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AE – 678 Aeroelasticity

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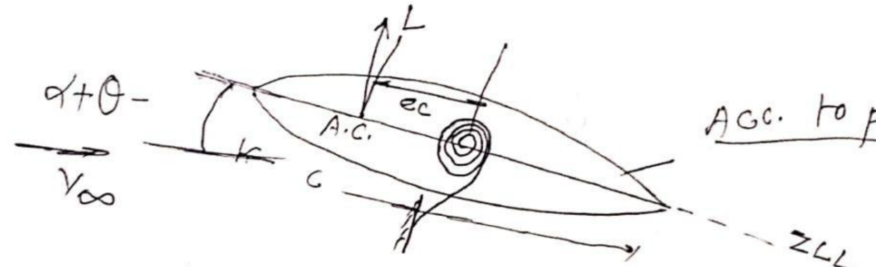


Acc. to problem Assumption

here α - Angle of Attack
θ - Rotation of Spring

Aerodynamic moment,

$$M_a = P_{dyn} S e c\, C_{L\alpha} \left[\alpha + \theta + \cancel{\frac{C_{m,ac}}{e C_{L\alpha}}}^{\,0} \right)$$

for Simplicity. $\therefore C_{m_{ac}} = 0$.

$$M_a = K_a \left(\alpha + \theta + \frac{C_{m,ac}}{e C_{L\alpha}} \right) \quad \therefore K_a = P_{dyn} S e c\, C_{L\alpha}$$

Structural moment $\Rightarrow M_S = K_\theta \theta$

For Equilibrium

$$M_S - M_a = 0 \quad -(1)$$

Non-linearity in both structural & aerodynamic moment

$$M_S = K_{\theta_1}\theta + K_{\theta_2}\theta^2 + K_{\theta_3}\theta^3$$

$$M_a = K_{a_1}(\alpha+\theta) + K_{a_2}(\alpha+\theta)^2 + K_{a_3}(\alpha+\theta)^3$$

from eqn (1) -

$$\Rightarrow K_{\theta_1}\theta + K_{\theta_2}\theta^2 + K_{\theta_3}\theta^3 = K_{a_1}(\alpha+\theta) + K_{a_2}(\alpha+\theta)^2 + K_{a_3}(\alpha+\theta)^3$$

$$K_\theta \theta \left[1 + \frac{K_{\theta_2}}{K_{\theta_1}}\theta + \frac{K_{\theta_3}}{K_{\theta_1}}\theta^2 \right] = K_a \left[(\alpha+\theta) + \frac{K_{a_2}}{K_{a_1}}(\alpha+\theta)^2 + \frac{K_{a_2}}{K_{a_1}}(\alpha+\theta)^3 \right]$$

Consider $\therefore\ \gamma_2 = \frac{k_{\theta 2}}{k_{\theta 1}}$ & $\gamma_3 = \frac{k_{\theta 3}}{k_{\theta 1}}$

$\therefore\ \beta_2 = \frac{k_{a2}}{k_{a1}}$ & $\beta_3 = \frac{k_{a3}}{k_{a1}}$

So,

$$[\theta + \gamma_2\theta^2 + \gamma_3\theta^3] = \frac{k_{a1}}{k_\theta}\left[(\alpha+\theta) + \beta_2(\alpha+\theta)^2 + \beta_3(\alpha+\theta)^3\right]$$

$$= \bar{P}_{dyn}\left[(\alpha+\theta) + \beta_2(\alpha^2+\theta^2+2\alpha\theta) + \beta_3\{\alpha^3+\theta^3+3\alpha\theta[\alpha+\theta)]\right\}$$

$$\theta + \gamma_2\theta^2 + \gamma_3\theta^3 = \bar{P}_{dyn}\{\alpha(1+\beta_3\alpha^2+\beta_2\alpha) + \theta(1+2\beta_2\alpha + 3\beta_3\alpha^2) + \theta^2(\beta_2+3\beta_3\alpha) + \beta_3\theta^3\}$$

$$\Rightarrow \boxed{\theta[\bar{P}_{dyn}(1+2\beta\alpha+3\beta_2\alpha^2)-1] + \theta^2[(\beta_2+3\beta_2\alpha)\bar{P}_{dyn} - \gamma_2] + \theta^3(\beta_2\bar{P}_{dyn}-\gamma_3) + \bar{P}_{dyn}\{\alpha(1+\beta_3\alpha^2+\beta_2\alpha]\} = 0}$$

This is general Equation.

pure quadratic non-linearity, $\gamma_3 = \beta_3 = 0$.

pure cubic non-linearity, $\gamma_2 = \beta_2 = 0$.

structural hardening $\Rightarrow$ r_2 & $r_3 > 0$.
softening $\Rightarrow$ r_2 & $r_3 < 0$.

<u>Stability</u>

For disturbance $(\Delta\theta)$, becomes

$$\theta = \theta_{eq} + \Delta\theta$$

$$\Delta \bar{M} = \Delta M_s - \Delta M_a$$

$$= K_{\theta_1}\left\{(\theta_{eq} + \Delta\theta) + \frac{K_{\theta_2}}{K_{\theta_1}}(\theta_{eq} + \Delta\theta)^2 + \frac{K_{\theta_3}}{K_{\theta_1}}(\theta_{eq} + \Delta\theta)^3\right\}$$

$$- \left\{K_{a_1}\left[(\alpha + \theta_{eq} + \Delta\theta) + \frac{K_{a_2}}{K_{a_1}}(\alpha + \theta_{eq} + \Delta\theta)^2\right.\right.$$

$$\left.\left. + \frac{K_{a_3}}{K_{a_1}}(\alpha + \theta_{eq} + \Delta\theta)^3\right\}\right\}.$$

$$\Rightarrow \Delta M = K_\theta\left\{(\theta_{eq} + r_2\theta_{eq}^2 + r_3\theta_{eq}^3) + \Delta\theta + 2r_2\theta_{eq}\Delta\theta\right.$$

$$\left. + 3r_3\theta_{eq}^2\Delta\theta\right] - K_a\left\{(\alpha + \theta_{eq} + \Delta\theta)\right.$$

$$+ \beta_2(\alpha^2 + \theta_{eq}^2 + \Delta\theta^2 + 2\alpha\theta_{eq} + 2\theta_{eq}\Delta\theta +$$

$$2\Delta\theta\alpha) + \beta_3(\alpha^3 + \theta_{eq}^3 + \Delta\theta^3 + 3\alpha^2\theta_{eq}$$

$$\cancel{+ 3\alpha\theta_{eq} + 3\alpha^2 + 6\alpha\theta_{eq}}.$$

$$+ 3\alpha^2\Delta\theta + 3\alpha\theta_{eq}^2 + 3\theta_{eq}^2\Delta\theta + 3\alpha\Delta\theta^2$$

$$\left. + 3\theta_{eq}\Delta\theta^2 + 6\alpha\theta_{eq}\Delta\theta\right\}$$

$\Delta\theta$ higher order can be neglected, $\Delta\theta^2, \Delta\theta^3 \to 0$.
we get final Result

$$\Delta M = K_\theta\Big[1 + 2\gamma_2\theta_{eq} + 3\gamma_3\theta_{eq}^2 - \bar{P}_{dyn} - 2\beta_2(\alpha+\theta_{eq})$$

always positive

$$\bar{P}_{dyn} - 3\bar{P}_{dyn}\beta_3(\alpha+\theta_{eq})^2\Big]\Delta\theta$$

Condition

$\Delta\bar{M} > 0$, Stable system.
$\Delta\bar{M} = 0$, Neutural system
$\Delta\bar{M} < 0$, unstable system.

So,

$$\Big[1 - \bar{P}_{dyn} + 2\gamma_2\theta_{eq} + 3\gamma_3\theta_{eq}^2 - 2\bar{P}_{dyn}\beta_2(\alpha+\theta_{eq}) - 3\bar{P}_{dyn}\beta_3(\alpha+\theta_{eq})^2\Big] = (\text{Coefficient of } \Delta\theta)$$

Coefficient of $\Delta\theta$ > 0 (stable)
,, ,, < 0 (unstable)

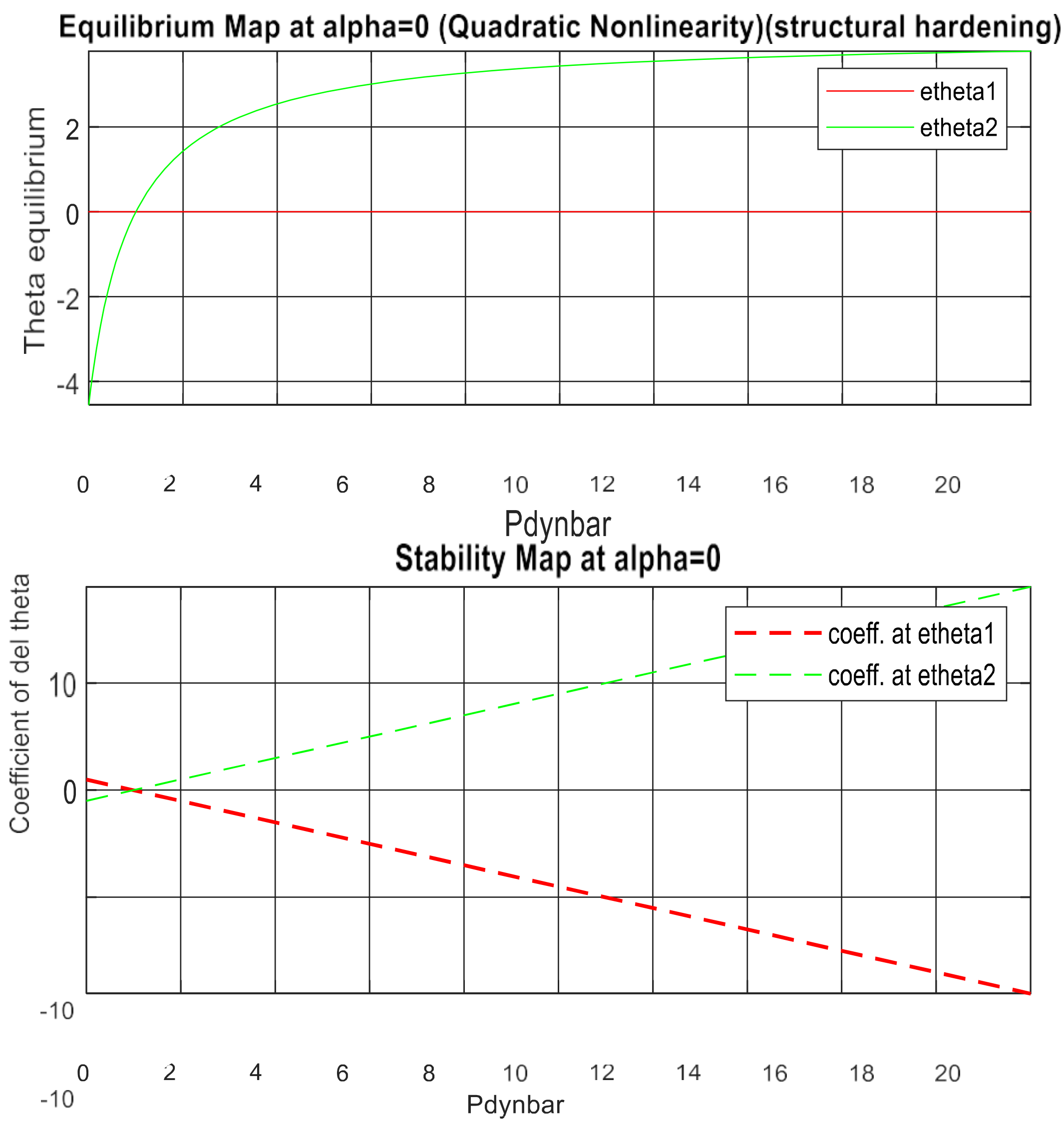
Cases of non- linearity
Case A : Structural & Aerodynamic both are quatratic.
Case B : ~~Cubic~~ Both are cubic (structure & Aerodynamic)
Case C : quadratic & cubic (Structural & Aerodynamic

Case(A): structural and aerodynamic both are in (Quadratic)**non-linearity.**

There are following cases in Quadratic($\gamma 3=0$, $\beta 3=0$)non-linearity:

Case(A1):

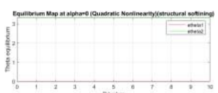
- (e>0) e=0.25
- alpha α =0
- structural hardening, $\gamma 2=0.30$
- $\beta 2=0.30$

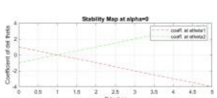


- (Red)etheta1 : stable for Pdynbar <1 ; neutral at Pdynbar=1; unstable forPdynbar >1
- (Green)etheta2 : stable for Pdynbar >1 ; neutral at Pdynbar=1; unstablefor Pdynbar <1

Case(A2):

- (e>0) e=0.25
- alpha α =0
- structural softining, $\gamma 2$=-0.30
- $\beta 2$=0.30

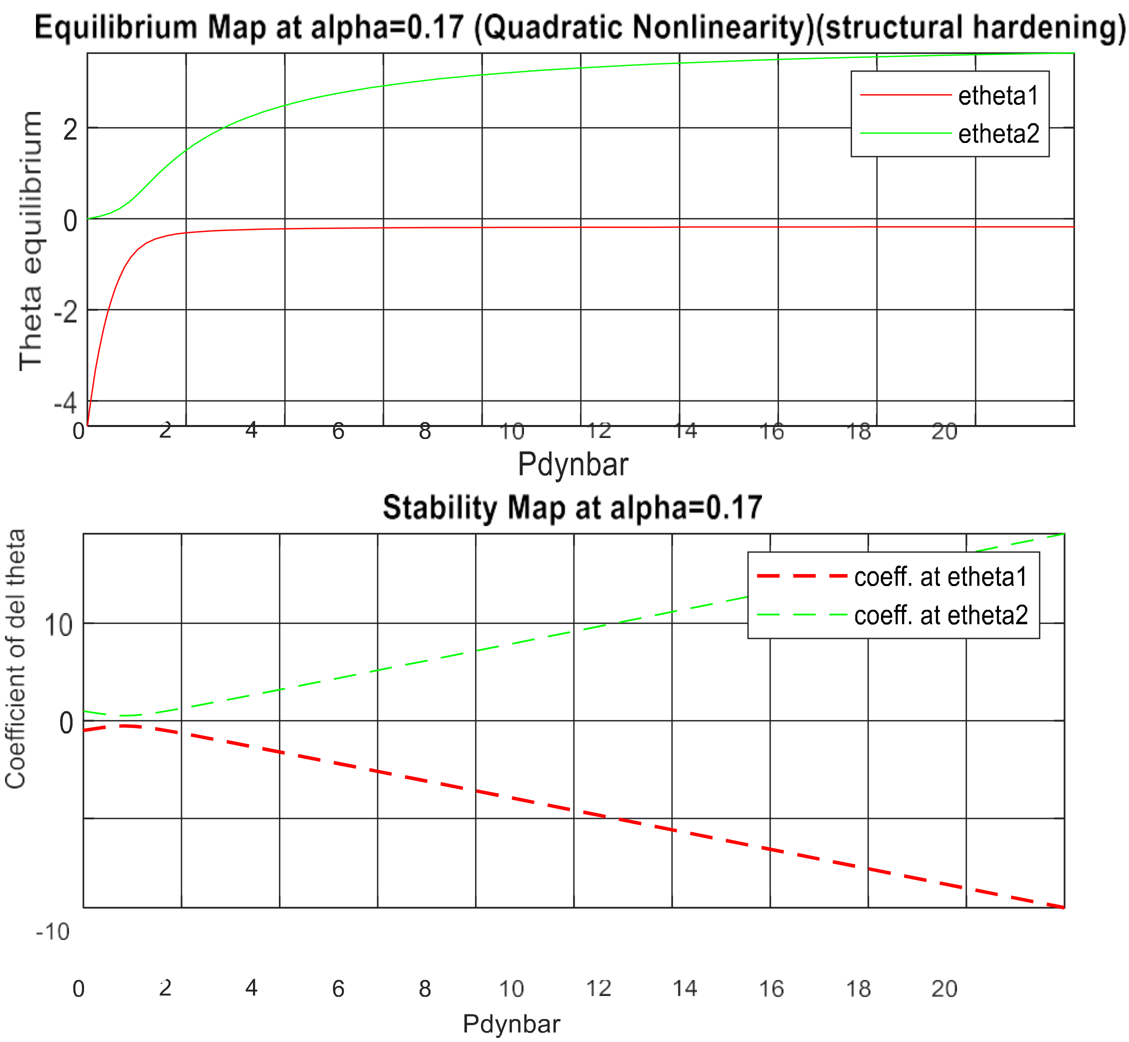




- (Red)etheta1 : stable for Pdynbar <1 ; neutral at Pdynbar=1; unstable forPdynbar >1
  - (Green)etheta2 : stable for Pdynbar >1 ; neutral at Pdynbar=1; unstablefor Pdynbar<1

Case(A3):

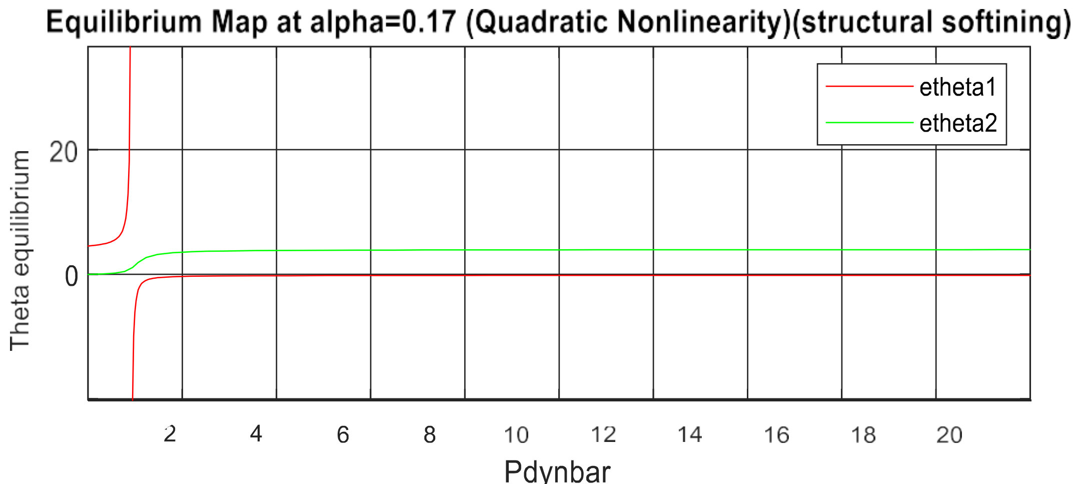
- (e>0) e=0.25
- alpha α =0.17
- structural hardening, $\gamma 2$=0.30
- $\beta 2$=0.30

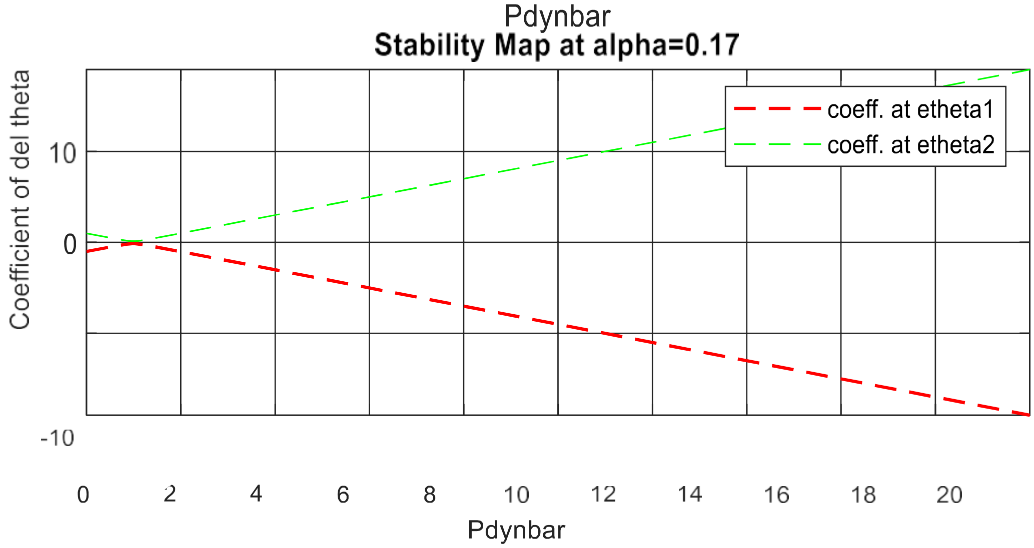


- (Red)etheta1 : unstable for all Pdynbar(0 to infinite)
- (Green)etheta2 : stable for all Pdynbar

Case(A4):

- (e>0) e=0.25
- alpha α =0.17
- structural softining, γ2=-0.30
- β2=0.30

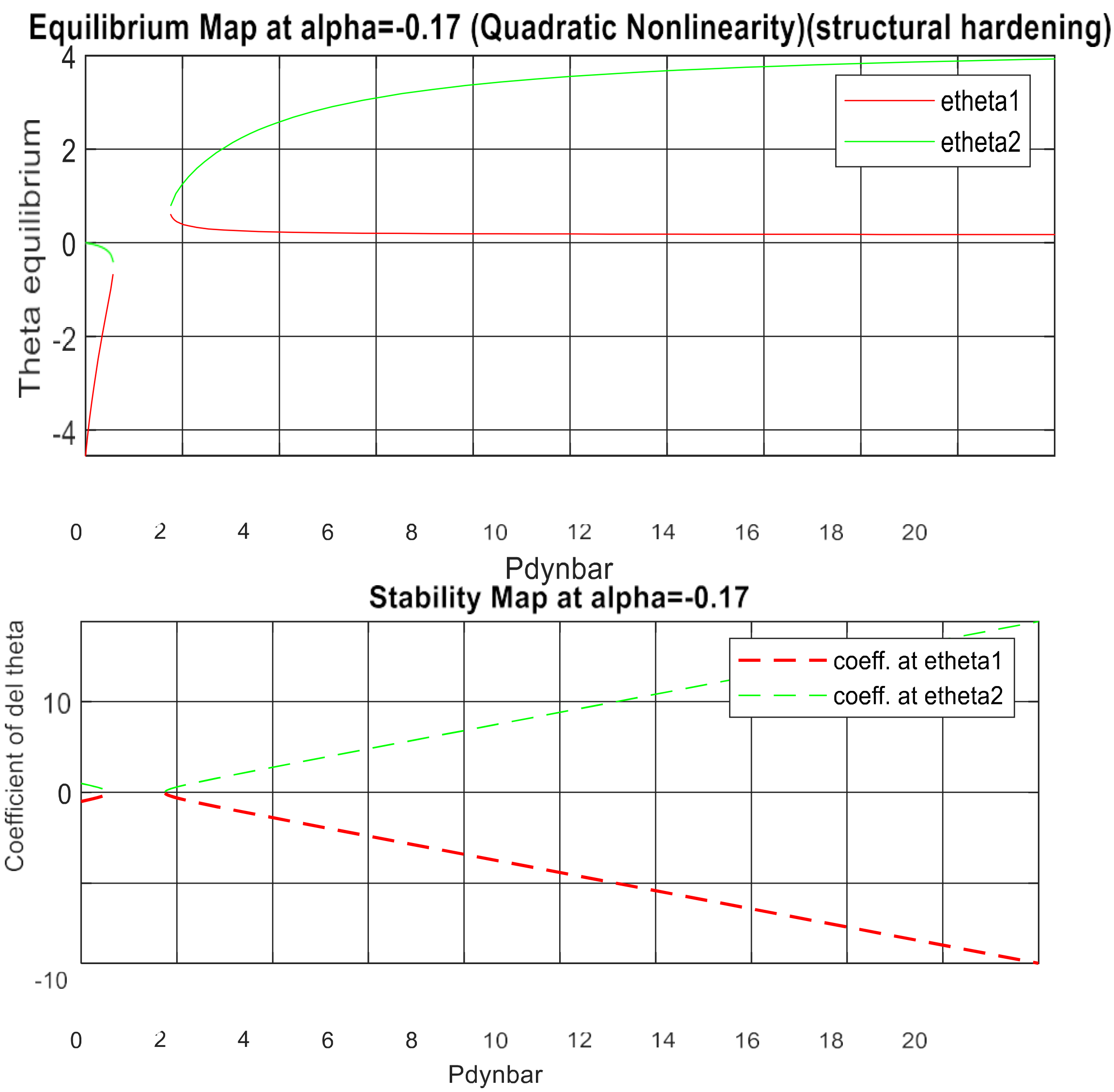




- (Red)etheta1 : unstable for Pdynbar =[0 1) U (1 ∞] & neutral for Pdynbar=1
- (Green)etheta2 : stable for Pdynbar =[0 1) U (1 ∞] & neutral forPdynbar =1

Case(A5):

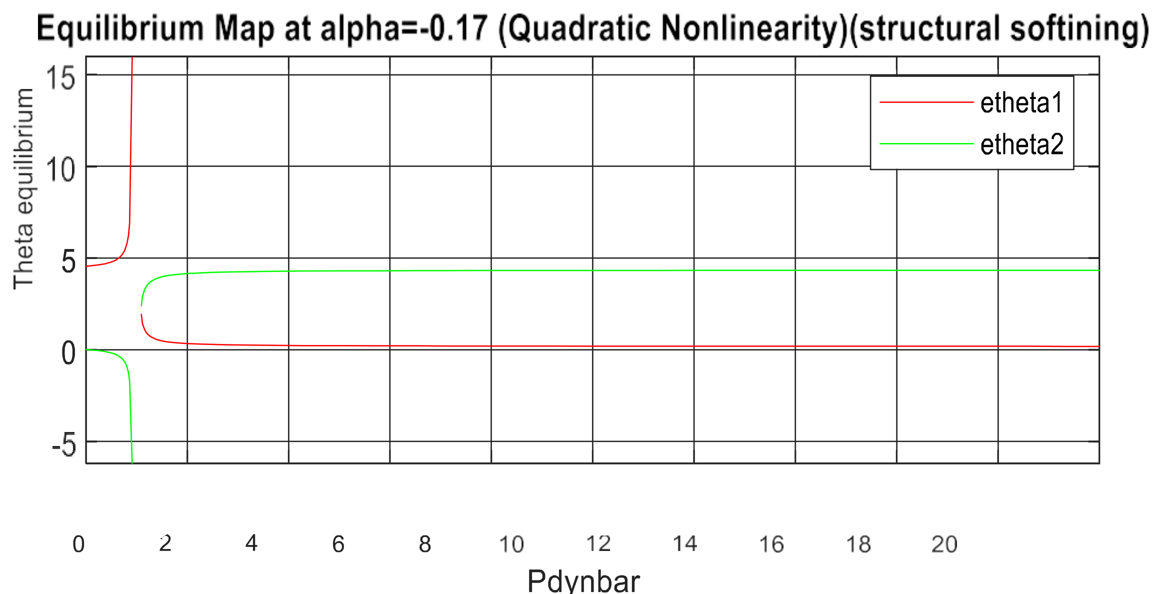
- (e>0) e=0.25
- alpha α =-0.17
- structural hardening, $\gamma 2$=0.30
- $\beta 2$=0.30

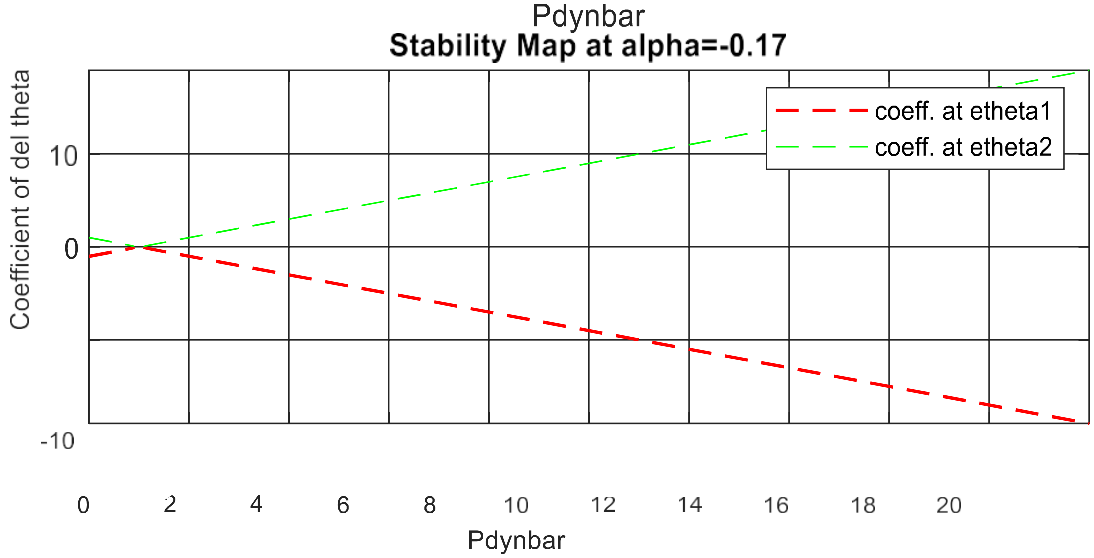


- (Red)etheta1 : stable for Pdynbar <0.4 ; neutral at Pdynbar=1.8 ; unstablefor Pdynbar >1.8
- (Green)etheta2 : stable for Pdynabr >1.8 ; neutral at Pdynbar=1.8;unstable for Pdynbar <0.4

Case(A6):

- (e>0) e=0.25
- alpha α =-0.17
- structural softining, $\gamma 2$=-0.30
- $\beta 2$=0.30

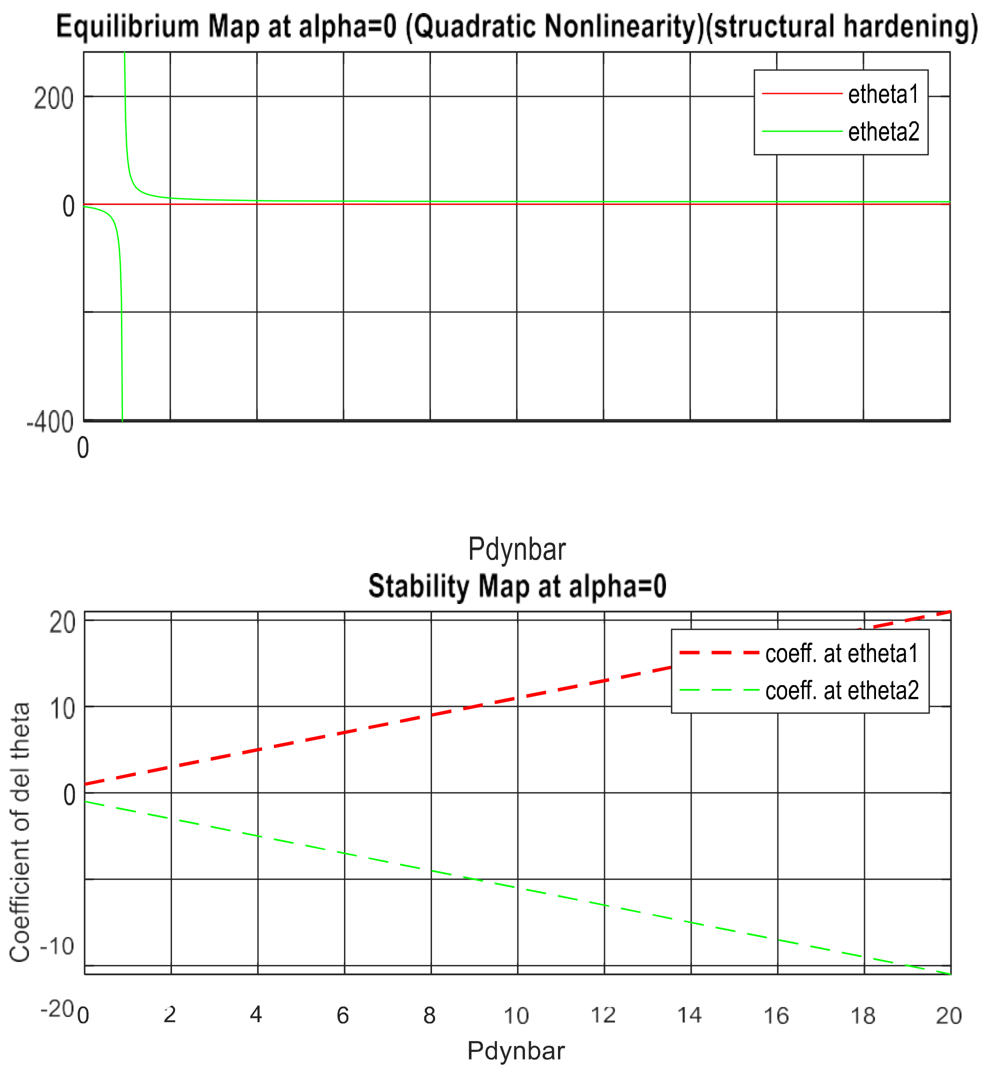




- (Red)etheta1 : unstable for Pdynabr =[0 1) U (1 ∞] & neutral for Pdynbar =1
- (Green)etheta2 : stable for Pdynabr =[0 1) U (1 ∞] & neutral forPdynbar =1

Case(A7):

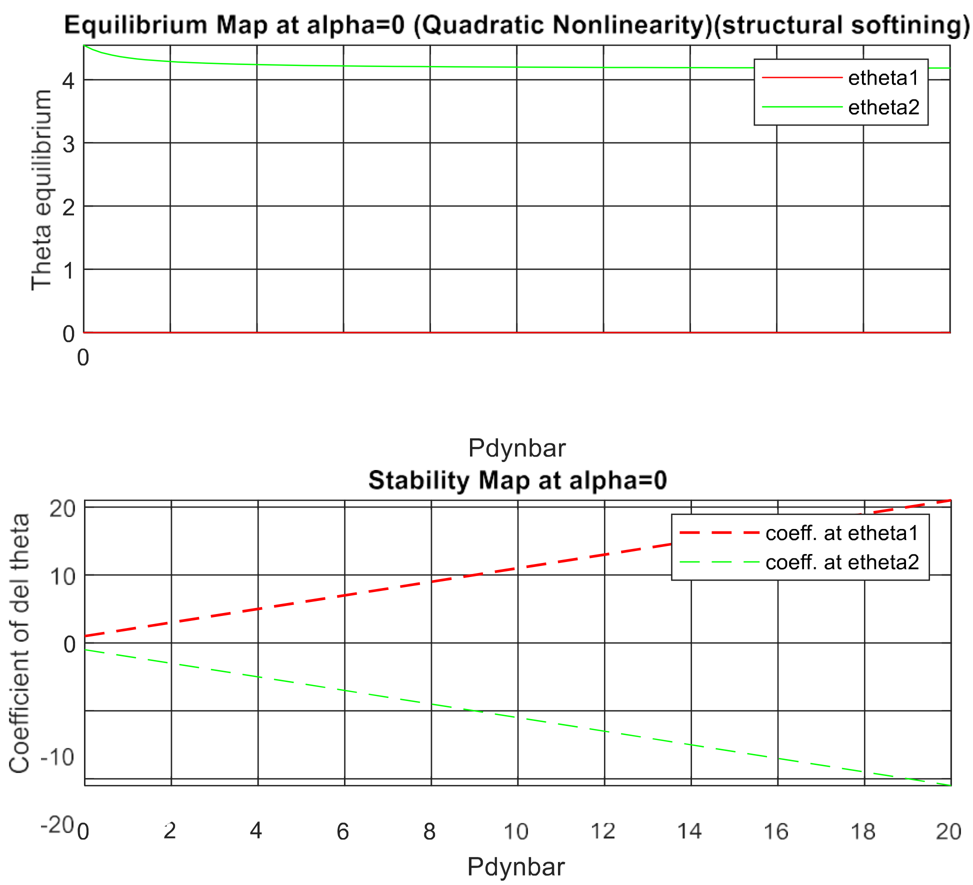
- (e<0) e=-0.25
- alpha α =0
- structural hardening, γ2=0.30
- β2=0.30



- (Red)etheta1 : stable for all Pdynbar =[0 ∞]
- (Green)etheta2 : unstable for all Pdynbar =[0 ∞]

Case(A8):

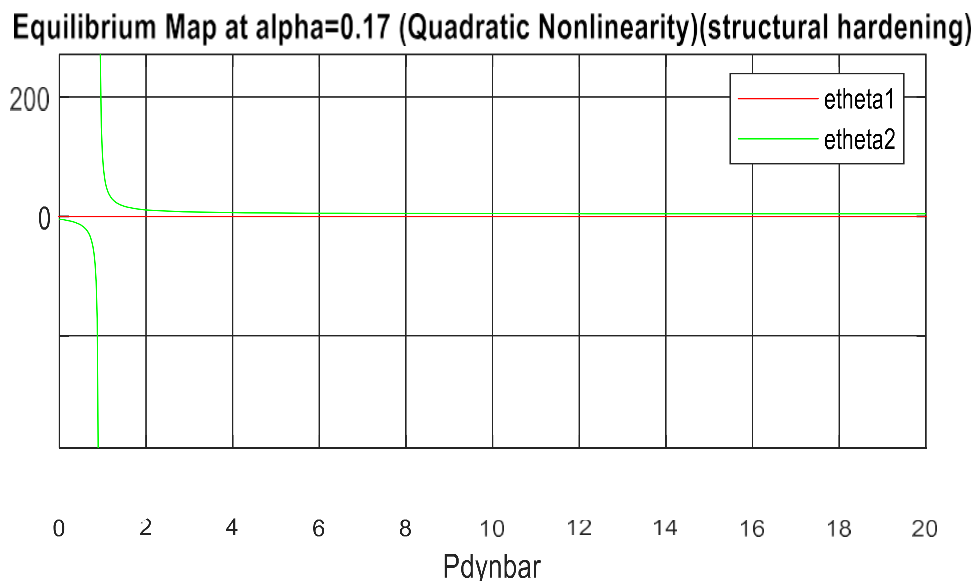
- (e<0) e=-0.25
- alpha α =0
- structural softining, $\gamma 2$=-0.30
- $\beta 2$=0.30

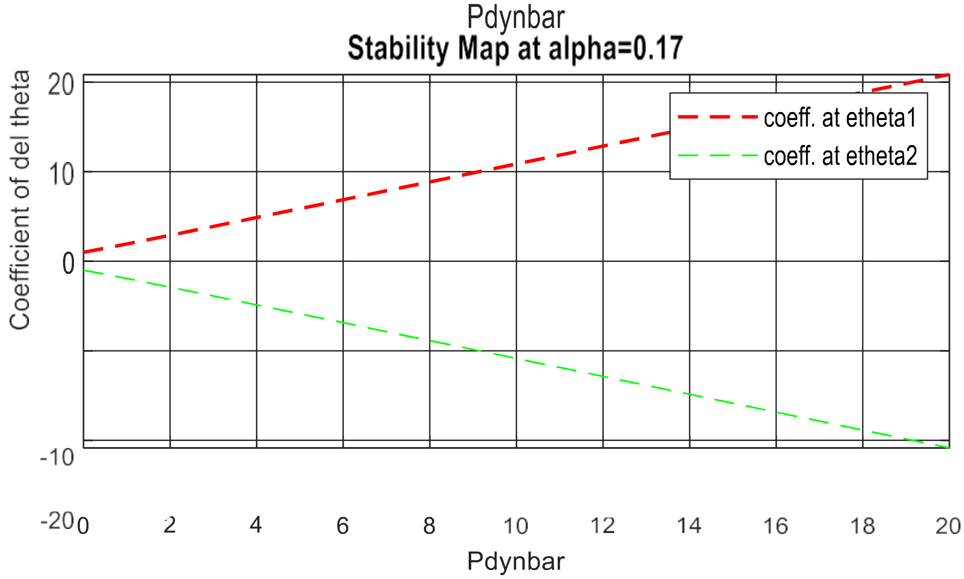


- (Red)etheta1 : stable for all Pdynbar =[0 ∞]
- (Green)etheta2 : unstable for all Pdynbar =[0 ∞]

Case(A9):

- (e<0) e=-0.25
- alpha α =0.17
- structural hardening, γ2=0.30
- β2=0.30

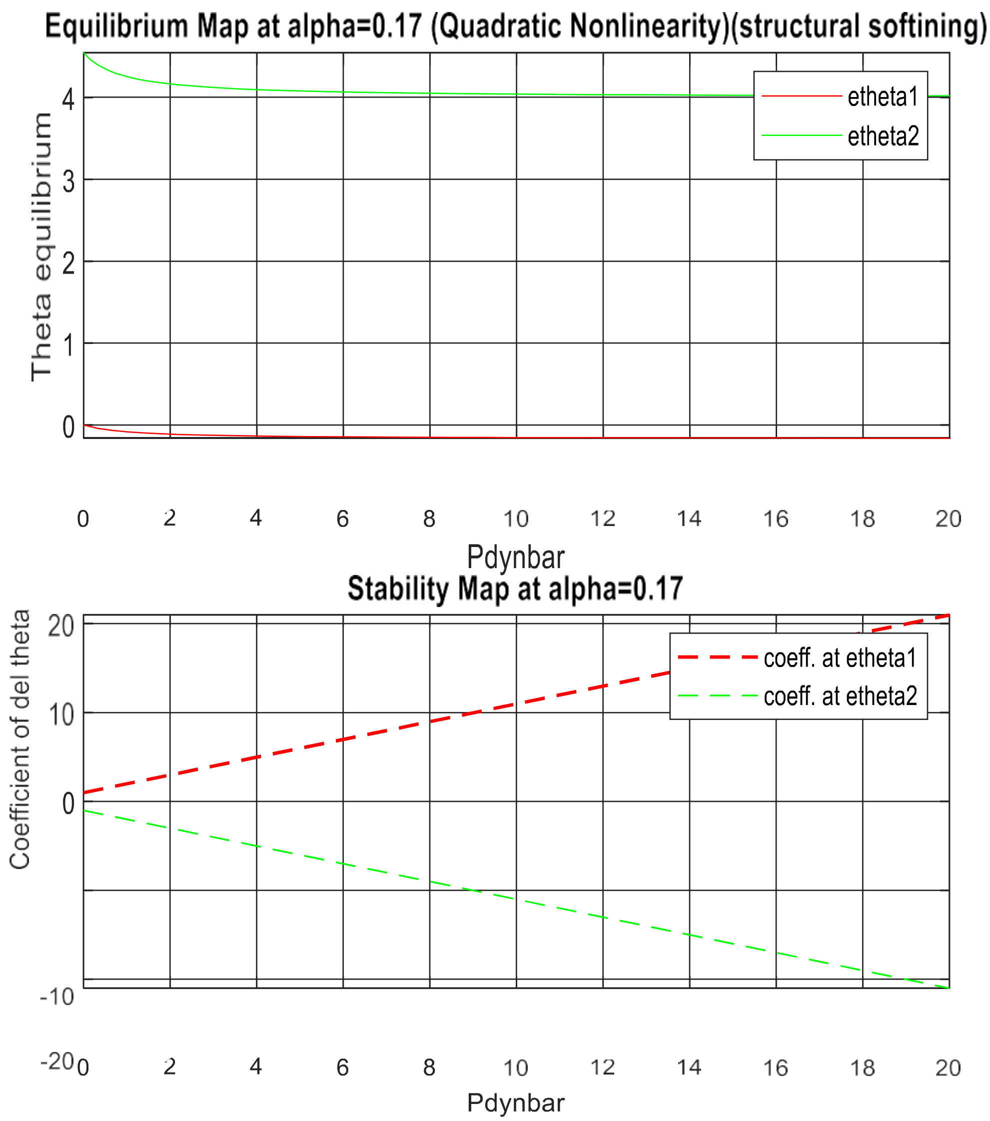




- (Red)etheta1 : stable for all Pdynbar =[0 ∞]
- (Green)etheta2 : unstable for all Pdynbar =[0 ∞]

Case(A10):

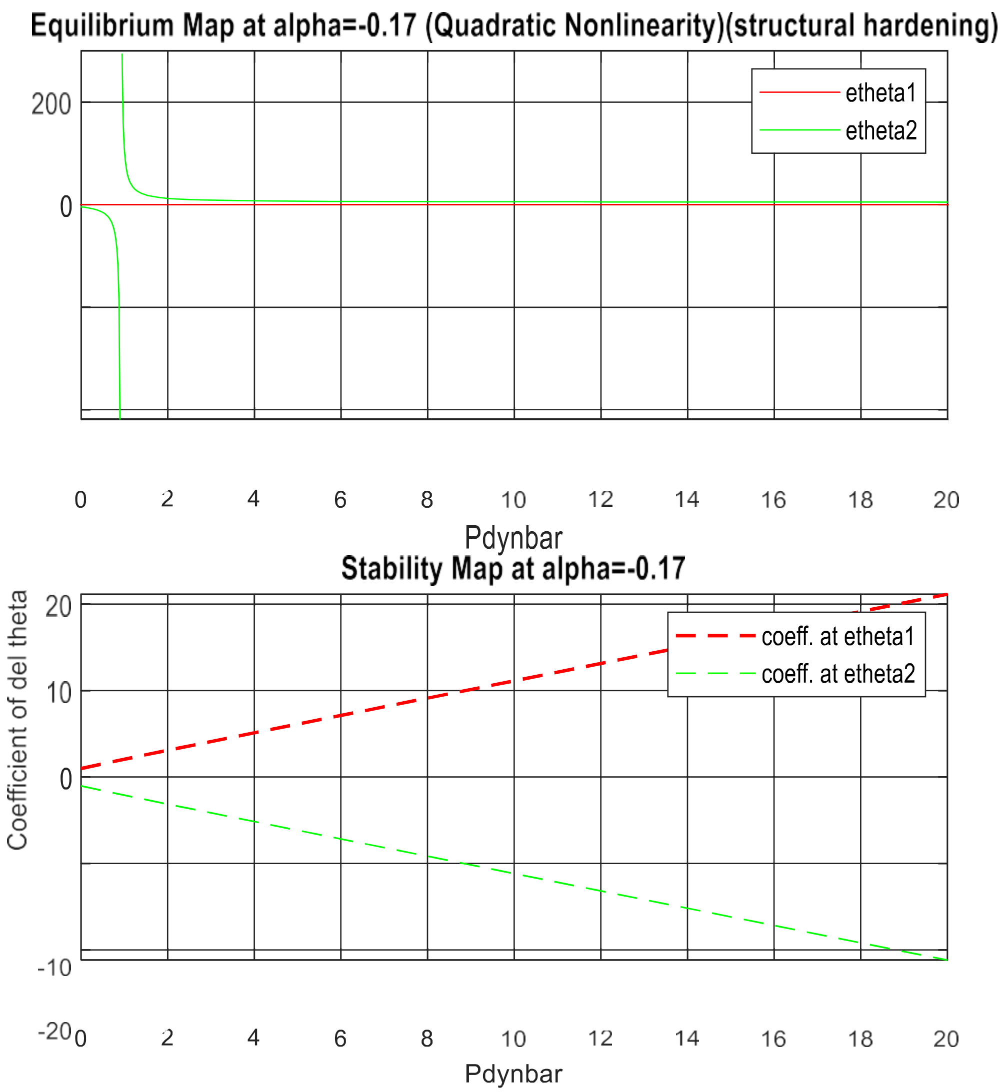
- (e<0) e=-0.25
- alpha α =0.17
- structural softining, γ2=-0.30
- β2=0.30



- (Red)etheta1 : stable for all Pdynbar =[0 ∞]
- (Green)etheta2 : unstable for all Pdynbar =[0 ∞]

Case(A11):

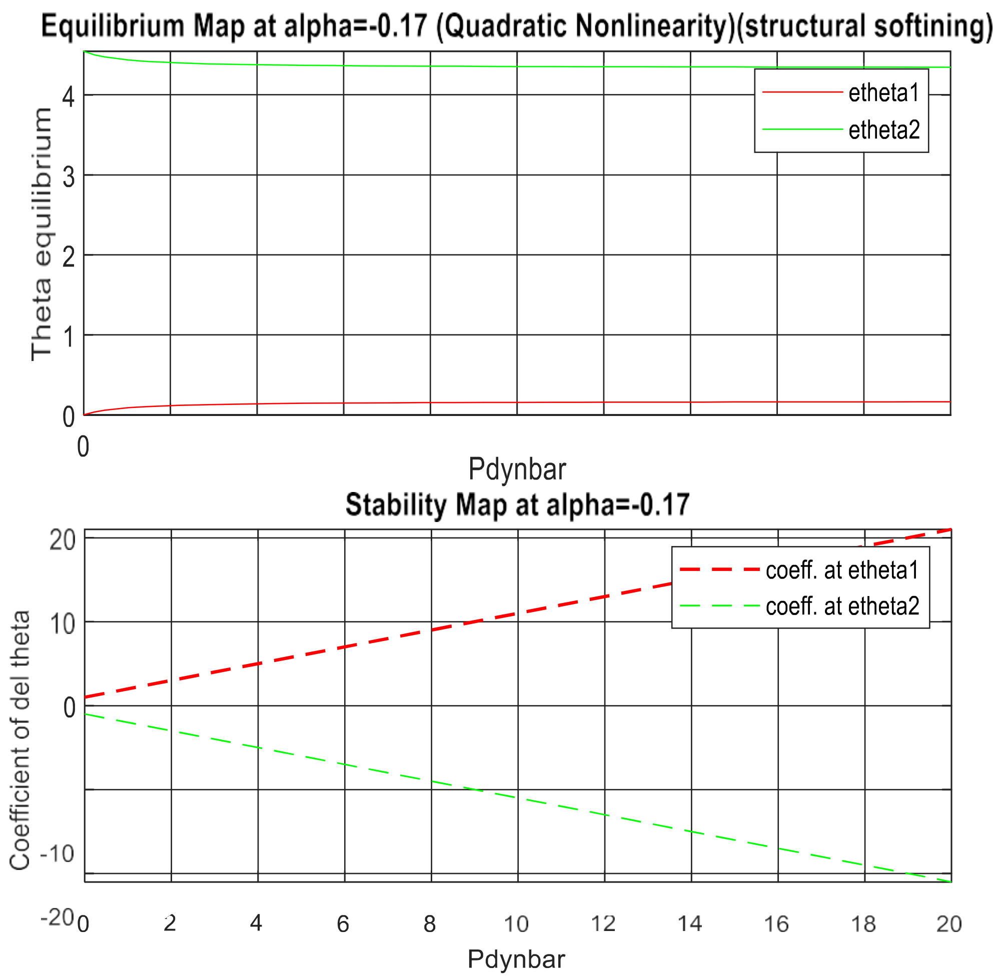
- ($e<0$) $e=-0.25$
- alpha $\alpha=-0.17$
- structural hardening, $\gamma 2=0.30$
- $\beta 2=0.30$



- (Red)etheta1 : stable for all Pdynbar =[0 ∞]
- (Green)etheta2 : unstable for all Pdynbar =[0 ∞]

Case(A12):

- (e<0) e=-0.25
- alpha α =-0.17
- structural softining, $\gamma 2$=-0.30
- $\beta 2$=0.30

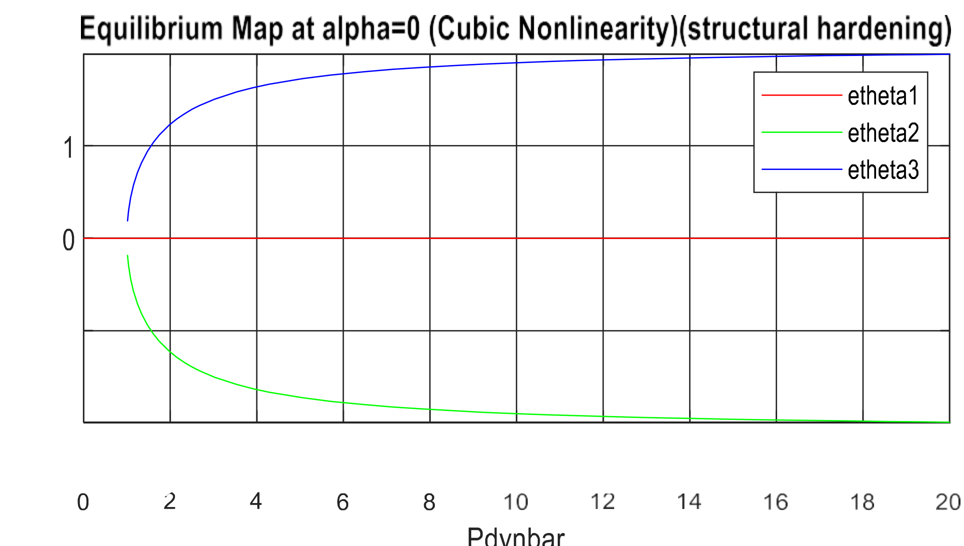


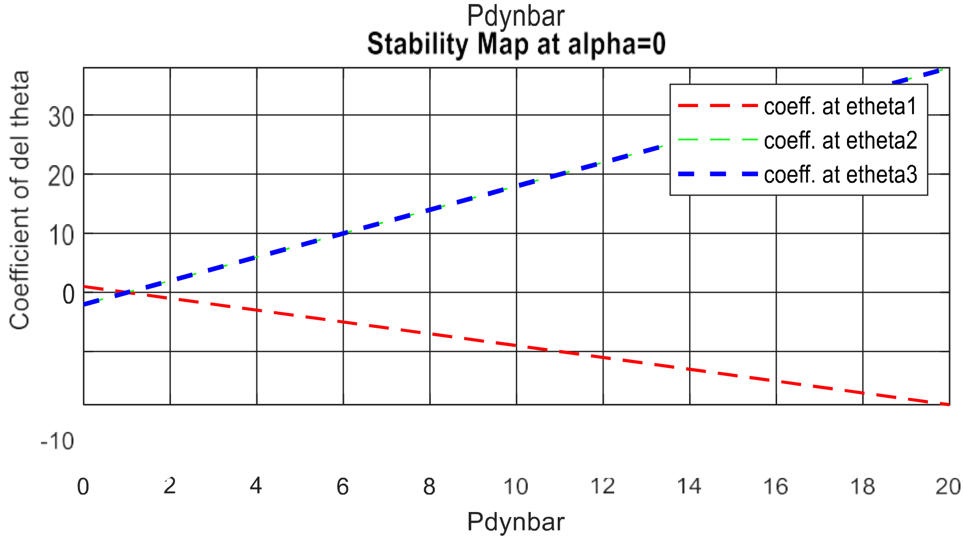
- (Red)etheta1 : stable for all Pdynbar =[0 ∞]
- (Green)etheta2 : unstable for all Pdynbar =[0 ∞]

Case(B): Both are Cubic (structural and aerodynamic)**non-linearity.**

There are following cases in Cubic($\gamma2=0$, $\beta2=0$)non-linearity: Case(B1):

- (e>0) e=0.25
- alpha α =0
- structural hardening, $\gamma3=0.20$
- $\beta3=0.23$

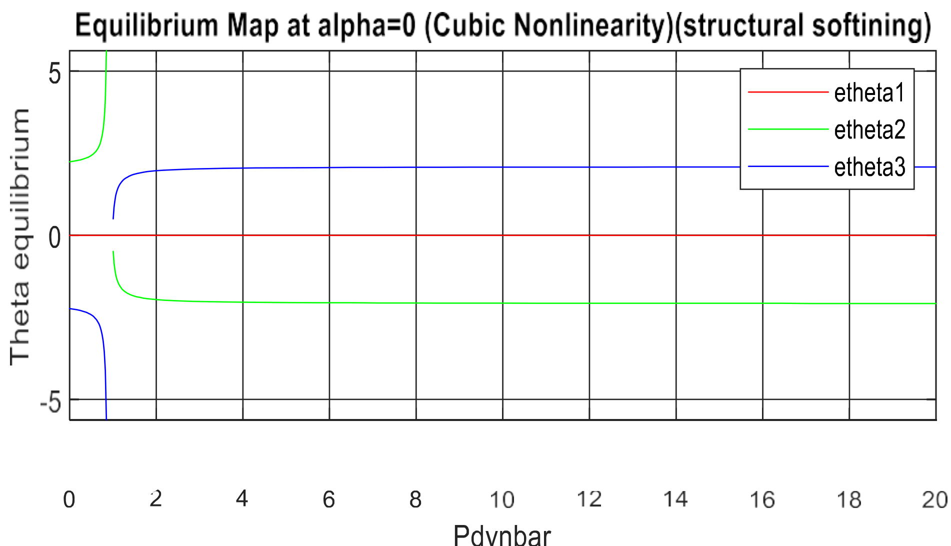


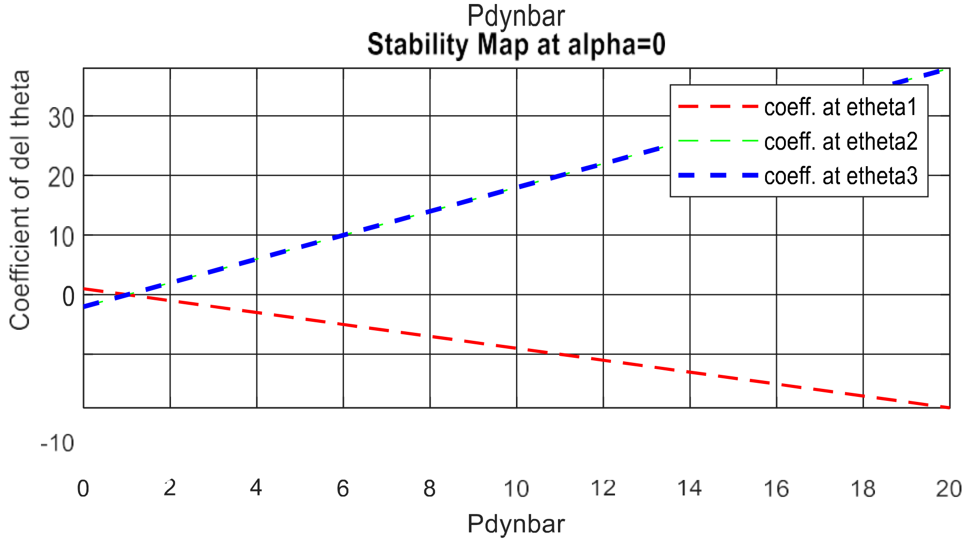


- (Red)etheta1 : stable for Pdynabr <1 ;neutral at Pdynabr=1; unstable forPdynbar >1
- (Green)etheta2 : stable for Pdynabr >1 ;neutral at Pdynbar=1; unstable forPdynbar <1
- (Blue)etheta3 : stable for Pdynabr >1 ;neutral at Pdynbar=1; unstable forPdynbar <1

Case(B2):

- (e>0) e=0.25
- alpha α =0
- structural softining, γ3=-0.21
- β3=0.22





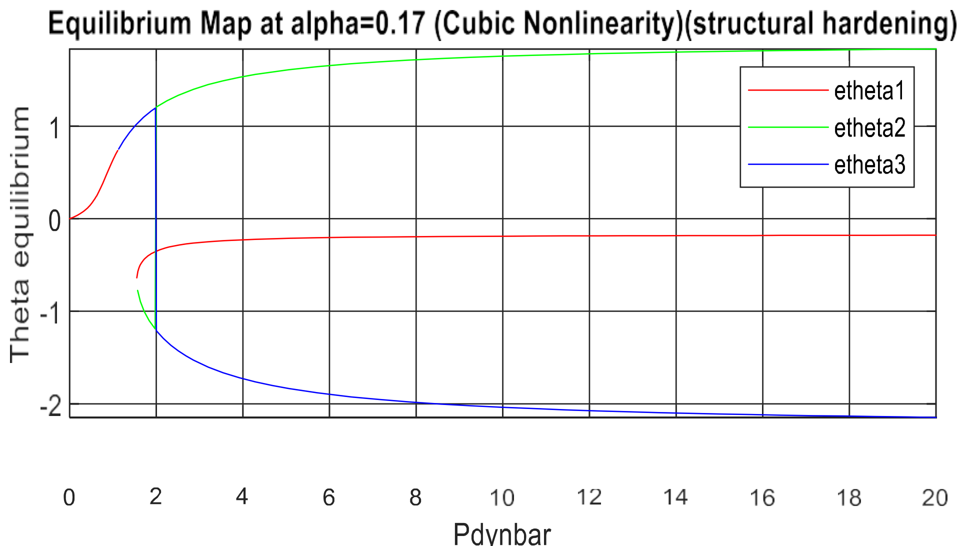
- (Red)etheta1 : stable for Pdynabr <1 ;neutral at Pdynabr=1; unstable forPdynbar >

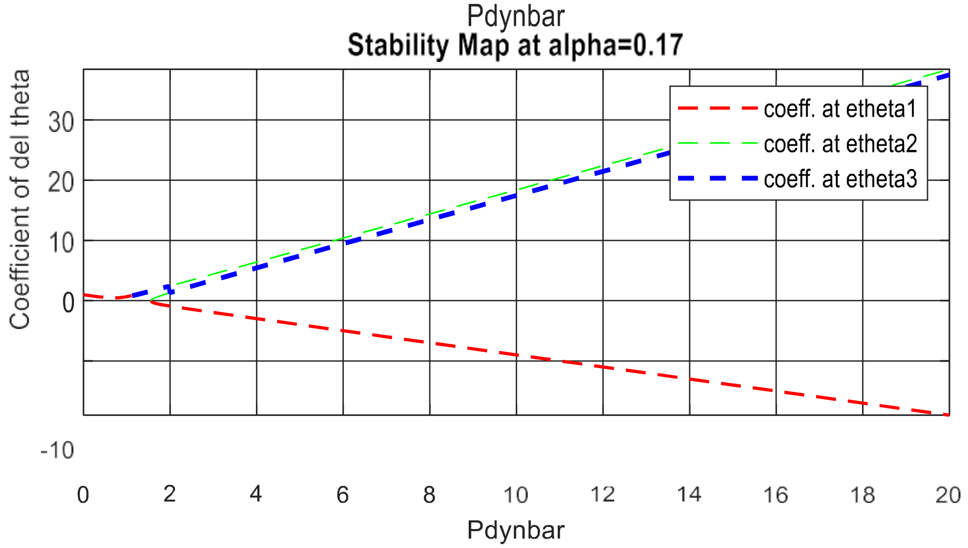
    - (Green)etheta2 : stable for Pdynabr >1 ;neutral at Pdynbar=1; unstable forPdynbar <1

- (Blue)etheta3 : stable for Pdynabr >1 ;neutral at Pdynbar=1; unstable forPdynbar <1

Case(B3):

- (e>0) e=0.25
- alpha α =0.17
- structural hardening, $\gamma 3$=0.30
- $\beta 3$=0.30

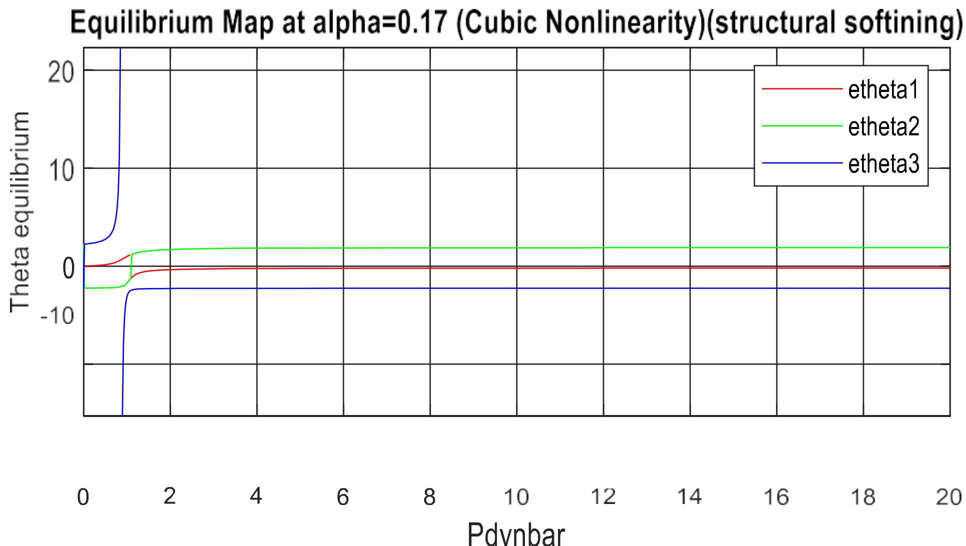


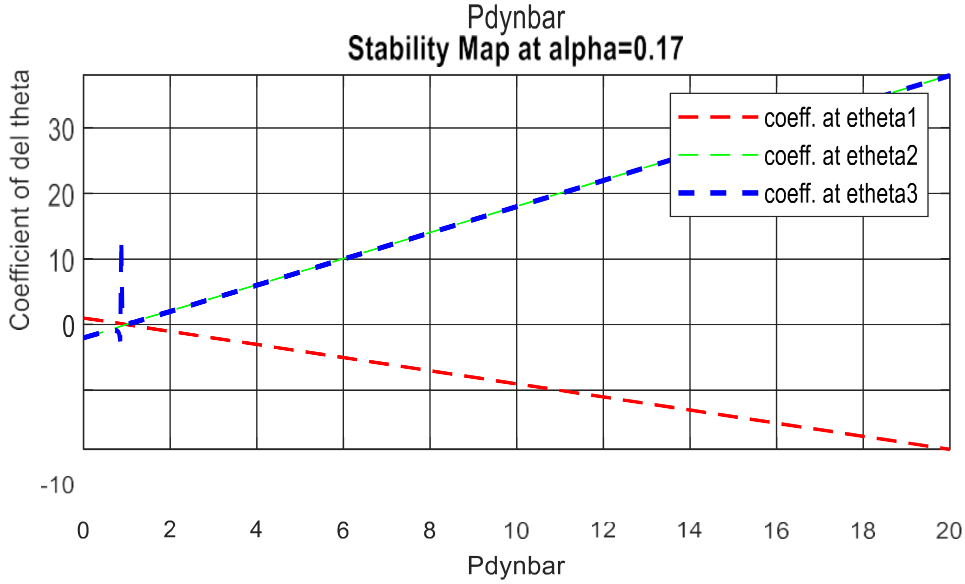


- (Red)etheta1 : stable for Pdynabr <1.12 ;neutral at Pdynabr=1.5; unstablefor Pdynba>1.5
- (Green)etheta2 : stable for Pdynabr >1.5 ;neutral at Pdynbar=1.5
- (Blue)etheta3 : stable for Pdynabr >1.12

Case(B4):

- (e>0) e=0.25
- alpha α =0.17
- structural softining, γ3=-0.30
- β3=0.30



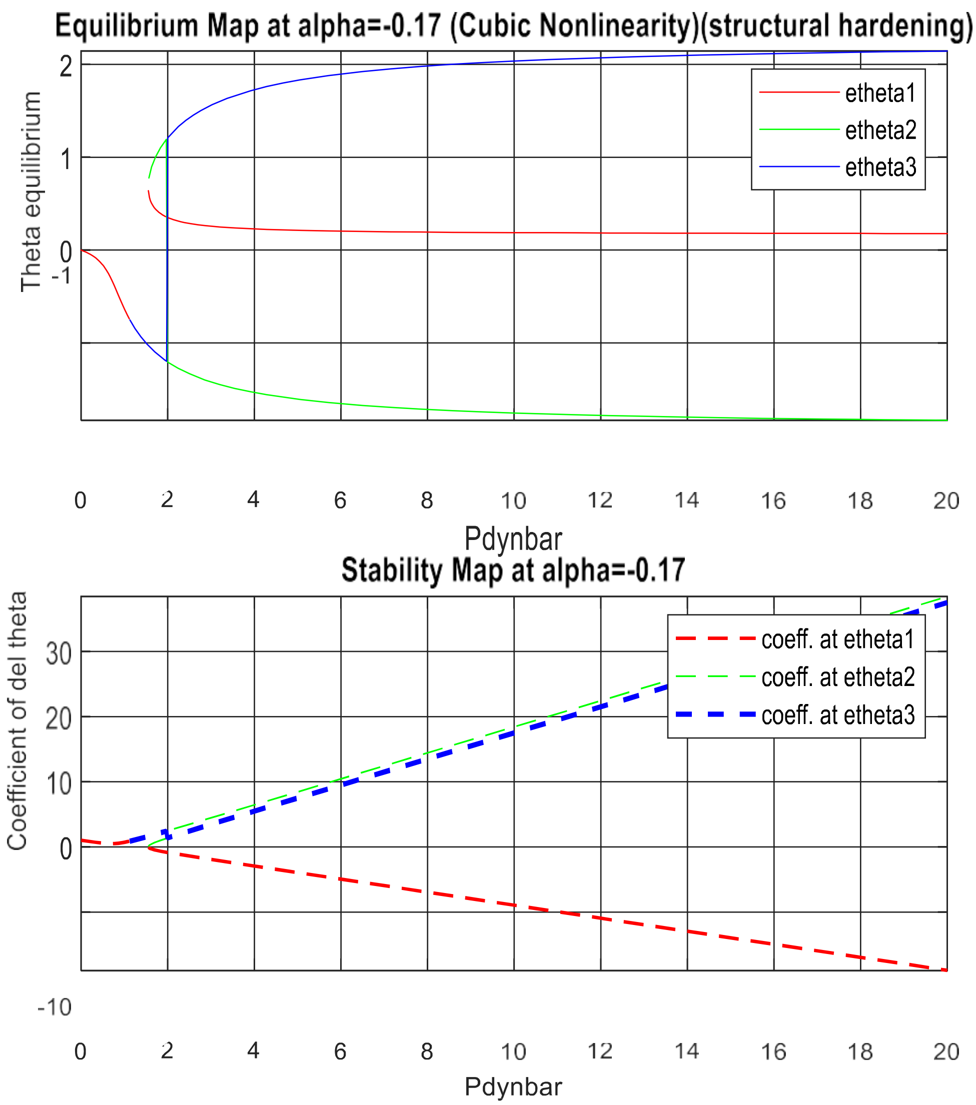


- (Red)etheta1 : stable for Pdynabr <1 ;neutral at Pdynabr=1; unstable forPdynbar >1
- (Green)etheta2 : stable for Pdynabr >1 ;neutral at Pdynbar=1; unstable forPdynbar <1
- (Blue)etheta3 : stable for Pdynabr >0.84 ; unstable for Pdynbar <0.84

Case(B5):

- (e>0) e=0.25
- alpha α =-0.17
- structural hardening, γ3=0.30
- β3=0.30

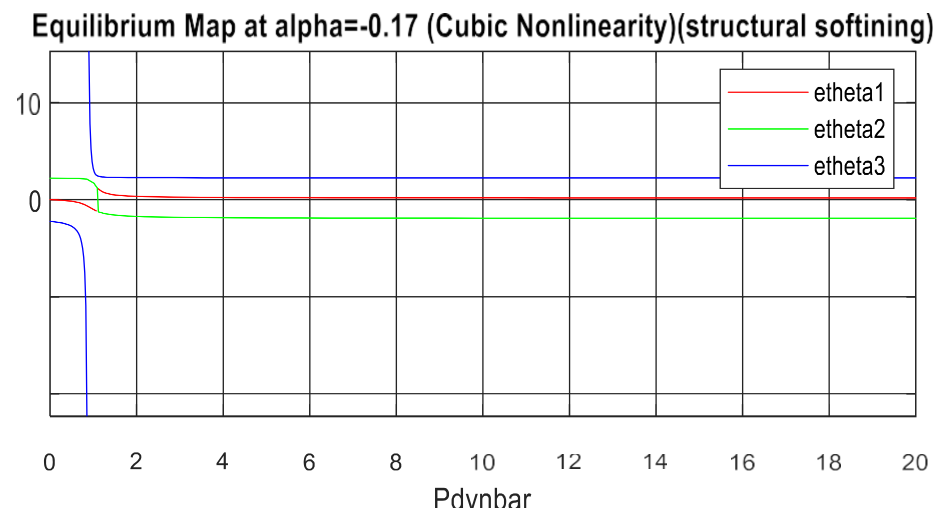
Roots: In this case ,roots is imaginary at some Pdynbar .

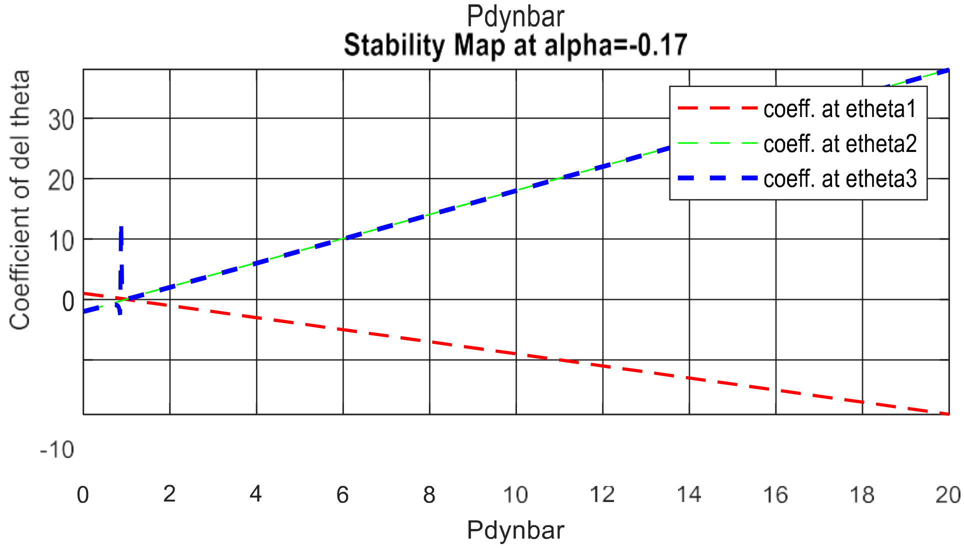


- (Red)etheta1 : stable for Pdynabr <1.12 ;neutral at Pdynabr=1.5; unstablefor Pdynba>1.5
- (Green)etheta2 : stable for Pdynabr >1.5 ;neutral at Pdynbar=1.5
- (Blue)etheta3 : stable for Pdynabr >1.12

Case(B6):

- (e>0) e=0.25
- alpha α =-0.17
- structural softining, γ3=-0.30
- β3=0.30

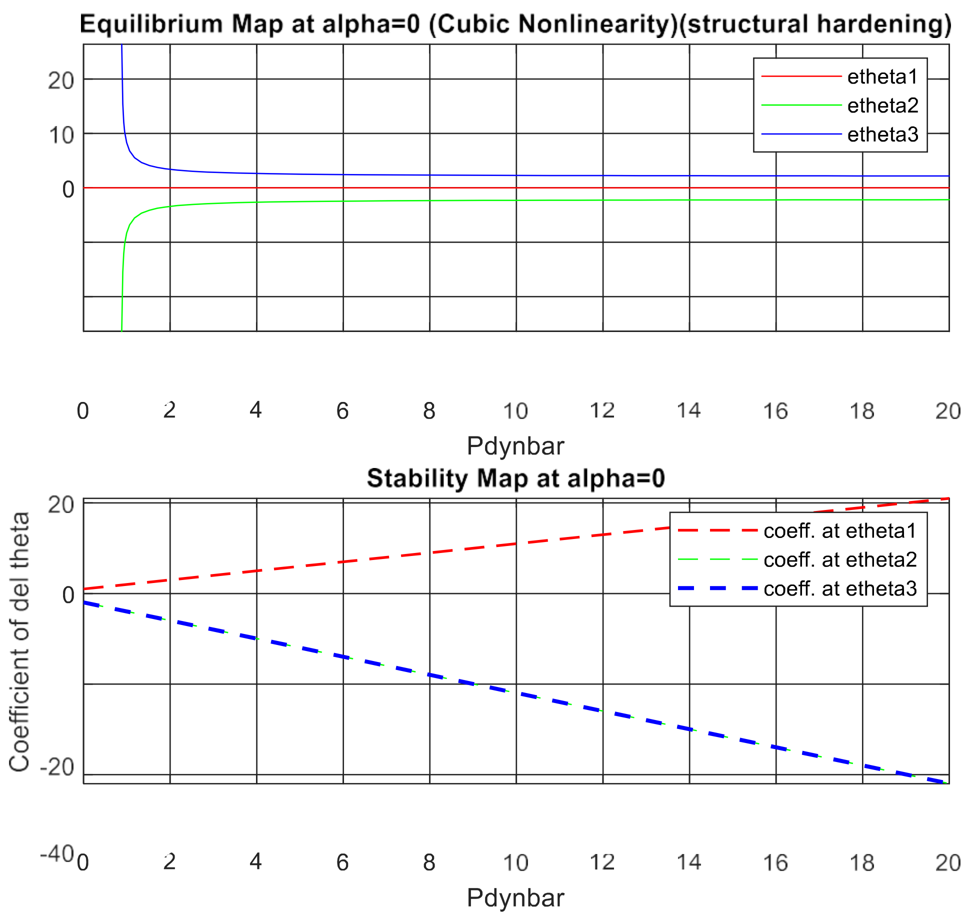




- (Red)etheta1 : stable for Pdynabr <1 ;neutral at Pdynabr=1; unstable forPdynbar >1
- (Green)etheta2 : stable for Pdynabr >1 ;neutral at Pdynbar=1; unstable forPdynbar <1
- (Blue)etheta3 : stable for Pdynabr >0.84 ; unstable for Pdynbar <0.84

Case(B7):

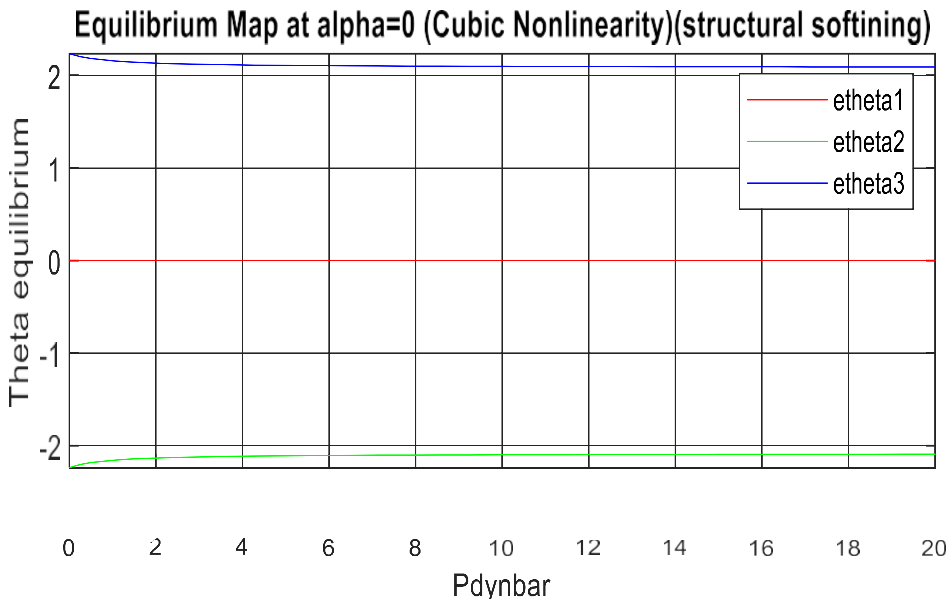
- (e<0) e=-0.25
- alpha α =0
- structural hardening, γ3=0.30
- β3=0.30

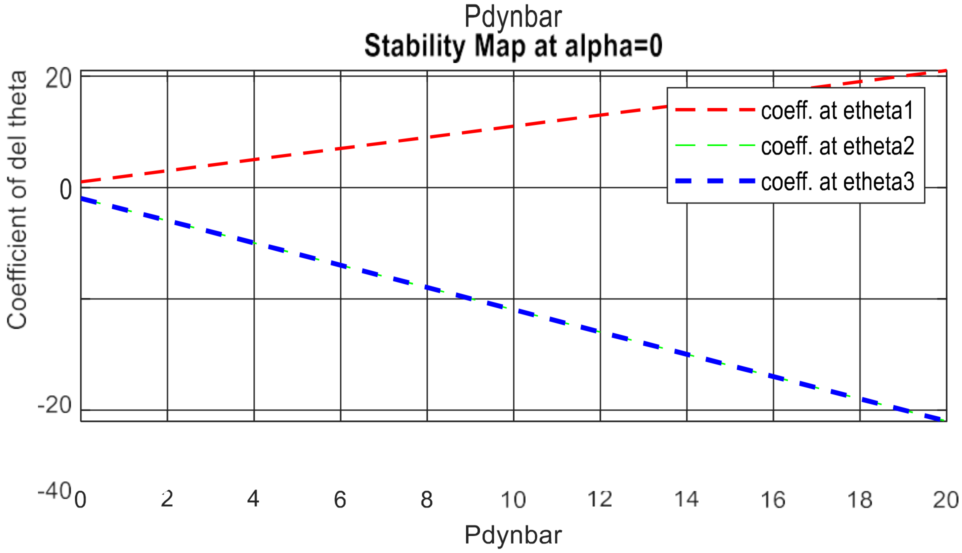


- (Red)etheta1 : stable for all Pdynbar
- (Green)etheta2 : unstable for all Pdynbar
- (Blue)etheta3 : unstable for all Pdynabr

Case(B8):

- (e<0) e=-0.25
- alpha α =0
- structural softining, $\gamma 3$=-0.30
- $\beta 3$=0.30

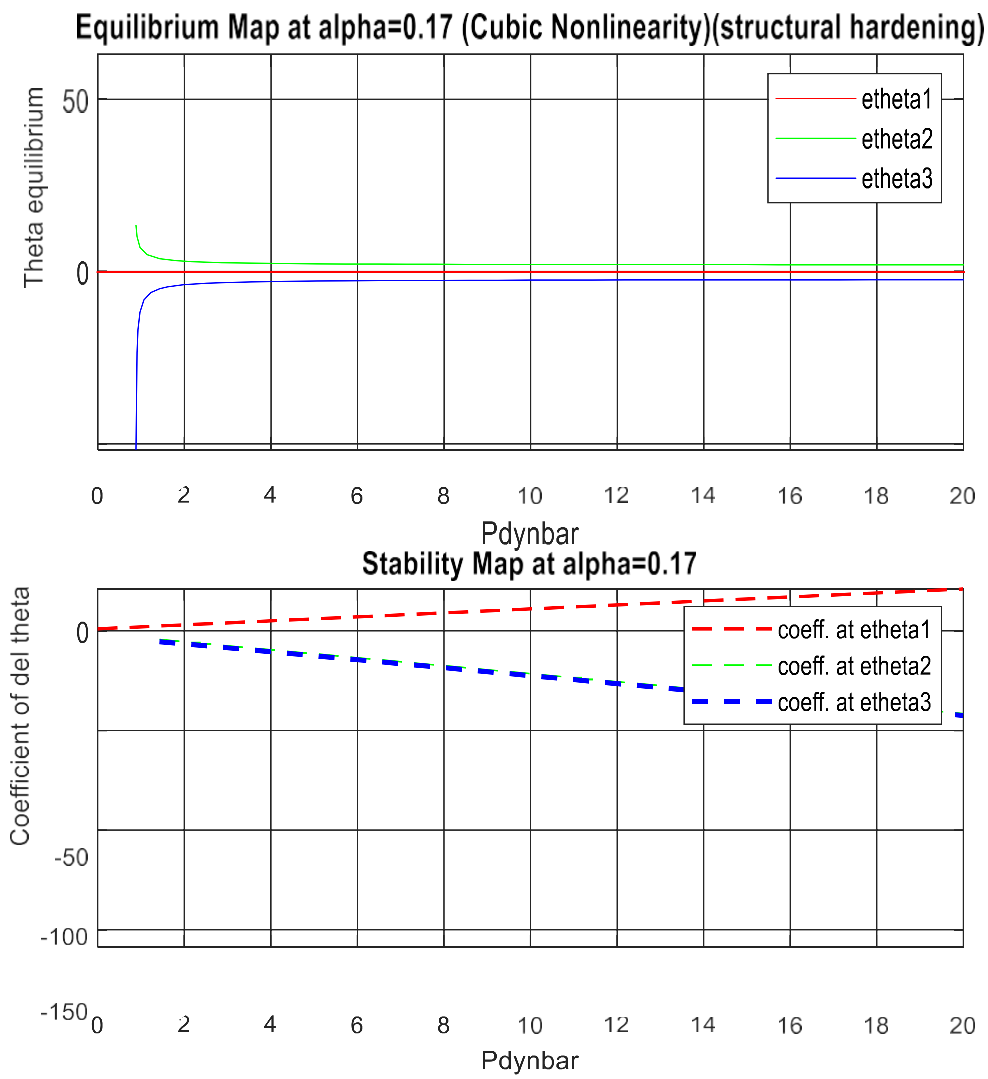




- (Red)etheta1 : stable for all Pdynbar
- (Green)etheta2 : unstable for all Pdynbar
  - (Blue)etheta3 : unstable for all Pdynabar

Case(B9):

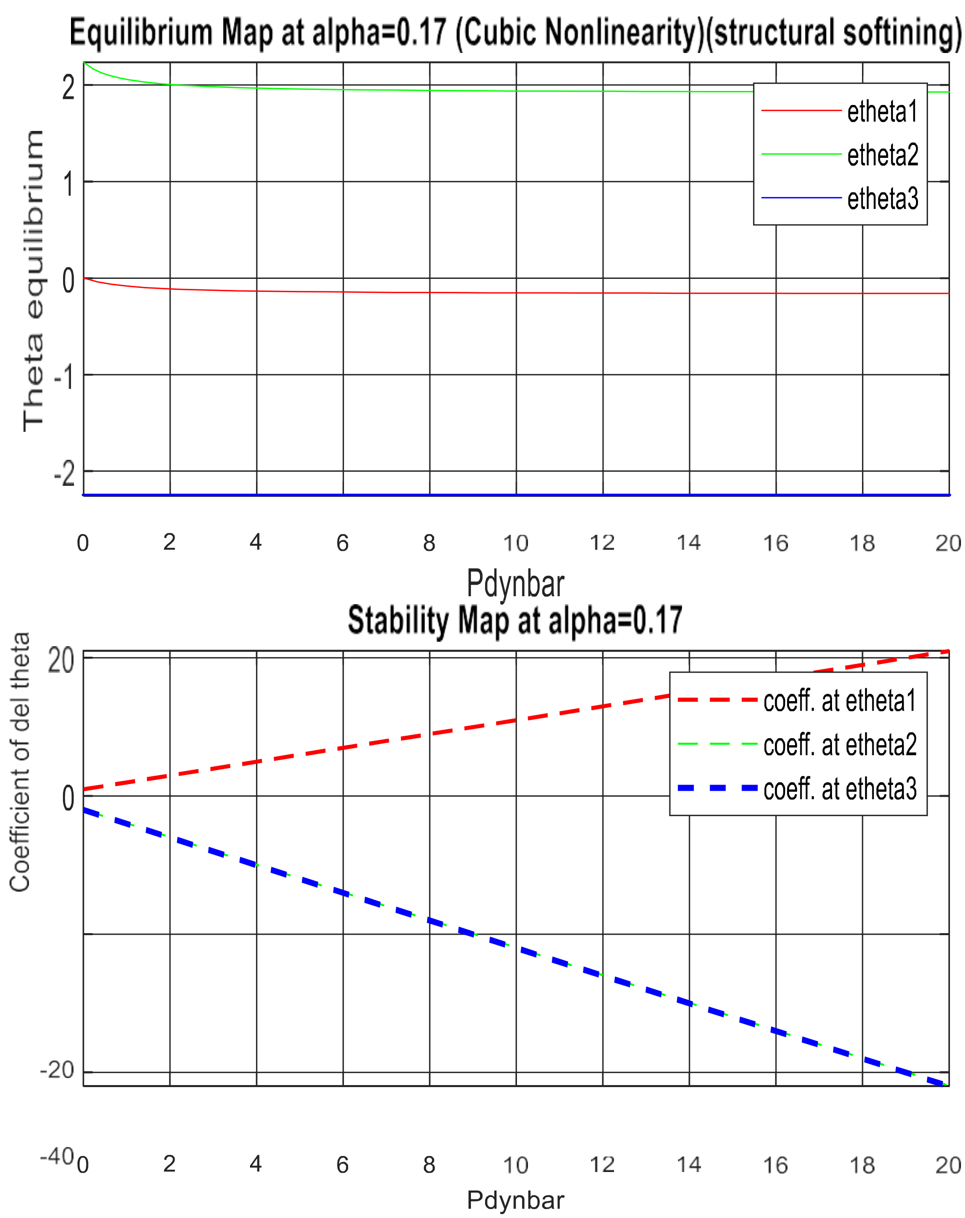
- (e<0) e=-0.25
- alpha α =0.17
- structural hardening, $\gamma 3$=0.30
- $\beta 3$=0.30



- (Red)etheta1 : stable for all Pdynbar
- (Green)etheta2 : unstable for Pdynbar>1.5
  - (Blue)etheta3 : unstable for for Pdynabr>1.5

Case(B10):

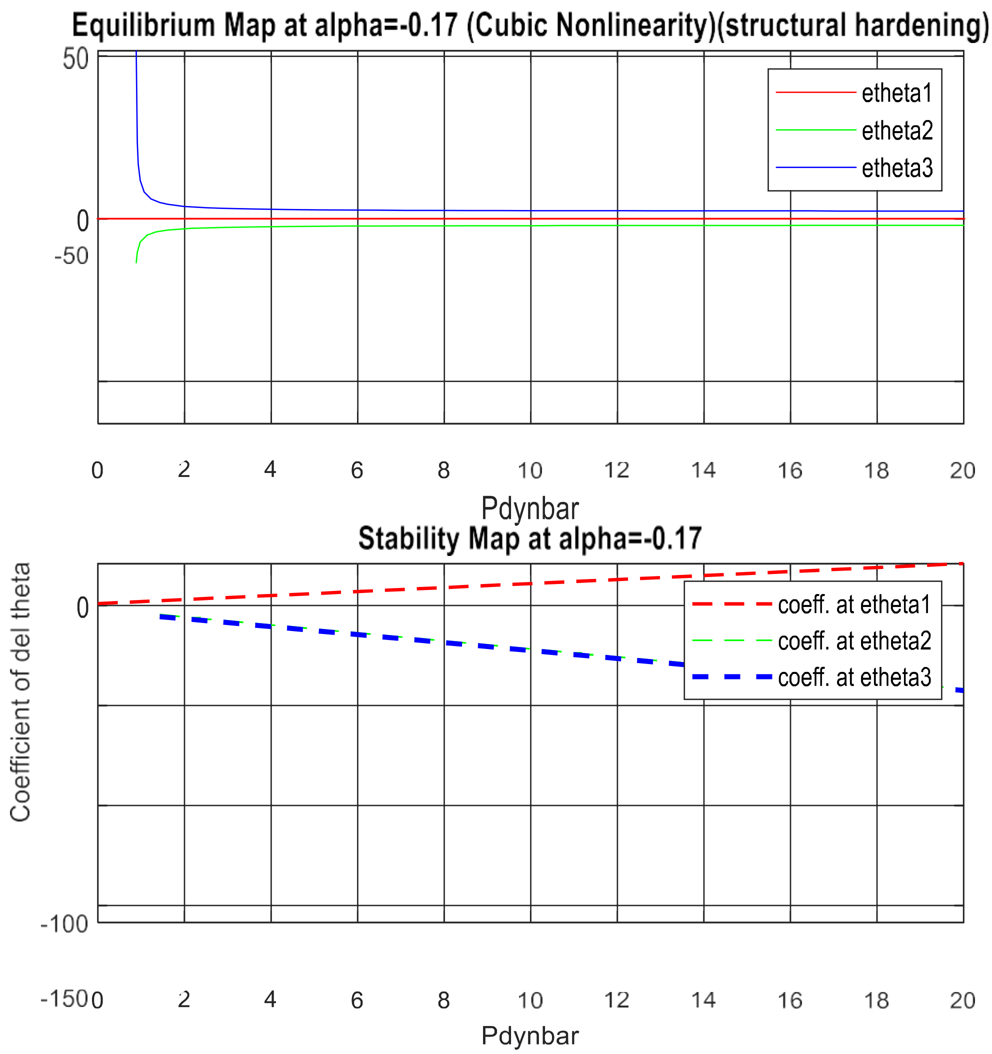
- (e<0) e=-0.25
- alpha α =0.17
- structural softining, γ3=-0.30
- β3=0.30



- (Red)etheta1 : stable for all Pdynbar
- (Green)etheta2 : unstable for all Pdynbar
    - (Blue)etheta3 : unstable for all Pdynabr

Case(B11):

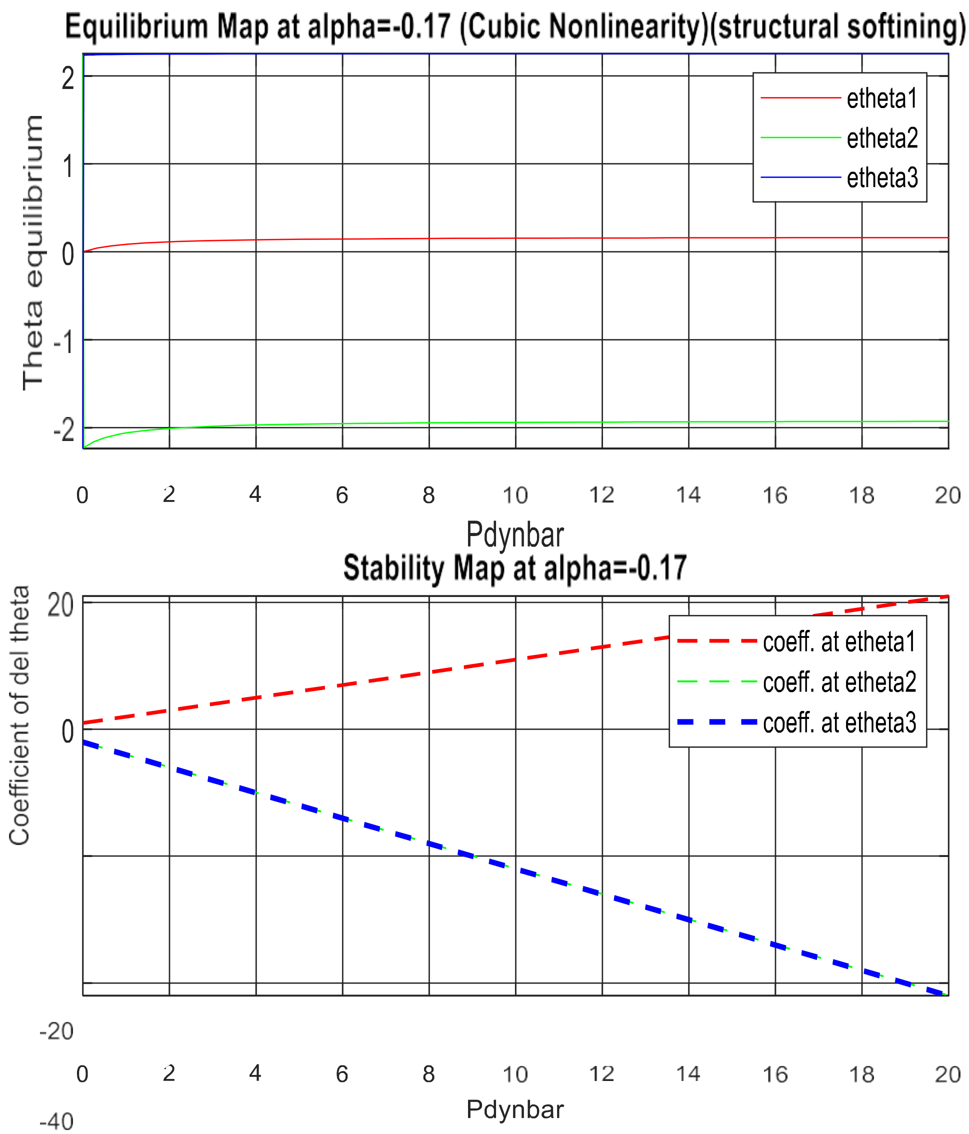
- (e<0) e=-0.25
- alpha α =0.17
- structural softining, γ3=0.30
- β3=0.30



- (Red)etheta1 : Stable for all Pdynbar
- (Green)etheta2 : Unstable for Pdynbar >1.5
    - (Blue)etheta3 : Unstable for Pdynabr >1.5

Case(B12):

- (e<0) e=-0.25
- alpha α =-0.17
- structural softining, γ3=-0.30
- β3=0.30



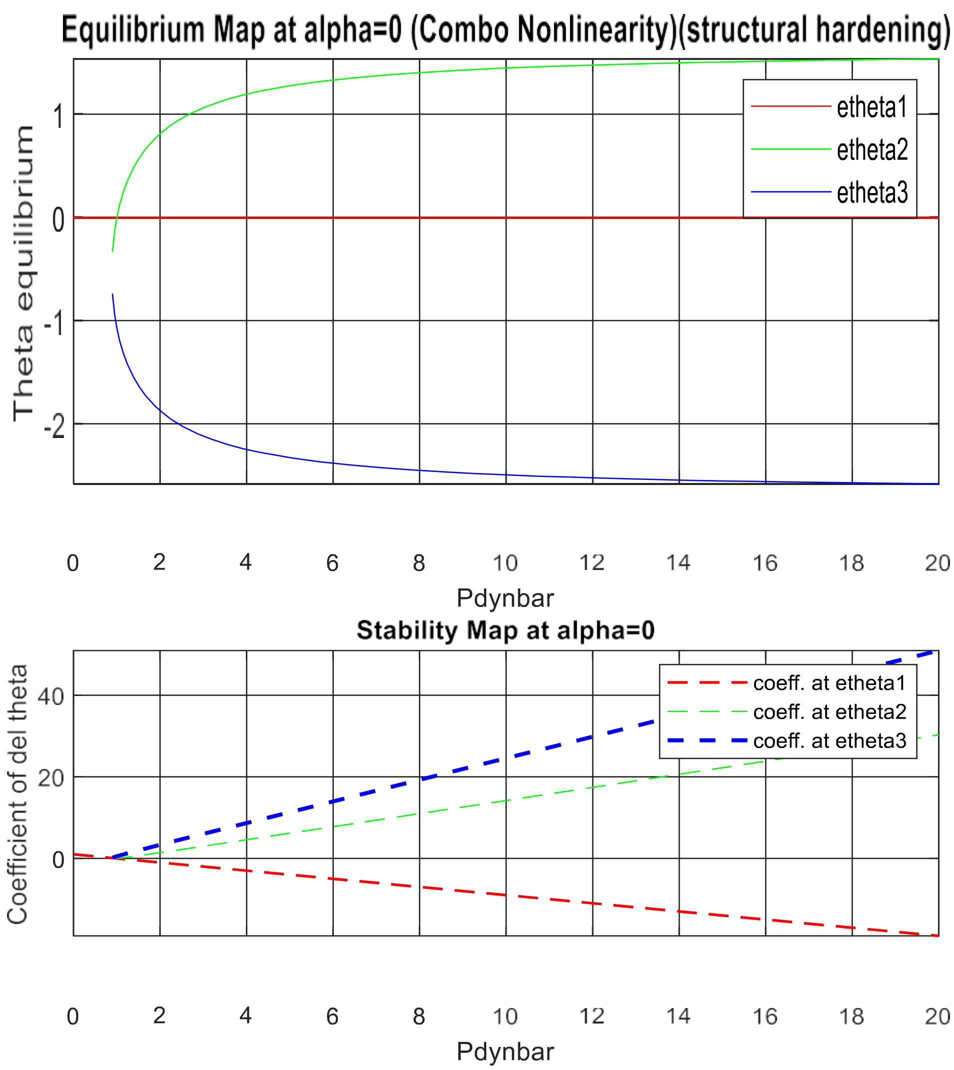
- (Red)etheta1 : stable for all Pdynbar
- (Green)etheta2 : unstable for all Pdynbar
- (Blue)etheta3 : unstable for all Pdynabr

Case(C): quad. &cubic (in both structural and aerodynamic) non-linearity.

There are following cases in Cubic non-linearity:

Case(C1):

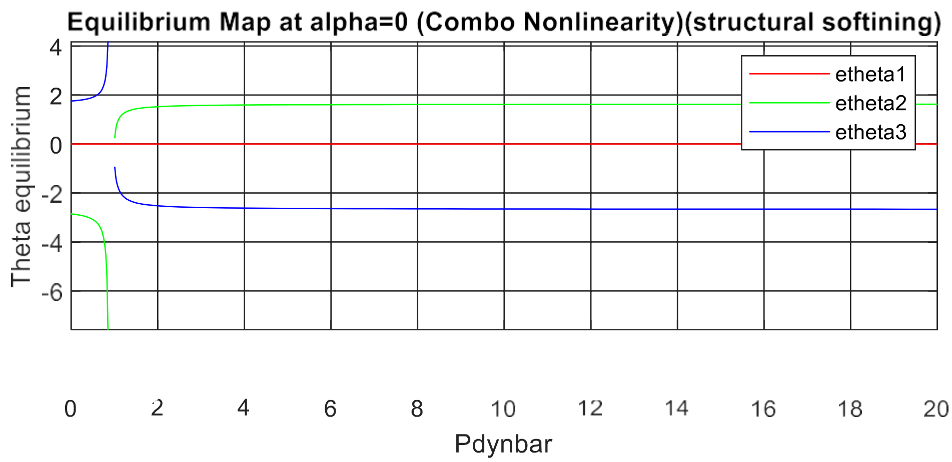
- (e>0) e=0.25
- alpha α =0
- structural hardening , $\gamma 2$=0.30, $\gamma 3$=0.30
- $\beta 2$=0.24, $\beta 3$=0.23

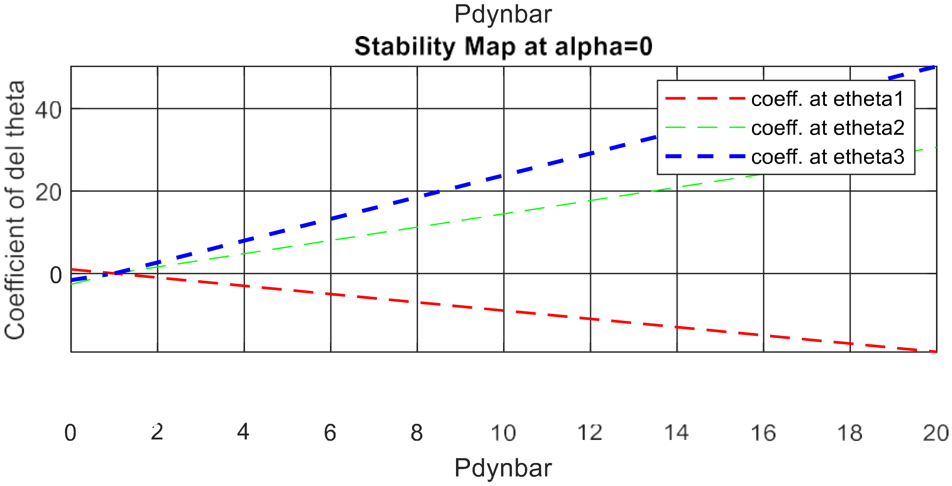


- (Red)etheta1 : stable for Pdynbar <1 ; neutral at Pdynbar=1; Unstable forPdynbar>1
- (Green)etheta2 : stable for Pdynbar =(1,∞) ;and neutral at Pdynbar=1
- (Blue)etheta3 : stable for Pdynbar =(1,∞) ;and neutral at Pdynbar=1

Case(C2):

- (e>0) e=0.25
- alpha α =0
- structural softining, $\gamma 2$=-0.20, $\gamma 3$=-0.20
- $\beta 2$=0.24, $\beta 3$=0.24



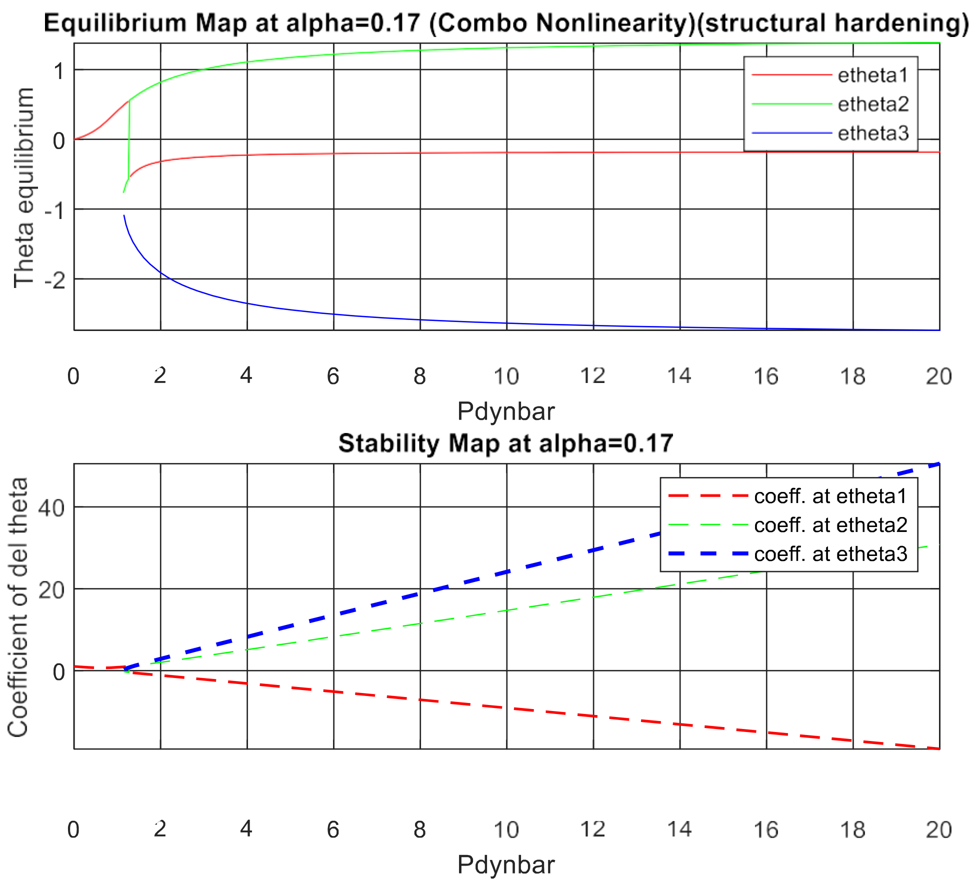


- (Red)etheta1 : stable for Pdynabr <1 ;neutral at Pdynabr=1; unstable forPdynbar >1
- (Green)etheta2 : stable for Pdynabr >1 ;neutral at Pdynbar=1; unstable forPdynbar <1
- (Blue)etheta3 : stable for Pdynabr >1 ;neutral at Pdynbar=1; unstable forPdynbar <1

Case(C3):

- (e>0) e=0.25
- alpha α =0.17
- structural hardening, $\gamma 2$=0.20, $\gamma 3$=0.20
- $\beta 2$=0.24, $\beta 3$=0.24

Roots: In this case ,roots is imaginary at some Pdynbar in the range of Pdynbar .

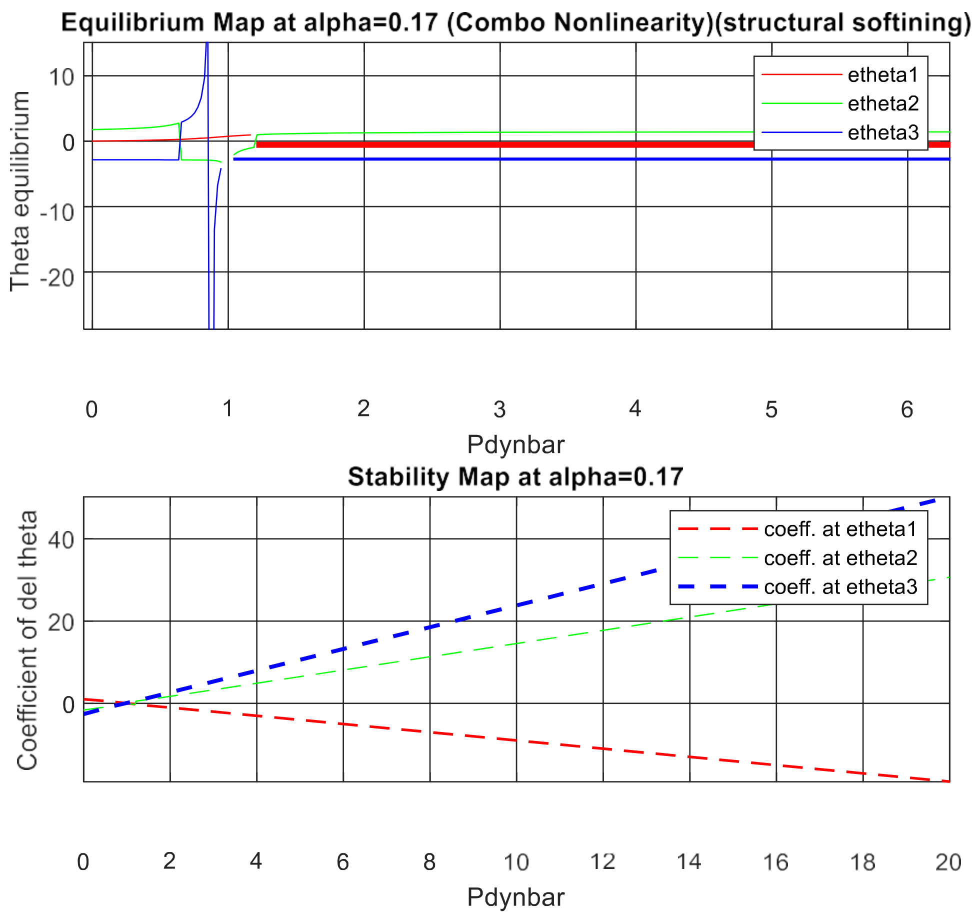


- (Red)etheta1 : stable for Pdynabr <1 ;neutral at Pdynabr=1; unstable forPdynbar >1
- (Green)etheta2 : stable for Pdynabr >1 ;neutral at Pdynbar=1
- (Blue)etheta3 : stable for Pdynabr >1 ;neutral at Pdynbar=1

Case(C4):

- (e>0) e=0.25
- alpha α =0.17
- structural softining, γ2=-0.22, γ3=-0.21
- β2=0.22, β3=0.22

Roots: In this case ,roots is imaginary at some Pdynbar in the range of Pdynbar

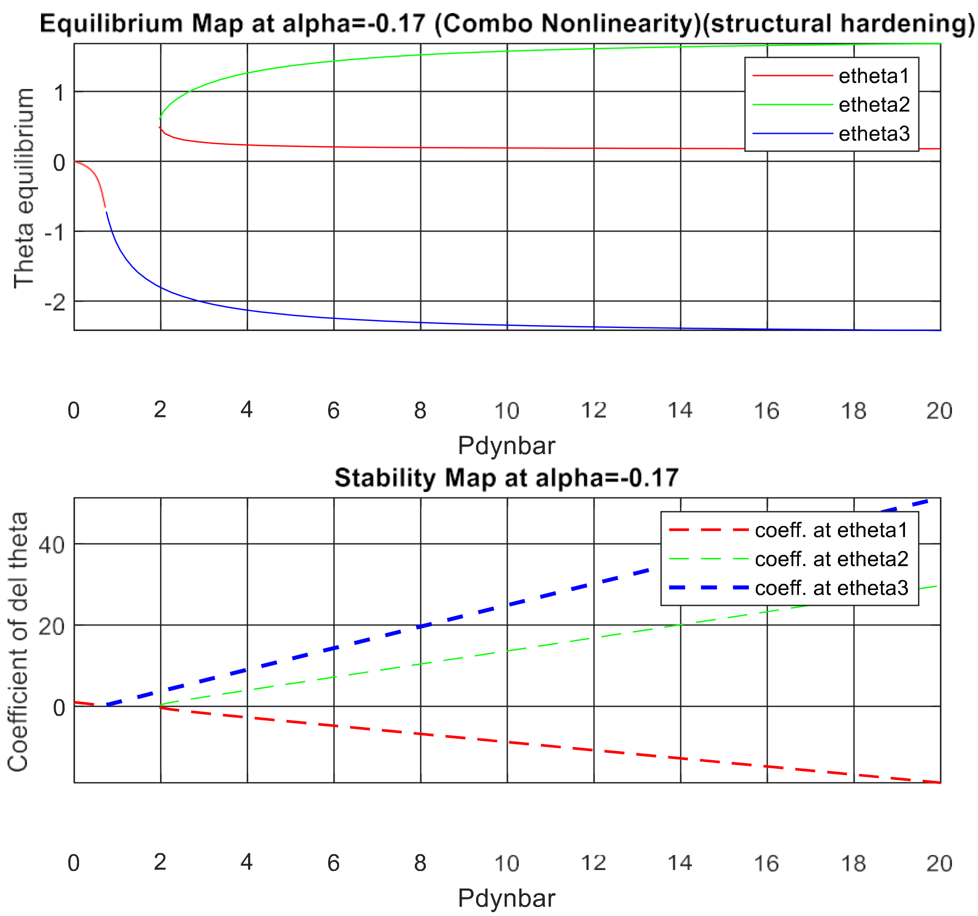


- (Red)etheta1 : stable for Pdynabr <1 ;neutral at Pdynabr=1; unstable forPdynbar >1
- (Green)etheta2 : stable for Pdynabr >1 ;neutral at Pdynbar=1; unstable forPdynbar <1
- (Blue)etheta3 : stable for Pdynabr >1 ;neutral at Pdynbar=1; unstable forPdynbar <1

Case(C5):

- (e>0) e=0.25
- alpha α =-0.17
- structural hardening, $\gamma 2$=0.22, $\gamma 3$=0.30
- $\beta 2$=0.24, $\beta 3$=0.23

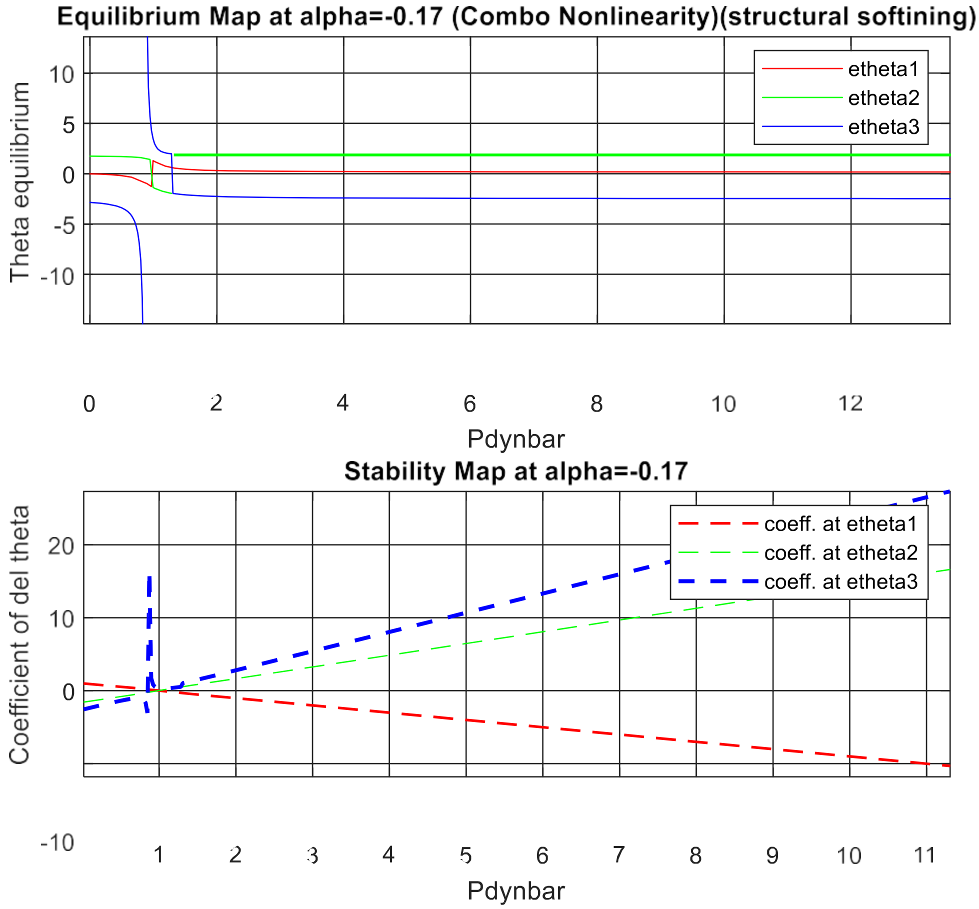
Roots: In this case ,roots is imaginary at some Pdynbar in the range of Pdynbar



- (Red)etheta1 : stable for Pdynabr <0.25 ;neutral at Pdynabr=1; unstablefor Pdynbar >1
- (Green)etheta2 : stable for Pdynabr >1 ;neutral at Pdynbar=1
- (Blue)etheta3 : stable for Pdynabr >0.75 ;neutral at Pdynbar=0.75

Case(C6):

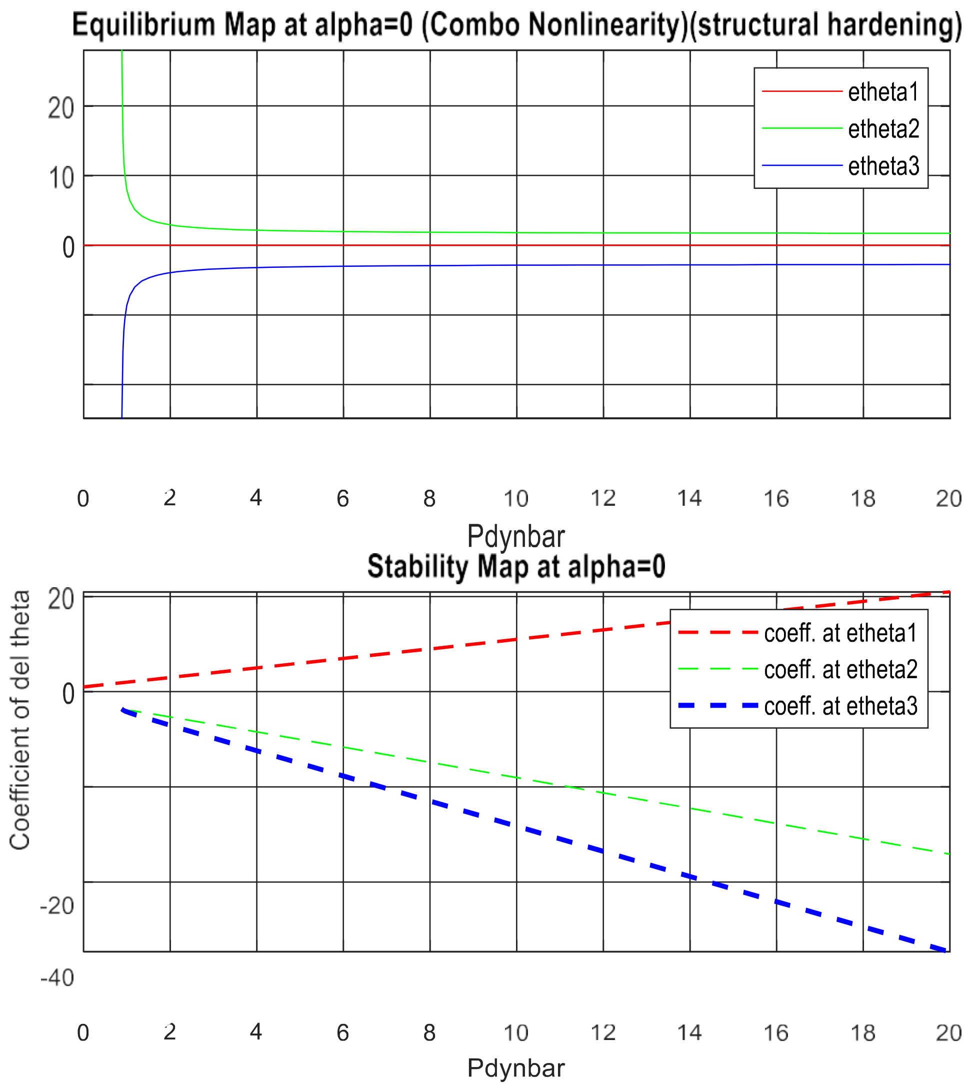
- (e>0) e=0.25
- alpha α =-0.17
- structural softining, γ2=-0.22, γ3=-0.22
- β2=0.24, β3=0.24



- (Red)etheta1 : stable for Pdynabr <1 ;neutral at Pdynabr=1; unstable forPdynbar >1
- (Green)etheta2 : stable for Pdynabr >1 ;neutral at Pdynbar=1; unstable forPdynbar <1
- (Blue)etheta3 : stable for Pdynabr >0.84 ; unstable for Pdynbar <0.84

Case(C7):

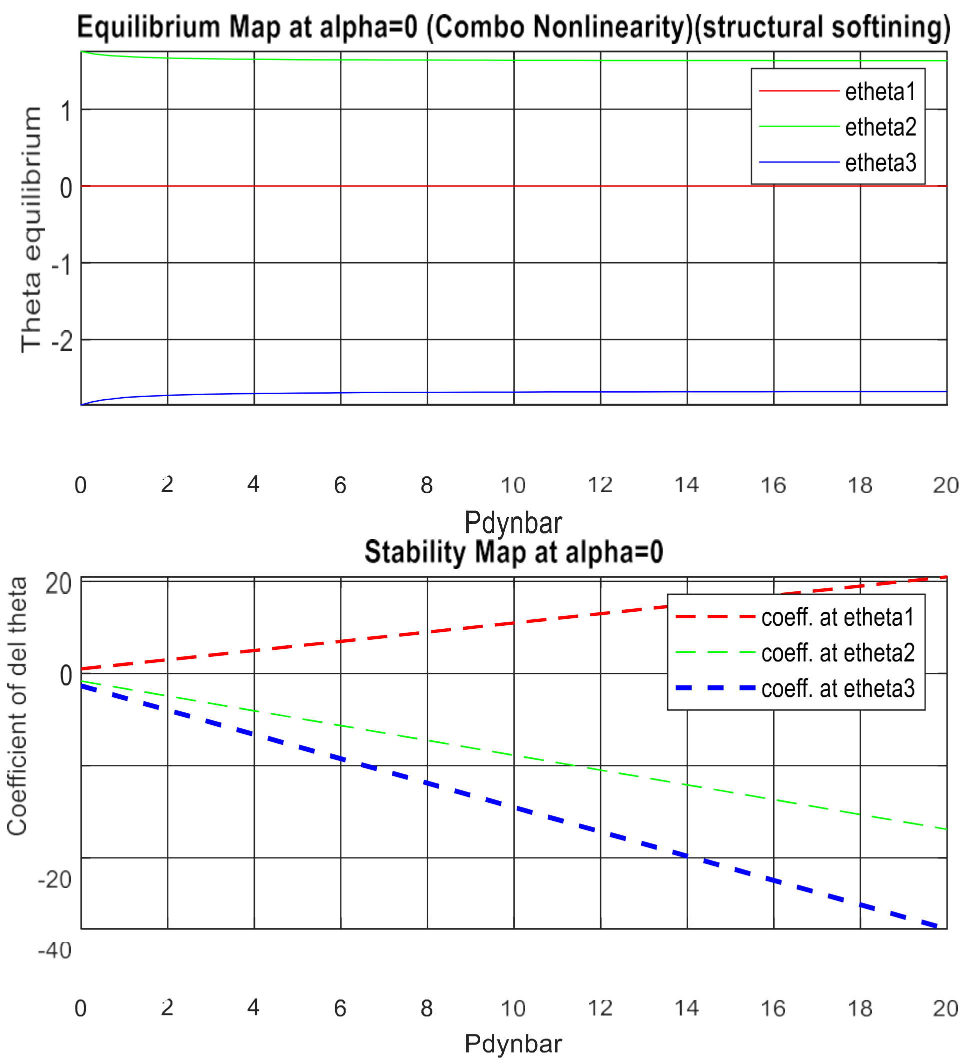
- (e<0) e=-0.25
- alpha α =0
- structural hardening , γ2=0.22, γ3=0.22
- β2=0.24, β3=0.24



- (Red)etheta1 : stable for all Pdynabr
- (Green)etheta2 : unstable for Pdynabr>0.8
- (Blue)etheta3 : unstable for Pdynabr>0.8

Case(C8):

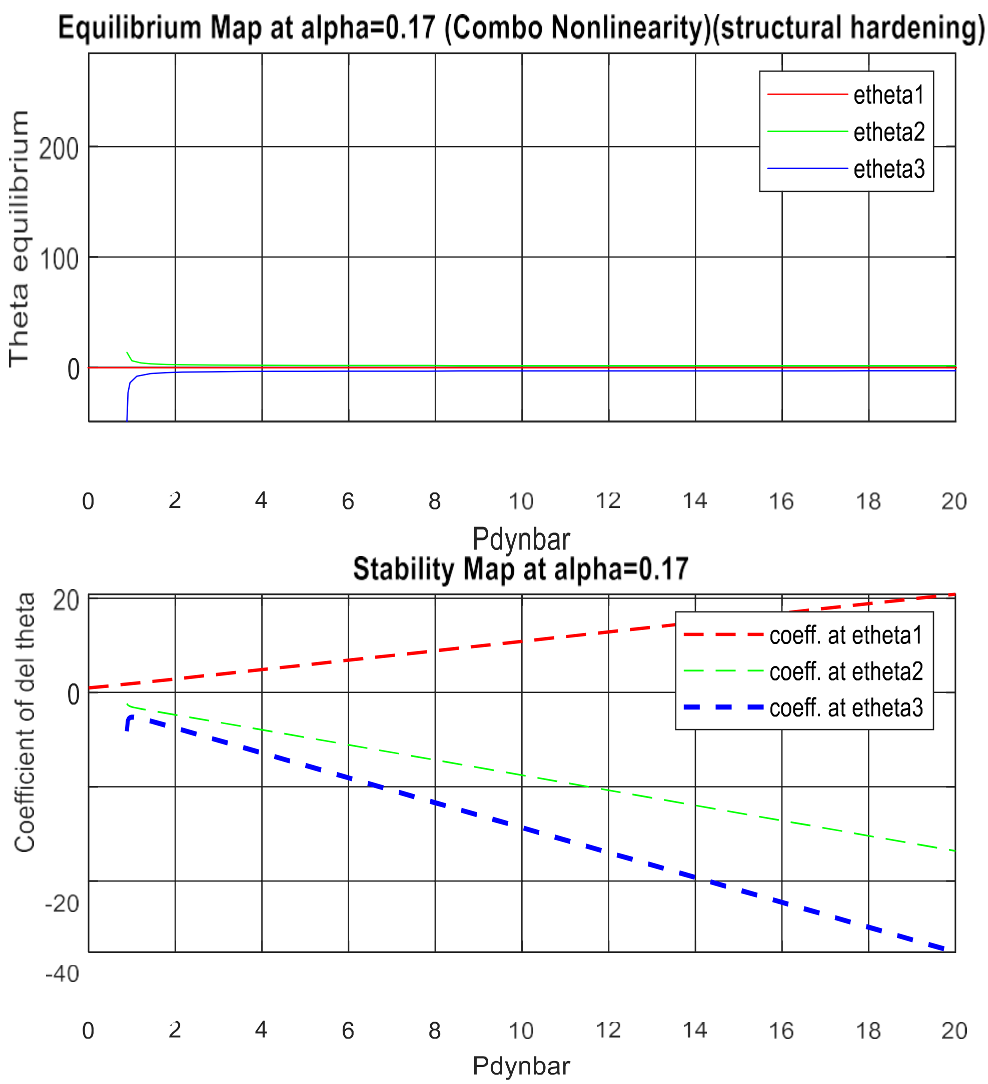
- (e<0) e=-0.25
- alpha α =0
- structural softining, γ2=-0.22, γ2=-0.22
- β2=0.24, β3=0.24



- (Red)etheta1 : stable for all Pdynabr
- (Green)etheta2 : unstable for all Pdynabr
- (Blue)etheta3 : unstable for all Pdynabr

Case(C9):

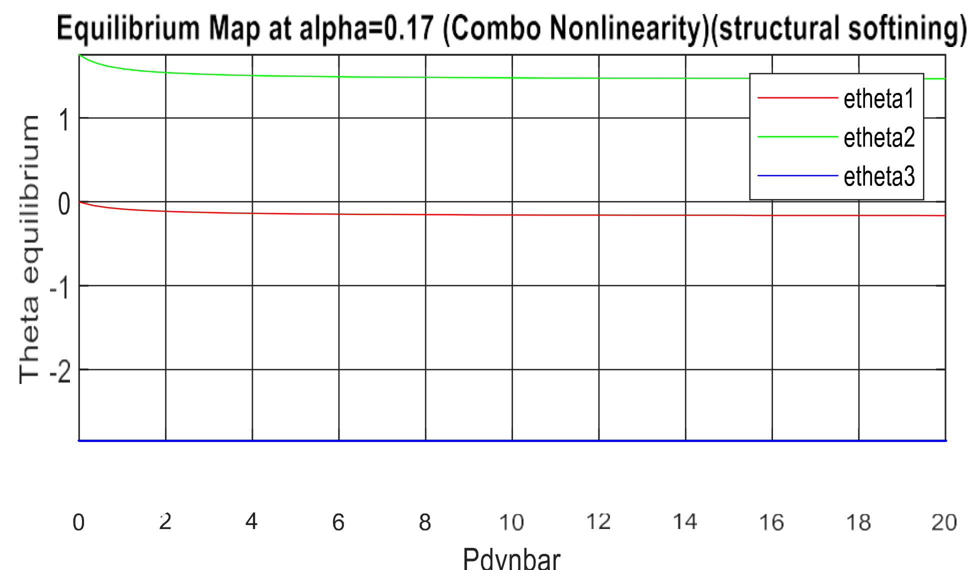
- (e<0) e=-0.25
- alpha α =0.17
- structural hardening, γ2=0.22, γ3=0.22
- β2=0.24, β3=0.24

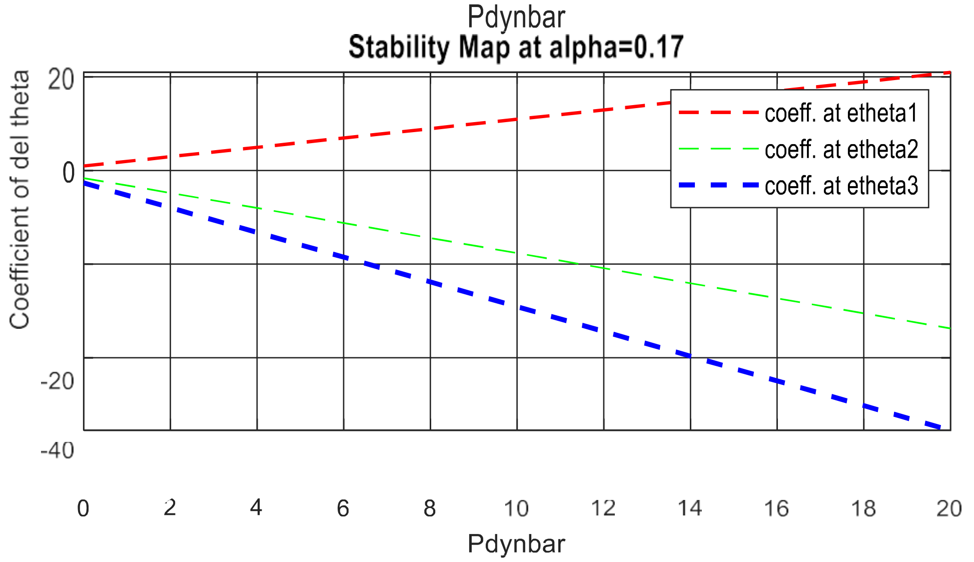


- (Red)etheta1 : stable for all Pdynabr
- (Green)etheta2 : unstable for Pdynabr>0.8
- (Blue)etheta3 : unstable for Pdynabr>0.8

Case(C10):

- (e<0) e=-0.25
- alpha α =0.17
- structural softining, γ2=-0.22, γ3=-0.22
- β2=0.24, β3=0.24

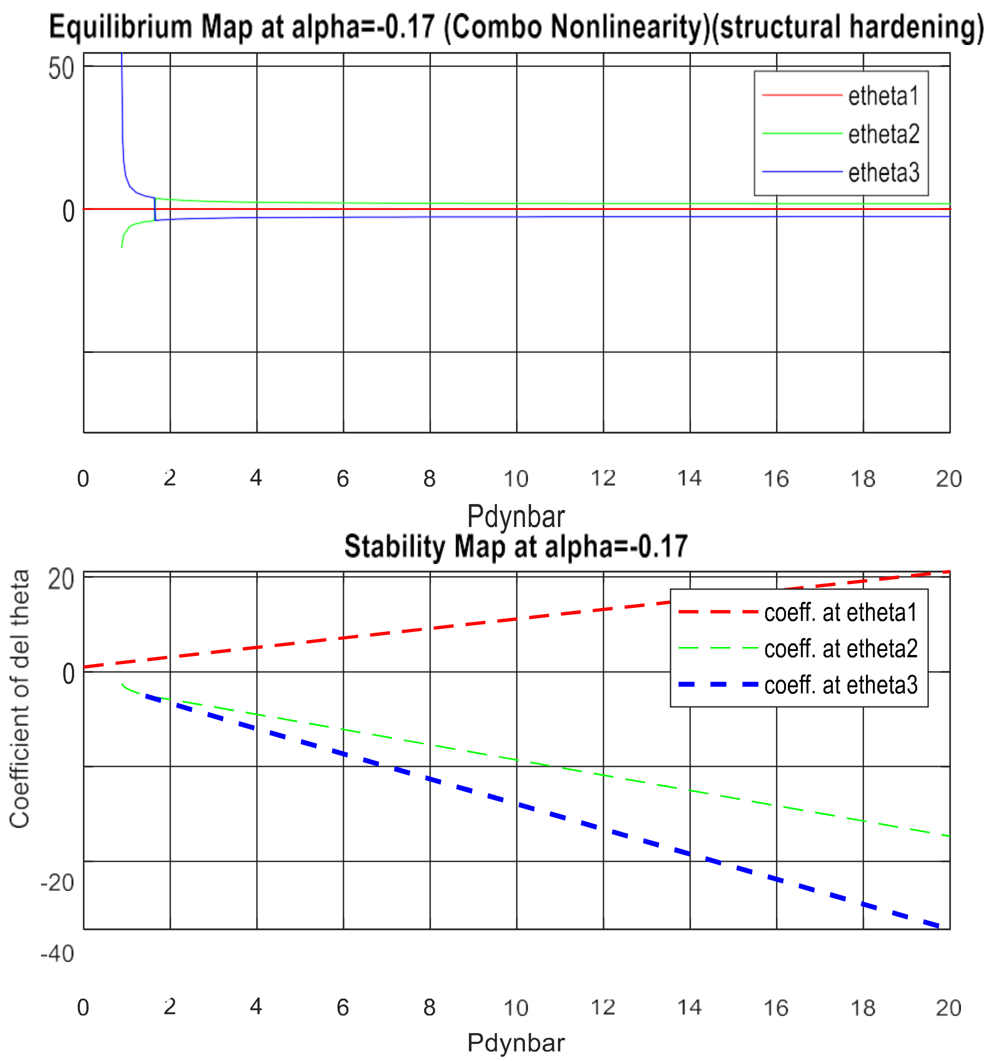




- (Red)etheta1 : stable for all Pdynabr
- (Green)etheta2 : unstable for all Pdynabr
- (Blue)etheta3 : unstable for all Pdynabr

Case(C11):

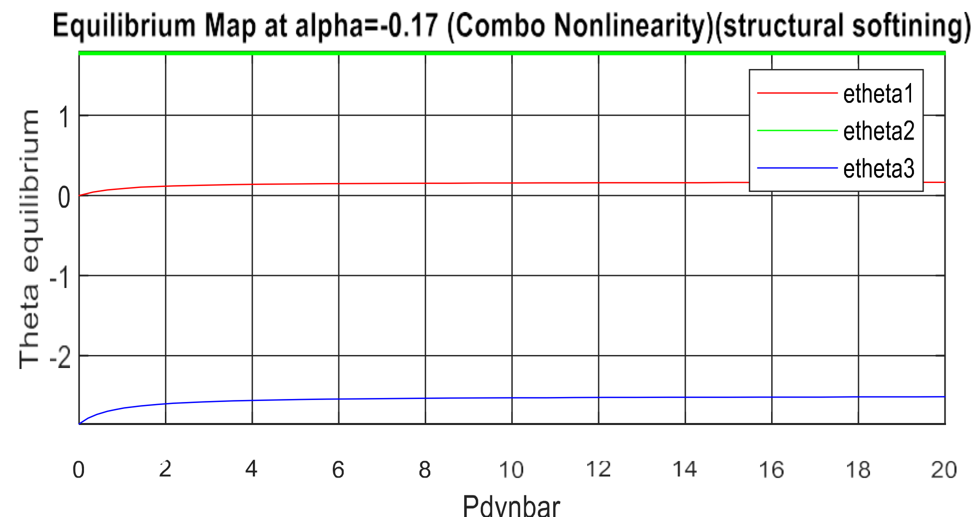
- (e<0) e=-0.25
- alpha α =-0.17
- structural hardening, γ2=0.22, γ3=0.22
- β2=0.24, β3=0.24

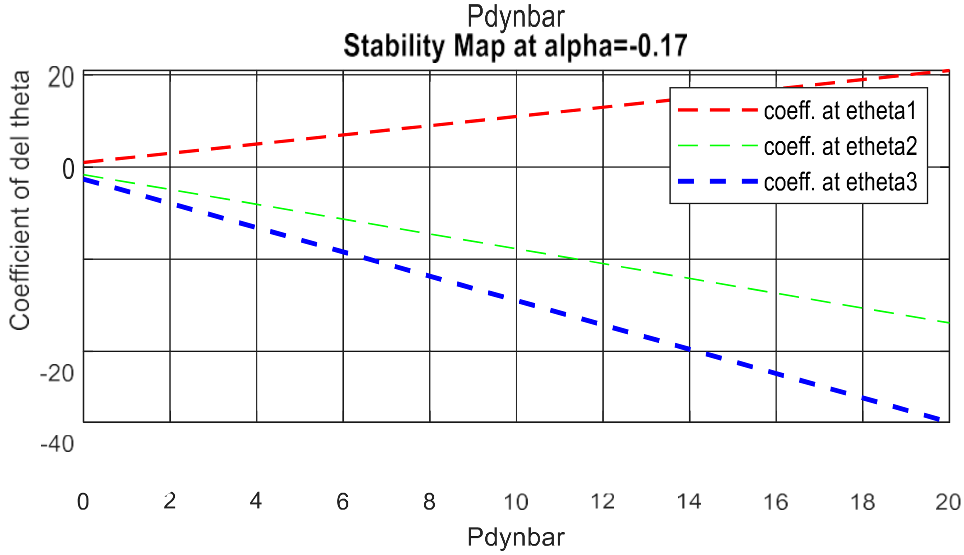


- (Red)etheta1 : stable for all Pdynabr
- (Green)etheta2 : unstable for Pdynabr>0.8
- (Blue)etheta3 : unstable for Pdynabr>1.5

Case(C12):

- (e<0) e=-0.25
- alpha α =-0.17
- structural softining, γ2=-0.22, γ3=-0.22
- β2=0.24, β3=0.24





- (Red)etheta1 : stable for all Pdynabr
- (Green)etheta2 : unstable for all Pdynabr
- (Blue)etheta3 : unstable for all Pdynabr

MATLAB CODE:-

```
clc
format compact
syms theta pdynbar etheta etheta1 etheta2 etheta3
gama2=input('input_value_of_Kt_2=');
beta2=input('input_value_of_Ka_2=');
gama3=input('input_value_of_Kt_3=');
beta3=input('input_value_of_Ka_3=');
a=input('input_value_of_alpha(in degree)='); %alpha:
angle of attack from Zero lift line(alphaZLL)
alpha=a*pi/180;
e=input('input_value_of_e(with sign)=');%distance
between elastic axis and aerodynamic center
if (gama2==0)&&(beta2==0)
    a=' (Cubic Nonlinearity)';
    if gama3>0
        b='(structural hardening)';
    else
        b='(structural softining)';
    end
elseif (gama3==0)&&(beta3==0)
    a=' (Quadratic Nonlinearity)';
    if gama2>0
        b='(structural hardening)';
    else
        b='(structural softining)';
    end
else
    a=' (Combo Nonlinearity)';
    if (gama2>0)&&(gama3>0)
        b='(structural hardening)';
    else
        b='(structural softining)';
    end
```

```
end
%equilibrium equation:
if e>0

eqn=(gama3+pdynbar*beta3)*theta^3+(gama2+pdynbar*(beta
2+3*beta3*alpha))*theta^2+(1-pdynbar*(1-2*beta2*alpha-
3*beta3*alpha^2))*theta-pdynbar*alpha*(1-beta2*alpha-
beta3*alpha^2)==0;
elseif e<0
    eqn=(gama3-pdynbar*beta3)*theta^3+(gama2-
pdynbar*(beta2+3*beta3*alpha))*theta^2+(1+pdynbar*(1-
2*beta2*alpha-
3*beta3*alpha^2))*theta+pdynbar*alpha*(1-beta2*alpha-
beta3*alpha^2)==0;
end
%roots of equilibrium equation:
roots=solve(eqn,theta);
disp('Roots:')
% roots in terms of pdynbar:
etheta1=roots(1) % etheta1 means equilibrium theta1
etheta2=roots(2) % etheta2 means equilibrium theta2
if (gama3~=0)&&(beta3~=0)
    etheta3=roots(3) % etheta3 means equilibrium
theta3
end
%coefficient of del_theta=cdt
if e>0
    cdt=1+2*gama2*etheta+3*gama3*etheta^2-pdynbar*(1-
2*beta2*(alpha+etheta)-3*beta3*(alpha+etheta)^2);
elseif e<0
    cdt=1+2*gama2*etheta+3*gama3*etheta^2+pdynbar*(1-
2*beta2*(alpha+etheta)-3*beta3*(alpha+etheta)^2);
end
cdt1=subs(cdt,etheta,etheta1); %cdt1= coefficient of
del_theta at etheta1
cdt2=subs(cdt,etheta,etheta2); %cdt2= coefficient of
del_theta at etheta2
```

```
if (gama3~=0)&&(beta3~=0)
    cdt3=subs(cdt,etheta,etheta3); %cdt3= coefficient
of del_theta at etheta3
end
figure
subplot(2,2,[1 2])
fplot(pdynbar,etheta1,[0,10],'r','DisplayName','etheta
1')
hold on ; grid on
fplot(pdynbar,etheta2,[0,10],'g','DisplayName','etheta
2')
if (gama3~=0)&&(beta3~=0)
fplot(pdynbar,etheta3,[0,10],'b','DisplayName','etheta
3')
end
title(['Equilibrium Map at
alpha=',num2str(alpha),a,b])
xlabel('Pdynbar')
ylabel('Theta equilibrium')
legend
subplot(2,2,[3 4])
fplot(pdynbar,cdt1,[0,5],'r--','DisplayName','coeff.
at etheta1')
hold on; grid on
fplot(pdynbar,cdt2,[0,5],'g--','DisplayName','coeff.
at etheta2')
if (gama3~=0)&&(beta3~=0)
fplot(pdynbar,cdt3,[0,5],'b--','DisplayName','coeff.
at etheta3')
end
title(['Stability Map at alpha=',num2str(alpha)])
xlabel('Pdynbar')
ylabel('Coefficient of del theta')
legend
```