# PROBLEM No.19

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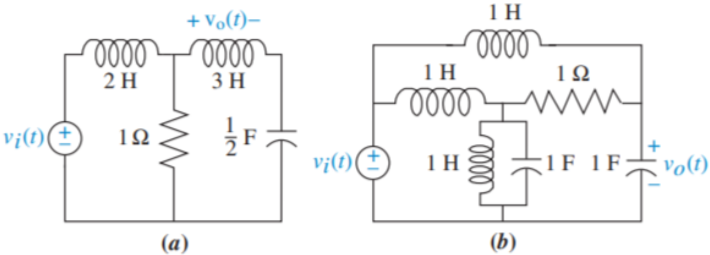
# Question

Find the transfer function, $G(s) = \frac{V_o(s)}{V_i(s)}$ for each of the figures shown here using Nodal equations.



# Solution (a)

a)The nodal equations we get are

$$\frac{V_R(s) - V_i(s)}{2s} + \frac{V_R(s)}{1} + \frac{V_R(s) - V_c(s)}{3s} = 0 \tag{0.1}$$

$$-\frac{1}{3s} V_R(s) + \left(\frac{1}{2}s + \frac{1}{3s}\right) V_C(s) = 0 \tag{0.2}$$

Rewriting and simplifying,

$$\frac{6s+5}{6s} V_R(s) - \frac{1}{3s} V_C(s) = \frac{1}{2s} V_i(s) \tag{0.3}$$

$$-\frac{1}{3s} V_R(s) + \left(\frac{3s^2+2}{6s}\right) V_c(s) = 0 \tag{0.4}$$

Solving for $V_R(s)$ and $V_C(s)$

$$V_R(s)=\frac{\begin{vmatrix}\frac{1}{2s}V_i(s) & -\frac{1}{3s}\\ 0 & \frac{3s^2+2}{6s}\end{vmatrix}}{\begin{vmatrix}\frac{6s+5}{6s} & -\frac{1}{3s}\\ -\frac{1}{3s} & \frac{3s^2+2}{6s}\end{vmatrix}};\ V_C(s)=\frac{\begin{vmatrix}\frac{6s+5}{6s} & \frac{1}{2s}V_i(s)\\ -\frac{1}{3s} & 0\end{vmatrix}}{\begin{vmatrix}\frac{6s+5}{6s} & -\frac{1}{3s}\\ -\frac{1}{3s} & \frac{3s^2+2}{6s}\end{vmatrix}}$$

$$G(s)=\frac{V_o(s)}{V_i(s)}=\frac{V_R(s)-V_C(s)}{V_i(s)}=\frac{3s^2}{6s^3+5s^2+4s+2} \tag{0.5}$$

# Solution (b)

The nodal equations we get are

$$\frac{(V_1(s) - V_i(s))}{s} + \frac{\left(s^2+1\right)}{s} V_1(s) + (V_1(s) - V_o(s)) = 0 \tag{0.6}$$

$$(V_o(s) - V_1(s)) + sV_o(s) + \frac{(V_o(s) - V_i(s))}{s} = 0 \tag{0.7}$$

Simplified equations are

$$\left(s + \frac{2}{s} + 1\right) V_1(s) - V_o(s) = \frac{1}{s} V_i(s) \tag{0.8}$$

$$V_1(s) + \left(s + \frac{1}{s} + 1\right) V_o(s) = \frac{1}{s} V_i(s) \tag{0.9}$$

Solving for $V_o(s)$

$$\mathrm{V_o(s)} = \frac{\left(s^2+2s+2\right)}{s^4+2s^3+3s^2+3s+2} V_i(s)$$

Hence,

$$\frac{V_o(s)}{V_i(s)} = \frac{\left(s^2+2s+2\right)}{s^4+2s^3+3s^2+3s+2}$$