Results and Discussion

Analysis of plots

The behavior of the temperature distribution with respect to time can be observed clearly in the plots for each grid resolution. The scheme is consistent and stable in the range of grid resolution taken and hence gives plots with consistent and stable values. It can be observed that as time increases, the diffusion term becomes dominant nearer the centre and temperature field rises up. But nearer the walls, the conduction term is dominant and the temperature stays close to the boundary values. This can be also seen when observing the temperature distribution along x=3pi/4 and y=pi/4. As the time increases, it can also be seen that temperature distribution plots tends to converge to a steady state value.

Error Analysis

AB2 Method

The code was run for grid sizes of 10, 20 40, 80 and dt=0.0005. The scheme was stable and the following L2 norms of errors were obtained assuming 80 to be the exact solution.
E10 = 0.006535246586311
E20 = 0.002102308949674
E40 = 5.240315064309386e-04

The grid resolution was further improved to 20, 40, 80, 160 and the code was run for the same dt. But the solution is unstable for N=160. The dt value was further reduced until stable result was obtained at dt= 0.0001. Following errors obtained assuming 160 to be exact solution
E20 = 0.002218985351091
E40 = 6.686055290228662e-04
E80 = 2.004299242970253e-04

RK4

The code was run for grid sizes of 10, 20 40, 80 and dt=0.0005. The scheme was stable and the following L2 norms of errors were obtained assuming 80 to be the exact solution.
E10 = 0.006535246702314
E20 = 0.002102309003005
E40 = 5.240315191225178e-04

The grid resolution was further improved to 20, 40, 80, 160 and the code was run for the same dt. Solution is found to be stable for N=160. Following errors obtained assuming 160 to be exact solution
E20 = 0.002218986848329
E40 = 6.686057242849385e-04
E80 = 2.004299314599854e-04

Error is becoming of the order of 10^{-4} at higher grid resolution for both the methods. This means solution is converging and it is becoming independent of the grid. It is also seen that RK4 gives

stable solution at higher *dt* values due to high stability region. It is also computationally less demanding than AB2 for similar accuracy. AB2 requires smaller time steps for stability and hence is time consuming and resource intensive. The order of both the schemes when the higher grid resolution of 160 is considered comes to about 1.78. This is the true order of accuracy of the schemes when the parameters are chosen as given here.

For both the methods used here, the stability region is small and hence the computational power required to get fast solutions are very high. This is because very low values of *dt* are required to keep the scheme stable. Hence a method with higher stability region would enable the use of larger *dt* values and hence gives a faster solution. Hence implicit schemes like Crank Nicolson can be used for time integration so as to improve the speed for same accuracy requirement.