In this challenge you are going to simulate the isentropic flow through a quasi 1D subsonic-supersonic nozzle. You will derive both the conservation and non-conservation forms of the governing equations and sovle them using the MacCormack's technique. You need to determine the steady-state temperature distribution for the flow-field variables and investigate the difference between the two forms of governing equations by comparing their solutions.
You need to show the following plots inside your report
1. Steady-state distribution of primitive variables inside the nozzle
2. Time-wise variation of the primitive variables
3. Variation of Mass flow rate distribution inside the nozzle at different time steps during the time-marching process
4. Comparison of Normalized mass flow rate distributions of both forms

Expected Results:

- Perform a grid independence test on the solution and expain all your findings.
- The code must be modular. You need to write separate functions to solve both forms.
- Discuss all the plots you are providing with observed inferences.

Objective : Numerical simulation of 1D supersonic nozzle flow using Macormack Method using conservative and non-conservative forms of equation

Assumptions:

1. Flow inside the nozzle is assumed to be isentropic.

2. The flow is considered Quasi-1D as properties vary along the x axis and not the y axis.

3. Compresible flow

4. No friction and heat transfer considerations.

5.Considerable pressure ratio is maintained between choked pressure and back pressure to allow expansion of the flow and avoid shock waves.

Problem Description:

1. Area profile along the direction of the flow :

$$A = 1 + 2.2 * (x - 1.5)^2$$

2. Initial non-dimnetionalized thermodynamic properties :

$$\rho = 1 - 0.3146 * x$$

$$T = 1 - 0.2314 * x$$

$$V = (0.1 + 1.09 * x)^{0.5}$$

Governing Equations:

1. Non-conservative form

- Continuity Equation : $\dfrac{\partial \rho}{\partial t} = -\rho \dfrac{\partial V}{\partial x} - \rho V \dfrac{\partial (ln A)}{\partial x} - V \dfrac{\partial \rho}{\partial x}$
- Momentum Equation : $\dfrac{\partial V}{\partial t} = -V \dfrac{\partial V}{\partial x} - \dfrac{1}{\gamma}\left(\dfrac{\partial T}{\partial x} + \dfrac{T \partial \rho}{\rho \partial x}\right)$
- Energy Equation : $\dfrac{\partial T}{\partial t} = -V \dfrac{\partial T}{\partial x} - (\gamma - 1)T\left(\dfrac{\partial V}{\partial x} + V \dfrac{\partial ln A}{\partial x}\right)$

2. Conservative Form

- Continuity Equation : $\dfrac{\partial (\rho A)}{\partial t} + \dfrac{\partial (\rho V A)}{\partial x} = 0$
- Momentum Equation : $\dfrac{\partial (\rho A V)}{\partial t} + \dfrac{\partial \left(\rho A V^2 + \frac{1}{\gamma} P A\right)}{\partial x} = \dfrac{P}{\gamma}\dfrac{\partial A}{\partial x}$
- Energy Equation:

$$\frac{\partial\left(\rho\left(\frac{T}{\gamma-1} + \frac{\gamma V^2}{2}\right) * A\right)}{\partial t} + \frac{\partial\left(\rho\left(\frac{T}{\gamma-1} + \frac{\gamma V^2}{2}\right) * A + PAV\right)}{\partial t} = 0$$

The equations were solved using Macormack Method.

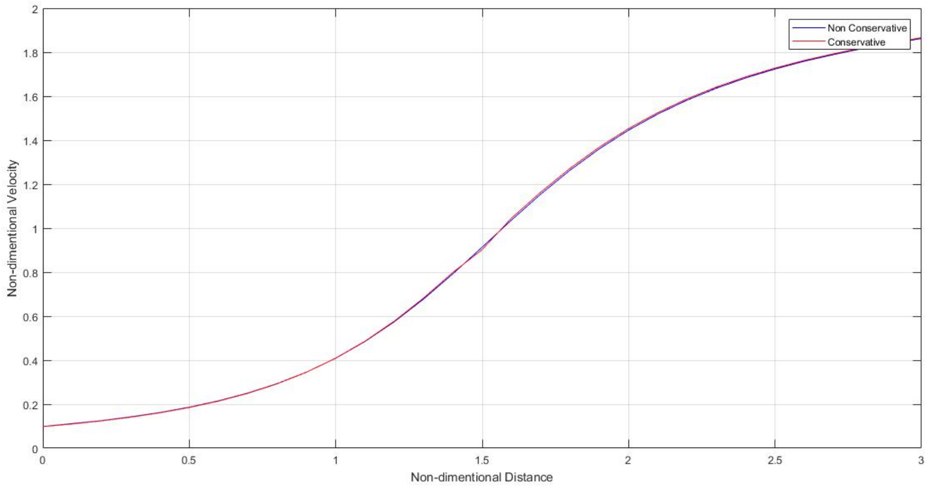
Result:

The simulation was run for a maximum of 5000 time-steps, with CFL=0.5 and n=31. The tolerance was kept to 1e-6, where the change in rho, v, and T had to fall under the tolerance. Net Mass Flow Rate tolerance has also been set at 1e-3 for non conservation form and 1e-2 for conservation form.

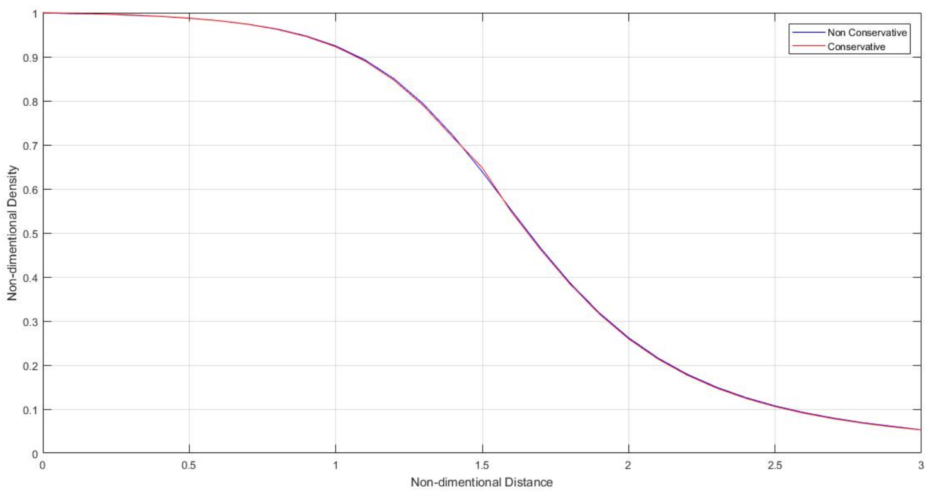
Non Conservation Form converged in 676 time steps, by converged it has reached a stable solution.

Conservation Form converged in 4515 time steps, by converged it has reached a stable solution.

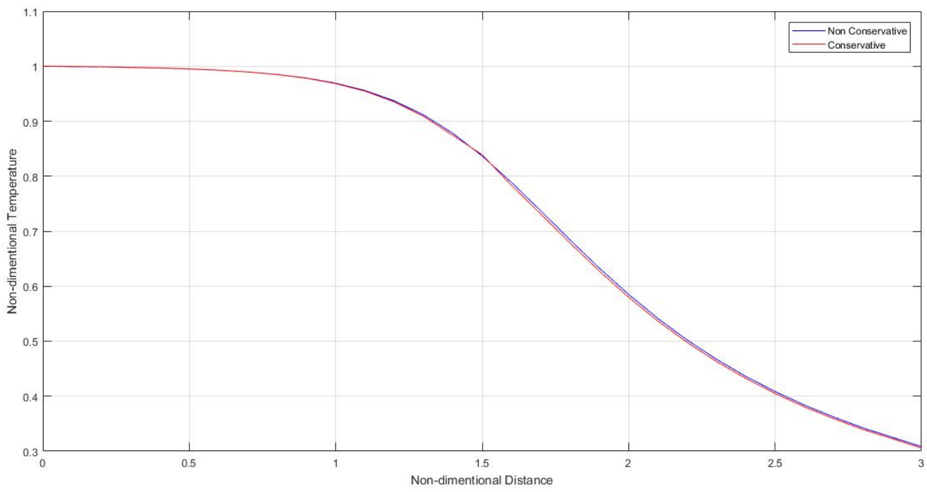
Velocity Plots:



Density Plots:

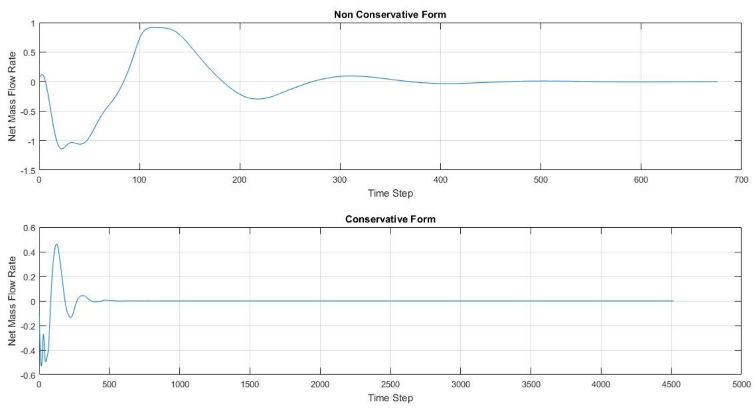


Temperature Plots:

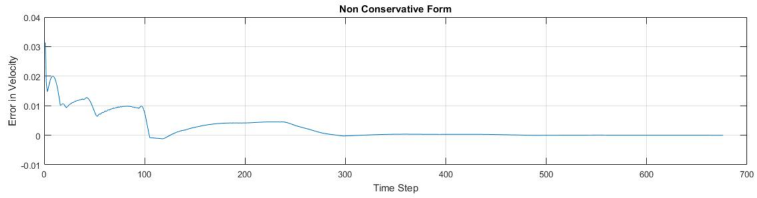


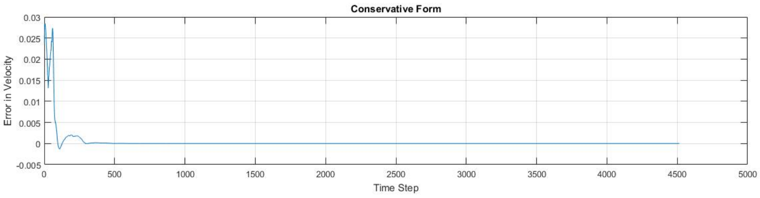
It has been observed that conservative form solves in less number of iterations than non-conservative form. However, the net mass flow rate hasn't stabilised in the current condition.

Net Mass Flow Rate :

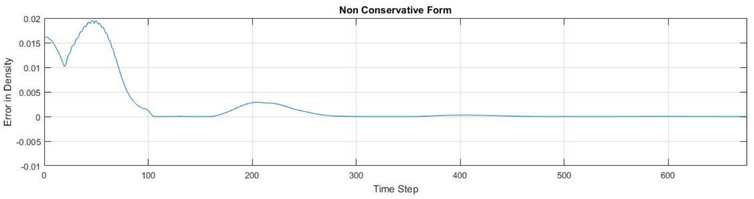


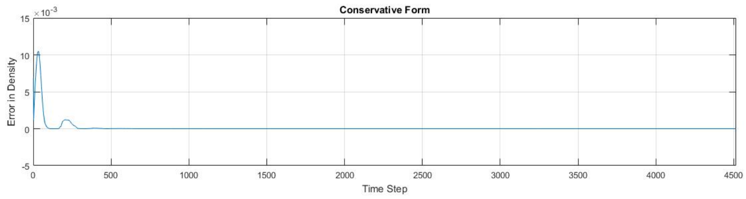
Net Change in Velocity:





Net Change in Density:





Net Change in Temperature: