \omega_{n_i}^2)} = \frac{-P_0 \omega_e}{\sqrt{k_{n_i} m_{n_i}}(\omega_e^2 - \omega_{n_i}^2)}$$

$$\eta_i(t) = \frac{-P_0 \omega_e}{\sqrt{k_{n_i} m_{n_i}}(\omega_e^2 - \omega_{n_i}^2)} \cdot \sin(\omega_{n_i} t) + \frac{P_0 \sin \omega_e t}{m_{n_i}(\omega_e^2 - \omega_{n_i}^2)}$$

$$m_{n_i} = \int_0^1 m \phi_i^2 \, d\bar{z}$$

$$k_{n_i} = \int_0^1 \frac{EA}{L^2} \phi_i'^2 \, d\bar{z}$$

Similar

$$m_{n_i} = m/2$$

$$k_{n_i} = \frac{EA}{2L^2} \lambda_n^2 .$$

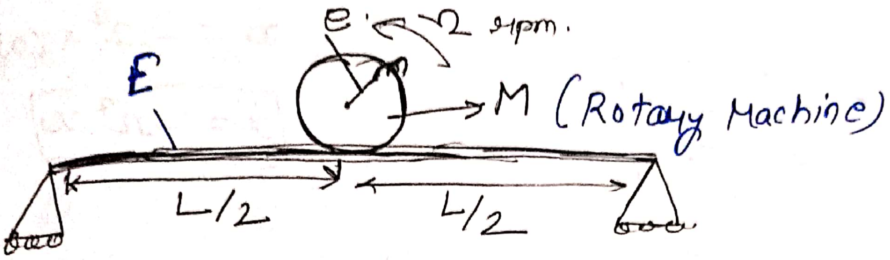
$$\therefore \boxed{\lambda_n = \left(\frac{2n+1}{2}\right)\pi = \omega_n L \sqrt{\frac{m}{EA}}}$$

$$\omega_{n_i} = \sqrt{\frac{\frac{EA}{2L^2}\lambda_n^2}{m/2}}$$

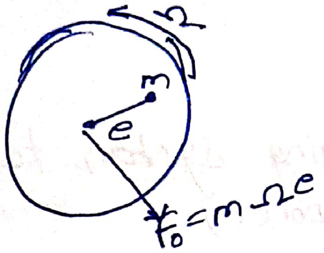
$$w(\bar{z}, t) = \sum_{i=1}^{4} \phi(z) \eta_i(t)$$

$$\boxed{\eta_j(t) = \sin\left[\frac{(2n+1)\pi \bar{z}}{2}\right]\left[\frac{-P_0\,\omega_c}{\sqrt{K_{nj}\,m_{nj}}\,(\omega_c^2-\omega_{nj}^2)}\cdot \sin(\omega_{nj} t) + \frac{P_0 \sin \omega_c t}{m_{nj}(\omega_c^2-\omega_{nj}^2)}\right]}$$

Sol^n



FBD

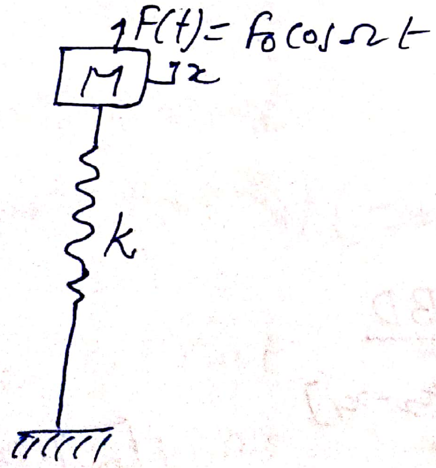


$$\boxed{F(t) = F_0 \cos \Omega t}$$

deflection of beam at mid point

$$\delta = \frac{WL^3}{48EI}$$

So, $K = \dfrac{48EI}{L^3}$ $\qquad\qquad K = \dfrac{mg}{\delta}$



Force balanced Eq^n.

$$M\ddot{x} + Kx = f_0 \cos \Omega t$$

$$M(-\Omega^2 x) + Kx = f_0 \cos \Omega t$$

$$\boxed{X_0 = \frac{F_0}{K - M\Omega^2}}$$

$x = X \cos \Omega t$ (harmonic function)

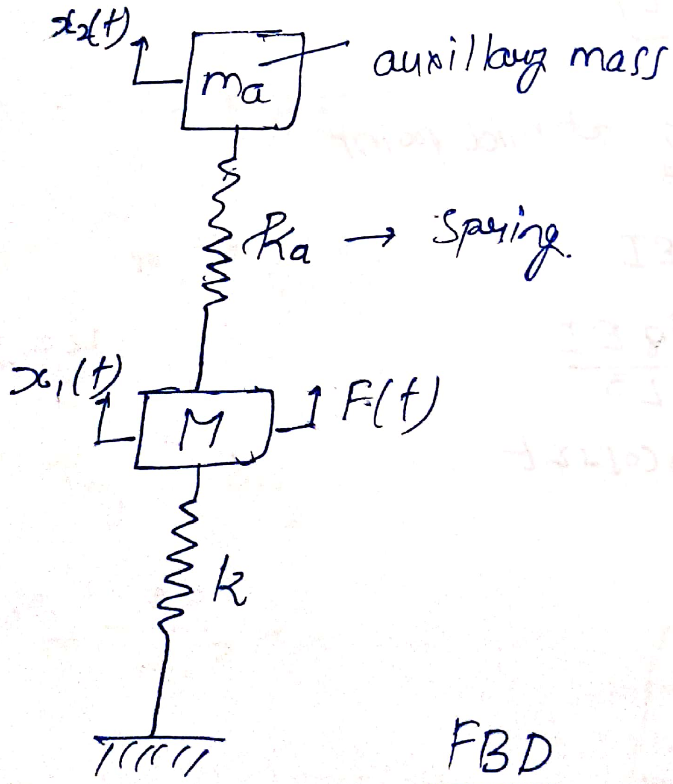
X → Amplitude

$\ddot{x} = -\Omega^2 X \cos \Omega t$

$$\boxed{\ddot{x} = -\Omega^2 x}$$

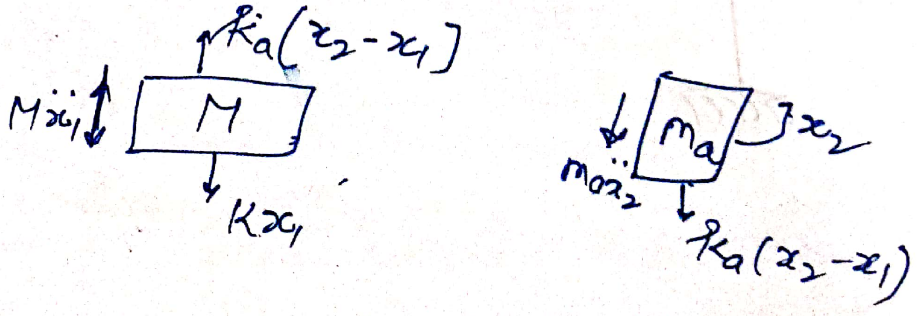
$$\Rightarrow \boxed{x = \frac{F_0}{K - M\Omega^2} \cos \Omega t}$$

add auxillary mass - spring system to reduce peak vibration (response)



$$\boxed{x_2 > x_1}$$

FBD



$$M\ddot{x}_1(t) + (k+k_a)x_1(t) - k_a x_2(t) = F(t) \quad -(1)$$

$$m_a \ddot{x}_2(t) + k_a x_2(t) - k_a(x_1)(t) = 0. \quad -(2)$$

$$\begin{bmatrix} M & 0 \\ 0 & m_a \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k+k_a & -k_a \\ -k_a & k_a \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{bmatrix} F(t) \\ 0 \end{bmatrix}$$

$$\cancel{\left| -\omega_n^2 [M] + [k] \right| = \begin{Bmatrix} F_0 \\ 0 \end{Bmatrix}}$$

$$\Rightarrow \begin{vmatrix} -\omega_n^2 M + k + k_a & -k_a \\ \cancel{0} + k_a & -\omega_n^2 m_a + k_a \end{vmatrix} = \begin{Bmatrix} F_0 \\ 0 \end{Bmatrix}$$

$$\cancel{k_a + k_a - \omega_n^2 m_a = 0}.$$

$$\cancel{\omega_n^2 = \frac{2k_a}{m_a}}$$

$$\cancel{\omega_n^2 = 2\omega_a^2}$$

$$\cancel{(-\omega_n^2 M + k + k_a)(-\omega_n^2 m_a + k_a) + k_a^2 = 0}.$$

$$\Rightarrow \quad x_1 = X_1 \cos\Omega t \quad \rightarrow \quad \ddot{x}_1 = -\Omega^2 X_1 \cos\Omega t$$

$$x_2 = X_2 \cos\Omega t \quad \rightarrow \quad \ddot{x}_2 = -\Omega^2 X_2 \cos\Omega t$$

putting the value in eq^n (1) and (2)

$$\begin{bmatrix} -\Omega^2 M + K + k_a & -k_a \\ -k_a & -\Omega^2 m_a + k_a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F_0 \\ 0 \end{bmatrix}$$

$$x_1 \left[-\Omega^2 M + K + k_a\right] - k_a x_2 = F_0$$

$$-k_a x_1 - \left[\Omega^2 m_a + k_a\right] x_2 = 0.$$

$$x_1 = \frac{\left[-\Omega^2 m_a + k_a\right] x_2}{k_a}$$

Subtitute the value in x_1.

$$\left[\frac{-\Omega^2 m_a - k_a}{k_a}\right] x_2 \left[-\Omega^2 M + K + k_a\right] - k_a x_2 = F_0.$$

$$x_2 = \frac{k_a F_0}{(K + k_a - M\Omega^2)(k_a - \Omega^2 m_a) - k_a^2}$$

<u>Similarly</u>

$$x_1 = \frac{F_0\left[k_a - \Omega^2 m_a\right]}{(K + k_a - M\Omega^2)(k_a - \Omega^2 m_a) - k_a^2}$$

<u>Subtitute</u>

$$\mu = \frac{m_a}{M}$$

$$\omega_m^2 = K/M$$

$$\omega_a^2 = \frac{k_a}{m_a}$$

$$\xi = \frac{\omega_m}{\omega_a}$$

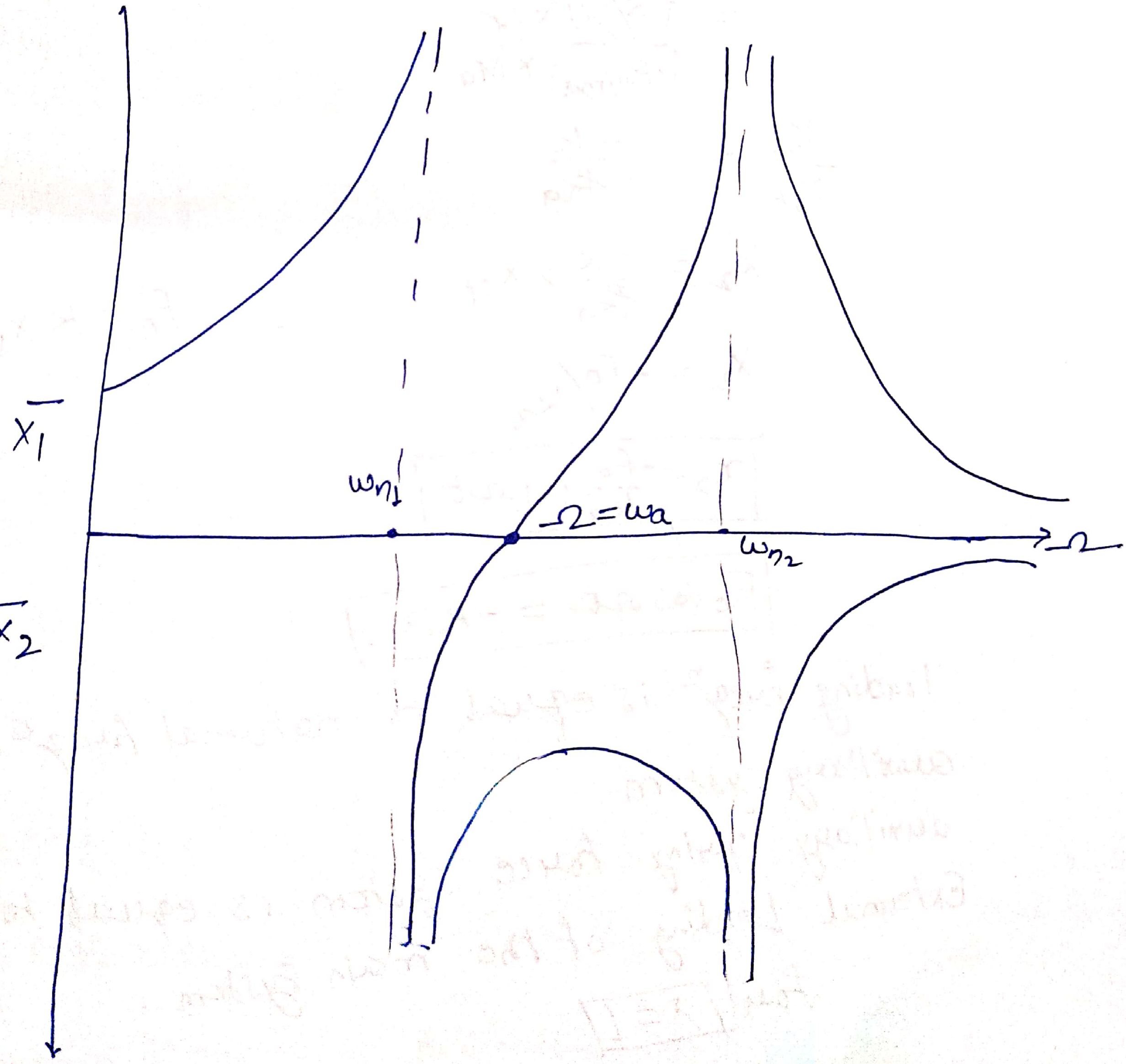
$$\lambda = \frac{\Omega}{\omega_a}$$

$$X_{st} = \frac{F_0}{K} \qquad \bar{X}_1 = \frac{X_1}{X_{st}}, \quad \bar{X}_2 = \frac{X_2}{X_{st}}$$

$$\bar{X}_1 = \frac{\xi_e^2(1-\lambda^2)}{\lambda^2-(1+\mu+\xi_e^2)\lambda^2+\xi_e^2}$$

$$\bar{X}_2 = \frac{\xi_e^2}{\lambda^2-(1+\mu+\xi_e^2)\lambda^2+\xi_e^2}$$

$\bar{X}_1 \Rightarrow 0 \cdot \quad 1-\lambda^2 = 0 \quad \lambda = 1 \Rightarrow \Omega = \omega_a$



If $\Omega = \omega_a$ then $\bar{X}_1 = 0$. $\lambda = 1$

$$\bar{X}_2 = \frac{\xi_1^2}{1-(1+\mu+\xi_1^2)\times 1+\xi_1^2}$$

$$= \frac{\xi_1^2}{\cancel{1}-\cancel{1}-\mu-\cancel{\xi_1^2}+\cancel{\xi_1^2}}$$

$$= \frac{-\xi_1^2}{\mu}.$$

$$= \frac{-\omega_m^2}{\omega_a^2} \times \frac{M}{m_a}$$

$$= \frac{(K/\cancel{M}) \times \cancel{M}}{(k_a/\cancel{m_a}) \times \cancel{M_a}}$$

$$\frac{X_2}{X_{st}} = \frac{-K}{k_a}$$

$$X_2 = \frac{-K}{k_a} \times X_{st} \qquad \because F_0 = K X_{st}$$

$$X_2 = -F_0/k_a$$

$$\boxed{x_2 = -\frac{F_0}{k_a} \cos \Omega t}$$

$$\boxed{F_0 \cos \Omega t = -k_a x_2}$$

loading freq^n is equal to natural freq^n of the auxillary system.

auxillary spring force system is equal to External loading of the main system

for $\boxed{\lambda = 1}$.

System response reduced by selected value of (k_a, m_a). These are related to main system response.

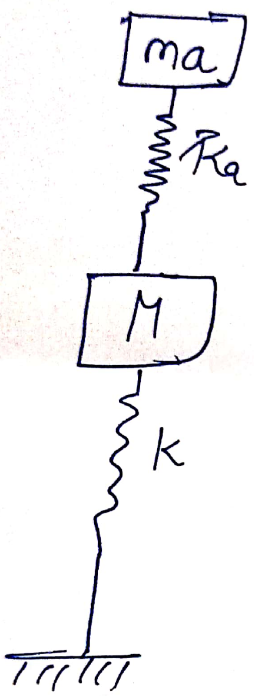
$$\mu = \frac{m_a}{M} \Rightarrow 0.1, 0.2, 0.25.$$

at $\Omega = \omega_a$

$$\bar{X_2} \propto \frac{1}{m_a}$$

or

$$\propto \frac{1}{k_a}$$

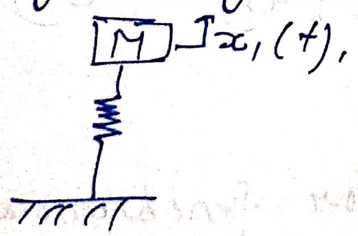


$$k_a = \frac{F_0}{A},$$

$$m_a = \frac{F_0}{A\Omega^2}$$

$$\therefore \boxed{\Omega = \omega_a} \text{. For } \lambda = 1$$

Original System - Free Vibration Response



$$M\ddot{x}_1 + Kx_1 = 0. \quad \text{(Free Vibration.}$$

$$x_1 = A\cos\omega_n t + B\sin\omega_n t.$$

$$\dot{x}_1(t) = (-A\sin\omega_n t + B\cos\omega_n t)\omega_n$$

Scanned with OKEN Scanner

ICs

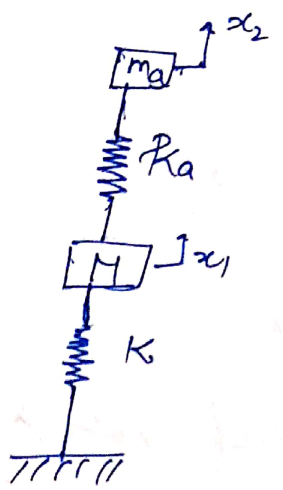
$\dot{x}_1(0) = u_0$

$B\omega_n = u_0$.

$$\boxed{B = \frac{u_0}{\omega_n}}$$

$$x_1(t) = \frac{u_0}{\omega_n} \sin \omega_n t$$

Final system by auxillary system.



previous eqn.

$$\begin{bmatrix} M & 0 \\ 0 & m_a \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} K+k_a & -k_a \\ -k_a & k_a \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ Free Vibration

$$\left| -\omega_n^2 [M] + [K] \right| = 0$$ For Synchronous motion

$$\begin{vmatrix} -M\omega_n^2 + K + k_a & -k_a \\ -k_a & -\omega_n^2 m_a + k_a \end{vmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0.$$

$$(-M\omega_n^2 + K + K_a)(-\omega_n^2 m_a + K_a) - K_a^2 = 0.$$

$$\Rightarrow +M\omega_n^4 m_a - M\omega_n^2 K_a - \omega_n^2 K m_a + K K_a - \omega_n^2 m_a K_a + \cancel{K_a^2} - \cancel{K_a^2}$$

$$\Rightarrow M\omega_n^4 m_a - \omega_n^2 [M K_a + K m_a + m_a K_a] + K K_a = 0.$$

$$\omega_n^4 - \omega_n^2\left[\frac{K_a}{m_a} + \frac{K}{M} + \frac{K_a}{M}\right] + \frac{K K_a}{M m_a} = 0.$$

$$\therefore \omega_a^2 = \frac{K_a}{m_a} \quad . \quad \omega_m^2 = \frac{K}{M}$$

$$\omega_n^4 - \omega_n^2\left[\omega_a^2 + \omega_n^2 + K_a/M\right] + \omega_m^2 \omega_a^2 = 0.$$

$$\omega_n^2 = \frac{[\omega_a^2 + \omega_n^2 + K_a/M] \pm \sqrt{(\omega_a^2 + \omega_n^2 + K_a/m)^2 - 4\,\omega_m^2 \omega_a^2}}{2}$$

Mode shape

$$M(-\omega_n^2)X_1 + (K + K_a)X_1 - K_a X_2 = 0.$$

$$\left(\frac{X_1}{X_2}\right) = \frac{K_a}{K + K_a - M\omega_n^2}$$

for ω_{n1} & ω_{n2}.

Final Sol^n

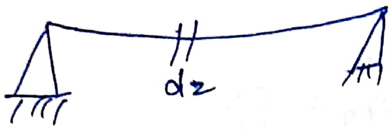
$$\{x\} = \{\phi\}\{\eta\}.$$

$$\{x\} = c_1\{\phi_1\}\cos(\omega_n t) + c_2\{\phi_2\}\cos(\omega_{n2} t)$$

$$\begin{Bmatrix} \eta_1 \\ \eta_2 \end{Bmatrix} = [\phi]^{-1} \begin{Bmatrix} 0 \\ x_0 \end{Bmatrix}.$$

$$\eta_1 = X_{12} x_0$$

$$\eta_2 = X_{22} x_0.$$



we use equivalance of kinetic Energy.

Kinetic Energy of all dz element sum over the entire span equal to the mass placed at the center ($z = L/2$) which is moving with same speed as that of beam.

$$\dot{u}(z,t) = U(\bar{z})\, \dot{g}(t)$$

$$U(\bar{z}) = A \sin \lambda_n \bar{z} + B \cos \lambda_n \bar{z} + C \sinh \lambda_n \bar{z} + D \cosh \lambda_n \bar{z}$$

<u>Boundary Conditions</u>

$$\left.\begin{array}{l} u(0) = 0 \\ u(1) = 0 \end{array}\right\} \text{Displacement Condition.}$$

$$\left.\begin{array}{l} u''(0) = 0 \\ u''(1) = 0 \end{array}\right\} \text{Moment Condition.}$$

$$u(0) = 0$$

$$\boxed{B + D = 0}$$

$$u''(0) = 0 \quad \Big| -\lambda_n^2 A \sin\lambda_n \bar{z} + \lambda_n^2 B \cos\lambda_n \bar{z} + \lambda_n^2 \sinh\lambda_n \bar{z} + \lambda_n^2 \cosh\lambda_n \bar{z}$$

$$-B + D = 0.$$

$$D = B.$$

$$\text{So,} \quad B = D = 0.$$

$$u(\bar{z}) = A \sin\lambda_n \bar{z} + C \sinh\lambda_n \bar{z}$$

@ $u(1) = 0$.

$$\ddot{u}(1) = 0.$$

From eq^n.

$$u(\bar{z}) = C_1 \sin\lambda_n \bar{z}$$

$$C_1 \neq 0.$$

$$\lambda_n = n\pi$$

$$\boxed{u(\bar{z}) = C \sin(n\pi \bar{z})}$$

Equating Kinetic Energy

$$\int_0^1 \frac{1}{2} \rho A \left[C \sin(n\pi \bar{z})\, \dot{g}(t)\right] d\bar{z} = \frac{1}{2} M_{eq} \left[C \sin(n\pi\bar{z})\, \dot{g}(t)\right]$$

$$\Rightarrow \rho A \int_0^1 \sin^2[n\pi\bar{z}]\, d\bar{z} = M_{eq} \sin\frac{n\pi}{2}.$$

uniform cross section area

$$\Rightarrow \frac{\rho A}{2}\left[\bar{z} - \frac{\sin[2n\pi\bar{z}]}{2n\pi}\right]_0^1 = M_{eq} \sin\left[\frac{n\pi}{2}\right]$$

$$\boxed{\frac{\rho A}{2} = M_{eq}} \quad \text{for } n = 1$$

$M_{eq} = \frac{1}{2}$ [Mass of the beam]

System response will be decrease as compaired to orginal massless beam assumption.

mass ratio $\mu = \frac{Mg}{M + 0.5 \rho A}$