Syntax Analyser- Bottom Up Parsing

Bottom-Up Parsing

- A **bottom-up parser** creates the parse tree of the given input starting from leaves towards the root.
- A bottom-up parser tries to find the right-most derivation of the given input in the reverse order.

  $S \Rightarrow \ldots \Rightarrow \omega$ (the right-most derivation of ω)

  $\leftarrow$ (the bottom-up parser finds the right-most derivation in the reverse order)
- Bottom-up parsing is also known as **shift-reduce parsing** because its two main actions are shift and reduce.
  - At each shift action, the current symbol in the input string is pushed to a stack.
  - At each reduction step, the symbols at the top of the stack (this symbol sequence is the right side of a production) will replaced by the non-terminal at the left side of that production.
  - There are also two more actions: accept and error.

Shift-Reduce Parsing

- A shift-reduce parser tries to reduce the given input string into the starting symbol.

  a string $\xrightarrow[\text{reduced to}]{}$ the starting symbol

- At each reduction step, a substring of the input matching to the right side of a production rule is replaced by the non-terminal at the left side of that production rule.
- If the substring is chosen correctly, the right most derivation of that string is created in the reverse order.

Rightmost Derivation: $S \overset{*}{\underset{rm}{\Rightarrow}} \omega$

Shift-Reduce Parser finds: $\omega \underset{rm}{\Leftarrow} \ldots \underset{rm}{\Leftarrow} S$

Shift-Reduce Parsing -- Example

$S \rightarrow aABb$

$A \rightarrow aA \mid a$

$B \rightarrow bB \mid b$

input string: aaabb

aaAbb

aAbb ⇓ reduction

aABb

S

$$S \underset{rm}{\Rightarrow} aABb \underset{rm}{\Rightarrow} aAbb \underset{rm}{\Rightarrow} aaAbb \underset{rm}{\Rightarrow} aaabb$$

Right Sentential Forms

- How do we know which substring to be replaced at each reduction step?

Handle

- Informally, a **handle** of a string is a substring that matches the right side of a production rule.
  - But not every substring matches the right side of a production rule is handle

- A **handle** of a right sentential form γ ($\equiv \alpha\beta\omega$) is

  a production rule $A \rightarrow \beta$ and a position of γ

  where the string β may be found and replaced by A to produce

  the previous right-sentential form in a rightmost derivation of γ.

$$S \overset{*}{\underset{rm}{\Rightarrow}} \alpha A \omega \underset{rm}{\Rightarrow} \alpha\beta\omega$$

- If the grammar is unambiguous, then every right-sentential form of the grammar has exactly one handle.
- We will see that ω is a string of terminals.

Handle Pruning

- A right-most derivation in reverse can be obtained by **handle-pruning**.

$$S=\gamma_0 \underset{rm}{\Rightarrow} \gamma_1 \underset{rm}{\Rightarrow} \gamma_2 \underset{rm}{\Rightarrow} \cdots \underset{rm}{\Rightarrow} \gamma_{n-1} \underset{rm}{\Rightarrow} \gamma_n = \omega$$

input string

- Start from γ_n, find a handle $A_n \rightarrow \beta_n$ in γ_n, and replace β_n in by A_n to get γ_{n-1}.
- Then find a handle $A_{n-1} \rightarrow \beta_{n-1}$ in γ_{n-1}, and replace β_{n-1} in by A_{n-1} to get γ_{n-2}.
- Repeat this, until we reach S.

A Shift-Reduce Parser

$E \rightarrow E+T \mid T$

$T \rightarrow T*F \mid F$

$F \rightarrow (E) \mid id$

Right-Most Derivation of `id+id*id`

$$E \Rightarrow E+T \Rightarrow E+T*F \Rightarrow E+T*id \Rightarrow E+F*id$$
$$\Rightarrow E+id*id \Rightarrow T+id*id \Rightarrow F+id*id \Rightarrow id+id*id$$

Right-Most Sentential Form	Reducing Production
`id+id*id`	$F \rightarrow id$
`F+id*id`	$T \rightarrow F$
`T+id*id`	$E \rightarrow T$
`E+id*id`	$F \rightarrow id$
`E+F*id`	$T \rightarrow F$
`E+T*id`	$F \rightarrow id$
`E+T*F`	$T \rightarrow T*F$
`E+T`	$E \rightarrow E+T$
`E`	

`Handles` are red and underlined in the right-sentential forms.

A Stack Implementation of A Shift-Reduce Parser

- There are four possible actions of a shift-parser action:

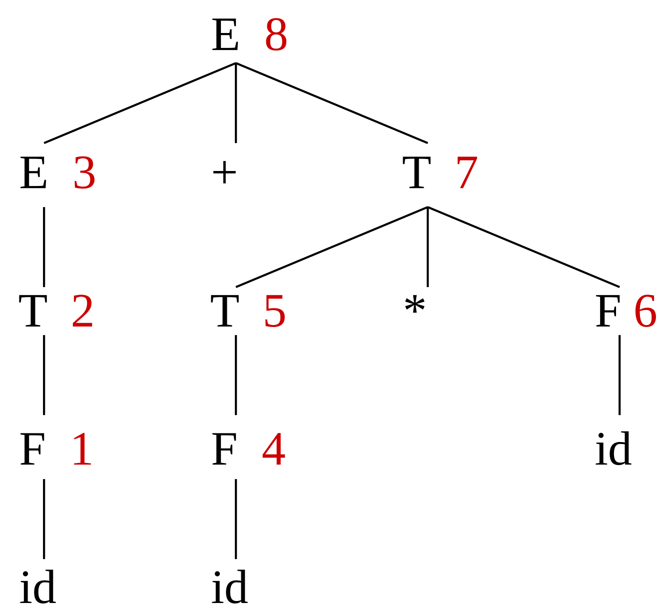
1. **Shift** : The next input symbol is shifted onto the top of the stack.
2. **Reduce**: Replace the handle on the top of the stack by the non-terminal.
3. **Accept**: Successful completion of parsing.
4. **Error**: Parser discovers a syntax error, and calls an error recovery routine.

- Initial stack just contains only the end-marker $.
- The end of the input string is marked by the end-marker $.

A Stack Implementation of A Shift-Reduce Parser

Stack	Input	Action
$	id+id*id$	shift
$id	+id*id$	reduce by F → id
$F	+id*id$	reduce by T → F
$T	+id*id$	reduce by E → T
$E	+id*id$	shift
$E+	id*id$	shift
$E+id	*id$	reduce by F → id
$E+F	*id$	reduce by T → F
$E+T	*id$	shift
$E+T*	id$	shift
$E+T*id	$	reduce by F → id
$E+T*F	$	reduce by T → T*F
$E+T	$	reduce by E → E+T
$E	$	accept

Parse Tree



Conflicts During Shift-Reduce Parsing

- There are context-free grammars for which shift-reduce parsers cannot be used.
- Stack contents and the next input symbol may not decide action:
  - **shift/reduce conflict**: Whether make a shift operation or a reduction.
  - **reduce/reduce conflict**: The parser cannot decide which of several reductions to make.
- If a shift-reduce parser cannot be used for a grammar, that grammar is called as non-LR(k) grammar.

left to right scanning

right-most derivation

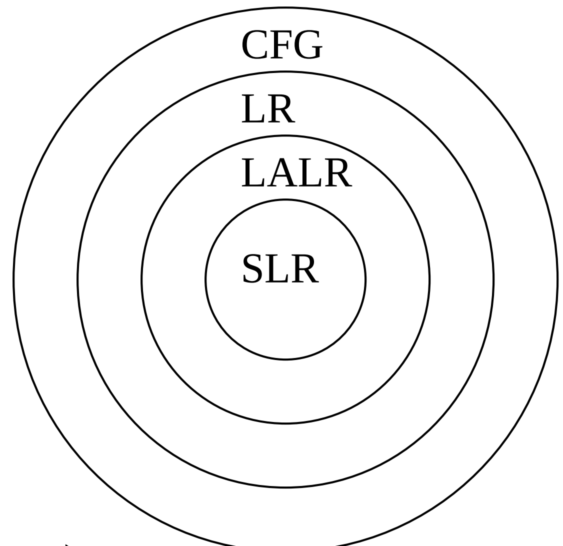
k lookhead

- An ambiguous grammar can never be a LR grammar.

Shift-Reduce Parsers

- There are two main categories of shift-reduce parsers

1. **Operator-Precedence Parser**
   - simple, but only a small class of grammars.



2. **LR-Parsers**
   - covers wide range of grammars.
     - SLR – simple LR parser
     - LR – most general LR parser
     - LALR – intermediate LR parser (lookhead LR parser)
   - SLR, LR and LALR work same, only their parsing tables are different.

LR Parsers

- The most powerful shift-reduce parsing (yet efficient) is:

LR(k) parsing.

left to right scanning | right-most derivation | k lookhead (k is omitted ➔ it is 1)

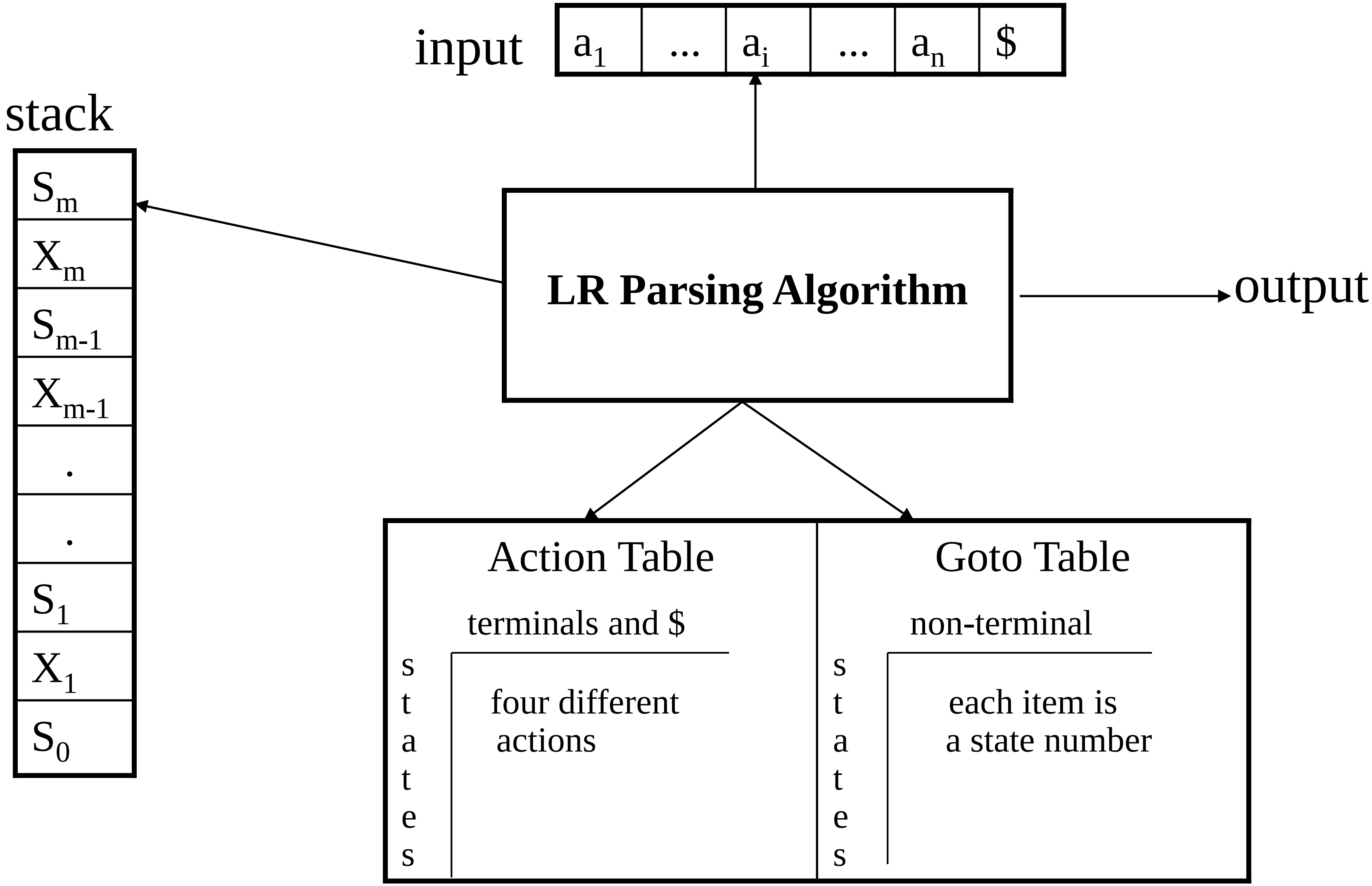
- LR parsing is attractive because:
  - LR parsing is most general non-backtracking shift-reduce parsing, yet it is still efficient.
  - The class of grammars that can be parsed using LR methods is a proper superset of the class of grammars that can be parsed with predictive parsers.

    LL(1)-Grammars $\subset$ LR(1)-Grammars
  - An LR-parser can detect a syntactic error as soon as it is possible to do so a left-to-right scan of the input.

LR Parsers

- **LR-Parsers**
  - covers wide range of grammars.
  - SLR – simple LR parser
  - LR – most general LR parser
  - LALR – intermediate LR parser (look-head LR parser)
  - SLR, LR and LALR work same (they used the same algorithm), only their parsing tables are different.

LR Parsing Algorithm



A Configuration of LR Parsing Algorithm

- A configuration of a LR parsing is:

$$(\underbrace{S_o\ X_1\ S_1\ \ldots\ X_m\ S_m}_{\text{Stack}},\ \underbrace{a_i\ a_{i+1}\ \ldots\ a_n\ \$}_{\text{Rest of Input}})$$

- S_m and a_i decides the parser action by consulting the parsing action table. (*Initial Stack* contains just S_o)

- A configuration of a LR parsing represents the right sentential form:

$$X_1\ \ldots\ X_m\ a_i\ a_{i+1}\ \ldots\ a_n\ \$$$

Actions of A LR-Parser

1. **shift s** -- shifts the next input symbol and the state **s** onto the stack

   $(S_o\ X_1\ S_1 \ldots X_m\ S_m,\ a_i\ a_{i+1} \ldots a_n\ \$) \Rightarrow (S_o\ X_1\ S_1 \ldots X_m\ S_m\ a_i\ s,\ a_{i+1} \ldots a_n\ \$)$

2. **reduce A→β** (or **`rn`** where n is a production number)
   - pop 2|**β**| (=r) items from the stack;
   - then push **A** and **s** where **s=goto[s_{m-r},A]**

   $(S_o\ X_1\ S_1 \ldots X_m\ S_m,\ a_i\ a_{i+1} \ldots a_n\ \$) \Rightarrow (S_o\ X_1\ S_1 \ldots X_{m-r}\ S_{m-r}\ A\ s,\ a_i \ldots a_n\ \$)$

   - Output is the reducing production reduce A→β

3. **Accept** – Parsing successfully completed

4. **Error** -- Parser detected an error (an empty entry in the action table)

Reduce Action

- pop 2|**β**| (=r) items from the stack; let us assume that **β** = $Y_1 Y_2 \ldots Y_r$
- then push **A** and **s** where **s=goto[s_{m-r},A]**

$$(S_o\ X_1\ S_1 \ldots X_{m-r}\ S_{m-r}\ Y_1\ S_{m-r} \ldots Y_r\ S_m,\ a_i\ a_{i+1} \ldots a_n\ \$)$$

$$\Rightarrow (S_o\ X_1\ S_1 \ldots X_{m-r}\ S_{m-r}\ A\ s,\ a_i \ldots a_n\ \$)$$

- In fact, $Y_1 Y_2 \ldots Y_r$ is a handle.

$$X_1 \ldots X_{m-r}\ A\ a_i \ldots a_n\ \$ \Rightarrow X_1 \ldots X_m\ Y_1 \ldots Y_r\ a_i\ a_{i+1} \ldots a_n\ \$$$

(SLR) Parsing Tables for Expression Grammar

1) $E \rightarrow E+T$
2) $E \rightarrow T$
3) $T \rightarrow T*F$
4) $T \rightarrow F$
5) $F \rightarrow (E)$
6) $F \rightarrow id$

Action Table | Goto Table

state	id	+	*	(	)	$		E	T	F
0	s5			s4				1	2	3
1		s6				acc				
2		r2	s7		r2	r2				
3		r4	r4		r4	r4				
4	s5			s4				8	2	3
5		r6	r6		r6	r6				
6	s5			s4					9	3
7	s5			s4						10
8		s6			s11					
9		r1	s7		r1	r1				
10		r3	r3		r3	r3				
11		r5	r5		r5	r5				

Actions of A (S)LR-Parser -- Example

stack	input	action	output
0	id*id+id$	shift 5	
0id5	*id+id$	reduce by $F \rightarrow id$	$F \rightarrow id$
0F3	*id+id$	reduce by $T \rightarrow F$	$T \rightarrow F$
0T2	*id+id$	shift 7	
0T2*7	id+id$	shift 5	
0T2*7id5	+id$	reduce by $F \rightarrow id$	$F \rightarrow id$
0T2*7F10	+id$	reduce by $T \rightarrow T*F$	$T \rightarrow T*F$
0T2	+id$	reduce by $E \rightarrow T$	$E \rightarrow T$
0E1	+id$	shift 6	
0E1+6	id$	shift 5	
0E1+6id5	$	reduce by $F \rightarrow id$	$F \rightarrow id$
0E1+6F3	$	reduce by $T \rightarrow F$	$T \rightarrow F$
0E1+6T9	$	reduce by $E \rightarrow E+T$	$E \rightarrow E+T$
0E1	$	accept	

Constructing SLR Parsing Tables – LR(0) Item

- An **LR(0) item** of a grammar G is a production of G a dot at the some position of the right side.
- Ex: $A \rightarrow aBb$ Possible LR(0) Items: $A \rightarrow \bullet aBb$

  (four different possibility) $A \rightarrow a \bullet Bb$

  $A \rightarrow aB \bullet b$

  $A \rightarrow aBb \bullet$

- Sets of LR(0) items will be the states of action and goto table of the SLR parser.
- A collection of sets of LR(0) items (**the canonical LR(0) collection**) is the basis for constructing SLR parsers.
- *Augmented Grammar*:

  G' is G with a new production rule $S' \rightarrow S$ where S' is the new starting symbol.

The Closure Operation

- If ***I*** is a set of LR(0) items for a grammar G, then ***closure(I)*** is the set of LR(0) items constructed from I by the two rules:

1. Initially, every LR(0) item in I is added to closure(I).
2. If $A \rightarrow \alpha \bullet B\beta$ is in closure(I) and $B \rightarrow \gamma$ is a production rule of G; then $B \rightarrow \bullet\gamma$ will be in the closure(I).
   We will apply this rule until no more new LR(0) items can be added to closure(I).

The Closure Operation -- Example

$E' \rightarrow E$
$E \rightarrow E+T$
$E \rightarrow T$
$T \rightarrow T*F$
$T \rightarrow F$
$F \rightarrow (E)$
$F \rightarrow id$

$closure(\{E' \rightarrow \bullet E\}) =$

$\{\ E' \rightarrow \bullet E$ ← kernel items
$E \rightarrow \bullet E+T$
$E \rightarrow \bullet T$
$T \rightarrow \bullet T*F$
$T \rightarrow \bullet F$
$F \rightarrow \bullet (E)$
$F \rightarrow \bullet id\ \}$

Goto Operation

- If I is a set of LR(0) items and X is a grammar symbol (terminal or non-terminal), then goto(I,X) is defined as follows:
  - If $A \to \alpha \bullet X\beta$ in I
    then every item in **closure({$A \to \alpha X \bullet \beta$})** will be in goto(I,X).

Example:

$I = \{ \quad E' \to \bullet E, \quad E \to \bullet E+T, \quad E \to \bullet T,$

$\qquad T \to \bullet T*F, \quad T \to \bullet F,$

$\qquad F \to \bullet (E), \quad F \to \bullet id \quad \}$

$goto(I,E) = \{ E' \to E \bullet, E \to E \bullet +T \}$

$goto(I,T) = \{ E \to T \bullet, T \to T \bullet *F \}$

$goto(I,F) = \{ T \to F \bullet \}$

$goto(I,() = \{ F \to (\bullet E), E \to \bullet E+T, E \to \bullet T, T \to \bullet T*F, T \to \bullet F,$

$\qquad F \to \bullet (E), F \to \bullet id \quad \}$

$goto(I,id) = \{ F \to id \bullet \}$

Construction of The Canonical LR(0) Collection

- To create the SLR parsing tables for a grammar G, we will create the canonical LR(0) collection of the grammar G'.

- ***Algorithm***:

  C is { closure({S'→ ● S}) }

  repeat the followings until no more set of LR(0) items can be added to ***C***.

      for each I in ***C*** and each grammar symbol X

          if goto(I,X) is not empty and not in ***C***

              add goto(I,X) to ***C***

- goto function is a DFA on the sets in C.

The Canonical LR(0) Collection -- Example

I_0: $E' \rightarrow .E$
$E \rightarrow .E+T$
$E \rightarrow .T$
$T \rightarrow .T*F$
$T \rightarrow .F$
$F \rightarrow .(E)$
$F \rightarrow .id$

I_1: $E' \rightarrow E.$
$E \rightarrow E.+T$

I_2: $E \rightarrow T.$
$T \rightarrow T.*F$

I_3: $T \rightarrow F.$

I_4: $F \rightarrow (.E)$
$E \rightarrow .E+T$
$E \rightarrow .T$
$T \rightarrow .T*F$
$T \rightarrow .F$
$F \rightarrow .(E)$
$F \rightarrow .id$

I_5: $F \rightarrow id.$

I_6: $E \rightarrow E+.T$
$T \rightarrow .T*F$
$T \rightarrow .F$
$F \rightarrow .(E)$
$F \rightarrow .id$

I_7: $T \rightarrow T*.F$
$F \rightarrow .(E)$
$F \rightarrow .id$

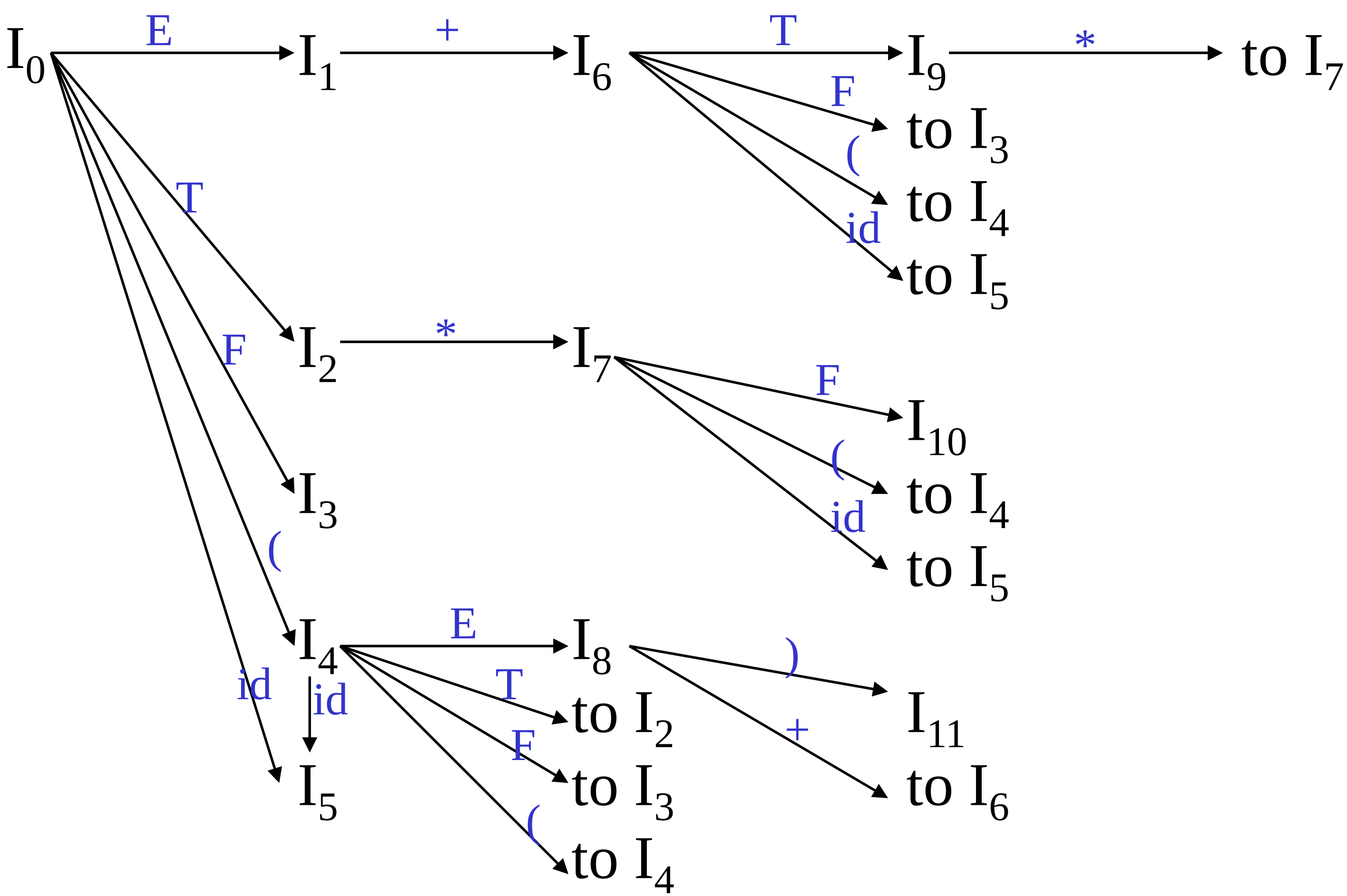
I_8: $F \rightarrow (E.)$
$E \rightarrow E.+T$

I_9: $E \rightarrow E+T.$
$T \rightarrow T.*F$

I_{10}: $T \rightarrow T*F.$

I_{11}: $F \rightarrow (E).$

Transition Diagram (DFA) of Goto Function



Constructing SLR Parsing Table

(of an augumented grammar G')

1. Construct the canonical collection of sets of LR(0) items for G'.

   $C \leftarrow \{I_0,...,I_n\}$

2. Create the parsing action table as follows
   - If a is a terminal, $A \rightarrow \alpha.a\beta$ in I_i and $goto(I_i,a)=I_j$ then action[i,a] is ***shift j.***
   - If $A \rightarrow \alpha.$ is in I_i, then action[i,a] is ***reduce*** $\boldsymbol{A \rightarrow \alpha}$ for all a in FOLLOW(A) where $A \neq S'$.
   - If $S' \rightarrow S.$ is in I_i, then action[i,\$] is ***accept***.
   - If any conflicting actions generated by these rules, the grammar is not SLR(1).

3. Create the parsing goto table
   - for all non-terminals A, if $goto(I_i,A)=I_j$ then goto[i,A]=j

4. All entries not defined by (2) and (3) are errors.

5. Initial state of the parser contains $S' \rightarrow .S$

Parsing Tables of Expression Grammar

	Action Table							Goto Table		
state	**id**	**+**	*****	**(**	**)**	**$**		**E**	**T**	**F**
0	s5			s4				1	2	3
1		s6				acc				
2		r2	s7		r2	r2				
3		r4	r4		r4	r4				
4	s5			s4				8	2	3
5		r6	r6		r6	r6				
6	s5			s4					9	3
7	s5			s4						10
8		s6			s11					
9		r1	s7		r1	r1				
10		r3	r3		r3	r3				
11		r5	r5		r5	r5				

SLR(1) Grammar

- An LR parser using SLR(1) parsing tables for a grammar G is called as the SLR(1) parser for G.
- If a grammar G has an SLR(1) parsing table, it is called SLR(1) grammar (or SLR grammar in short).
- Every SLR grammar is unambiguous, but every unambiguous grammar is not a SLR grammar.

shift/reduce and reduce/reduce conflicts

- If a state does not know whether it will make a shift operation or reduction for a terminal, we say that there is a **shift/reduce conflict**.

- If a state does not know whether it will make a reduction operation using the production rule `i` or `j` for a terminal, we say that there is a **reduce/reduce conflict**.

- If the SLR parsing table of a grammar G has a conflict, we say that that grammar is not SLR grammar.

Conflict Example

$S \rightarrow L=R$
$S \rightarrow R$
$L \rightarrow *R$
$L \rightarrow id$
$R \rightarrow L$

I_0: $S' \rightarrow .S$
$S \rightarrow .L=R$
$S \rightarrow .R$
$L \rightarrow .*R$
$L \rightarrow .id$
$R \rightarrow .L$

I_1: $S' \rightarrow S.$

I_2: $S \rightarrow L.=R$
$R \rightarrow L.$

I_3: $S \rightarrow R.$

I_4: $L \rightarrow *.R$
$R \rightarrow .L$
$L \rightarrow .*R$
$L \rightarrow .id$

I_5: $L \rightarrow id.$

I_6: $S \rightarrow L=.R$
$R \rightarrow .L$
$L \rightarrow .*R$
$L \rightarrow .id$

I_7: $L \rightarrow *R.$

I_8: $R \rightarrow L.$

I_9: $S \rightarrow L=R.$

Problem

FOLLOW(R)={=,$}

= → shift 6
= → reduce by $R \rightarrow L$

shift/reduce conflict

Conflict Example2

$S \rightarrow AaAb$
$S \rightarrow BbBa$
$A \rightarrow \varepsilon$
$B \rightarrow \varepsilon$

I_0: $S' \rightarrow .S$
$S \rightarrow .AaAb$
$S \rightarrow .BbBa$
$A \rightarrow .$
$B \rightarrow .$

Problem

FOLLOW(A)={a,b}

FOLLOW(B)={a,b}

a $\rightarrow$ reduce by $A \rightarrow \varepsilon$
$\searrow$ reduce by $B \rightarrow \varepsilon$

reduce/reduce conflict

b $\rightarrow$ reduce by $A \rightarrow \varepsilon$
$\searrow$ reduce by $B \rightarrow \varepsilon$

reduce/reduce conflict

Constructing Canonical LR(1) Parsing Tables

- In SLR method, the state i makes a reduction by $A \rightarrow \alpha$ when the current token is a:
  - if the $A \rightarrow \alpha \bullet$ in the I_i and a is FOLLOW(A)
- In some situations, βA cannot be followed by the terminal a in a right-sentential form when $\beta\alpha$ and the state i are on the top stack. This means that making reduction in this case is not correct.

$S \rightarrow AaAb$	$S \Rightarrow AaAb \Rightarrow Aab \Rightarrow ab$	$S \Rightarrow BbBa \Rightarrow Bba \Rightarrow ba$
$S \rightarrow BbBa$		
$A \rightarrow \varepsilon$	$Aab \Rightarrow \varepsilon\, ab$	$Bba \Rightarrow \varepsilon\, ba$
$B \rightarrow \varepsilon$	$AaAb \Rightarrow Aa\, \varepsilon\, b$	$BbBa \Rightarrow Bb\, \varepsilon\, a$

LR(1) Item

- To avoid some of invalid reductions, the states need to carry more information.
- Extra information is put into a state by including a terminal symbol as a second component in an item.

- A LR(1) item is:

$A \rightarrow \alpha \bullet \beta, a$ where **a** is the look-head of the LR(1) item

(**a** is a terminal or end-marker.)

LR(1) Item (cont.)

- When β (in the LR(1) item $A \to \alpha \bullet \beta, a$) is not empty, the look-head does not have any affect.
- When β is empty ($A \to \alpha \bullet, a$), we do the reduction by $A \to \alpha$ only if the next input symbol is **a** (not for any terminal in FOLLOW(A)).

- A state will contain $A \to \alpha \bullet, a_1$ where $\{a_1,...,a_n\} \subseteq$ FOLLOW(A)

$$\begin{array}{l} A \to \alpha \bullet, a_1 \\ \quad ... \\ A \to \alpha \bullet, a_n \end{array}$$

Canonical Collection of Sets of LR(1) Items

- The construction of the canonical collection of the sets of LR(1) items are similar to the construction of the canonical collection of the sets of LR(0) items, except that *closure* and *goto* operations work a little bit different.

closure(I) is: (where I is a set of LR(1) items)

- every LR(1) item in I is in closure(I)
- if $A \rightarrow \alpha \bullet B\beta, a$ in closure(I) and $B \rightarrow \gamma$ is a production rule of G; then $B \rightarrow .\gamma, b$ will be in the closure(I) for each terminal b in FIRST(βa) .

goto operation

- If I is a set of LR(1) items and X is a grammar symbol (terminal or non-terminal), then goto(I,X) is defined as follows:
  - If $A \rightarrow \alpha.X\beta,a$ in I
    then every item in **closure($\{A \rightarrow \alpha X.\beta,a\}$)** will be in goto(I,X).

Construction of The Canonical LR(1) Collection

- ***Algorithm***:

  C is { closure({S'→.S,$}) }

  repeat the followings until no more set of LR(1) items can be added to ***C***.

      for each I in ***C*** and each grammar symbol X

          if goto(I,X) is not empty and not in ***C***

              add goto(I,X) to ***C***

- goto function is a DFA on the sets in C.

A Short Notation for The Sets of LR(1) Items

- A set of LR(1) items containing the following items

$$A \rightarrow \alpha \bullet \beta, a_1$$

...

$$A \rightarrow \alpha \bullet \beta, a_n$$

can be written as

$$A \rightarrow \alpha \bullet \beta, a_1/a_2/\ldots/a_n$$

Canonical LR(1) Collection -- Example

$S \to AaAb$
$S \to BbBa$
$A \to \varepsilon$
$B \to \varepsilon$

I_0: $S' \to .S\ ,\$$
$S \to .AaAb\ ,\$$
$S \to .BbBa\ ,\$$
$A \to .\ ,a$
$B \to .\ ,b$

$I_0 \xrightarrow{S} I_1$: $S' \to S.\ ,\$$

$I_0 \xrightarrow{A} I_2$: $S \to A.aAb\ ,\$$ $\xrightarrow{a}$ to I_4

$I_0 \xrightarrow{B} I_3$: $S \to B.bBa\ ,\$$ $\xrightarrow{b}$ to I_5

I_4: $S \to Aa.Ab\ ,\$$
$A \to .\ ,b$
$\xrightarrow{A}$ I_6: $S \to AaA.b\ ,\$$ $\xrightarrow{a}$ I_8: $S \to AaAb.\ ,\$$

I_5: $S \to Bb.Ba\ ,\$$
$B \to .\ ,a$
$\xrightarrow{B}$ I_7: $S \to BbB.a\ ,\$$ $\xrightarrow{b}$ I_9: $S \to BbBa.\ ,\$$

Canonical LR(1) Collection – Example2

$S' \to S$
1) $S \to L=R$
2) $S \to R$
3) $L \to *R$
4) $L \to id$
5) $R \to L$

I_0: $S' \to .S,\$$
$S \to .L=R,\$$
$S \to .R,\$$
$L \to .*R,\$/=$
$L \to .id,\$/=$
$R \to .L,\$$

$I_0 \xrightarrow{S} I_1$: $S' \to S.,\$$

$I_0 \xrightarrow{L} I_2$: $S \to L.=R,\$$ $\longrightarrow$ to I_6
$R \to L.,\$$

$I_0 \xrightarrow{R} I_3$: $S \to R.,\$$

$I_0 \xrightarrow{*} I_4$: $L \to *.R,\$/=$
$R \to .L,\$/=$
$L \to .*R,\$/=$
$L \to .id,\$/=$

$\xrightarrow{R}$ to I_7
$\xrightarrow{L}$ to I_8
$\xrightarrow{*}$ to I_4
$\xrightarrow{id}$ to I_5

$I_0 \xrightarrow{id} I_5$: $L \to id.,\$/=$

I_6: $S \to L=.R,\$$
$R \to .L,\$$
$L \to .*R,\$$
$L \to .id,\$$

$\xrightarrow{R}$ to I_9
$\xrightarrow{L}$ to I_{10}
$\xrightarrow{*}$ to I_{11}
$\xrightarrow{id}$ to I_{12}

I_7: $L \to *R.,\$/=$

I_8: $R \to L.,\$/=$

I_9: $S \to L=R.,\$$

I_{10}: $R \to L.,\$$

I_{11}: $L \to *.R,\$$
$R \to .L,\$$
$L \to .*R,\$$
$L \to .id,\$$

$\xrightarrow{R}$ to I_{13}
$\xrightarrow{L}$ to I_{10}
$\xrightarrow{*}$ to I_{11}
$\xrightarrow{id}$ to I_{12}

I_{12}: $L \to id.,\$$

I_{13}: $L \to *R.,\$$

I_4 and I_{11}

I_5 and I_{12}

I_7 and I_{13}

I_8 and I_{10}

Construction of LR(1) Parsing Tables

1. Construct the canonical collection of sets of LR(1) items for G'.

   $C \leftarrow \{I_0,...,I_n\}$

2. Create the parsing action table as follows
   - If a is a terminal, $A \rightarrow \alpha \bullet a\beta,b$ in I_i and $goto(I_i,a)=I_j$ then action[i,a] is ***shift j***.
   - If $A \rightarrow \alpha \bullet ,a$ is in I_i , then action[i,a] is ***reduce*** $\boldsymbol{A \rightarrow \alpha}$ where $A \neq S'$.
   - If $S' \rightarrow S \bullet ,\$$ is in I_i , then action[i,$] is ***accept***.
   - If any conflicting actions generated by these rules, the grammar is not LR(1).

3. Create the parsing goto table
   - for all non-terminals A, if $goto(I_i,A)=I_j$ then goto[i,A]=j

4. All entries not defined by (2) and (3) are errors.

5. Initial state of the parser contains $S' \rightarrow .S,\$$

LR(1) Parsing Tables – (for Example2)

	id	*	=	$	S	L	R
0	s5	s4			1	2	3
1				acc			
2			s6	r5			
3				r2			
4	s5	s4				8	7
5			r4	r4			
6	s12	s11				10	9
7			r3	r3			
8			r5	r5			
9				r1			
10				r5			
11	s12	s11				10	13
12				r4			
13				r3			

no shift/reduce or
no reduce/reduce conflict

$\Downarrow$

so, it is a LR(1) grammar

LALR Parsing Tables

- **LALR** stands for **LookAhead LR.**

- LALR parsers are often used in practice because LALR parsing tables are smaller than LR(1) parsing tables.
- The number of states in SLR and LALR parsing tables for a grammar G are equal.
- But LALR parsers recognize more grammars than SLR parsers.
- ***yacc*** creates a LALR parser for the given grammar.
- A state of LALR parser will be again a set of LR(1) items.

Creating LALR Parsing Tables

Canonical LR(1) Parser $\rightarrow$ LALR Parser

shrink # of states

- This shrink process may introduce a **reduce/reduce** conflict in the resulting LALR parser (so the grammar is NOT LALR)
- But, this shrink process does not produce a **shift/reduce** conflict.

The Core of A Set of LR(1) Items

- The core of a set of LR(1) items is the set of its first component.

Ex: $S \rightarrow L \bullet =R,\$$ ➔ $S \rightarrow L \bullet =R$ ← Core

$R \rightarrow L \bullet ,\$$ $R \rightarrow L \bullet$

- We will find the states (sets of LR(1) items) in a canonical LR(1) parser with same cores. Then we will merge them as a single state.

$I_1 : L \rightarrow id \bullet ,=$ A new state: $I_{12}: L \rightarrow id \bullet ,=$

➔ $L \rightarrow id \bullet ,\$$

$I_2 : L \rightarrow id \bullet ,\$$ have same core, merge them

- We will do this for all states of a canonical LR(1) parser to get the states of the LALR parser.
- In fact, the number of the states of the LALR parser for a grammar will be equal to the number of states of the SLR parser for that grammar.

Creation of LALR Parsing Tables

- Create the canonical LR(1) collection of the sets of LR(1) items for the given grammar.
- Find each core; find all sets having that same core; replace those sets having same cores with a single set which is their union.

  $C=\{I_0,...,I_n\} \Rightarrow C'=\{J_1,...,J_m\}$ where $m \leq n$
- Create the parsing tables (action and goto tables) same as the construction of the parsing tables of LR(1) parser.
  - Note that: If $J=I_1 \cup ... \cup I_k$ since $I_1,...,I_k$ have same cores

    $\Rightarrow$ cores of $goto(I_1,X),...,goto(I_2,X)$ must be same.
  - So, $goto(J,X)=K$ where K is the union of all sets of items having same cores as $goto(I_1,X)$.

- If no conflict is introduced, the grammar is LALR(1) grammar.
  (We may only introduce reduce/reduce conflicts; we cannot introduce a shift/reduce conflict)

Shift/Reduce Conflict

- We say that we cannot introduce a shift/reduce conflict during the shrink process for the creation of the states of a LALR parser.
- Assume that we can introduce a shift/reduce conflict. In this case, a state of LALR parser must have:

$$A \to \alpha \bullet ,a \qquad \text{and} \qquad B \to \beta \bullet a\gamma ,b$$

- This means that a state of the canonical LR(1) parser must have:

$$A \to \alpha \bullet ,a \qquad \text{and} \qquad B \to \beta \bullet a\gamma ,c$$

But, this state has also a shift/reduce conflict. i.e. The original canonical LR(1) parser has a conflict.

(Reason for this, the shift operation does not depend on lookaheads)

Reduce/Reduce Conflict

- But, we may introduce a reduce/reduce conflict during the shrink process for the creation of the states of a LALR parser.

$$I_1 : A \rightarrow \alpha \bullet ,a \qquad\qquad I_2 : A \rightarrow \alpha \bullet ,b$$

$$B \rightarrow \beta \bullet ,b \qquad\qquad B \rightarrow \beta \bullet ,c$$

$$\Downarrow$$

$$I_{12} : A \rightarrow \alpha \bullet ,a/b$$

$$B \rightarrow \beta \bullet ,b/c$$

➔ reduce/reduce conflict

Canonical LALR(1) Collection – Example2

$S' \rightarrow S$
1) $S \rightarrow L=R$
2) $S \rightarrow R$
3) $L \rightarrow *R$
4) $L \rightarrow id$
5) $R \rightarrow L$

$I_0: S' \rightarrow \bullet S,\$$
$S \rightarrow \bullet L=R,\$$
$S \rightarrow \bullet R,\$$
$L \rightarrow \bullet *R,\$/=$
$L \rightarrow \bullet id,\$/=$
$R \rightarrow \bullet L,\$$

S: $I_1: S' \rightarrow S \bullet,\$$

L: $I_2: S \rightarrow L \bullet =R,\$$ → to I_6
$R \rightarrow L \bullet,\$$

R: $I_3: S \rightarrow R \bullet,\$$

*: $I_{411}: L \rightarrow * \bullet R,\$/=$
$R \rightarrow \bullet L,\$/=$
$L \rightarrow \bullet *R,\$/=$
$L \rightarrow \bullet id,\$/=$

R: to I_{713}
L: to I_{810}
*: to I_{411}
id: to I_{512}

id: $I_{512}: L \rightarrow id \bullet,\$/=$

$I_6: S \rightarrow L= \bullet R,\$$
$R \rightarrow \bullet L,\$$
$L \rightarrow \bullet *R,\$$
$L \rightarrow \bullet id,\$$

R: to I_9
L: to I_{810}
*: to I_{411}
id: to I_{512}

$I_9: S \rightarrow L=R \bullet,\$$

$I_{713}: L \rightarrow *R \bullet,\$/=$

$I_{810}: R \rightarrow L \bullet,\$/=$

Same Cores
I_4 and I_{11}

I_5 and I_{12}

I_7 and I_{13}

I_8 and I_{10}

LALR(1) Parsing Tables – (for Example2)

	id	*	=	$	S	L	R
0	s5	s4			1	2	3
1				acc			
2			s6	r5			
3				r2			
4	s5	s4				8	7
5			r4	r4			
6	s12	s11				10	9
7			r3	r3			
8			r5	r5			
9				r1			

no shift/reduce or
no reduce/reduce conflict

$\Downarrow$

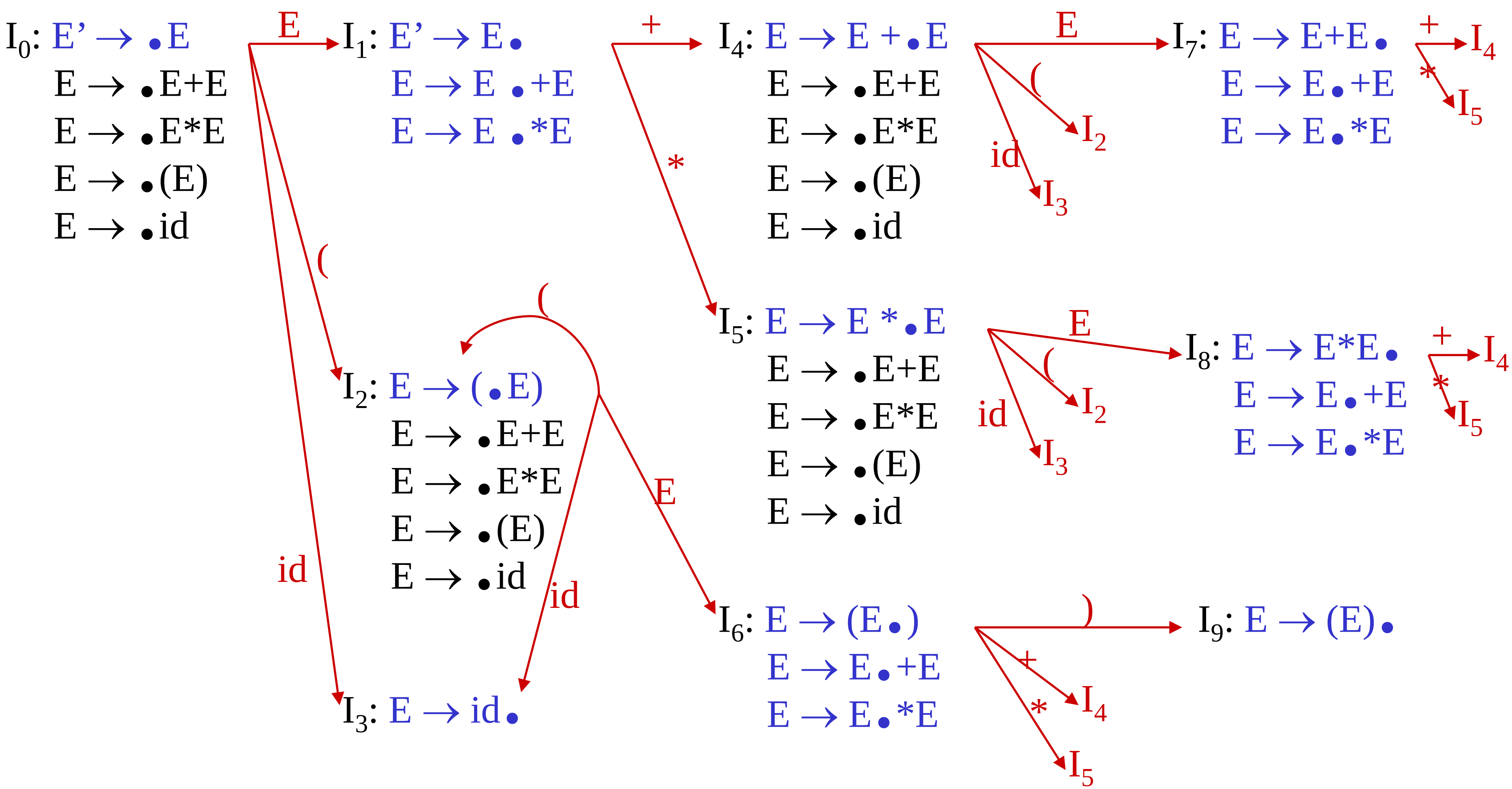
so, it is a LALR(1) grammar

Using Ambiguous Grammars

- All grammars used in the construction of LR-parsing tables must be un-ambiguous.
- Can we create LR-parsing tables for ambiguous grammars ?
  - Yes, but they will have conflicts.
  - We can resolve these conflicts in favor of one of them to disambiguate the grammar.
  - At the end, we will have again an unambiguous grammar.
- Why we want to use an ambiguous grammar?
  - Some of the ambiguous grammars are **much natural**, and a corresponding unambiguous grammar can be very complex.
  - Usage of an ambiguous grammar may **eliminate unnecessary reductions**.
- Ex.

$$E \rightarrow E+E \mid E*E \mid (E) \mid id \quad \Rightarrow \quad \begin{aligned} E &\rightarrow E+T \mid T \\ T &\rightarrow T*F \mid F \\ F &\rightarrow (E) \mid id \end{aligned}$$

Sets of LR(0) Items for Ambiguous Grammar



SLR-Parsing Tables for Ambiguous Grammar

FOLLOW(E) = { $, +, *,) }

State I_7 has shift/reduce conflicts for symbols + and *.

$$I_0 \xrightarrow{E} I_1 \xrightarrow{+} I_4 \xrightarrow{E} I_7$$

when current token is +
- shift ➔ + is right-associative
- reduce ➔ + is left-associative

when current token is *
- shift ➔ * has higher precedence than +
- reduce ➔ + has higher precedence than *

SLR-Parsing Tables for Ambiguous Grammar

FOLLOW(E) = { $, +, *,) }

State I_8 has shift/reduce conflicts for symbols + and *.

$$I_0 \xrightarrow{E} I_1 \xrightarrow{*} I_5 \xrightarrow{E} I_7$$

when current token is *
- shift ➔ * is right-associative
- reduce ➔ * is left-associative

when current token is +
- shift ➔ + has higher precedence than *
- reduce ➔ * has higher precedence than +

SLR-Parsing Tables for Ambiguous Grammar

	Action							Goto
	id	**+**	*****	**(**	**)**	**$**		**E**
0	s3			s2				1
1		s4	s5			acc		
2	s3			s2				6
3		r4	r4		r4	r4		
4	s3			s2				7
5	s3			s2				8
6		s4	s5		s9			
7		r1	s5		r1	r1		
8		r2	r2		r2	r2		
9		r3	r3		r3	r3		

Error Recovery in LR Parsing

- An LR parser will detect an error when it consults the parsing action table and finds an error entry. All empty entries in the action table are error entries.
- Errors are never detected by consulting the goto table.
- An LR parser will announce error as soon as there is no valid continuation for the scanned portion of the input.
- A canonical LR parser (LR(1) parser) will never make even a single reduction before announcing an error.
- The SLR and LALR parsers may make several reductions before announcing an error.
- But, all LR parsers (LR(1), LALR and SLR parsers) will never shift an erroneous input symbol onto the stack.

Panic Mode Error Recovery in LR Parsing

- Scan down the stack until a state **s** with a goto on a particular nonterminal **A** is found. (Get rid of everything from the stack before this state s).
- Discard zero or more input symbols until a symbol **a** is found that can legitimately follow A.
  - The symbol a is simply in FOLLOW(A), but this may not work for all situations.
- The parser stacks the nonterminal **A** and the state **goto[s,A]**, and it resumes the normal parsing.
- This nonterminal A is normally is a basic programming block (there can be more than one choice for A).
  - stmt, expr, block, ...

Phrase-Level Error Recovery in LR Parsing

- Each empty entry in the action table is marked with a specific error routine.
- An error routine reflects the error that the user most likely will make in that case.
- An error routine inserts the symbols into the stack or the input (or it deletes the symbols from the stack and the input, or it can do both insertion and deletion).
  - missing operand
  - unbalanced right parenthesis