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SMILING SURFACE DYNAMICS

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Abstract

Understanding volatility of financial assets is considered important in academics courses and among practitioners since the usage of Black Scholes formula for option pricing. Whether in risk management, portfolio hedging, or option pricing, a precise notion of the market's assessment and expectation of volatility is clearly inevitable. This report is centric around analyzing how well volatility surface models used in practice behaves with respect to empirical database for the option chains. Data and volatility models analysis is carried out with implementation of few of popular models as a part of VolSurface UI tool package.

The organization of this paper is as follows. It starts with relevent theory description for volatility surfaces covering volatility related topics that influence the successful construction of IVS in practice. Futher it covers implementation of least squares kernel estimator which smoothes implied volatility in the option price space. Detail for how to perform smoothing, choice of calibrating functional and selection of numerical optimization algorithms are covered along with working on relevent quantitative routines. Another implementation for caliberation of widely-used SVI parameterization is coverd as well for implied volatility surface usage in the absence of static arbitrage. Software suite which is developed as a part of development testing is also described along with relevent code and data used. Final report work is summarized with some definination for future scope of work on extension on the work.

I would like to thanks Director, NUS MFE Program for giving us opportunity to work on project as part of FE5110 module. The analysis of market data for empirical study of implied volatility surfaces behavior, implementation and testing for the surface models certainly boosted my interest and confidence for future work in the area of project within financial industry.

Contents

Black Scholes Pricing Model

As per Black-Scholes option-pricing model all options on the same underlying should have the same implied volatility, regardless of strike or term, but it is well known that this is not the case in reality. Black Scholes option prices in terms of implied volatility are expressed as ‘‘the wrong number to plug into the wrong formula to get the right price.’’ It is established market practice to quote the price of vanilla options using this volatility ‘metric’. This practice is followed in the equity, in the FX and in the interest-rate area. The market chooses to retain the simplicity and convenience of a Black-like quote by assigning different implied volatilities for options with identical underlying and expiration, but different strikes. The dependence on the strike of the implied volatility for options of the same maturity is referred to in what follows as ‘the smile’.

A basic options pricing model uses six inputs to calculate the theoretical price of an option: asset value, strike price, time left until expiration, dividends (if applicable), risk-free interest rates, and, arguably, the most important factor, the volatility of the asset. When the volatility input increases, the theoretical option premiums go up. When the volatility input decreases, the theoretical option premiums go down.

Price of the European call option and put options are given by 1.1 and 1.2 respectively.

$$C(S,\tau) = SN(d_1) - Ke^{-rT}\mathrm{N}(d_2) \quad 1.1$$

$$P(S,\tau) = Ke^{-rT}\mathrm{N}(-d_2) - SN(-d_1) \quad 1.2$$

$$d_1 = \frac{log\frac{S}{K} + \left(r + \frac{\sigma^2}{2}\right)\mathcal{T}}{\sigma\sqrt{\mathcal{T}}} \quad 1.3$$

$$d_2 = d_1 - \sigma\sqrt{\mathcal{T}} \quad 1.4$$

S is the current price of the underlying asset, K is the strike or exercise price, r is the continuously compounded risk-free interest rate, and τ is the time to option maturity.

N (d1) is the cumulative probability distribution of a standard normal distribution for the area below d1, and N (−d1) =1 − N (d1).

Black Scholes has several assumptions including short sale is permitted, no transaction costs or taxes, no dividend before option maturity, no arbitrage, continuous trading, constant risk-free interest rate and constant volatility etc. Empirical findings suggest that option pricing is not sensitive to the assumption of a constant interest rate. Apart from the constant volatility assumption, the violation of any of the remaining assumptions will result in the option price being traded within a band instead of at the theoretical price.

European call option price C(S,τ) for a contract with strike price K and expiration date, the implied volatility σ iv is defined as the input value of the volatility parameter to the Black–Scholes formula such that

$CBS\ (t,S;\ K,\mathcal{T}\ ;\ \sigma iv)\ =\ Cobs$ **(1.6)**

The option implied volatility σiv is often interpreted as a market's expectation of volatility over the option's maturity, i.e. the period from t to T. Given the true (unconditional) volatility is σ over period T. If Black– Scholes is correct, then

$CBS\ (t,S;\ K,\mathcal{T}\ ;\ \sigma iv)\ =\ (t,S;\ K,\mathcal{T}\ ;\ \sigma)$ **(1.7)**

for all strikes. That is the function (or graph) of σiv (K) against K for fixed t, S, T and r , observed from market option prices is supposed to be a straight horizontal line.

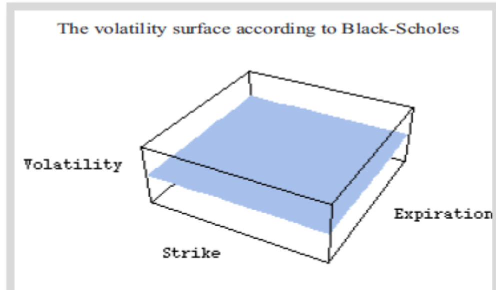

Figure 1

But, it is well known that the Black–Scholes σiv, differ across strikes. There is plenty of documented empirical evidence to suggest that implied volatilities are different across options of different strikes, and the shape is like a smile when we plot Black–Scholes implied volatility σiv against strike price K, the shape is anything but a straight line.

Implied Volatility

Alternatively one can describe the observed market price of an option at a specific time point with the help of the BS formula using the so called "implied" volatility. In doing this one typically finds a U shaped form for the resulting surface of the volatility for different times to maturity and strike prices. This phenomenon is also referred to as the Volatility Smile"

Given an observed European call option price C^{obs} for a contract with strike price K and expiration date T, the implied volatility σiv is defined as the input value of the volatility parameter to the Black–Scholes formula such that

$$CBS\ (t,S;\ K,\mathcal{T}\ ;\ \sigma iv) =\ C^{obs} \qquad \textbf{(1.8)}$$

The option implied volatility σiv is often interpreted as a market's expectation of volatility over the option's maturity, i.e. the period from t to T. There is a one-to-one correspondence between prices and implied volatilities. Since $\frac{\partial CBS}{\partial \sigma}$> 0, the condition

$C^{obs} = CBS\ (t,S;\ K,\mathcal{T}\ ;\ \sigma iv) >\ CBS\ (t,S;\ K,\mathcal{T}\ ;\ 0)$ **(1.9)**

means σiv > 0; i.e. implied volatility is always greater than zero. The implied volatilities from put and call options of the same strike price and time to maturity are the same because of put–call parity. Traders often quote derivative prices in terms of σiv rather than dollar prices, the conversion to price being made through the Black–Scholes formula. Given the true (unconditional) volatility is σ over period T. If Black–Scholes is correct, then

$$CBS\ (t, S;\ K, \mathcal{T}\ ;\ \sigma iv) = CBS\ (t, S;\ K, \mathcal{T}\ ;\ \sigma) \qquad \textbf{(1.10)}$$

for all strikes. That is the function (or graph) of σiv (K) against K for fixed t, S, T and r, observed from market option prices is supposed to be a straight horizontal line. There is plenty of documented empirical evidence to suggest that implied volatilities are different across options of different strikes, and the shape is like a smile when we plot Black– Scholes implied volatility σiv against strike price K, the shape is anything but a straight line.

Moneyness

Term moneyness $\frac{\mathrm{S}}{\mathrm{K}}$ refers to the ratio of the actual price S of the financial underlying and the strike price K of the respective option. It is also considered as measure of degree to which derivative is likely to have positive monetary value at its expiration. Using the moneyness m = $\frac{\mathrm{S}}{\mathrm{K}}$ of the option, one can also represent the implied volatility surface in relative coordinates, as a function of moneyness and time to maturity. This representation is convenient since there is usually a range of moneyness around m = 1 for which the options are liquid and therefore empirical data are most readily available.

Volatility Smile

Prior to the stock market crash of October 1987, the volatility surface σiv (K) against K was often observed to be U-shaped, with the minimum located at or near at-the-money options$\mathrm{K} = \mathrm{Se}^{-\mathrm{r}(\mathrm{T}-\mathrm{t})}$. It give rise to the term 'smile effect'. After the stock market crash in 1987, σiv (K) is typically downward sloping at and near the money and then curves upward at high strikes. This curve is known as a 'smirk'. The smile/smirk usually 'flattens' out as T gets longer. Moreover, implied volatility from option is typically higher than historical volatility and often decreases with time to maturity.

The smile/smirk curve, tells us that there is a premium charged for options at low strikes (OTM puts and ITM calls) above their BS price as compared with the ATM options. Although the market uses Black–Scholes implied volatility, σiv , as pricing units, the market itself prices options as though the constant volatility lognormal model fails to capture the probabilities of large downward stock price movements and so supplement the Black–Scholes price to account for this. The empirical relation between implied volatilities and exercise prices is also known as the "volatility skew". The volatility skew can be represented graphically in 2 dimensions (strike versus volatility).

Forward Skew

This is the case when implied volatility increases as the strike price increases. This suggests that out-of-the-money calls and in-the-money puts are in greater demand compared to in-the-money calls and out-of-the-money puts.

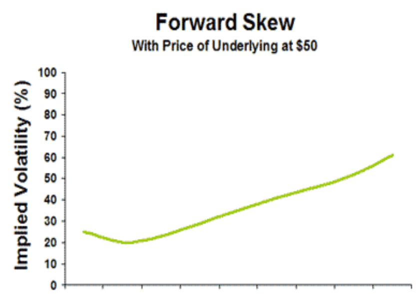

Figure 2

The forward skew pattern is more apparent for options in the commodities market. When supply is low, businesses would rather pay more to secure supply than to risk supply disruption.

For example, if weather reports indicate a heightened possibility of an impending frost, fear of supply disruption will cause businesses to drive up demand for out-of-the-money calls for the affected crops.

Reverse Skew

Another type of volatility smirk is the reverse skew. The reverse skew is a more common skew pattern. It usually appears for longer term equity options and index options. The implied volatilities for options with lower strikes are higher than those with higher strikes.

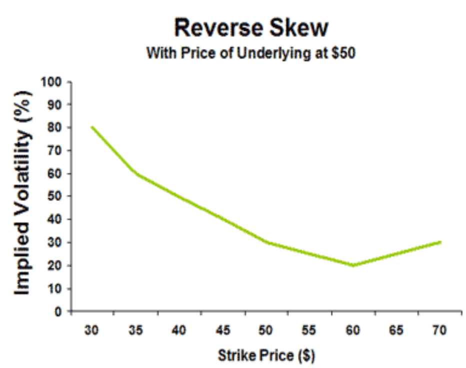

Figure 3

One explanation for the reverse volatility skew is that investors are usually worried about market crashes, so they buy puts for protection. This notion is supported by the fact that the reverse skew was not apparent until after the Crash of 1987 (Crash-o-phobia).

Buying in-the-money calls have become popular alternatives to buying stocks since they offer leverage, which increases the rate of interest. This causes a greater demand for in-the-money calls, which therefore increases implied volatilities at the lower strikes.

Reasons for volatility smile/skew

1. Black–Scholes assumes stock returns have normal distribution but empirically it is evident that risky financial asset returns have leptokurtic tails. In the case where the strike price is very high, the call option is deep-out-of-the-money with very low probability to be exercised. Leptokurtic right tail will give this option a higher probability to exceed the strike price to finish in the money which leads to a higher Black–Scholes implied volatility at high strike.
2. An OTM put option has a close to nil intrinsic value and the put option price is due mainly to time value. Because of the thicker tail on the left, we expect the probability that the OTM put option finishes in the money to be higher than that for a normal distribution and hence the put

option price (and hence the call option price through put–call parity) should be greater than that predicted by Black–Scholes.

3. Investor risk preference: In some situations, investor risk preference may override the risk neutral valuation relationship. Investors are willing to pay a price that is higher than the fair price because they like the potential payoff and the option premium is so low that mispricing becomes negligible. On the other hand, we also have fund managers who are willing to buy comparatively expensive put options for fear of the collapse of their portfolio value. Both types of behavior could cause the market price of options to violate arbitrage arguments.

4. Leverage effects: A firm's value of equity can be seen as the net present value of all its future income plus its assets minus its debt. These constituents have very different relative volatilities which gives rise to a leverage related skew.
5. Supply and demand: Equivalently, downwards risk insurance is more desired due to the intrinsic asymmetry of positions in equity: by their financial purpose it is more natural for equity to be held long than short, which makes downwards protection more important.
6. Liquidity and Market imperfections: Usually, a limited number of market prices are available. Because of the illiquidity in the option prices and market imperfections and frictions, such as transaction costs and other trading restrictions cause the prices deviate from Black Scholes value.
7. Model Risk: Given that some of the Black–Scholes assumptions have been shown to be invalid, there is also a model risk. If option writers are aware of this model risk and mark up option prices accordingly, the Black–Scholes implied volatility will be greater than the true volatility.
8. Declining stock prices are more likely to give rise to massive portfolio rebalancing (and thus volatility) than increasing stock prices. This asymmetry arises naturally from the existence of thresholds below which positions must be cut unconditionally for regulatory reasons.
9. Risk managers use stress scenarios defined on the IVS to visualize and quantify the risk inherent to option portfolios.

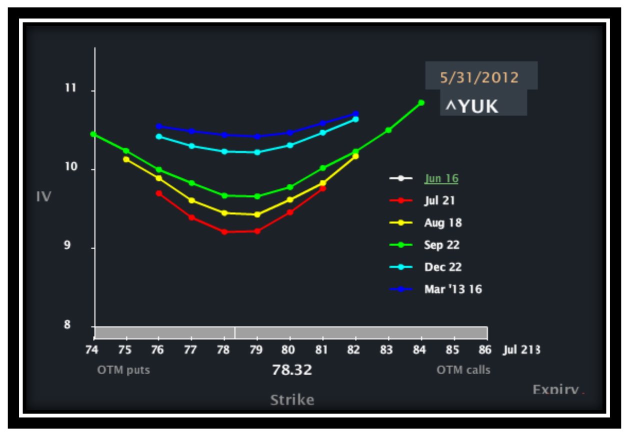

Figure 4 APPL Vol Skewness

Disturbances in the price process of the underlying assets that do not follow a constant volatility

Specific reasons for FX and interest rate skews

Economic effects giving rise to an FX skew and smile

1. Anticipated government intervention to stabilize FX rates.
2. Government changes that are expected to change policy on trade deficits, interest rates, and other economic factors that would give rise to a market bias.
3. Foreign investor FX rate protection.

Economic effects giving rise to an interest rate skew and smile

1. Elasticity of variance and/or mean reversion. Unlike equity or FX, interest rates cannot be split, bought back or re-valued and it is this intrinsic difference that connects volatilities to absolute levels of interest rates.
2. Anticipated central bank action.

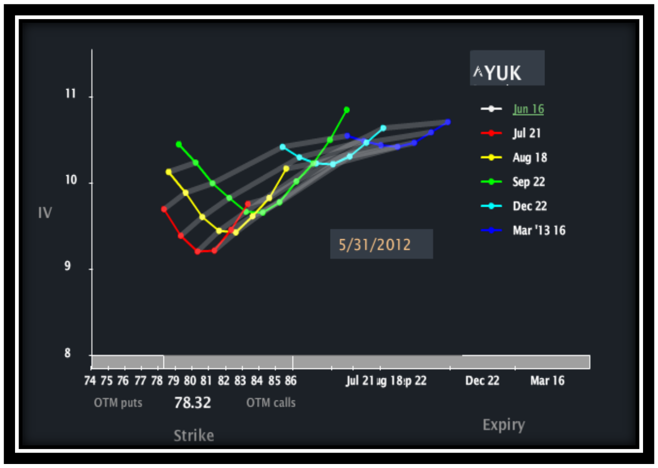

Figure 5 JPYUSD Vol Curves

Term structure of volatility

Collection of smiles for different maturities is considered as smiling surface. Most derivative markets exhibit persistent patterns of volatilities varying by strike. In some markets, those patterns form a smile curve. In others, such as equity index options markets, they form more of a skewed curve. It shows following general characteristics

- Implies volatilities are steepest for the shorter expirations and shallower for long expiration.
- Lower strike and higher strikes has higher volatilities than the ATM. implied volatilities.
- Implied volatilities tend to rise fast and decline slowly.
- Implied volatility is usually greater than recent historical volatility.

Characteristics of the equity implied volatility smile details

1. Volatilities are steepest for small expirations as a function of strike, shallower for longer expirations as evident in figure 6.
2. The minimum volatility as a function of strike occurs near atm strikes o strikes corresponding to slightly OTM call options.
3. Low strike volatilities are usually higher than high-strike volatilities, but high strike volatilities can also have.
4. The term structure is usually increasing but can change depending on views of the future. After large sudden market declines, the implied volatility out-of the-money calls may be greater than for ATM calls, reflecting an expectation that the market may rebound.

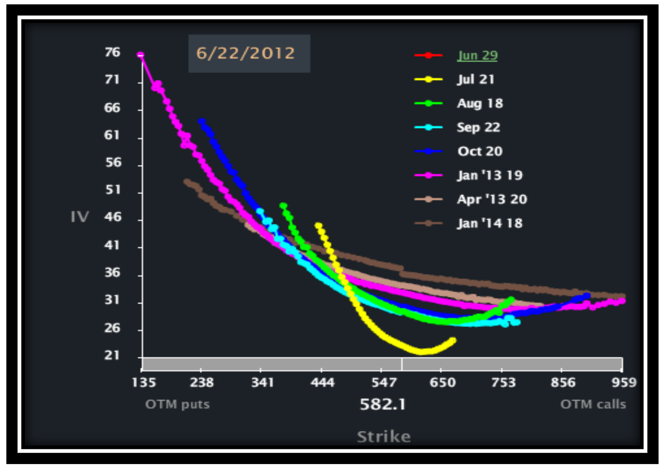

Figure 6 SPX options Vol Term Structure

5. The volatility of implied volatility is greatest for short maturities, as with Treasury rates.
6. There is a negative correlation between changes in implied ATM volatility and changes in the underlying asset itself.

Figure 7 SNP 500 vs SPX 3m Implied Vol Inverted relation

7. Implied volatility appears to be mean reverting with a life of about 60 days.
8. Implied volatility tends to rise fast and decline slowly.
9. Shocks across the surface are highly correlated. There are a small number of principal components or driving factors.
10. The IV surface term structure is upward sloping in calm times and typically downward sloping in times of crisis. During the financial crisis the term structure slope achieved unprecedented levels.

Figure 8 SPX 1M Implied Vol vs 3m Implied Vol

Building a Implied Volatility Smile Surface

Volatility surface can be built from observed option quotes. However most options markets do not have liquid quotes for many OTM options. In practice, it is common that the only observed volatilities in the market are coming from a limited set of broker quotes which are quite restrictive in terms of coverage of strike prices and forward contracts. For longer term maturities and deep OTM options, the data tends to be quite scattered and sometimes unreliable unless it is updated regularly and there are actual traded contracts at those prices.

In order to build and update the surface, one needs to get quoted options and prevailing forward prices. Many firms operating in options markets build a volatility surface with end-of-day closing prices to perform mark-to-market calculations. An options book may have options with different strikes than the ones currently traded. For example, the ATM options at some point in time may become well ITM or OTM in the future. A similar problem is faced by option market makers and option traders. An options market-maker needs to be able to provide bid and offer prices on the options that he is making a market for.

Option pricing with volatility surface is worked out with following steps

- Convert Option prices to Implied Volatility Surface from observed market option prices
- Fit Models to IVS
- Convert IVS based models back to option prices

Updating Volatility Smiles and Surfaces for changes in market prices

After the volatility surface has been calibrated, we need to be able to "update" it in order to take into account changes in market forward prices and implied volatilities for certain traded contracts.

Therefore, instead of working with volatiles as a function of strikes (for a fixed contract expiry) internally we convert those into volatilities as a function of a moneyness parameter, given a specified forward curve.

Once that implied volatility surface has been updated, we may be interested in extracting implied volatilities as a function of strikes, “moneyness” or expiration dates. In order to update the volatility surface, we need to define what constitutes a reasonable model to describe the evolution of implied volatilities for a particular strike as market forward prices change. There are three main approaches used by practitioners:

Sticky Moneyness (or log-Moneyness)

The basic assumption of this approach is that, while the implied volatility as a function of strike does not adequately capture volatility market movements, the implied volatility as a function of “moneyness” parameter does. In essence, due to a move in the market forward curve we should move along the smile. Many traders express moneyness based on the log of the forward price vs. the strike. For a strike K and a forward price FT with expiry T we define the associated moneyness as log(FT/K).

The implied volatility for a particular combination of strike price and maturity is calculated from the “moneyness” surface by translating that combination of strike and current market prices into “moneyness” terms and using a particular set of interpolation or extrapolation schemes.

Sticky Delta

Another common approach is to define the degree of moneyness in terms of the delta of the options. This implies that the volatility is stuck to the delta of a particular strike.

This approach has the advantage that we can take into account the passage of time when building the moneyness surface, as time enters the calculation of the delta. However, this comes with an added pitfall because deltas depend on the volatility parameter, and therefore we need to assume a starting level of volatility to determine the delta that would give the right implied volatility in the surface.

Sticky Strike

Another approach assume that the implied volatility for an option with a given strike and maturity does not react to changes in forward prices. This approach is used in equity markets under certain conditions, but it is not very applicable to the energy world. This would be equivalent of not using the prevailing forward curve to update the volatility surface.

Volatility Surface Modeling

Volatility Surface fitting is required due to above mentioned complications in the presence of smile/skewness. Though original market data set does not have arbitrage, the constructed volatility surface may not be arbitrage free. The trading desks need to price European options for strikes and maturities not quoted in the market, as well as pricing and hedging more exotic options by taking the smile into account. There are several practical reasons to have a smooth and well-behaved implied volatility surface (IVS).

Typical approaches used by financial institutions are based on:

1. Local / stochastic volatility models
2. Direct modeling of dynamics of the implied volatility
3. Parametric or semi-parametric representations
4. Specialized interpolation methodologies

In order to obtain an implied volatility for arbitrary strikes and maturities, the practice is to interpolate or smooth the discrete data. This can be done either with a parametric form or in a non-parametric way. It is common practice in many banks to use (piecewise) polynomial functions to fit the implied volatility smile. These choices are driven sometime by convenience over fundamental consideration.

To analyze how well these models behave with empirical dataset we are going to discuss both parametric and non parametric approach widely used in financial institutions for volatility surface modeling. Before discussing the implementation details will touch some parts of theoretical details for following.

1. SVI parameterization for Fitting of SPX Options
2. Least squares kernel estimator which smoothes implied volatility in the option price space

Parametric model of implied volatility

Gatheral SVI parameterization

The ***Stochastic volatility inspired*** or SVI parameterization of the implied volatility surface was originally devised at Merrill Lynch in 1999 and subsequently publicly disseminated. Jim Gatheral introduced a parameterization (SVI) model motivated by the asymptotic behavior of the implied volatility smile at extreme strikes. He demonstrated that this parameterization fits option prices generated by many theoretical models. These models include stochastic volatility and pure jump models.

For a given parameter set $\chi R = \{a, b, \rho, m, \sigma\}$, the raw SVI parameterization of total implied variance reads:

$$\mathrm{var}(k; a, b, \sigma, \rho, m) = a + b\{\rho (k - m) + \sqrt{(k - m)^2 + \sigma^2}\} \qquad \textbf{(1.11)}$$

With left and right asymptotes given by:

$$\mathrm{var}_L(k; a, b, \sigma, \rho, m) = a - b(1 - \rho)(k - m) \qquad \textbf{(1.12)}$$

$$\mathrm{var}_R(k; a, b, \sigma, \rho, m) = a + b(1 + \rho)(k - m) \qquad \textbf{(1.13)}$$

Variance is always positive
Variance increases linearly with $|k|$ for k very positive or very negative
Intuition is that the more out-of-the-money an option is, the more volatility convexity it has.

Parameter	**Description**
var()	the implied variance of prices of vanilla options at particular time to maturity(T),
k	log of the moneyness,
a	the overall level of variance,
b	the angle between the left and the right asymptotes,
σ	describes the smoothness of the vertex,
ρ	describes the orientation of the graph
M	describes how the graph can be translated

Table 1 – SVI parameters

In practice, if SVI is fitted to actual option price data, negative vertical spreads never arise. In terms of the SVI (implied variance fit) parameters, the condition is

The condition that should be satisfied to prevent arbitrage is

$$b(1 + |\rho|) \leq 4/T \qquad \textbf{(1.14)}$$

It turns out that if there are no negative vertical spreads, negative butterflies are also excluded. In contrast, it is not obvious how to prevent negative time spreads. Gatheral, also shown what conditions a parameterization needs to satisfy to exclude arbitrage between expirations (calendar spread arbitrage).

Non Parametric model of implied volatility

Least squares kernel estimator

Once we have the option data for market implied volatilities these are filter to certain condition to keep only ATM/OTM options data and remove extreme illiquid option data. The filtered data set is used to construct for each day a smooth estimator of the implied volatility surface, defined on a fixed grid, using a non-parametric Nadaraya–Watson estimator.

Let $m(x)$ be the function which gives the relationship between an explanatory variable X and a dependent variable Y.

$$m(x) = E[\, Y|X = (x)] \qquad \textbf{(1.15)}$$

Other than certain regularity and smoothing assumptions no special assumption on the form of function m is made. One way to estimate m is to use the method of local polynomial regression (LP Method). The idea is based on the fact that the function m can be locally approximated with a Taylor polynomial, i.e., in a neighborhood around a given point x_0 it holds that

$$m(x) \approx \sum_{k=0}^{p} \frac{m^{(k)}(x_0)}{k!}(x - x_0)^k \qquad \textbf{(1.16)}$$

In order to find an estimate for m at point x_0, one therefore tries to find a polynomial based on observations, $(X_1, Y_1) \dots (X_n, Y_n)$ that is a good approximation of m around x_0. As a measure of the quality of the approximation one usually chooses a LS criterion, i.e., one wants to minimize the expression

$$\sum_{i=1}^{n} \left\{ Y_i - \sum_{j=0}^{p} \beta_j (X_i - x_0)^j \right\}^2 \qquad \textbf{(1.17)}$$

with respect to $\beta = \left(\beta_0, \dots, \beta_p\right)^T$. Since the representation holds only locally, one still has to take into consideration that some of the observations X_i may not be close enough to x_0 and thus above equation no longer applies to them. One must then sufficiently localize the observations, i.e., only consider those observations that lie close enough to x_0.

One of the classical methods for localization is based on weighting the data with the help of a kernel. A kernel is a function $K : \mathbb{R} \to [0, \infty)$ with $\int K(u)du = 1$. The most useful kernels are also symmetric and disappear outside of a suitable interval around the zero point.

If K is a kernel and h > 0

$$K_h(u) = \frac{1}{h} K\left(\frac{u}{h}\right) \qquad \textbf{(1.18)}$$

and the kernel K_h is re-scaled with the bandwidth h, which again integrates to 1. If, for example, the initial kernel K disappears outside of the interval [−1, 1], then $K_h$is zero outside of the interval [−h, h]. By weighting the i_{th} term in with $K_h(x - X_i)$, one has a minimization problem which, due to the applied localization, can be formulated to be independent of the point x_0.The coefficient vector $\hat{\beta} = \hat{\beta}(x) = (\hat{\beta}_0(x), .., \hat{\beta}_p(x))^\intercal$ that determines the polynomial of the point x is thus given by

$$\hat{\beta} = \arg\min_{\beta} \sum_{i=1}^{n} \left\{Y_i - \sum_{j=0}^{p} \beta_j (x - X_i)^j\right\}^2 K_h(x - X_i) \qquad \textbf{(1.19)}$$

It is obvious that $\hat{\beta}$ depends heavily on the choice of kernel and the bandwidth. With the representation the solution $\hat{\beta}$ to the weighted least squares problem can be explicitly written as

$$\hat{\beta}(x) = (X^\intercal W X)^{-1} X^\intercal W Y \qquad \textbf{(1.20)}$$

The estimation $\hat{m}(x)$ for $m(x)$ can be obtained only by calculating the approximating polynomial at x:

$$\hat{m}(x) = \hat{\beta}_0(x) \qquad \textbf{(1.21)}$$

In order to apply the LP Method mentioned above, consider a given sample Y_0 … Y_n. as observations of the form$(Y_0, Y_1), \ldots, (Y_{n-1}, Y_n)$. The process (Y_i) must fulfill certain conditions, so that these observations are identically distributed and in particular so that the function m is independent of the time index i. Such is the case when (Y_i) is stationary. By substituting $X_i = Y_{i-1}$ into above equation and replacing Y_i with $\lambda(Y_i)$, we obtain in this situation

$$\hat{\beta} = \arg\min_{\beta} \sum_{i=1}^{n} \left\{\lambda(Y_i) - \sum_{j=0}^{p} \beta_j (x - Y_{i-1})^j\right\}^2 K_h(x - Y_{i-1}) \qquad \textbf{(1.22)}$$

and the estimate for m(x) is again given by $\hat{\beta}_0(x)$

$$X = \begin{pmatrix} 1 & (X_i - x) & (X_i - x)^2 & \cdots & (X_i - x)^p \\ & \vdots & & \ddots & \vdots \\ 1 & (X_n - x) & (X_n - x)^2 & \cdots & (X_n - x)^p \end{pmatrix}$$

$$Y = \begin{pmatrix} (Y_i \\ \vdots \\ (Y_n \end{pmatrix}$$

$$W = \begin{pmatrix} K_h(x - X_i) & & 0 \\ & \ddots & \\ 0 & & K_h(x - X_n) \end{pmatrix}$$

The important parameters are the bandwidth parameters h1, h2, which determine the degree of smoothing. Too small values will lead to a bumpy surface; too large ones will smooth away important details. The bandwidth can be determined using a cross-validation criterion. A more efficient method is to use an adaptive bandwidth estimator in order to obtain an ‘optimal’ bandwidth h.

The kernel regression procedures employed in order to obtain our implied volatility surface time series on a given grid of maturities $\{\mathcal{K}_i\}$ and moneyness$\{\mathcal{T}_j\}$.

For a partition of explanatory variables $(x_1, x_2) = (\mathcal{K}, \mathcal{T})$ i.e. of moneyness and maturity, a two-dimensional Nadaraya-Watson kernel estimator of is given by

$$\hat{\sigma}(x_{1,}x_2) = \frac{\sum_{i=1}^{n} K_1\left(\frac{x_1 - x_{1i}}{h_1}\right) K_2\left(\frac{x_2 - x_{2i}}{h_2}\right) \hat{\sigma}_i}{\sum_{i=1}^{n} K_1\left(\frac{x_1 - x_{1i}}{h_1}\right) K_2\left(\frac{x_2 - x_{2i}}{h_2}\right)} \qquad \mathbf{(1.23)}$$

For implementation we have used quadratic kernel of order 2 is used, i.e.

$$K_i(u) = \frac{15}{16}(1 - u^2)^2 I(|u| \leq 1) \qquad \mathbf{(1.24)}$$

As from an empirical point of view, the choice of the kernel function has little influence on the results this choice was lead by reasoning that the quadratic kernel generally behaves well in practical applications.

Programming Language Choice

MATLAB is chosen for the model development and charting since it as a high-performance language which integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation.

Dataset

Options dataset used is taken from end of prices of European style call and put options on SP500 Index. Daily historical data for SPX option chain is taken from IVolatility.com (paid service) for a period of 1 year. For the purpose of our analysis, I reviewed daily data from 2-May-11 –30- Mar-2012. We used this data to model implied volatility surface using non parametric regression. The data has irregularities which are first fitted a smooth parametric surface to each set of observations before investigating how this surface fluctuates over time.

Data Sources

Data Description	**Data Source**	**Website Link**
S&P 500 and other contracts option chains	Yahoo Finance	http://finance.yahoo.com/q/hp?s=SPX
VIX: Daily Closing Values	Chicago Board of Options Exchange (CBOE)	http://www.cboe.com/micro/VIX/historical.aspx
History data for S&P 500	IVolatility.com	http://www.ivolatility.com/info/aboutus.html

Implementation part A - Volatility Surface with kernel smoothing

Volatility Surface matlab based GUI is developed (explained in upcoming section) which allows to load the historical volatility surfaces for analysis. VolSurface display raw surface vs. smoothened surface using non parametric regression smoothing approach over a grid of option moneyness, maturities and implied Volatilities

Working Explanation

1. VolSurface pulls SPX options data for selected date to Matlab vectors.
2. It then filters it for OTM calls and puts. We filter away in and at the money options in our data processing module.
3. The filtered data set is used to construct for each day a smooth estimator of the implied volatility surface, defined on a fixed grid, using a non-parametric Nadaraya–Watson estimator considering only actively traded options on given day.
4. Time to maturity τ range between a month and a year.
5. Moneyness values outside the interval [0.8, 1.2] are filtered out since the numerical uncertainty on implied volatility may be too high and the liquidity very low.
6. All options used are out of the money options: Calls are used for $m > 1$, puts for $m < 1$. These are precisely the options which contain the most information about implied volatility movements.
7. The standardized option implied volatilities in the Volatility Surface are calculated using a kernel smoothing technique
8. We use quadratic kernels of order 2, i.e. As from an empirical point of view, the choice of the kernel function has little influence on the results, this choice was lead by reasoning that the quadratic kernel generally behaves well in practical applications.
9. A kernel smoother is then used to generate a smoothed volatility value at each of the specified interpolation grid points. At each grid point j on the volatility surface, the smoothed volatility i is calculated as a weighted sum of option implied volatilities.
10. Please refer to attached VolSurface package from attached CD in directory SurfaceGUI\src as explained in VolSurface GUI Section.
11. Generated charts can be located under SurfaceGUI\charts\surface. It will generate Charts for volatility surfaces both raw and smoothened surface for the selected date. For historical dates up to a year back we can trace both market volatility surface vs smoothened volatility surface using non parametric regression.
12. Some amount of back testing is carried over to validate smoothened volatility surface by validating surface prices to the actual option prices generating Monte Carlo simulation for asset price. For calibration of the model the important parameters are the bandwidth parameters h1, h2, which determine the degree of smoothing. Too small values will lead to a bumpy surface; too large ones smooth away important details.
13. For bandwidth selection we employed the procedure as suggested by Fengler, Hardle and Villa. And found model behaves well when h1=0.1, h2=0.35 (h2=0.35 maturity <= 3m and h2=1.1 maturity > 3m)

Implementation part B - SVI parameterization of the implied volatility surface

As opposed to non parametric model SVI parameterization of the implied volatility surface was considered for analysis and implementation. Stochastic volatility inspired or SVI was originally devised at Merrill Lynch in 1999 and subsequently publicly disseminated.

Working Explanation

1. Again SPX options are used for analysis. Volatility Surface matlab based GUI is developed to support (explained in upcoming section) SVI parameterization of the implied volatility surface. Please refer to attached VolSurface package from attached CD in directory SurfaceGUI\src as explained in Vol Surface GUI description section.
2. Generated charts can be located under SurfaceGUI\charts\svi\date.
3. Given mid implied volatilities $\sigma ij = \sigma BS(ki, tj)$, compute mid option prices using the Black-Scholes formula.
4. Fit the square-root SVI surface by minimizing sum of squared distances between the fitted prices and the mid option prices taking it as initial guess.
5. Starting with the square-root SVI initial guess, change SVI parameters slice-by slice so as to minimize the sum of squared distances between the fitted prices and the mid option prices with a big penalty for crossing either the previous slice or the next slice

Arbitrage-free SVI volatility surface model with a simple closed-form representation is implemented. Again good quality of SVI fits is observed using recent SPX options data for the purpose of analysis and testing.

VolSurface GUI Description

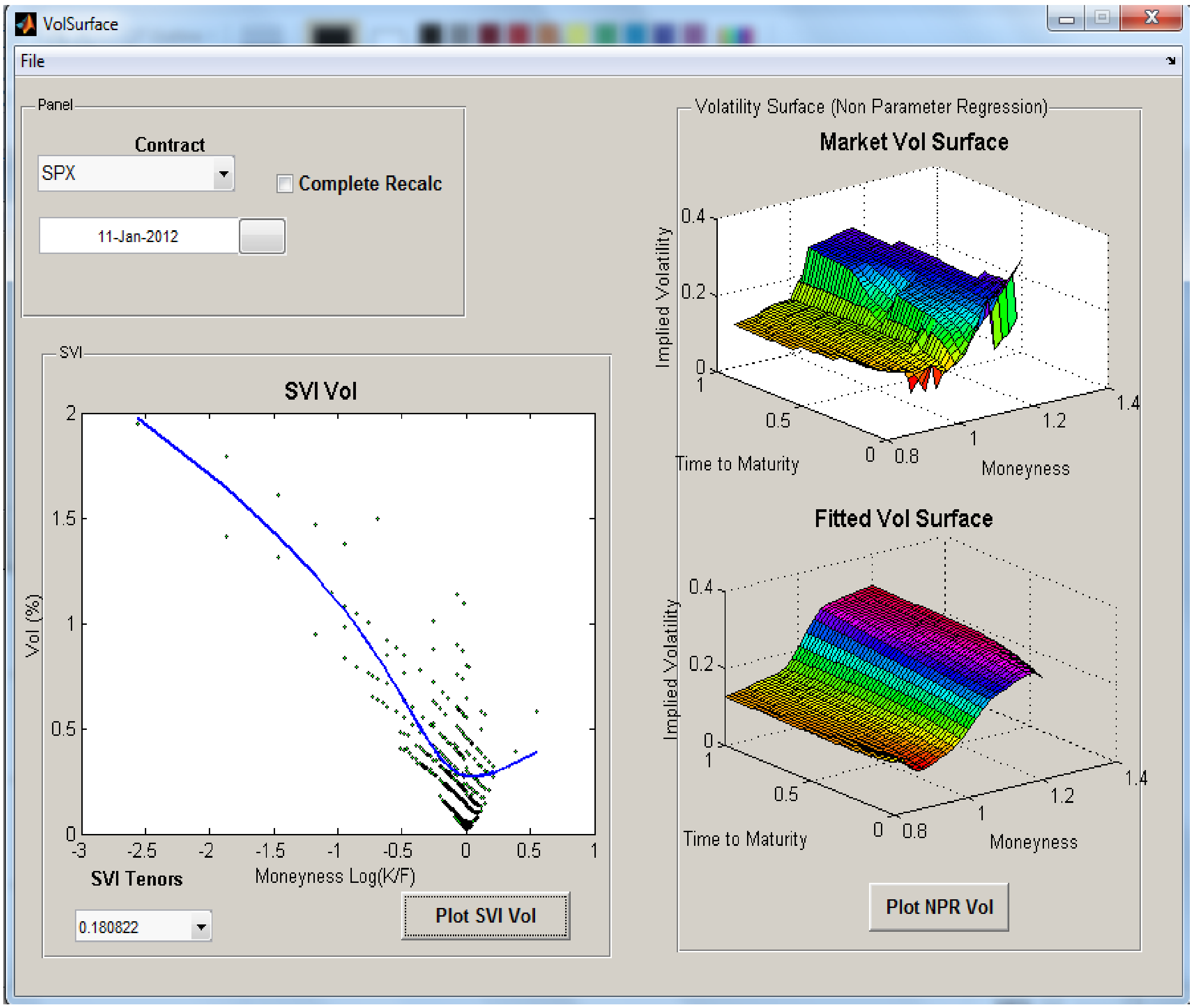

Figure 9 VolSurface GUI

How to run VolSurface GUI:

1. Extract the VolSurface package from attached CD and unzip it.
2. Go to directory extraction directory SurfaceGUI\src.
3. Matlab version 7.9 is used for development/testing.
4. Run the matlab file VolSurface.
   Please give it a minute to come up as it loads the history db for SPX options for last one year.

Modelling and plotting Market vs Smoothened Volatility Surface

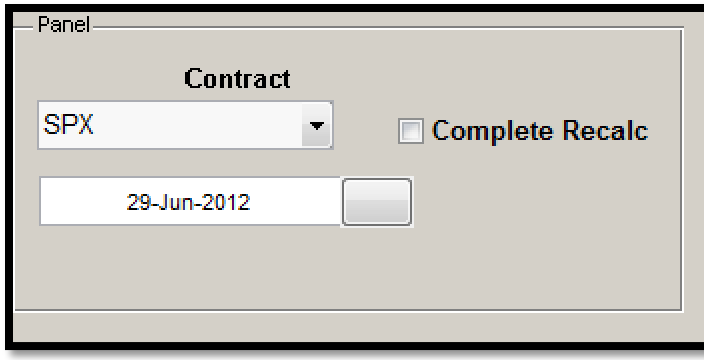

Figure 10

1. Contract SPX is pre selected since VolSurface CD contains data for SPX only.
2. Please select any business date between 2-May-11 to 30-Mar-12
3. Click on button Plot NPR Vol button.
4. It will generate Charts for volatility surfaces both raw and smoothened surface for the selected date. For historical dates up to a year back we can trace both market volatility surface vs smoothened volatility surface using non parametric regression.
5. It uses the data from the mat file db which pre computed database for volatility surface information. It saves significant time for computation in case we need to analyze it several times.
6. In case we want complete computation check the "Complete recalc" tab and again click on button

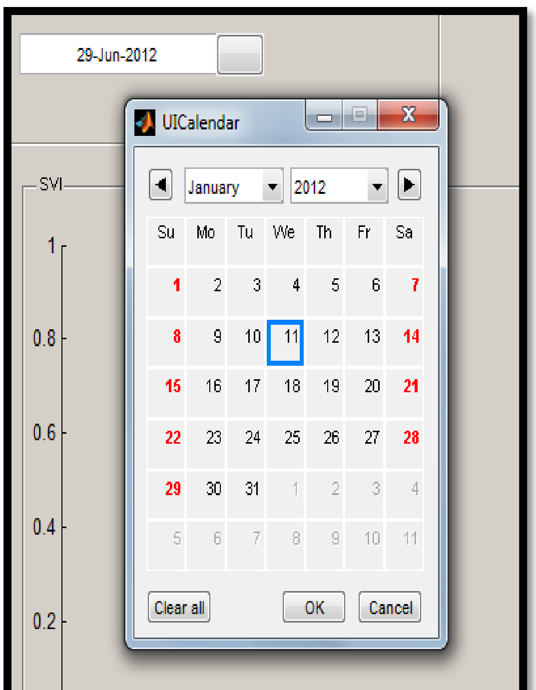

Figure 11 Date Picker

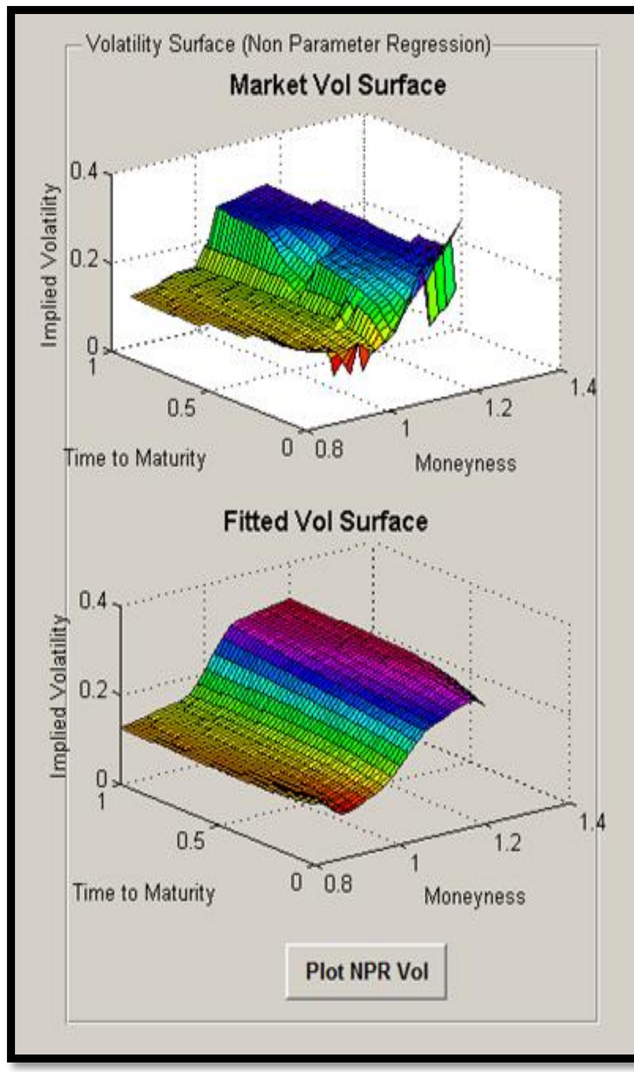

Figure 12 Market vs Smoothened Plots

Plot NPR Vol.

7. Please note it take generally more than a minute to re compute the volatility surface depending on the machine configuration.

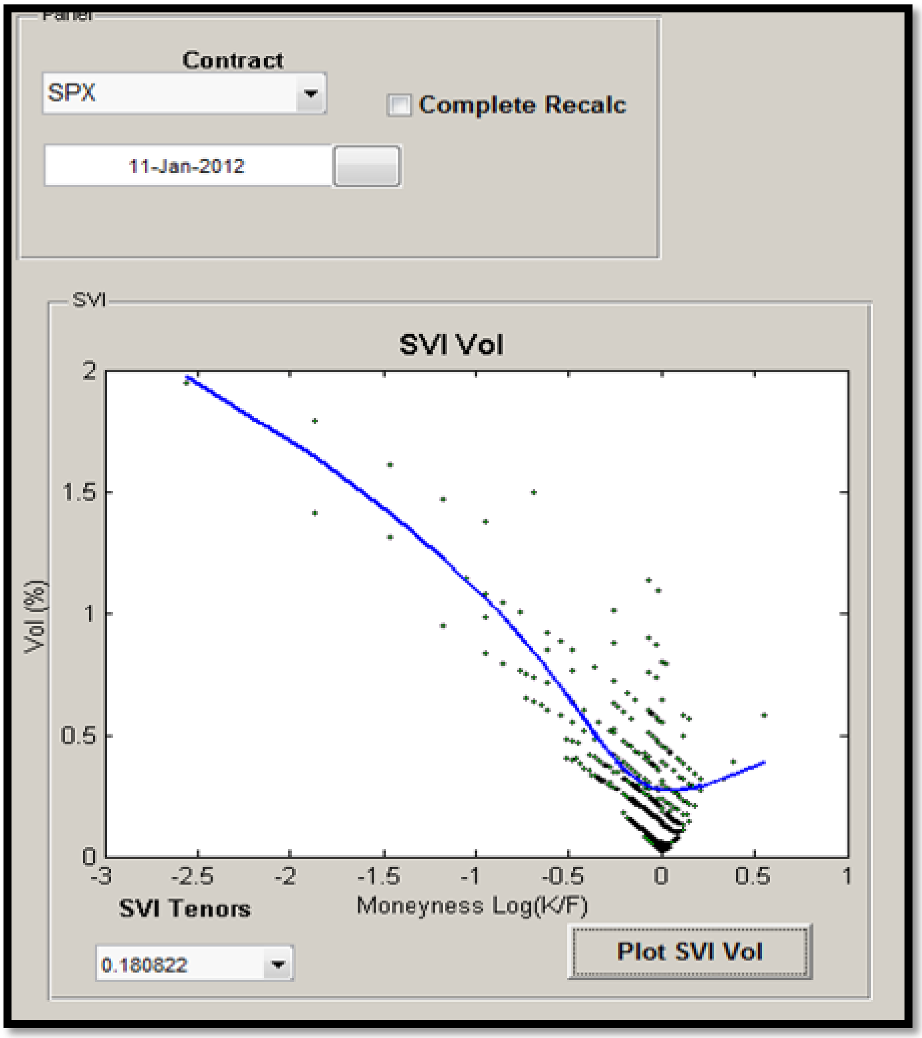

Figure 13 SVI Vol Plotter

Fitting and plotting SVI Vol

1. For SVI Volatility fitting again select the appropriate date
2. In the lower left corner there is tenor selection for maturities available for specific date.
3. Generally there are 14 maturities available for SPX options.
4. Click on Plot SVI Vol button.
5. Date specific graphs for fitted square-root SVI wrt market vols for implied volatilities tenors for all 14 expirations can plot selecting the different tenors.

Software Files Description

Source File s

Directory	Matlab File	Description/Functionality
	VolSurface	Volatility Surface Controller - Allows to load the historical volatility surfaces for analysis - Display raw surface over a grid of option moneyness, maturities and implied Volatilities - Display smooth surface using non parametric regression smoothing approach over a grid of option moneyness, maturities and implied Volatilities - Display SVI Fitted vs. market Volatility using SVI fitting proposed by Jim Gatheral - Allows skipping the complete computation by using back the pre computed surface data being stored in special matlab format files.
src	VolSurface.fig	VolSurface GUI window layout
src\datasrv	loadDataFromYahoo.m	Load Option Chain data from Yahoo web service for given contract
	loadSurface.m	Used by VolSurface controller for loading maturities, moneyness and implied vols
	DataLoader.m	Helps to load the option data from excel db.
src\iv	bisection.m	This function uses bisection method to find implied volatility.
	impliedVol.m	Implied vol helper using blsimpv function in finance package of matlab. Optionally helps to filter options based on moneyness, market volume etc
src\iv	npregression.m	The kernel regression procedures employed in order to obtain implied volatility surface time series on a given grid of maturities MAT and moneyness MON. We use quadratic kernels of order 2 for non parametric regression for surface smoothing
src\iv	sviVol.m	It takes SPX option quotes as of given date and compute implied volatilities for all 14 expirations passed as tenor The result of fitting square-root SVI passed back for plotting. Given mid implied volatilities compute mid option prices using the Black-Scholes formula. It fits the square-root SVI surface by minimizing sum of squared
src\svi	SVIFit.m	This function generates two vectors for plotting the fitting curve. Parameters is a vector of SVI parameters in the order of a, b, sigma,

		rho and m. Moneyness is a vector containing the log of K over F in the same number of the observations.
src\svi	SVIParams.m	This function generates two vectors for plotting the fitting curve. Parameter is a vector of SVI parameters in the order of a, b, sigma, rho and m. Moneyness is a vector containing the log of K over F in the same number of observations.
src\svi	SVIPCalc.m	This function is defined as the objective function in the least square process.
src\vix	SPXvsVIX.m	It takes SNP 500 Index prices and VIX prices from 1990 - 2012 to find relations between index volatility vs. VIX.
src\plot	PlotSVI	Helper to generate SVI vol fitting curves for specific date for available tenors
	PlotVolSurfaces	Helper to generate smooth Vol Surfaces based on non parametric regression for specific date

Table 2 Source Files

Data/DB Files

Directory	Matlab File	Description/Functionality
db	SPX-optionData	Excel file containing SPX option chain history data from 2-May-11 to 30-Mar-12
db	optionData	Non numeric values are changed to Boolean/numeric counterparts for easy access using MATLAB
db	recDates	Helper class to establish relation between dates and numeric values to be used in conjunction with option Data.

Table 3 Data/DB Files

Mat File

Directory	Matlab File	Description/Functionality
mat	npr (example SPX-01-Feb-2012.mat)	Special matlab format files containing both raw and smoothened surface from 2-May-11 to 30-Mar-12.
mat	svi	Placeholder for special matlab files for SVI vol fitting to avoid re computation in case of no parameter change.

Table 4 Mat Files

Charts

Directory	Matlab File	Description/Functionality
Chart	surface	Charts for volatility surfaces both raw and smoothened surface from 2-May-11 to 30-Mar-12. For historical dates up to a year back we can trace both market volatility surfaces vs. smoothened volatility surface using non parametric regression.
chart	svi/date	Date specific graphs for fitted square-root SVI wrt market vols for implied volatilities tenors for all 14 expirations
chart	VIX	Plot for SNP 500 Index prices and VIX prices from 1990 - 2012 to show relations between index volatility vs. VIX.

Table 5 Charts

Conclusions and Further Work

This project report work is kept focused around empirical analysis for implied volatilities on volatility surfaces. Analysis work shown us the grid of observation for moneyness and time to maturity is both irregular and changing with time as the level of the underlying fluctuates. Characteristics study for implied volatility surface is carried out with implementation of 2 specific modeling methods for generating smooth and well-behaved implied volatility surfaces. Implementation is carried with VolSurface package for smoothing volatilities using least squares kernel estimator which smoothes implied volatility in the option price space over a grid of option moneyness, maturities. Also arbitrage-free SVI volatility surface model with a simple closed-form representation is implemented. Both these models are tested on SPX options historical data which shows good quality of fits.

The potential applications of implied volatility surface needs to be tested in the field of arbitrage free pricing of derivative products that it covers for the shortcoming of Black Scholes. Work can be carried over to test the volatility surfaces for arbitrage opportunities in market and observe the behavior of the surface to work out profitable trading strategies for given portfolios. Another potential area of work could be to work on derivative sensitivities and hence quantify and hedge volatility risk of the portfolio. I hope to pursue some of these research topics in the near future.

References

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4. R. Lee, “Implied Volatility: Statics, Dynamics, and Probabilistic Interpretation”. Recent Advances in Applied Probability, 2005.
5. CHICAGO BOARD OPTIONS EXCHANGE. 2003. The New CBOE Volatility Index – VIX. www.cboe.com/micro/vix/vixwhite.pdf
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7. Xu, X. and S. J. Taylor, 1994, “The Term Structure of Volatility Implied by Foreign Exchange Options,”
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9. ”Journal ofHeynen, R., A. Kemna and T. C. F. Vorst, 1994, “Analysis of the Term Structure of Implied Volatilities,”
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11. Kamal, Michael, and Emanuel Derman (1997). The patterns of change in implied index volatilities, in Goldman Sachs Quantitative Re- search Notes, Goldman Sachs, New York.
12. http://finance.yahoo.com/q/hp?s=SPX
13. http://www.cboe.com/micro/VIX/historical.aspx
14. http://www.ivolatility.com

APPENDIX

SPX Options Market vs Smoothened Surfaces (least squares kernel estimator)

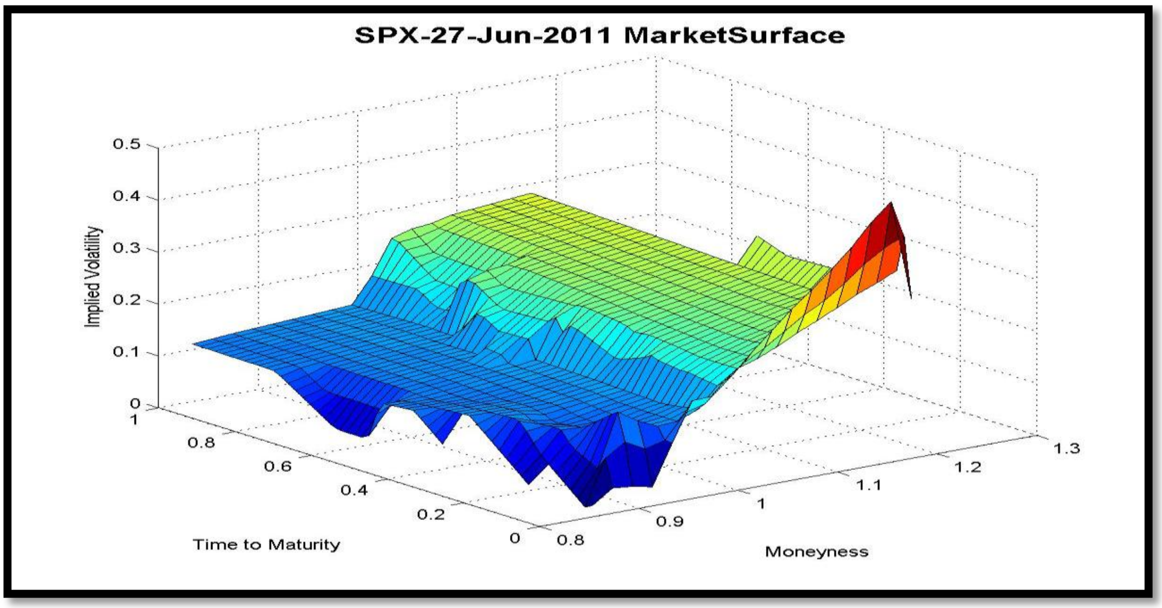

Figure 14 SPX-27-Jun-2011 Market Vol Surface

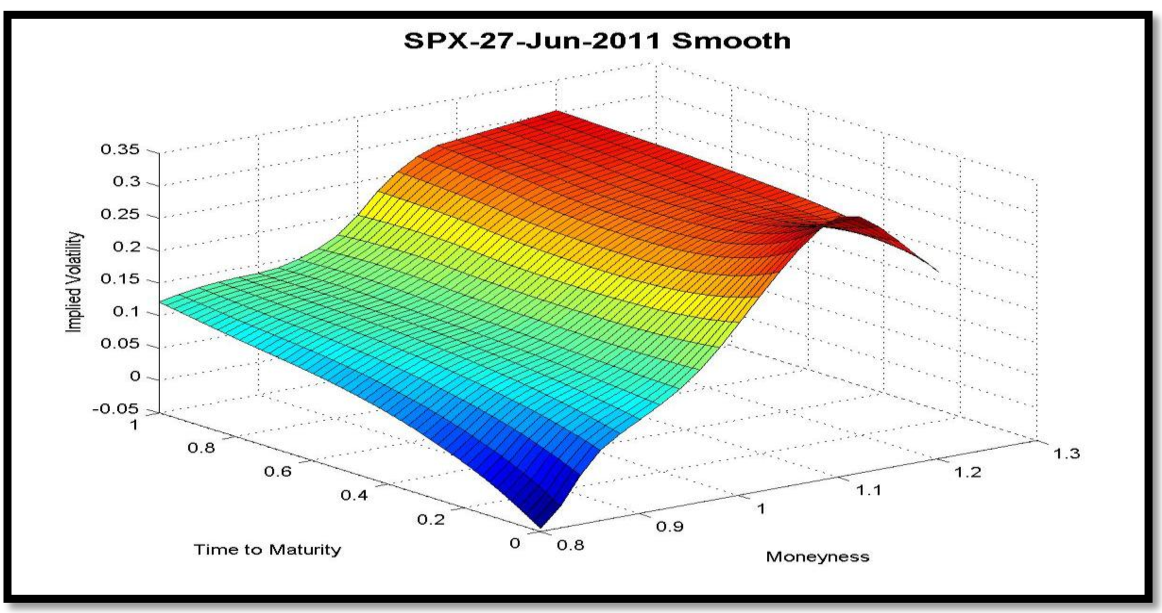

Figure 15, SPX-27-Jun-2011 Smooth Vol Surface

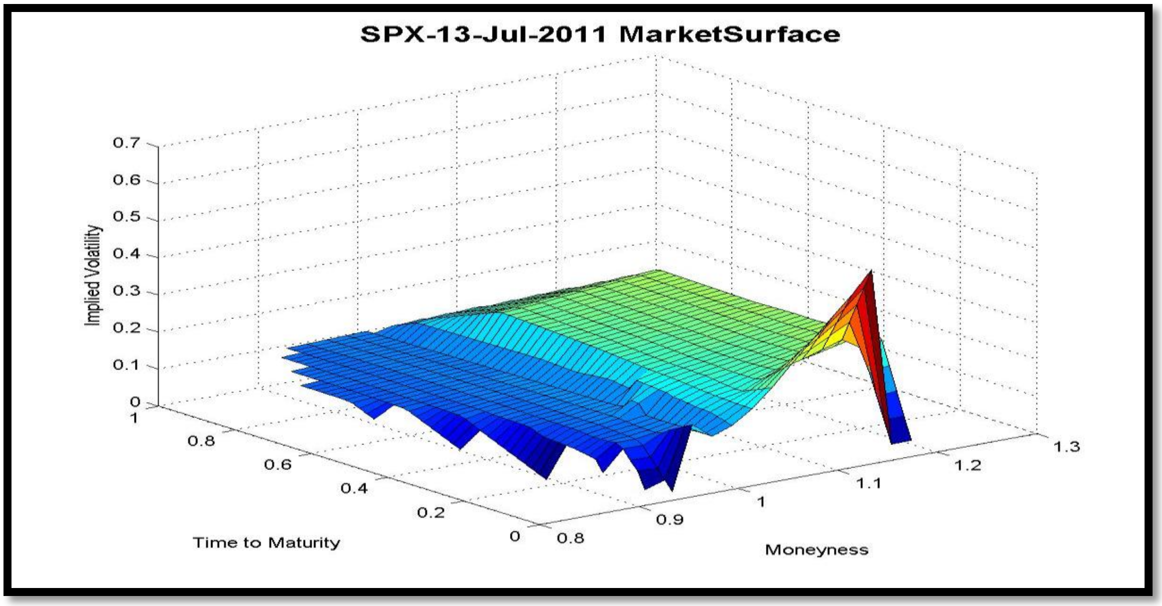

Figure 16, SPX-13-Jul-2011 Market Vol Surface Vol Surface

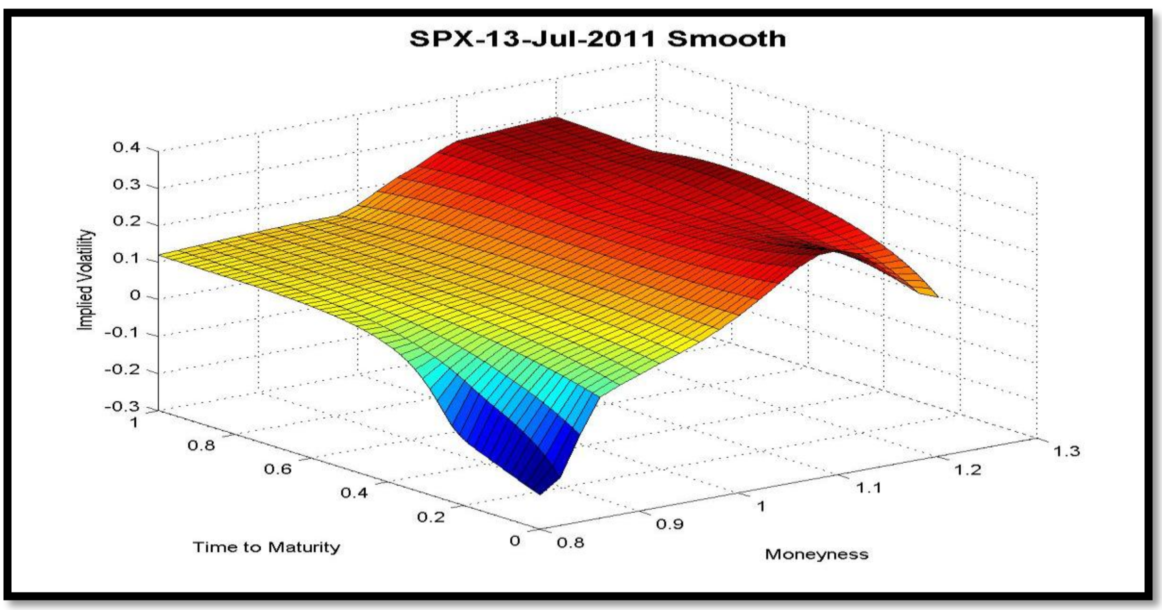

Figure 17, SPX-13-Jul-2011 Smooth

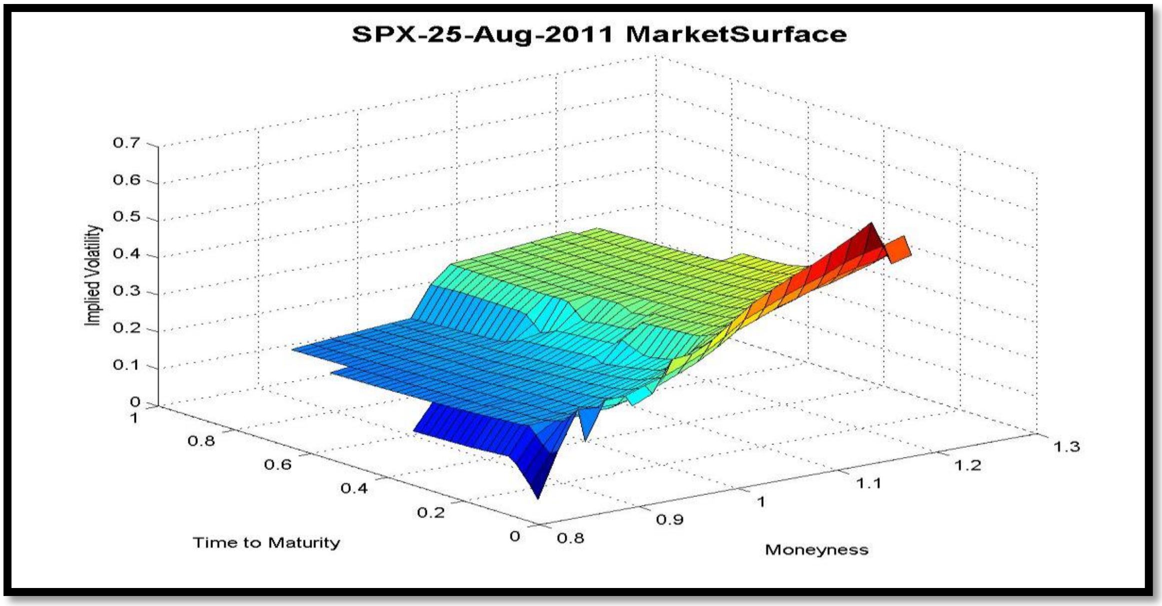

Figure 18, SPX-25-Aug-2011 Market Vol Surface

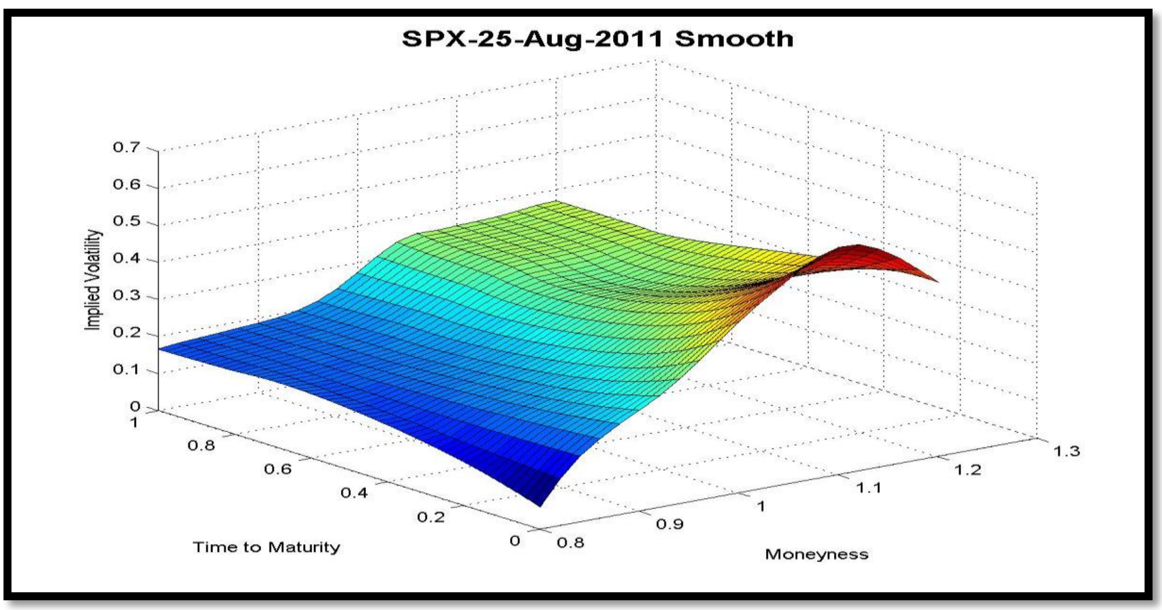

Figure 19, SPX-25-Aug-2011 Smooth Vol Surface

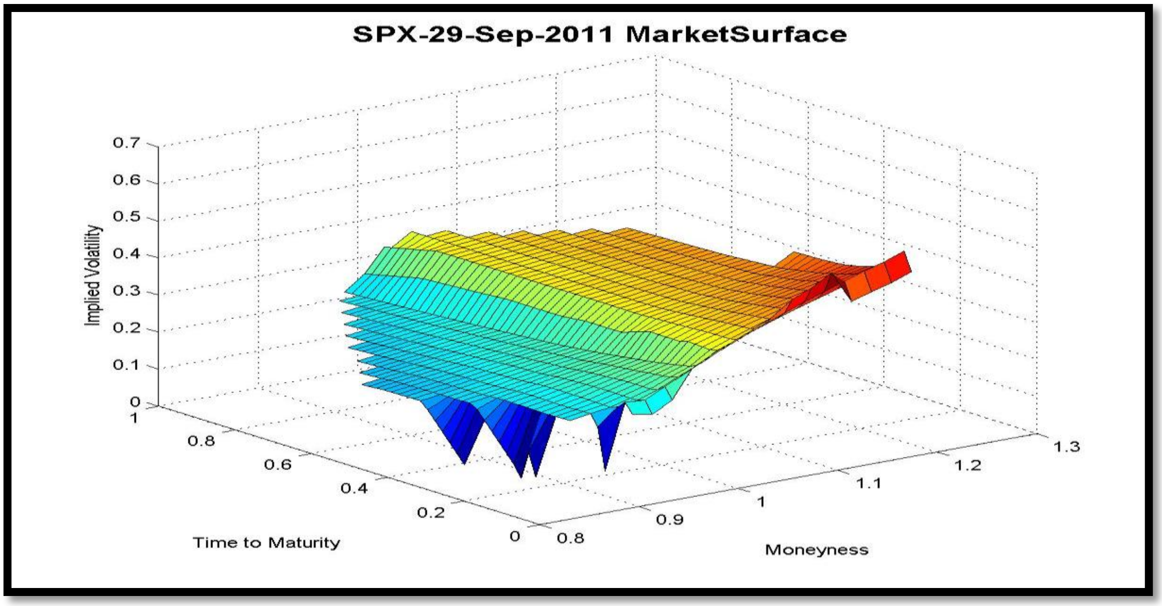

Figure 20, SPX-29-Sep-2011 Market Vol Surface

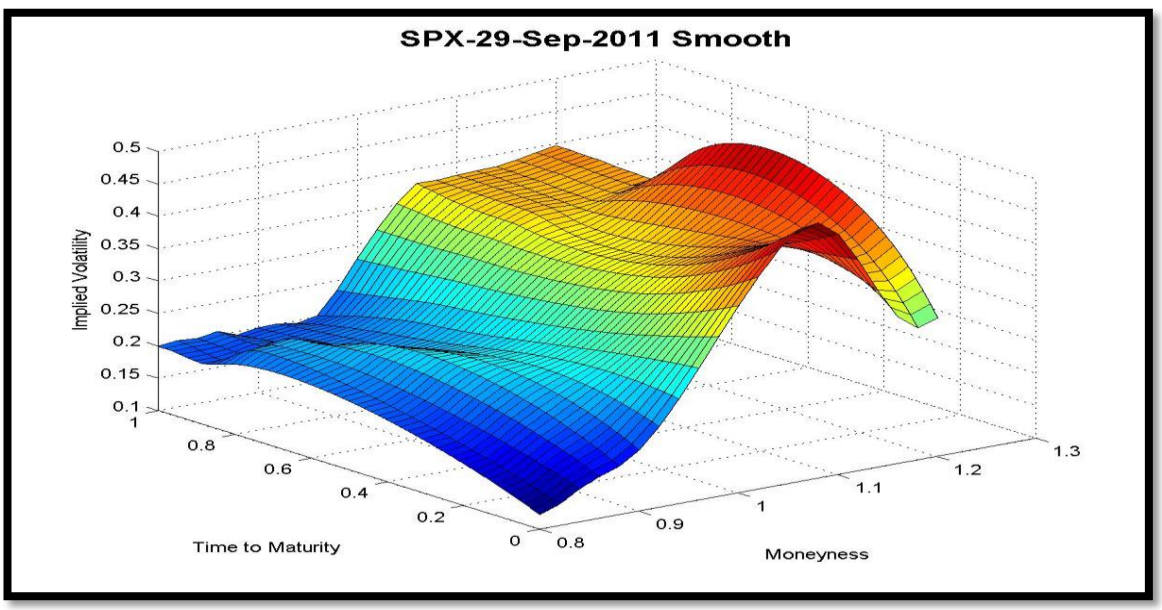

Figure 21, SPX-29-Sep-2011 Smooth Vol Surface

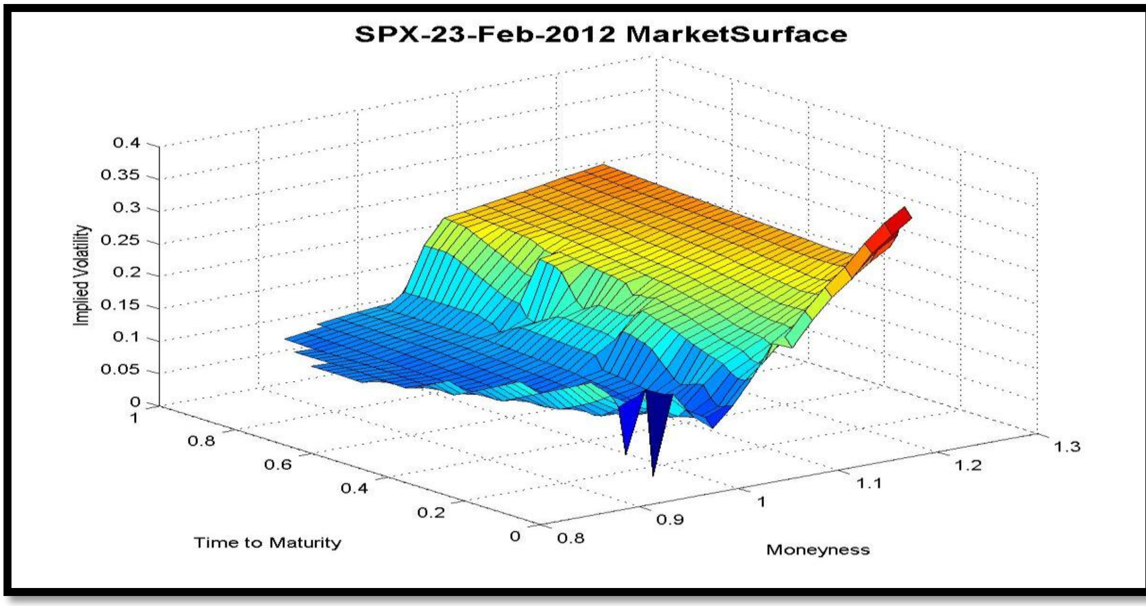

Figure 22, SPX-23-Feb-2012 Market Vol Surface

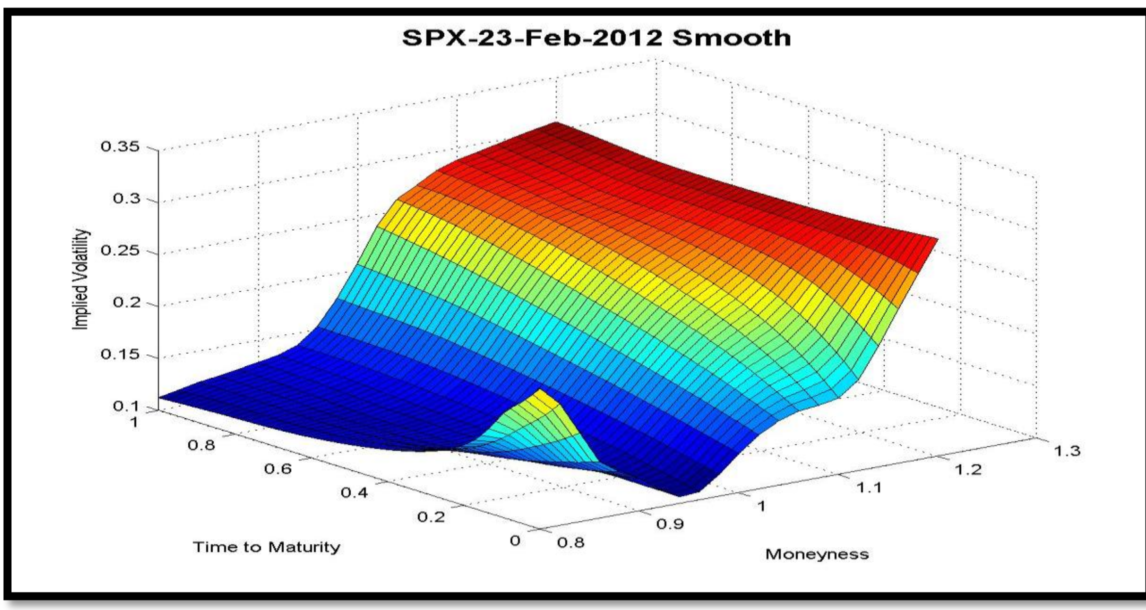

Figure 23, SPX-23-Feb-2012 Smooth Vol Surface

SPX Option SVI Fitting plots for different Tenors

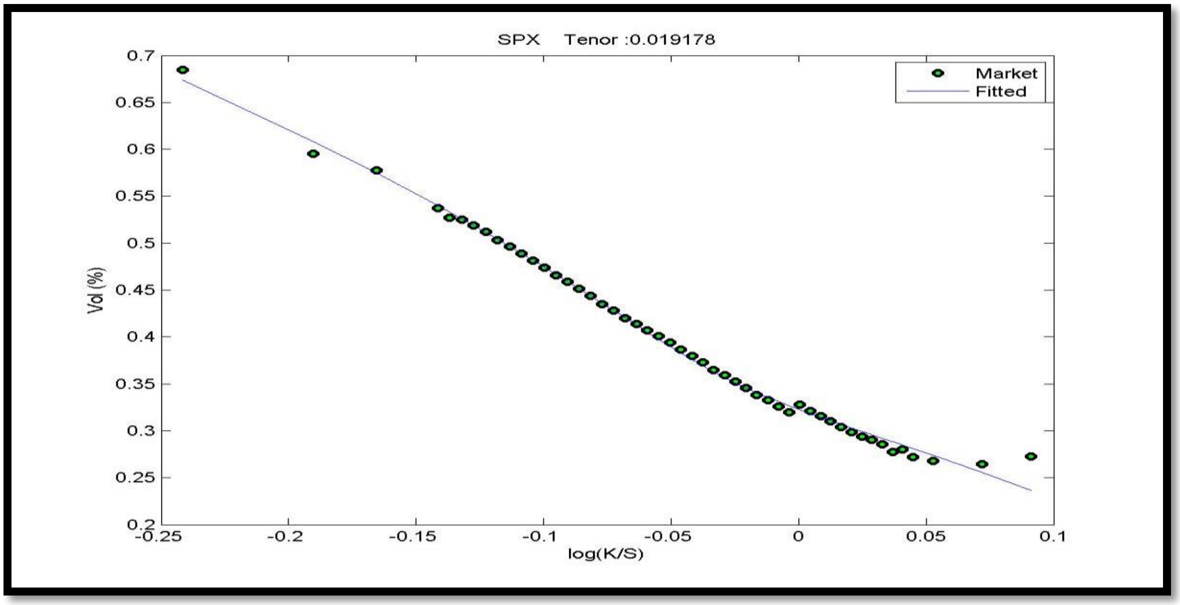

Figure 24 **SPX Option SVI fitting (Tenor: 0.019)**

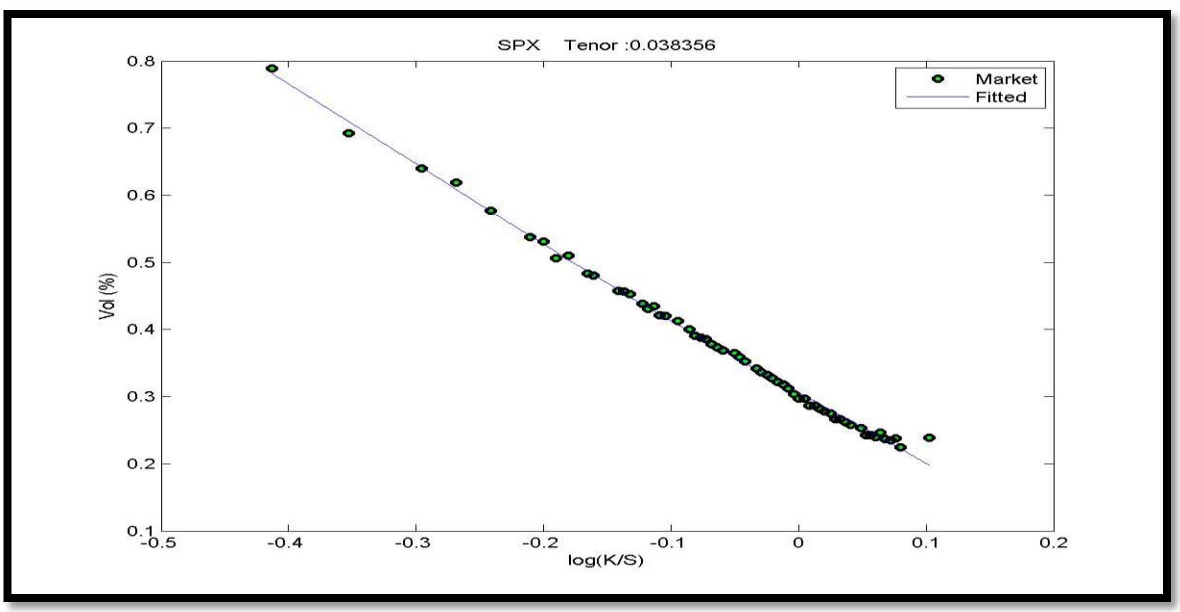

Figure 25 **SPX Option SVI fitting (Tenor: 0.038)**

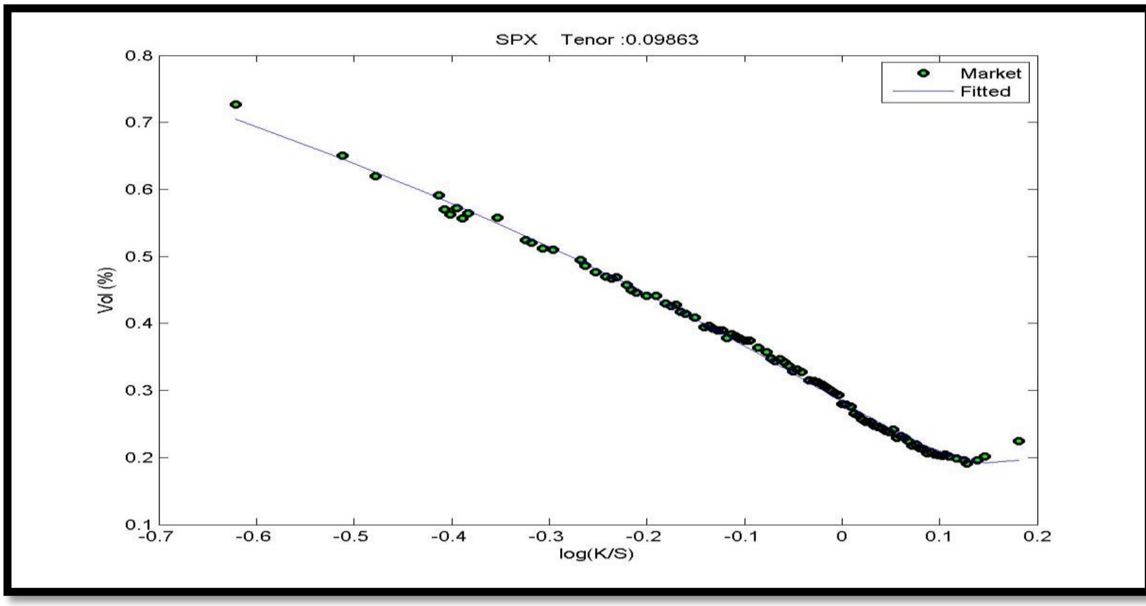

Figure 26 **SPX Option SVI fitting (Tenor-0.099)**

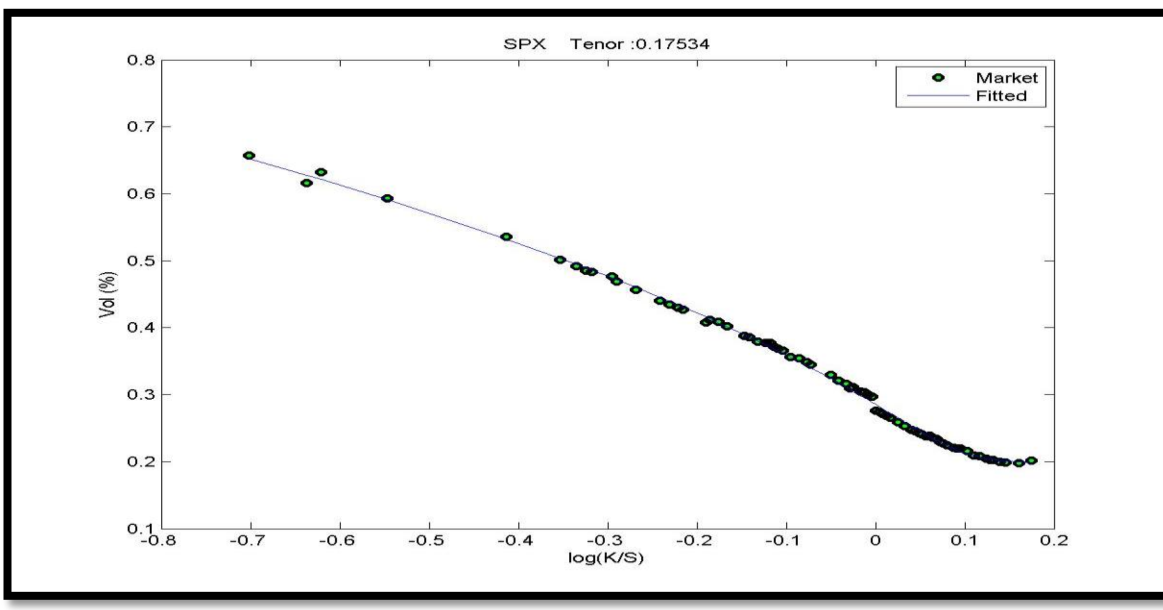

Figure 27 **SPX Option SVI fitting (Tenor-0.175)**

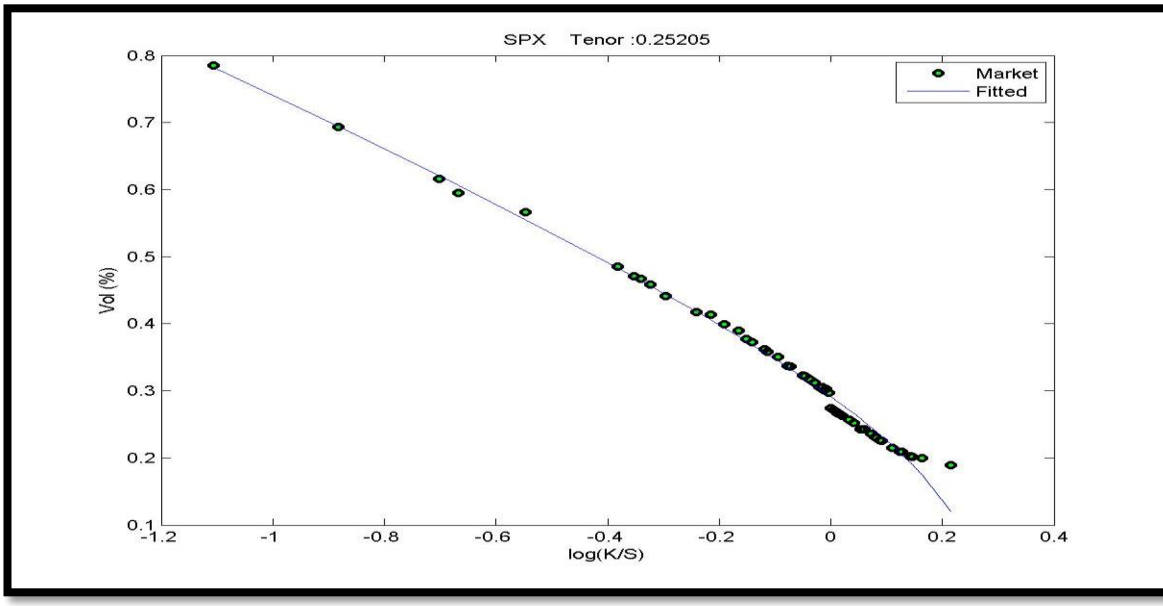

Figure 28 **SPX Option SVI fitting (Tenor-0.252)**

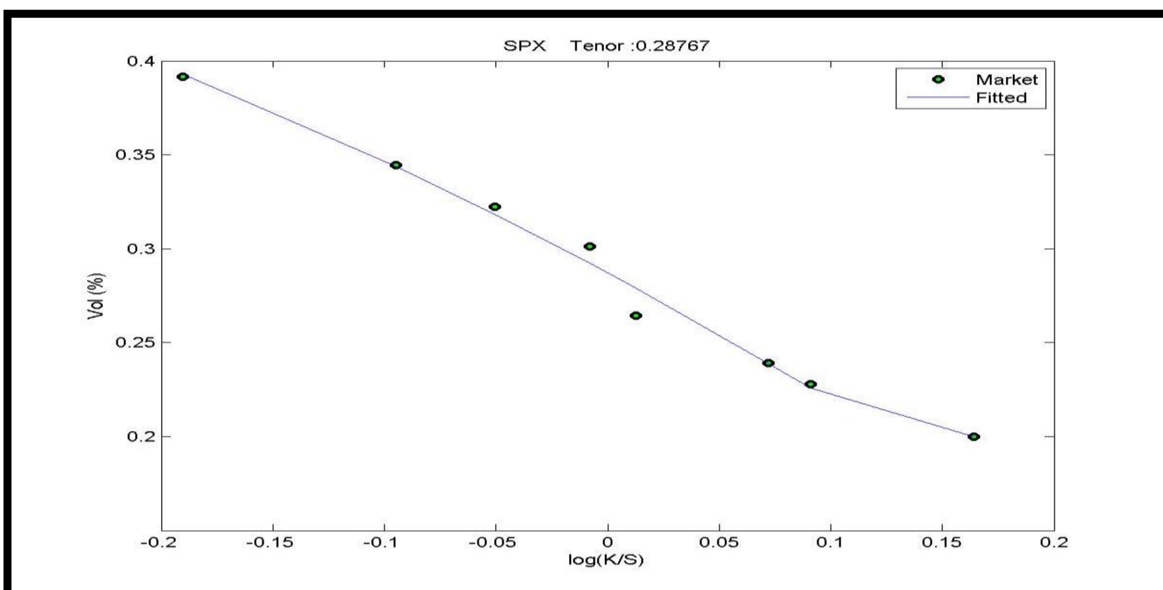

Figure 29 **SPX Option SVI fitting (Tenor-0.29)**

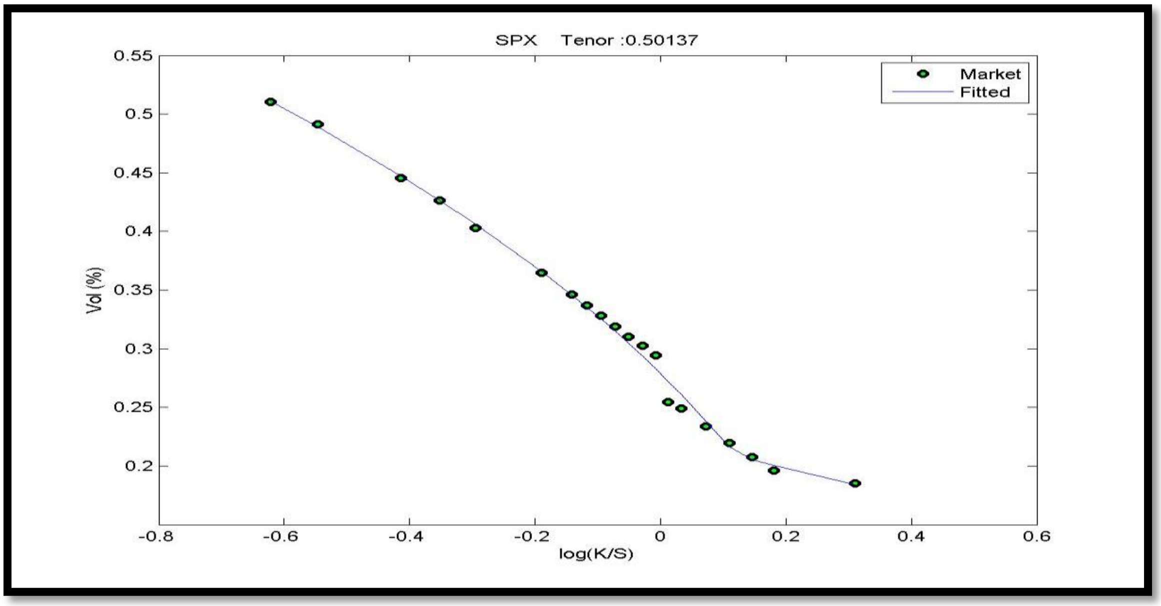

Figure 30 **SPX Option SVI fitting (Tenor-0.501)**

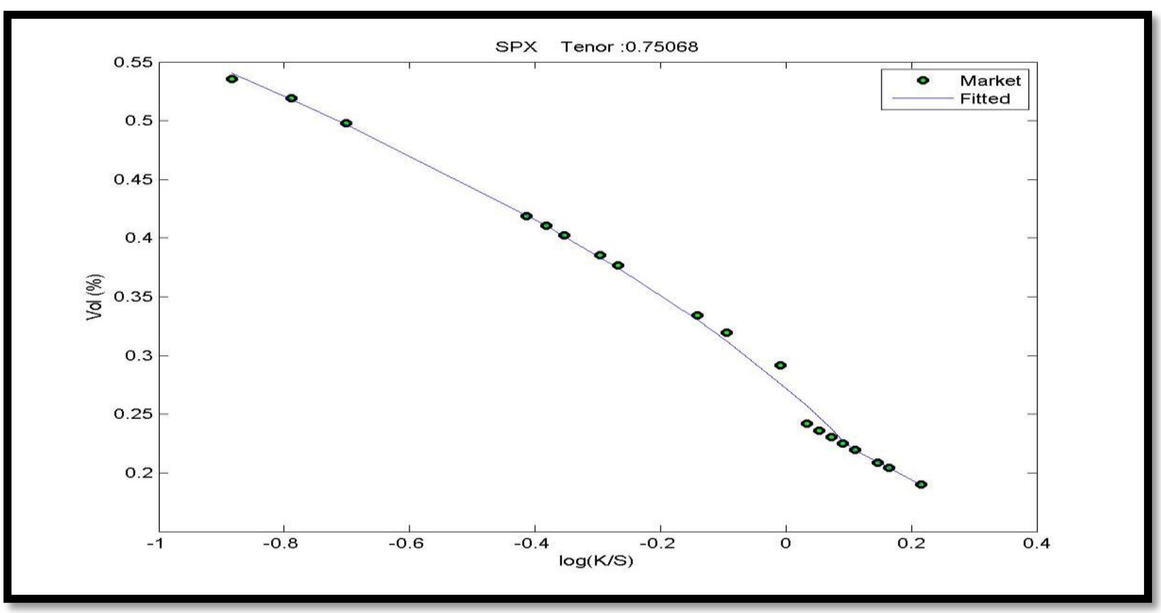

Figure 31 **SPX Option SVI fitting (Tenor-0.75)**

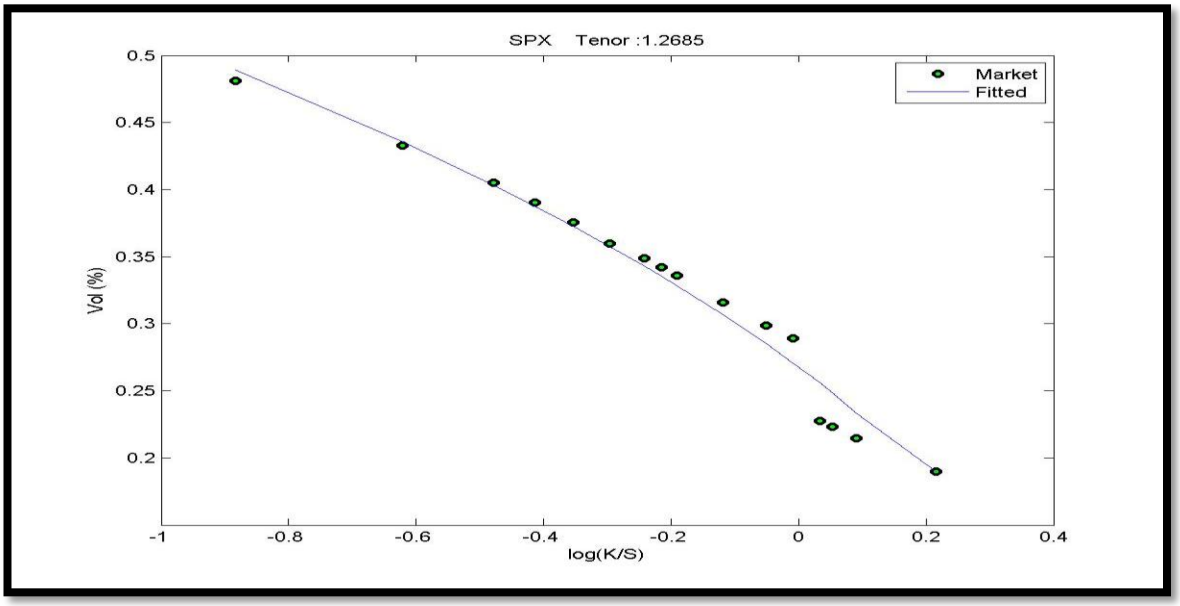

Figure 32 **SPX Option SVI fitting (Tenor-1.269)**

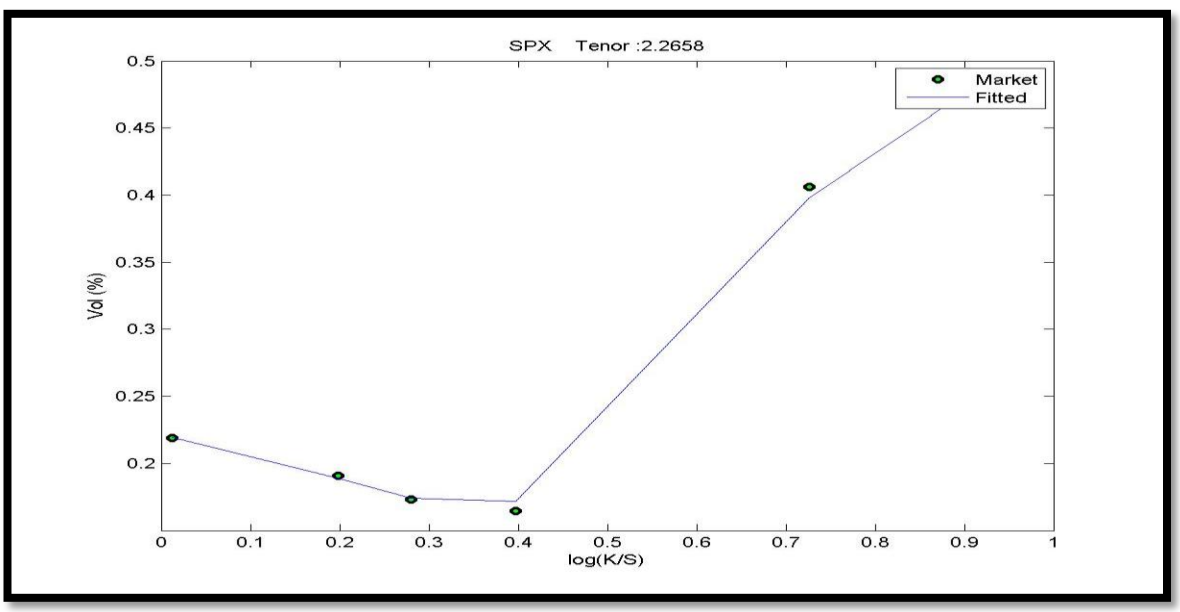

Figure 33 **SPX Option SVI fitting (Tenor-2.266)**

Inverted relationship of index and volatility

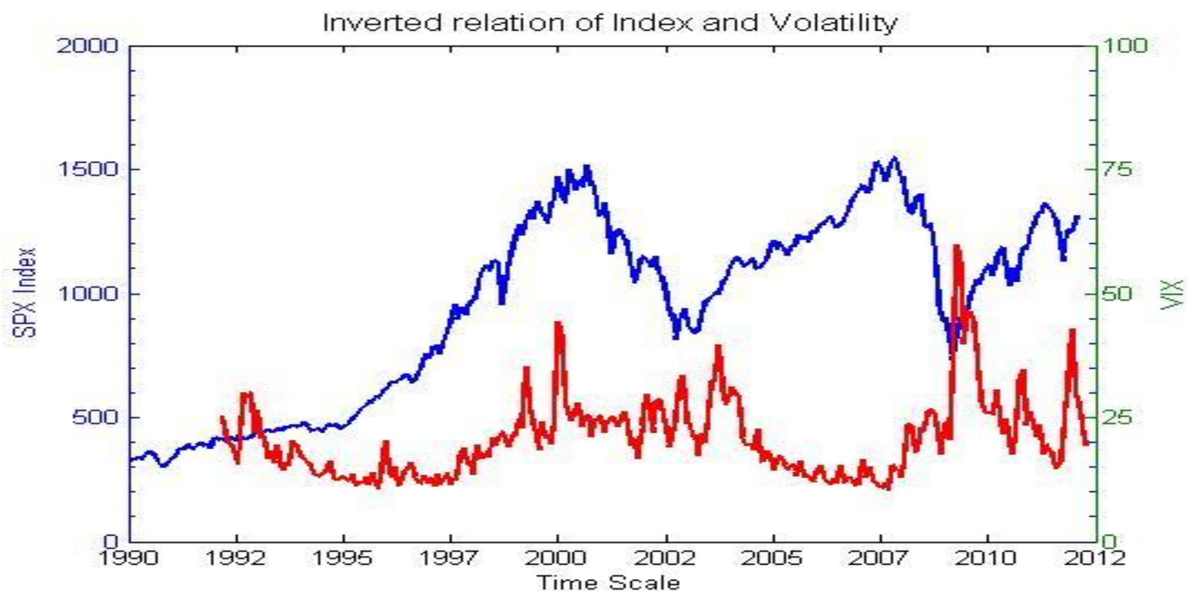

Figure 34 **INVERTED RELATIONSHIP OF INDEX AND VOLATILITY**

Source Code Snapshots (Only selected files)

For complete reference please refer to attached CD, under SurfaceGUI\src.

```
function [MON,MAT,IV] = npregression(data,iv)
  ------------------------------------------------------------------
Auther : Nirmaljit
 Description :  the kernel regression procedures employed in order
to obtain
 implied volatility surface time series on a given grid of
 maturities MAT  and moneyness MON.We use quadratic kernels of order
2
 Important parameters are the bandwidth parameters h1, h2,
 which determine the degree of smoothing
  ------------------------------------------------------------------
Inputs:
   data - option data required for np regression
   iv - pre computed implied volatility from option prices
 Output:
    [MON,MAT,IV] -  grid of moneyness, maturities and smoothened
Implied
    Vols.
 MON - Moneyness
 MAT - Maturires
 IV  - Smooth volatilities
 -------------------------------------------------------------------

optionChain = data;

firstmon=0.8;
lastmon=1.2;
firstmat=0;
lastmat=1;
stepwidth=[0.02 1/52];
lengthmon=ceil((lastmon-firstmon)/stepwidth(1));
lengthmat=ceil((lastmat-firstmat)/stepwidth(2));
mongrid=linspace(0.8,1.2,lengthmon+1);
matgrid=linspace(0,1,lengthmat+1);

[MON, MAT]=meshgrid(mongrid,matgrid);
gmon=lengthmon+1;
gmat=lengthmat+1;
uu=size(optionChain);
v=uu(1,1);
beta=zeros(gmat,gmon);
```

```
j=1;

nph = waitbar(0,'Kernel smoothing in progress. pls wait');

while (j<gmat+1);
    k=1;
    while (k<gmon+1);
        Y=iv;
        h1=0.1;
        h2=0.75;
        W=zeros(v,v); Kernel matrix
        i=1;
        X=zeros(v,3);
        while (i<v+1);
            u1=(optionChain(i,7)-MON(j,k));
            u2=(optionChain(i,4)-MAT(j,k));
            X(i,:)=[1,u1, u2];
            u1=u1/h1;
            u2=u2/h2;
            aa=15/16*(1-u1^2)^2*(abs(u1) <= 1)/h1;
            bb=15/16*(1-u2^2)^2*(abs(u2) <= 1)/h2;
            W(i,i)=aa*bb;
            i=i+1;
        end
        est=(X'*W*X)\(X'*W*Y);
        beta(j,k)=est(1);
        k=k+1;
    end
    j=j+1;
    waitbar(j/(gmat+1))
end
close(nph)

IV=beta;
end
```

```
function  [optionChain,FitX,FitY] =
sviVol(optionData,tenor,usePrecomputedIV)
  ------------------------------------------------------------------
 Auther : Nirmaljit
 Description :  We take SPX option quotes as of given date and
 compute implied volatilities for all 14 expirations passed as tenor
 The result of fitting square-root SVI passed back for plotting.
 Data was extracted from the excel file. For the calculation of the
implied volatility, out of money data and close to the money data was
used and the in the money data discarded. This is done as it is seen
that in practice out of the money options are found to give better
values of implied volatility.
 -------------------------------------------------------------------
 Given mid implied volatilities ?ij = ?BS(ki, tj), compute mid option
prices using the Black-Scholes formula.
 Fit the square-root SVI surface by minimizing sum of squared
  ------------------------------------------------------------------
 Inputs:
   optionData - option data required for np regression
   iv - precomputed implied volatility from option prices
 Output:
    [optionChain,FitX,FitY] -
 FitX - fitted X for plotting
 FitY - fitted Y for plotting
 usePrecomputedIV  - Always true. (can cache the computation to
matlab files)
   -----------------------------------------------------------------
  RowSize=length(optionData(:,1));
    optionChain=zeros(RowSize,3);
    optionChain(:,1)=optionData(:,2); % Strike
    optionChain(:,2)=optionData(:,5);% Option Price
    optionChain(:,3)=(optionData(:,6)==1);% Call Put
    optionChain(:,4)=optionData(:,8);% precomputed IV
    optionChain(:,5)=optionData(:,9);% vol
    spot =optionData(1:1,1);
    rate =optionData(1:1,3);
    rate = 0.01;
    ex1=find(optionChain(:,3)>0 & optionChain(:,1)<spot);
    ex2=find(optionChain(:,3)<1 & optionChain(:,1)>spot);
    ex3=find(optionChain(:,2)<=0);
    ex4=find(optionChain(:,5)<1);

    ex=[ex1;ex2;ex3;ex4];
    optionChain(ex,:)=[];

    [Parameters,optionChain] =
SVIParams(optionChain,tenor,spot,rate,usePrecomputedIV);

    [FitX FitY]=SVIFit(Parameters,optionChain(:,5));
end
```

```
function [SVIx,SVIy]=SVIFit(Parameters,Moneyness)
  -----------------------------------------------------------------
---------------------------------


 Description : This function generates two vectors for plotting the
fitting curve.
Paramters is a vector of SVI paramters in the order of a,b,sigma,rho
and m.
Moneyness is a vector containing the log of K over F in the same
number of
the observations.
 Inputs:
   Parameters - pre computed parameters for SVI Fitting
   Moneyness - Option Moneyness
 Output:
    [optionChain,FitX,FitY] -
 FitX - fitted X for plotting
 FitY - fitted Y for plotting

  -----------------------------------------------------------------
---------------------------------

SVIvol=sqrt(Parameters(1)+Parameters(2).*(Parameters(4).*(Moneyness-
Parameters(5))+sqrt((Moneyness-Parameters(5)).^2+Parameters(3)^2)));

    SVItemp(:,1)=Moneyness;
    SVItemp(:,2)=SVIvol;
    SVInew=sortrows(SVItemp);
    SVIx=SVInew(:,1);
    SVIy=SVInew(:,2);
end
-------------------------------------------------------------------
-------------------------------
```

```
function [Parameters,data] =
SVIParams(data,tenor,S0,Rate,usePrecomputedIV)
  -------------------------------------------------------------------
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 Description : This function generates two vectors for plotting the
fitting curve.
Paramters is a vector of SVI paramters in the order of a,b,sigma,rho
and m.
Moneyness is a vector containing the log of K over F in the same
number of
the observations.
 Inputs:
data - option data
tenor - tenor for option maturity
S0 - spot
Rate - risk free rate of return
usePrecomputedIV - not used
  -------------------------------------------------------------------
---------------------------------

    for j = 1:size(data,1)
        data(j,4) = bisection(data(j,2), S0, data(j,1), Rate, tenor,
data(j,3));
    end

    data(:,5) = log(data(:,1)./S0);


Parameters=(fminsearch(@(SVIparameters)SVIPCalc(data,SVIparameters),
[0.015 0.05 0.127 -0.568
0.165],optimset('MaxIter',1000000,'MaxFunEvals',1000000000)));
End
  -------------------------------------------------------------------
---------------------------------
```

```
function[SLS]=SVIPCalc(optionChain,params)

 Description : This function is defined as the objective function in
the least square
process.
 Inputs:
optionChain - option data
params - intermediate params for curve fitting



    SLS=0;
    DSize=length(optionChain(:,1));
    for i=1:DSize
        Temp=sqrt(params(1)+params(2)*(params(4)*(optionChain(i,5)-
params(5))+sqrt((optionChain(i,5)-params(5))^2+params(3)^2)))-
optionChain(i,4);
        Temp=params(1)+params(2)*(params(4)*(optionChain(i,5)-
params(5))+sqrt((optionChain(i,5)-params(5))^2+params(3)^2))-
optionChain(i,4)^2;
        SLS=SLS+Temp^2;
    end
```