CS251 Fall 2025

(cs251.stanford.edu)

Consensus in the Internet Setting

Recap of the Last Lecture

- Byzantine Generals Problem and Byzantine Broadcast
  - Required security properties: agreement, validity, termination
- Definition of Byzantine adversary
  - **Byzantine:** Adversarial nodes can deviate from the protocol arbitrarily
- Synchronous and asynchronous networks
  - **Synchronous network:** known upper bound Δ on max network delay
- The Dolev-Strong protocol (1983):
  - Assumes a PKI. Runs in $f + 1$ rounds for any $f < n$.
- State Machine Replication (SMR): continuous stream of transactions.
  - Required security properties: Safety and Liveness

Sybil Attack

How to select the nodes that participate in consensus?

Two variants:

- *Permissioned:* There is a *fixed* set of nodes (previous lecture – historic consensus).
- *Permissionless*: Anyone is free to join the protocol at any time -- Bitcoin.

Can we accept any node that has a signing key to participate in consensus?

How can an attacker maximize its disruptive power?

Sybil Attack!

Sybil Attack

How to select the nodes that participate in consensus?

Two variants:

- *Permissioned:* There is a *fixed* set of nodes (previous lecture).
- *Permissionless*: Anyone is free to join the protocol at any time.

In a **sybil attack**, a single adversary impersonates many different nodes, outnumbering the honest nodes and potentially disrupting consensus.

Sybil Resistance

Consensus protocols with Sybil resistance are based on a bounded resource:

	Resource dedicated to the protocol	Some Example Blockchains
Proof-of-Work	Total computational power	Bitcoin
Proof-of-Stake	Total number of coins	PoS Ethereum, Cardano, Cosmos, …
Proof-of-Space/Time	Total storage across time	Chia, Filecoin, …

How does Proof-of-Work prevent Sybil attacks?

We assume that the adversary controls a small fraction of the scarce resource

⇒ Adversary has less influence than the totality of honest nodes.

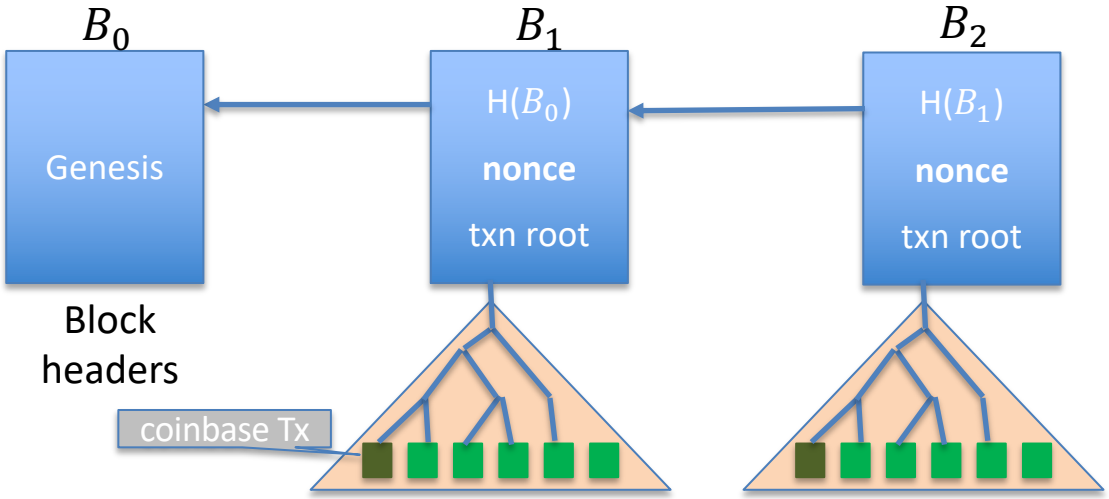
Bitcoin: Mining

To mine a new block, a miner must find *nonce* such that

$$H(h_{prev}, \text{txn root}, \text{nonce}) < \text{Target} = {}^{2^{256}}/_{D} \qquad (D = \text{difficulty})$$

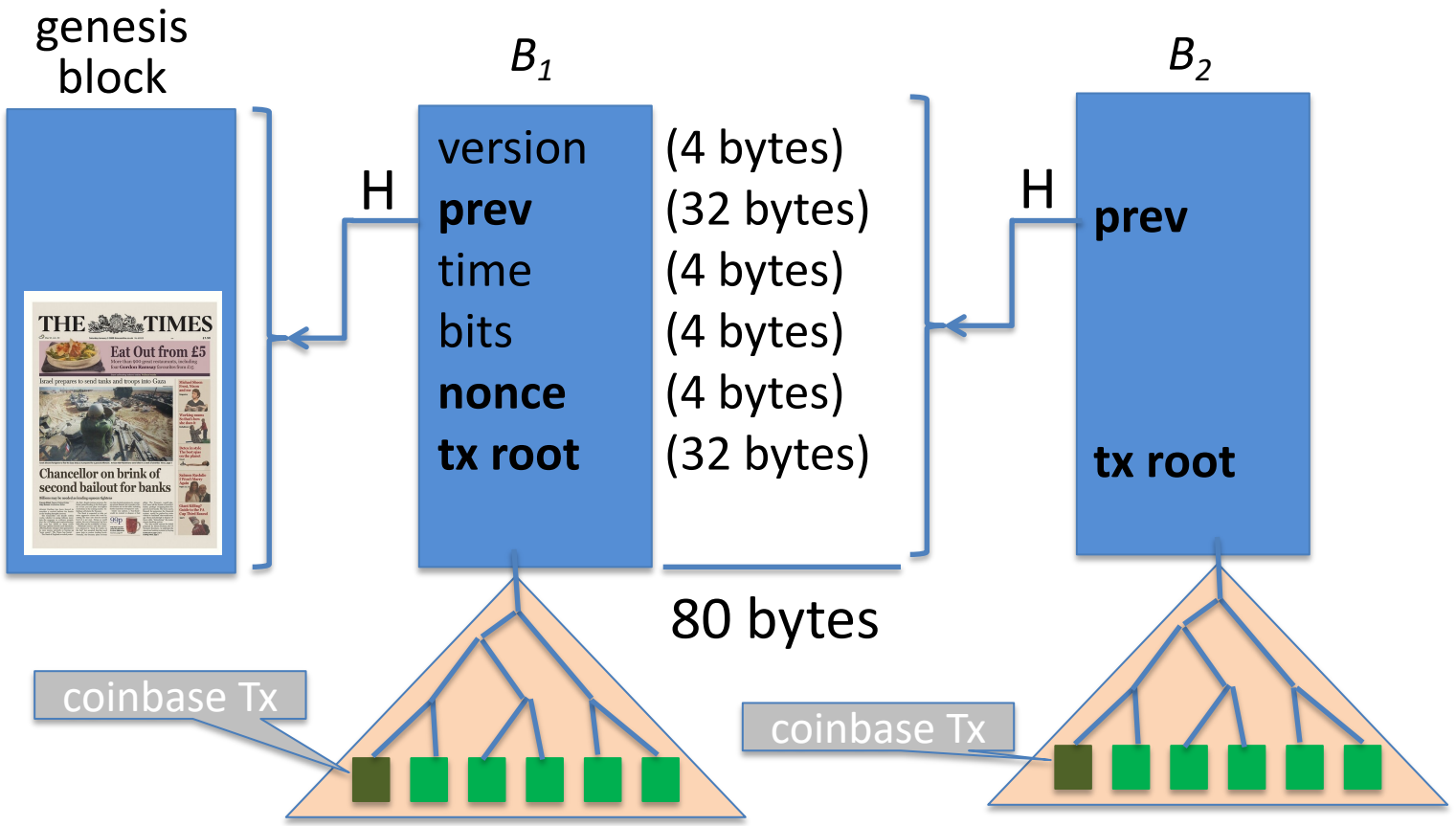
Every miner tries random nonces until a miner finds a nonce that satisfies the above equation.

- Expected total work is D hashes. Bitcoin uses $H(x) = SHA256(SHA256(x))$.

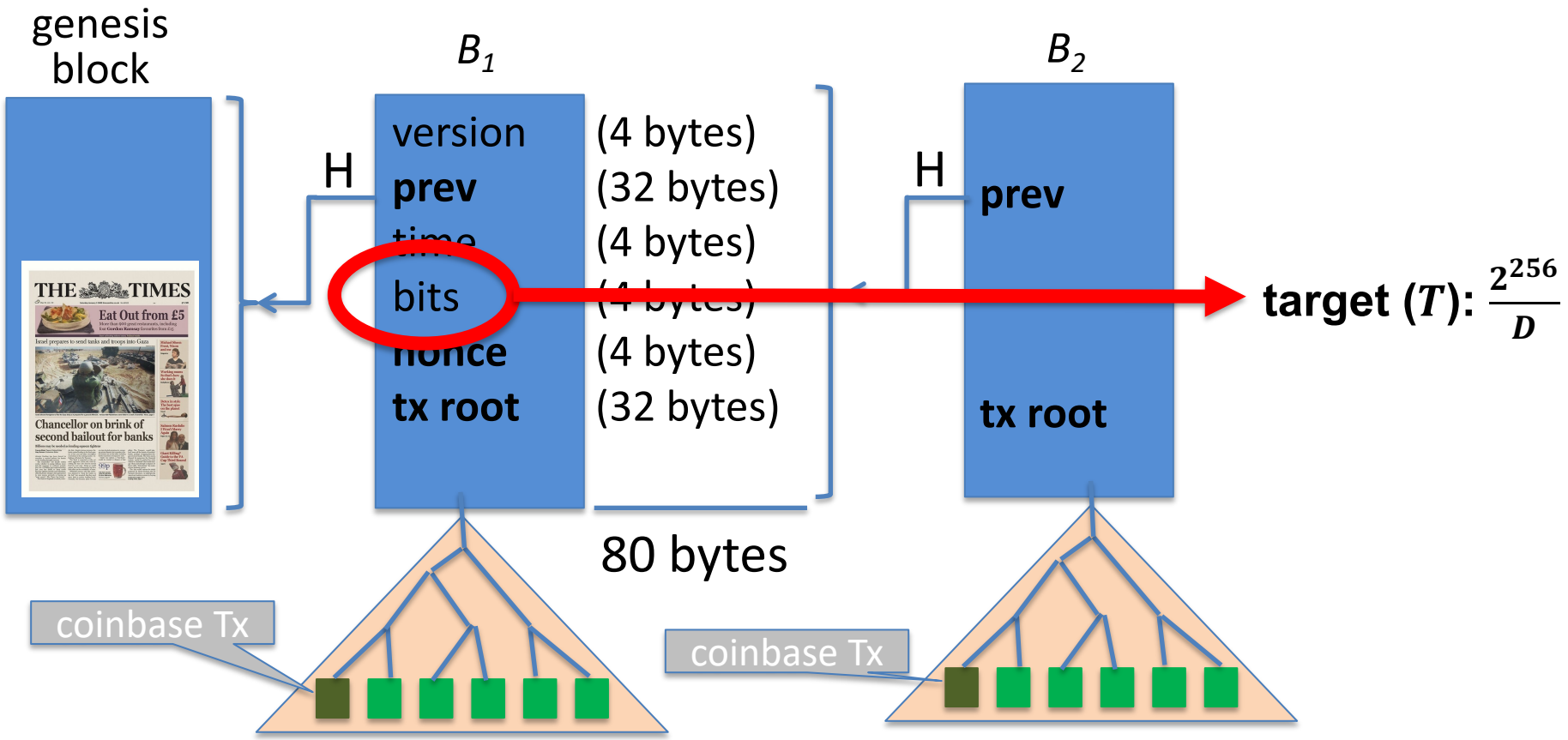


New block: a random process but one every ≈ 10 minutes

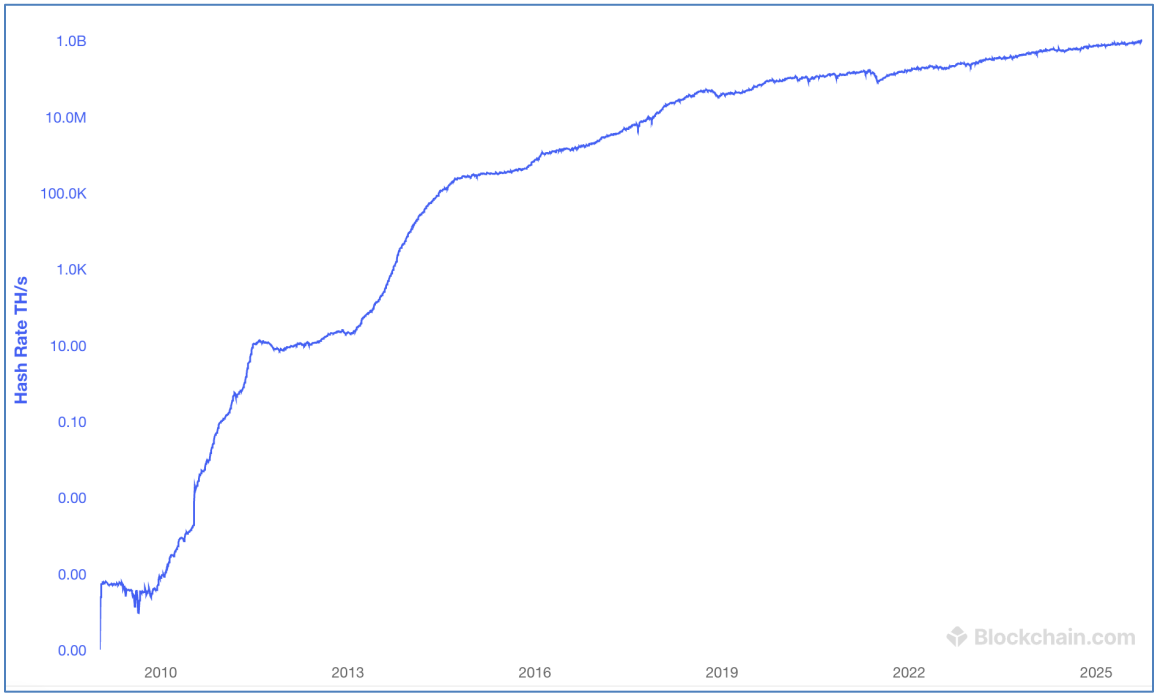
Bitcoin: Mining



Bitcoin: Mining



Bitcoin hash rate over time



Sep. 2025:
10^{19} hash/sec.

MacBook: 15M/sec

ANTMINER

- SHA256 Hydro-cooling Miner
- 5.5 kW, 580 TeraHash/sec, 13.5 Kg

Bitcoin mining data center

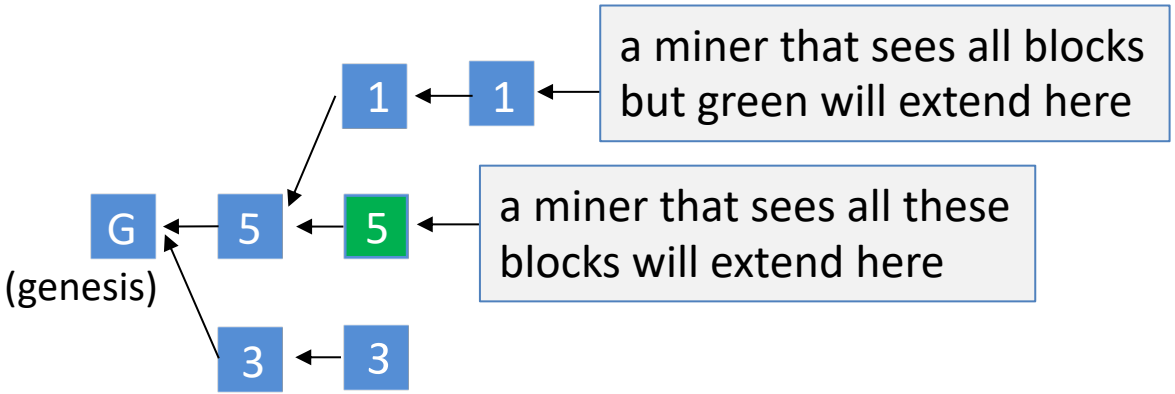
Nakamoto Consensus

Chain with the highest difficulty, i.e, largest sum of the difficulty D within blocks!

Bitcoin uses **Nakamoto consensus**:

- **Fork-choice / proposal rule:** At any given time, every honest miner attempts to extend (i.e., mines on the tip of) the heaviest chain *held in its view.* (ties broken arbitrarily)

Numbers indicate difficulty values on each block



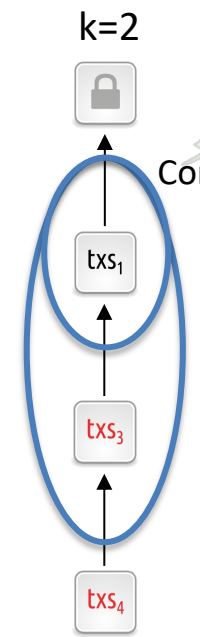
A block B will be discarded if a heavier chain that does not contain B appears.

Nakamoto Consensus

Bitcoin uses **Nakamoto consensus**:

- **Fork-choice / proposal rule:** At any given time, each honest miner attempts to extend (i.e., mines on the tip of) the heaviest (longest for us) chain *held* in its view (ties broken arbitrarily).
- **Confirmation rule:** Each miner confirms a block (along with its prefix) that is k-deep within the longest chain in its view. In practice, $k = 6$.
  - Miners and clients (wallets) accept the transactions in the latest confirmed block and its prefix as their log.
  - Note that *confirmation* is different from *finalization*.
- **Leader selection rule:** Proof-of-Work. Anyone who solves the proof-of-work can post a block.

Nakamoto Consensus



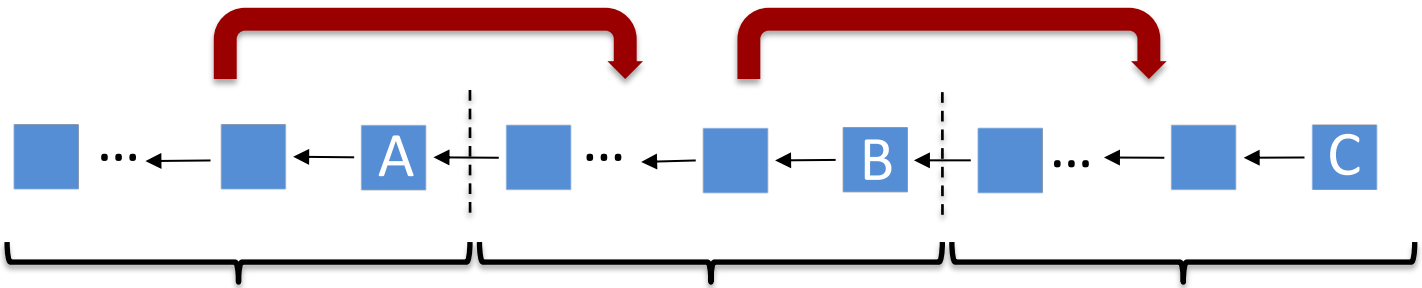
Alice's log: $txs_1\ txs_3$

Available under dynamic participation

Bitcoin: difficulty adjusts every two weeks

New difficulty: $D_2 = D_1 \frac{2016 \times 10\ mins}{t_1}$

New difficulty: $D_3 = D_2 \frac{2016 \times 10\ mins}{t_2}$



2016 blocks (≈2 weeks)
Time it took to mine: t_1(min)
Difficulty: D_1

2016 blocks (≈2 weeks)
Time it took to mine: t_2(min)
Difficulty: D_2

2016 blocks (≈2 weeks)
Time it took to mine: t_3(min)
Difficulty: D_3

New difficulty is not allowed to be more than 4x old target.
New difficulty is not allowed to be less than ¼ x old target.

t_2: difference between the timestamps in B and A
t_3: difference between the timestamps in C and B

The Bitcoin Backbone Protocol with Chains of Variable Difficulty (2016)

Consensus in the Internet Setting

Characterized by *open participation*. Challenges:

- Adversary can create many Sybil nodes to take over the protocol.
- Honest nodes can come and go at will.

Achieved by Bitcoin!

Requirements:

- Limit adversary's participation.
  - **Sybil resistance (e.g., by Proof-of-Work)**
- Maintain availability (liveness) when the honest nodes come and go at will, resulting in changes in the number of nodes.
  - **Dynamic availability**

Security?

Can we show that Bitcoin is a secure state machine replication (SMR) protocol (satisfies safety and liveness) under synchrony against a Byzantine adversary?

$\beta(t)$ $\in [0,1]$ for all t

Fraction of the mining power controlled by the adversary at time t.

What is the highest $\beta(t)$ for which Bitcoin is secure??

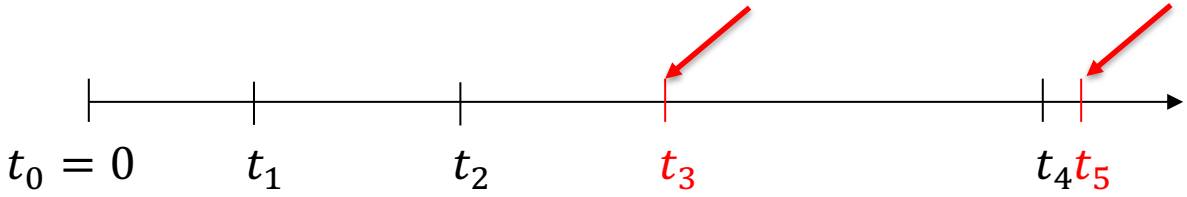
Model for Bitcoin

- Many different miners, each with *infinitesimal* power.

  Total mining rate (growth rate of the chain): λ (1/minutes). In Bitcoin, $\lambda = 1/10$.

- Suppose Adversary is Byzantine and controls $\beta < 1$ fraction of the mining power.
  - **Adversarial mining rate**: $\lambda_a = \beta\lambda$
  - **Honest mining rate**: $\lambda_h = (1-\beta)\lambda$

- Network is **synchronous** with a known upper bound Δ on delay.

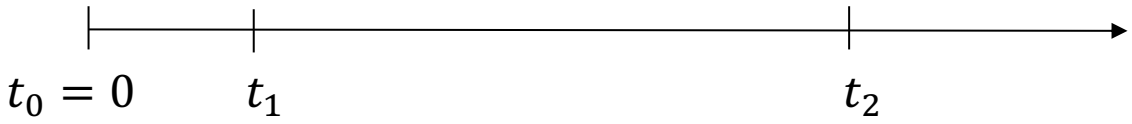


times when blocks are mined (red indicates Byzantine miner)

Reminder: Why is safety important?

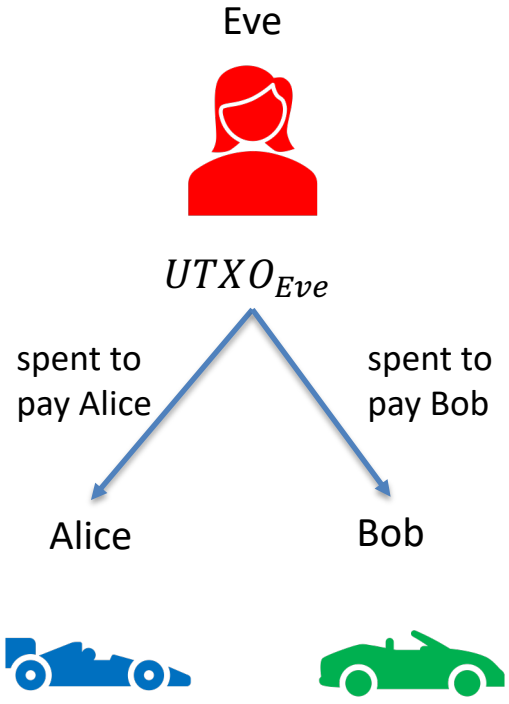
Suppose Eve has a UTXO.

- tx_1: transaction spending Eve's UTXO to pay to car vendor Alice.
- tx_2: transaction spending Eve's UTXO to pay to car vendor Bob.



- Alice's ledger at time t_1 contains tx_1:
  $LOG_{t_1}^{Alice} =< tx_1 >$
- Alice thinks it received Eve's payment and sends over the car.

- Bob's ledger at time t_2 contains tx_2:
  $LOG_{t_2}^{Bob} =< tx_2 >$
- Bob thinks it received Eve's payment and sends over the car.



Reminder: Why is safety important?

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safety violation

- Alice's ledger at time t_1 contains tx_1: $LOG_{t_1}^{Alice} =< tx_1 >$
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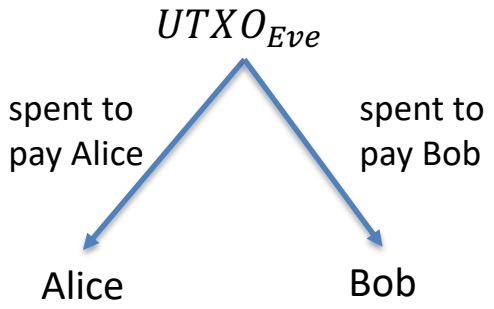
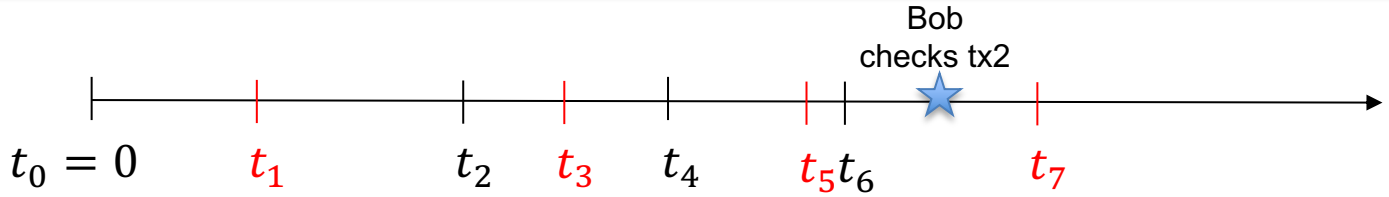
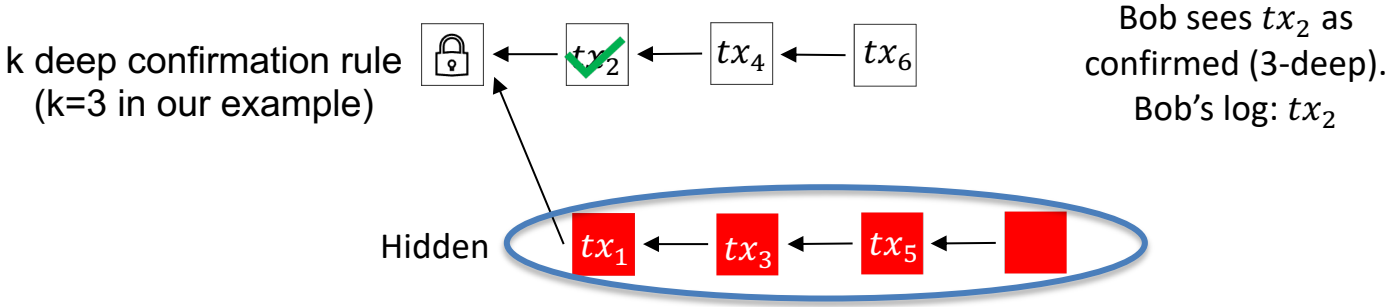


When safety is violated, Eve can double-spend!

Nakamoto's Private Attack: $\beta \geq 1/2$



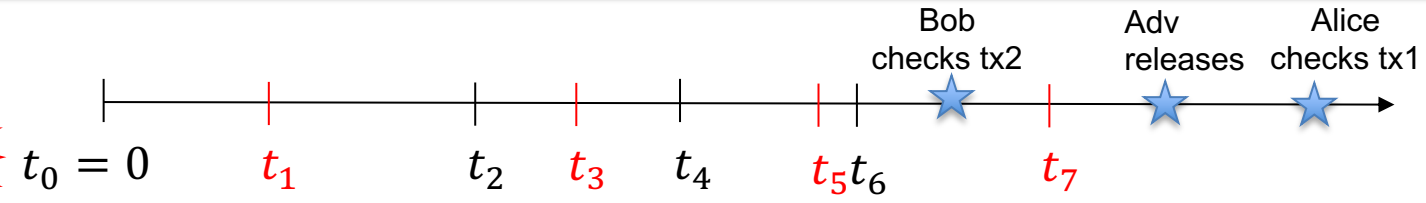


Let's show that Bitcoin is insecure if $\beta(t) \geq 1/2$

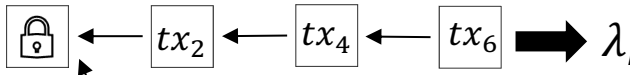
A Peer-to-Peer Electronic Cash System (2008)

Nakamoto's Private Attack: $\beta \geq 1/2$

safety violation! double spend!



k deep confirmation rule (k=3 in our example)



λ_h Honest mining rate

Bob sees tx_2 as confirmed.
Bob's log: tx_2

λ_a Adversarial mining rate

Alice sees the red chain as the longest chain.
tx_2 is not confirmed!
Alice's log: tx_1 tx_3

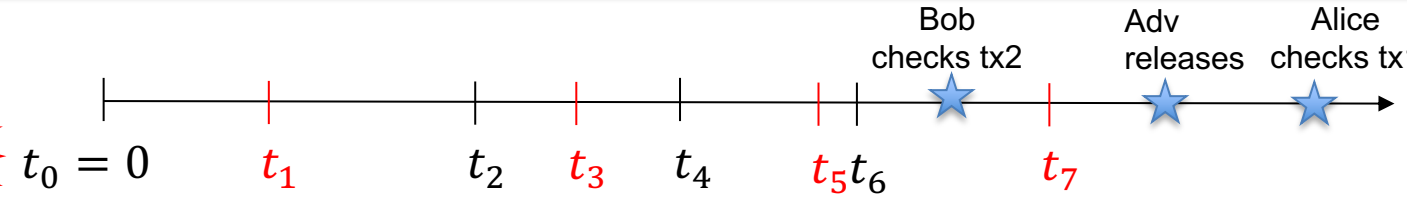
Private attack succeeds!

tx_2 got 'reorged': It was part of the longest chain before but not anymore!!

A Peer-to-Peer Electronic Cash System (2008)

Nakamoto's Private Attack: $\beta \geq 1/2$

safety violation! double spend!



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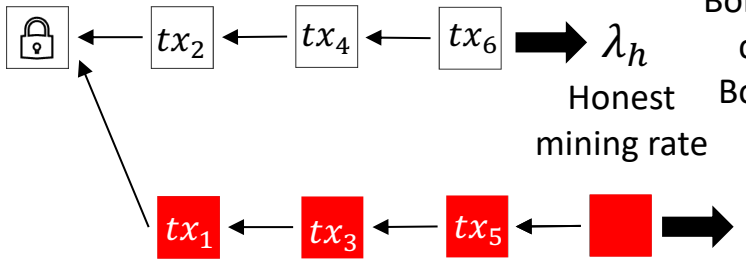


Bob sees tx_2 as confirmed.
Bob's log: tx_2

Alice sees the red chain as the longest chain.
tx_2 is not confirmed!
Alice's log: tx_1 tx_3

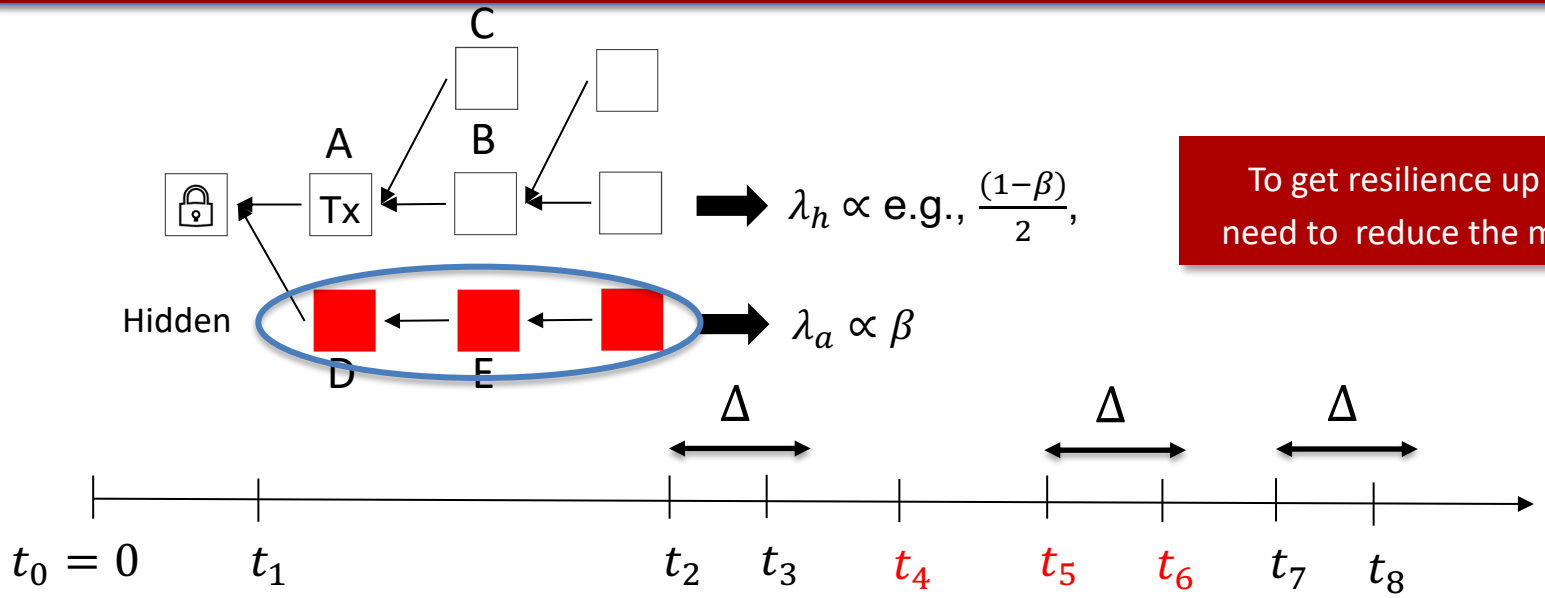
Private attack succeeds!

Private attack succeeds (w.h.p) if $\lambda_a \geq \lambda_h$, i.e., if $\beta \geq 1-\beta$, i.e., if $\beta \geq \frac{1}{2}$.

Private attack fails (w.h.p) if $\lambda_a < \lambda_h$, i.e., if $\beta < 1-\beta$, i.e., if $\beta < \frac{1}{2}$.

Is there a more powerful attack?

Problem: forking slows down honest miners' chain growth



To get resilience up to $\beta < \frac{1}{2}$, need to reduce the mining rate!

Multiple honest blocks at the same height due to network delay.

Adversary's chain grows at rate proportional to β!

… but honest miners' chain grows at rate less than $1 - \beta$ because of forking!

Now, adversary succeeds if $\beta \geq \frac{(1-\beta)}{2}$, which implies $\beta \geq \frac{1}{3}$!!

Reminder for SMR Security

Let's recall the security definition for state machine replication (SMR) protocols. Let ch_t^i denote the confirmed Tx (i.e., k-deep) of a client i at time t.

Safety (Consistency):

- For any two clients i and j, and times t and s: $\boldsymbol{ch_t^i \preccurlyeq ch_s^j}$ (prefix of) or vice versa, i.e., chains are consistent.

No double spend

Liveness:

- If a transaction tx is input to an honest miner at some time t, then for all clients i, and times $s \geq t + T_{conf}$: $tx \in \boldsymbol{ch_s^i}$.

No censorship

Security Theorem

Theorem: If $\beta < 1/2$, there exists a small enough mining rate $\lambda_{\text{safe}}(\Delta, \beta) = \lambda_a + \lambda_h$ such that Bitcoin satisfies safety and liveness except with error probability $\epsilon = e^{-\Omega(k)}$ under Δ-synchronous network (recall that k is used in the k deep confirmation rule).

- Latest result for bounding the error probability as a function of k:

$$\epsilon \leq \left(2 + 2\sqrt{\frac{1-\beta}{\beta}}\right)\left(4\beta(1-\beta)\right)^k \qquad \text{(exp. small in } k\text{)}$$

- We say 'confirmation' instead of finalization because when you *confirm* a block or transaction, you *confirm* it with an error probability…
  …unlike *finalizing* a block where there is no error probability.

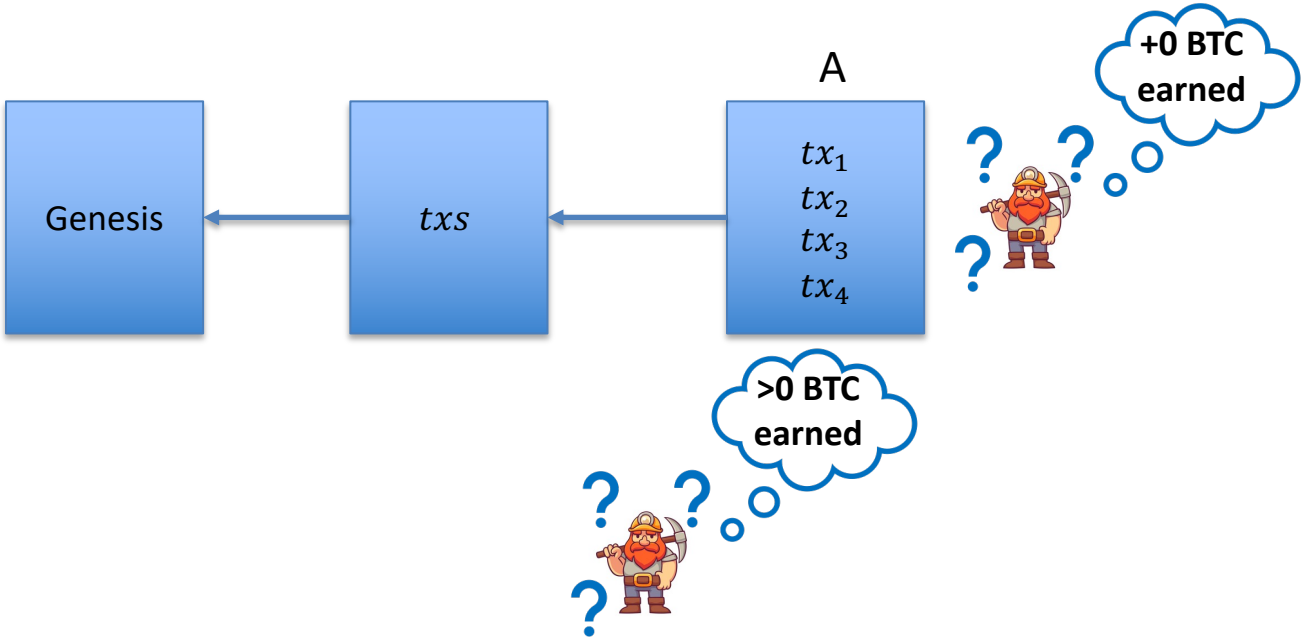
The Bitcoin Backbone Protocol: Analysis and Applications (2015)
Analysis of the Blockchain Protocol in Asynchronous Networks (2016)
Analysis of Nakamoto Consensus (2019)
Everything is a Race and Nakamoto Always Wins (2022)
Bitcoin's Latency–Security Analysis Made Simple (2022)

Now, we see why Bitcoin has 1 block every 10 minutes, instead of 1 block every second…

Proof of the Security Theorem

See the optional slides at the end of the deck …

Would $\beta < 1/2$ hold in practice?



Transaction fees (paid to the miner):

tx_1: 4 BTC

tx_2: 3 BTC

tx_3: 2 BTC

tx_4: 1 BTC

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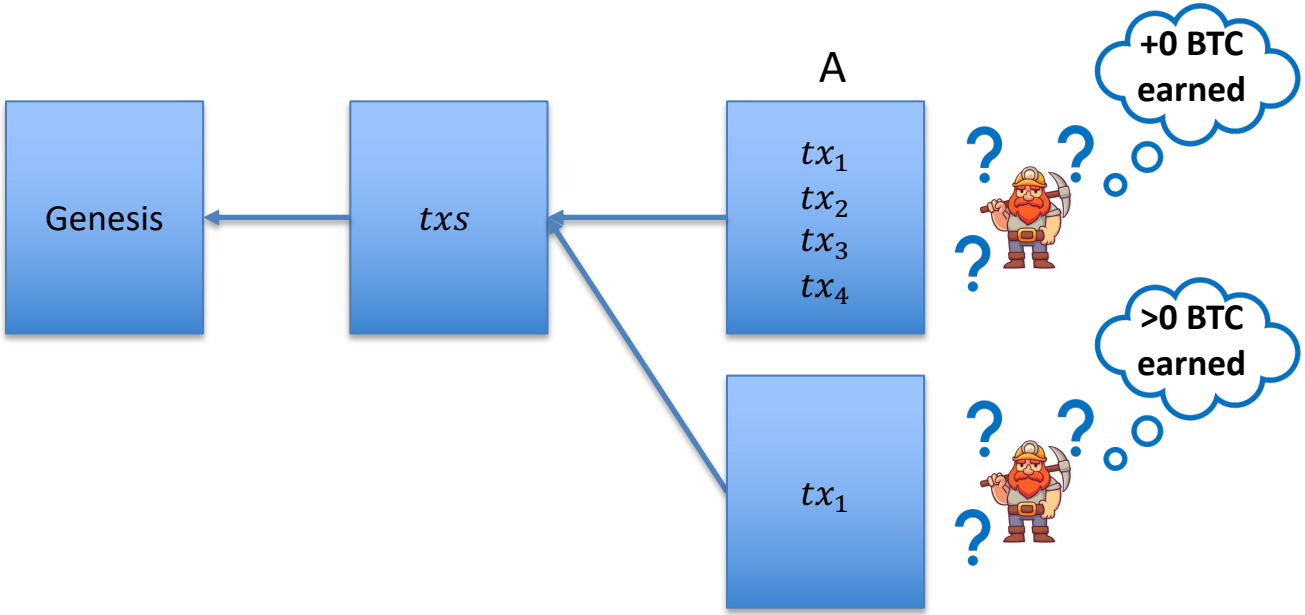

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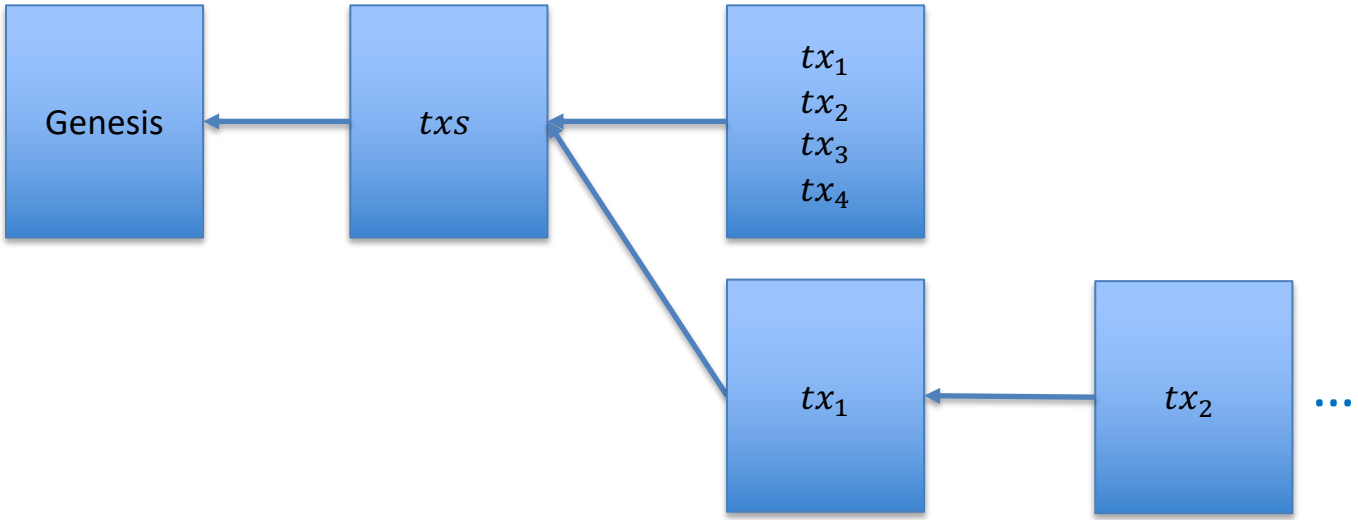

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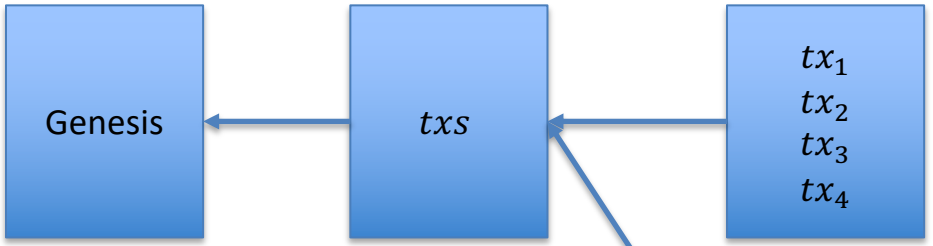

A

tx_1
tx_2
tx_3
tx_4

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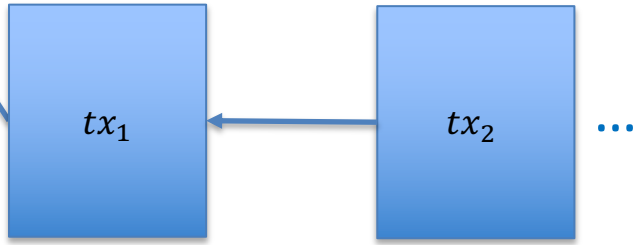
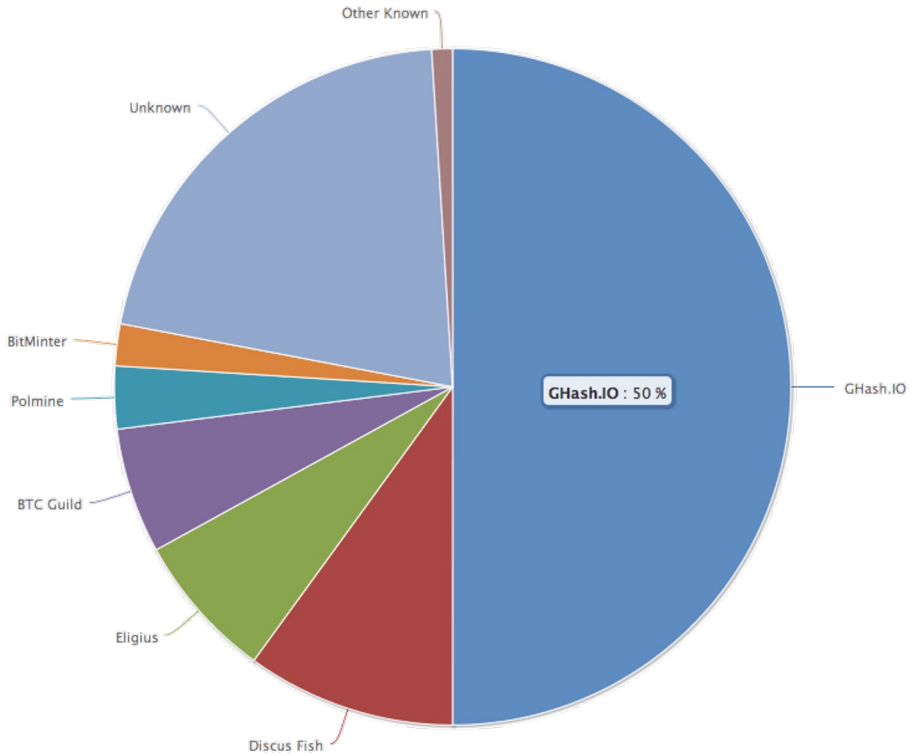


Miner (maximal) extractable value (MEV): a measure of miner's profit via inclusion, exclusion or re-ordering of transactions within its block

- Miners have incentive to violate the protocol!
- Can lead to double spends!

MEV gives miners incentive to violate the protocol!!

No Attacks on Bitcoin?



Ghash.IO had >50% in 2014

- Gave up mining power

Why are visible attacks not more frequent?

- Miners care about the Bitcoin price?
- Not quite a valid argument.
- They can ‘short’ the chain for profit!

… but then no more block rewards

No guarantees for the future!

Is Bitcoin the Endgame?

Bitcoin provides Sybil resistance and dynamic availability.

Is it the Endgame for consensus?

No!

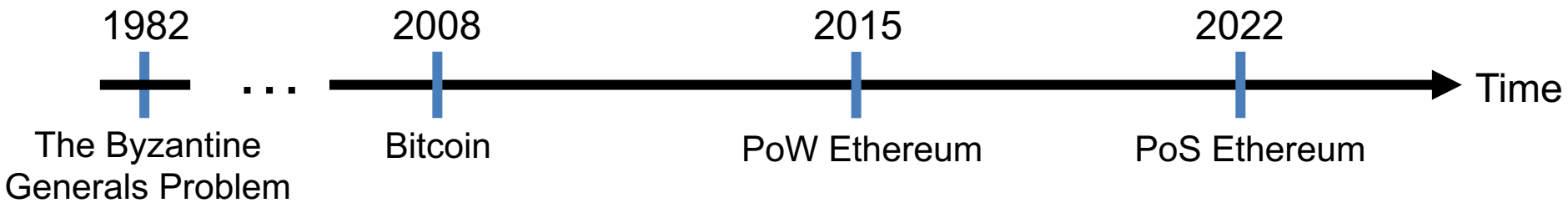
Bitcoin is secure only under synchrony and loses security during periods of asynchrony.

It *confirms* blocks with an error probability depending on *k*, i.e., blocks are not finalized.

Energy consumption?

Can we have low-energy consensus with finality? Answer: yes! Using proof-of-stake

From Bitcoin to Proof-of-Stake



Consensus in the Internet Setting
- Sybil resistance
- Dynamic availability
  - (Liveness under changing part.)

Block rewards (carrot)
- ➢ to incentivize participation!

- ➢ Consensus in the Internet Setting
  - ➢ Sybil resistance
  - ➢ Dynamic availability
- ➢ Block rewards (carrot)
- ➢ Finality and accountable safety
- ➢ Slashing (stick)
  - ➢ to punish protocol violation!

The Byzantine Generals Problem (1982)
Bitcoin: A Peer-to-Peer Electronic Cash System (2008)
Ethereum: A Next-Generation Smart Contract and Decentralized Application Platform. (2015)
Combining GHOST and Casper (2020)

A few words on Proof-of-Stake (PoS)

In a Proof-of-Stake protocol, nodes lock up (i.e., stake) their coins in the protocol to become eligible to participate in consensus.

The more coins staked by a node…

- Higher the probability that the node is elected as a leader.
- Larger the weight of that node's actions.

If a node is caught doing an adversarial action (e.g., signing two blocks), it can be punished by burning its locked coins (stake)! This is called *slashing*.

Thus, in a Proof-of-Stake protocol, nodes can be held *accountable* for their actions (unlike in Bitcoin, where nodes do not lock up coins).

END OF LECTURE

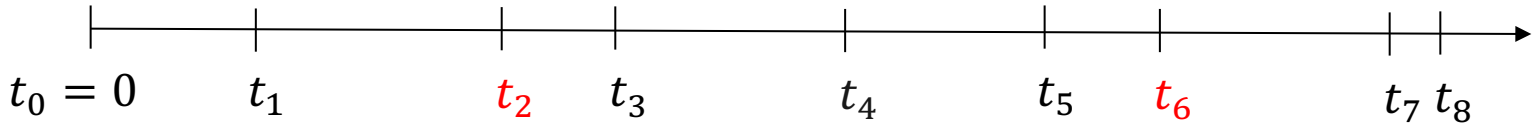
How?

Next lecture: Incentives and Accountability in Consensus

Optional: Security Proof

Loner block:

- An honest block such that no other honest block is mined within Δ time of the loner block.



Analysis of Nakamoto Consensus (2019)
The Sleepy Model of Consensus (2016)

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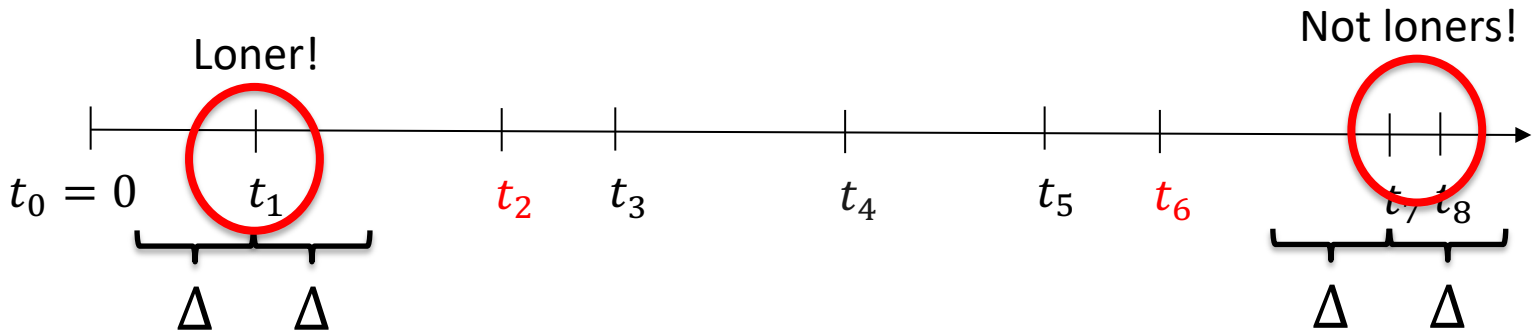

Length of the shortest chain among the longest chains observed by the clients at time t:

$$L(t)$$

Analysis of Nakamoto Consensus (2019)
The Sleepy Model of Consensus (2016)

Optional: Security Proof

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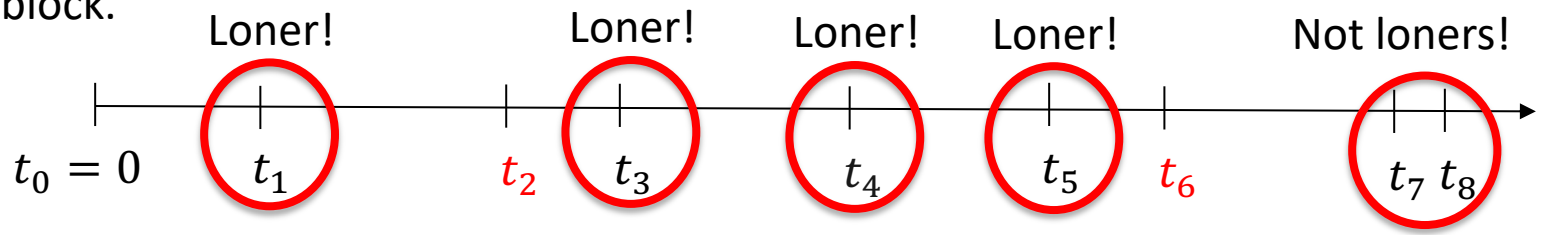
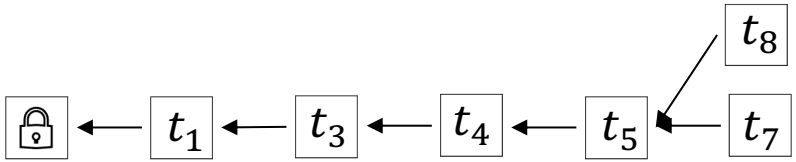


Lemma: For any $s > t$, $L(s) - L(t) \geq$ "number of loners mined in the interval $(t + \Delta, s - \Delta]$".

Proof sketch: Each loner increases the length of the longest chains observed by the clients by one block. For instance;

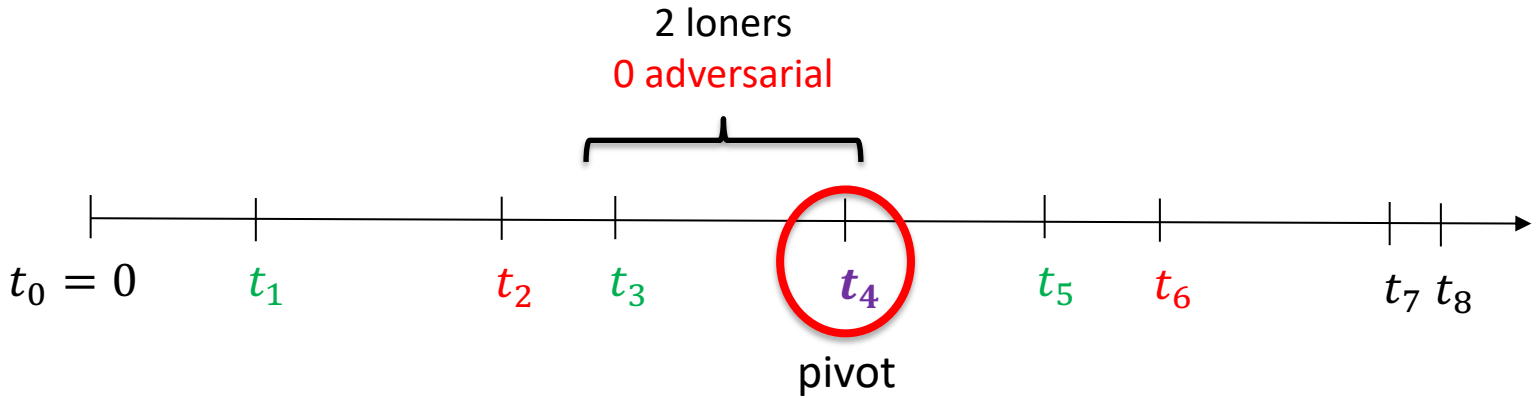


Analysis of Nakamoto Consensus (2019)
The Sleepy Model of Consensus (2016)

Optional: Security Proof

Pivot block:

- In any interval covering the mining time of the pivot block, more loner blocks are mined than adversarial blocks.
- Pivot block is a loner.



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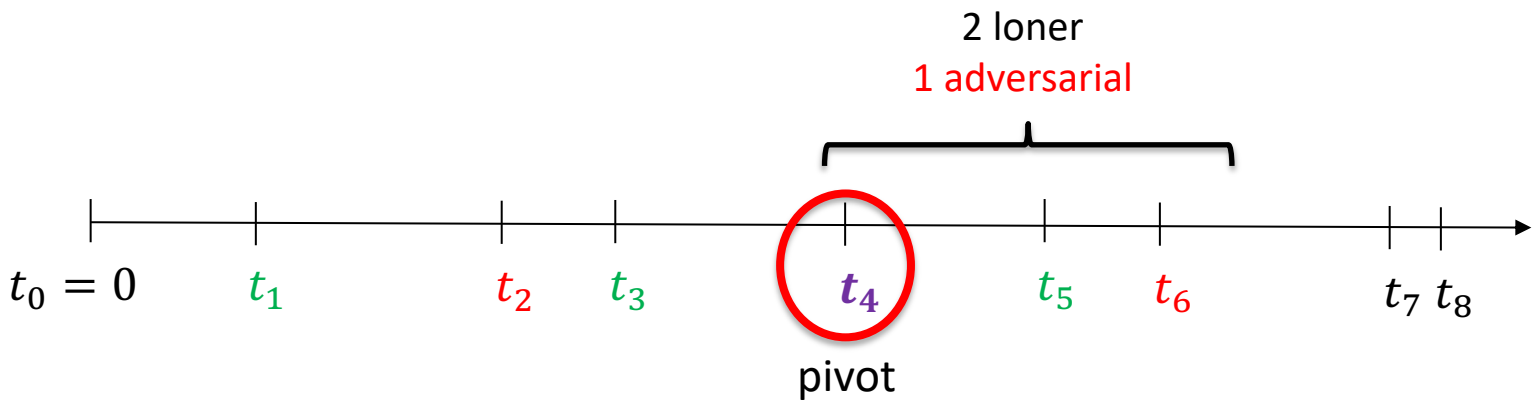

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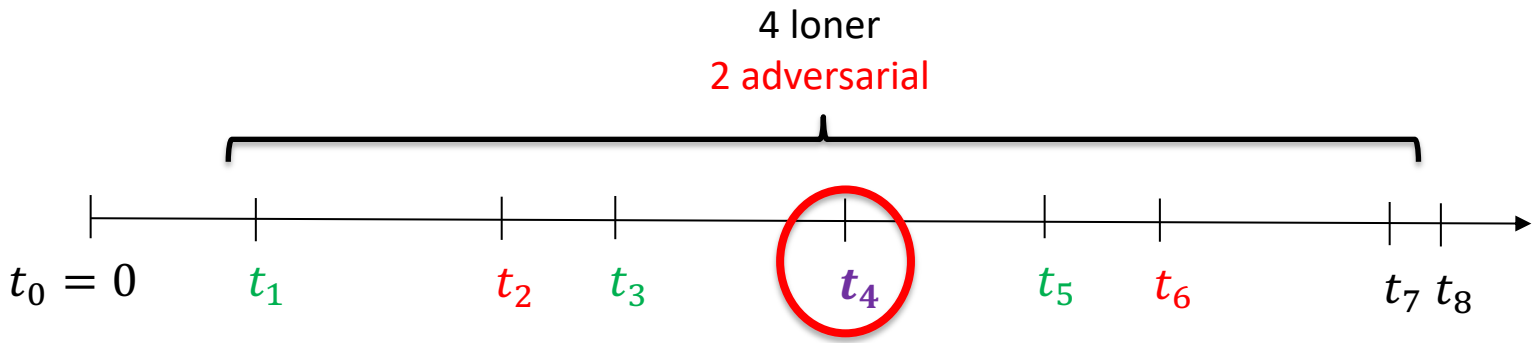


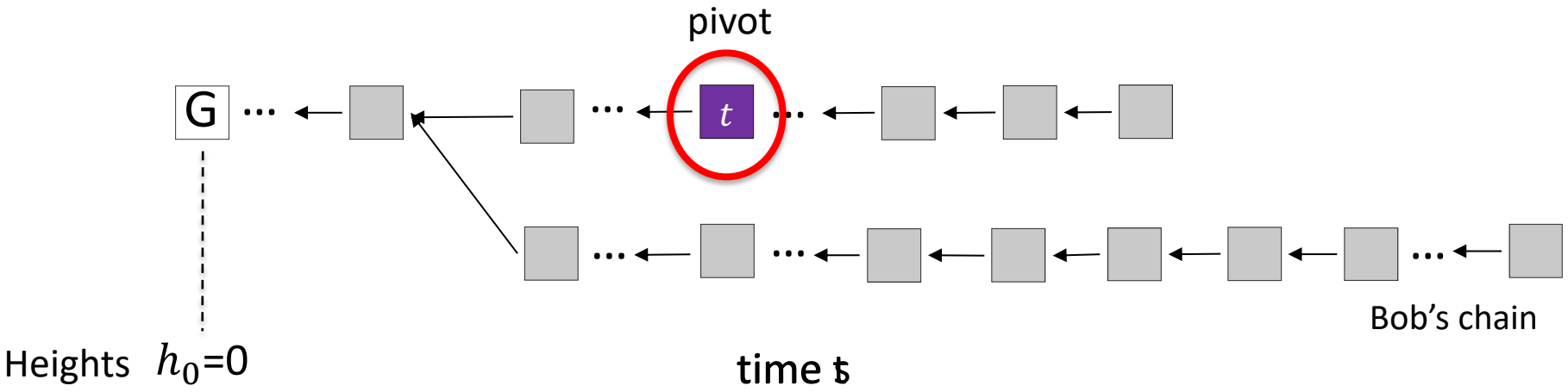
Theorem: If $\beta < 1/2$, there exists a small enough mining rate $\lambda(\Delta, \beta)$ such that any time interval of T have a pivot except with probability $e^{-\Omega(\sqrt{T})}$.

Proof: Probability theory

Optional: Security Proof

Theorem: Suppose a block mined at time t is a pivot. Then, the pivot block is on every (longest) chain held by any client at all times $\geq t$.

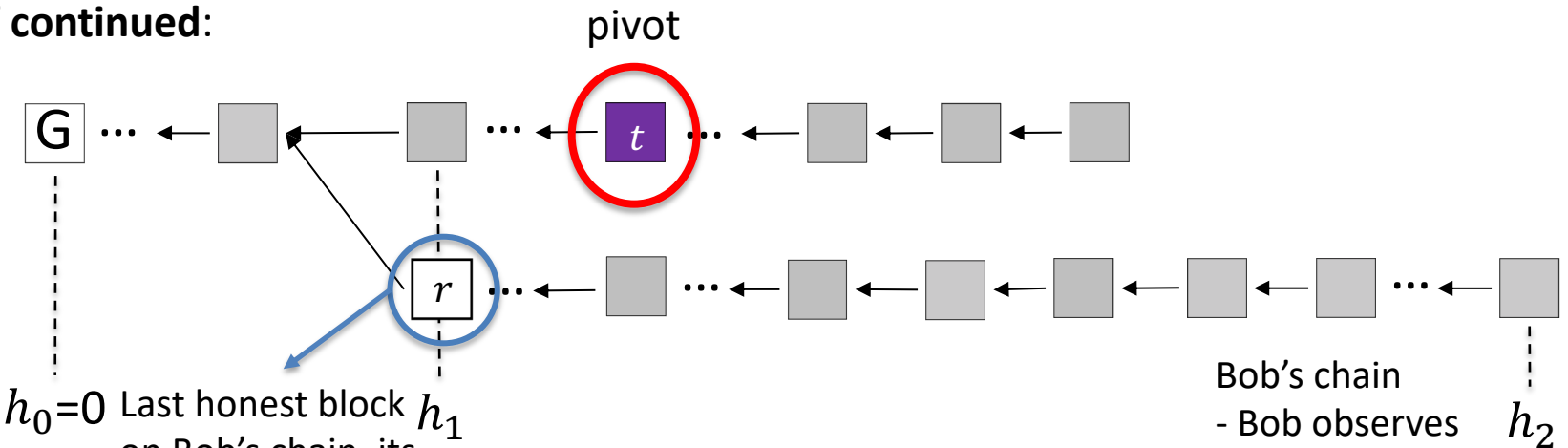
Proof: For contradiction, suppose there exists a minimum time $s \geq t$ such that a client Bob holds a chain conflicting with the pivot block.



Optional: Security Proof

Theorem: If a client holds a chain containing a pivot block, then no client can hold a chain conflicting with the pivot block after the pivot block is mined.

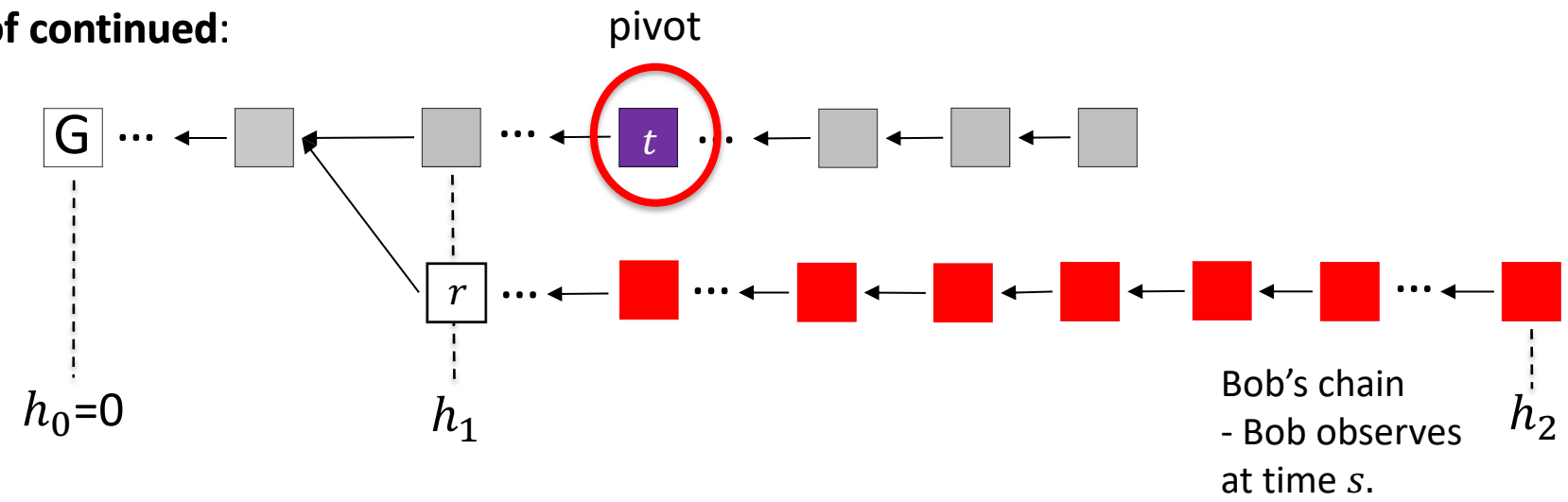
Proof continued:



- $r < s$ by definition.
- $r < t$ because otherwise s is not the first time a conflicting chain is held a client or honest miner.

Optional: Security Proof

Proof continued:

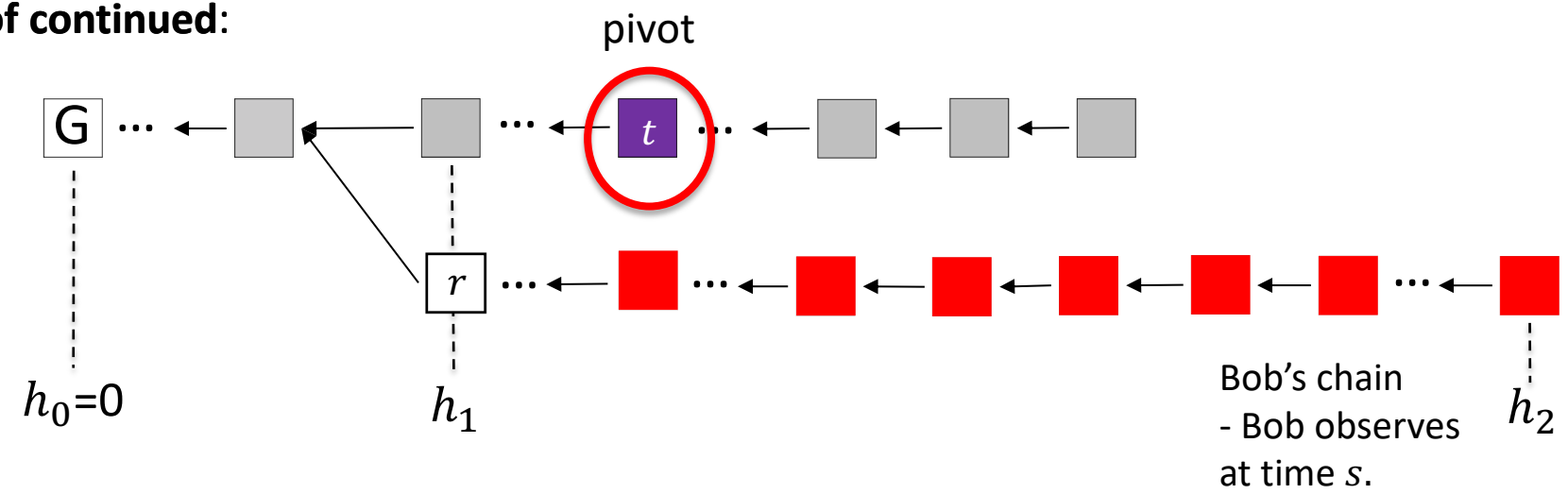


- $h_2 - h_1 <$ "blocks mined by the adversary in the interval $(r, s]$"
- length of the shortest 'longest chain' held by any client at time r, $L(r) \leq h_1$
- length of Bob's chain at time s, $h_2 \geq L(s)$

Hence, $h_2 - h_1 \geq L(s) - L(r) \geq$ "number of loners mined in the interval $(r + \Delta, s - \Delta]$" by the lemma.

Optional: Security Proof

Proof continued:



Finally, "blocks mined by the adversary in the interval $(r, s]$" $> h_1 - h_2$

$h_1 - h_2 \geq L(s) - L(r) \geq$ "number of loners mined in the interval $(r + \Delta, s - \Delta]$".

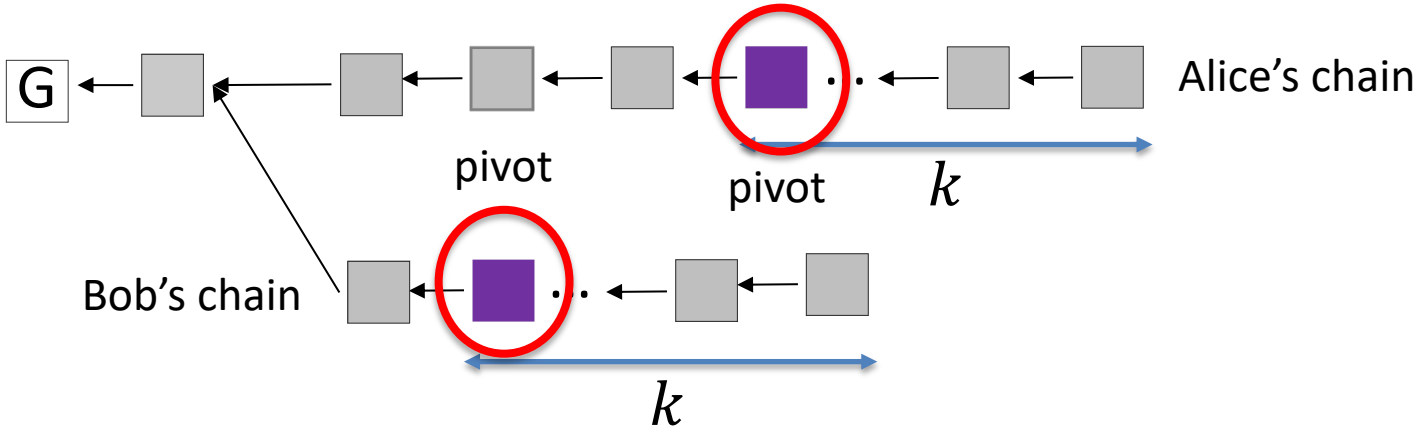
In the interval $(r, s]$ covering t, more adversary blocks are mined than loners!

Contradiction with the definition of pivot!!

Optional: Security Proof

Proof Sketch of Liveness: The pivot is mined by an honest miner and contains all transactions input to the honest miners. Since it is on all chains held by all clients at all times, liveness is satisfied.

Proof Sketch of Safety: Consider two clients that confirm two chains after chopping off the last k blocks on their chains. One of the last k blocks is a pivot on both chains except with probability $e^{-\Omega(\sqrt{k})}$ (follows from probability theory). Thus,



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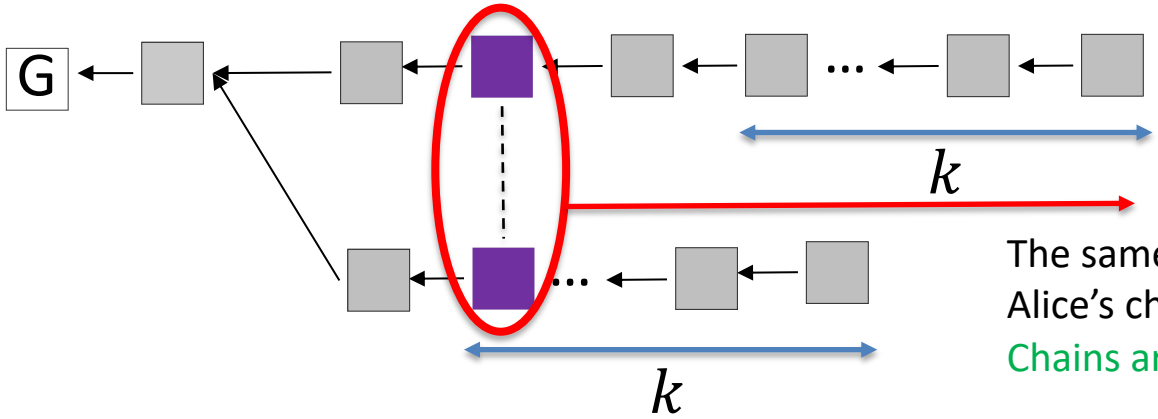

Pivots:
The same block appears in Alice's chain by the Theorem.
Chains are consistent!!