01204211 Discrete Mathematics
Lecture 11b: Context-free languages and grammars (2)[1]

Jittat Fakcharoenphol

September 11, 2025

[1] Based on lecture notes of *Models of Computation* course by Jeff Erickson.

Review: Definition

A **context-free grammer** consists of the following components:

- a finite set Σ, a set of *symbols* (or *terminals*),
- a finite set Γ disjoint from Σ, a set of *non-terminals* (you can think of them as variables),
- a finite set R of *production rules* of the form $A \to w$ where $A \in \Gamma$ and $w \in (\Sigma \cup \Gamma)^*$ is a string of symbols and variable, and
- a *starting* non-terminal (usually the non-terminal of the first production rule).

Review: Applying the rules

If you have strings $x, y, z \in (\Sigma \cup \Gamma)^*$ and the production rule

$$A \to y,$$

You can apply the rule to the string xAz. This yields the string

$$xyz.$$

We use the notation

$$xAz \rightsquigarrow xyz$$

to describe this application.

Review: Derivation

We say that z derives from x if we can obtain z from x by production rule applications, denoted by $x \rightsquigarrow^* z$.
Formally, for any string $x, z \in (\Sigma \cup \Gamma)^*$, we say that $x \rightsquigarrow^* z$ if either

- $x = z$, or
- $x \rightsquigarrow y$ and $y \rightsquigarrow^* z$ for some string $y \in (\Sigma \cup \Gamma)^*$.

Review: $L(w)$

The *language* $L(w)$ of string $w \in (\Sigma \cup \Gamma)^*$ is the set of all strings in Σ^* that derive from w, i.e.,

$$L(w) = \{x \in \Sigma^* \mid w \leadsto^* x\}.$$

The language **generated by** a context-free grammar G, denoted by $L(G)$ is the language of its starting non-terminal.
A language L is **context-free** if there exists some context-free grammar G such that $L(G) = L$.

Review: Parse tree

- 00011

$$
\begin{aligned}
S &\rightarrow A \mid B \\
A &\rightarrow 0A \mid 0C \\
B &\rightarrow B1 \mid C1 \\
C &\rightarrow \varepsilon \mid 0C1
\end{aligned}
$$

Ambiguity

- $1 + 1 + 1 + 1 + 1$

$$S \quad \rightarrow \quad 1 \mid S + S \mid S * S$$

- A string w is **ambiguous** with respect to a grammar G if more than one parse tree for w exists.
- A grammar G is **ambiguous** if some string is ambiguous with respect to G.

More example

11011

$S \to \varepsilon \mid 0S0 \mid 1S1 \mid 0 \mid 1$

Palindrome in $\{0,1\}^*$

More example

$S \to 11$

$L = \{ w \mid w = w^R,\ w \in \{0,1\}^* \}$

Palindrome in $\{0,1\}^*$

Grammar $\boxed{G}$

$$S \to 0S0 \mid 1S1 \mid 1 \mid 0 \mid \varepsilon$$

อยากจะพิสูจน์ว่า $\boxed{L(G)} = L$

- ▷ $L(G) \subseteq L$
- ▷ $L \subseteq L(G)$

Consider the following grammar

$$S \longrightarrow \mathtt{0}S\mathtt{1} \mid \varepsilon$$

To show that

$$L(S) = \{\mathtt{0}^n\mathtt{1}^n \mid n \geq 0\},$$

we have to prove

Consider the following grammar

$$S \longrightarrow 0S1 \mid \varepsilon$$

To show that

$$L(S) = \{0^n 1^n \mid n \geq 0\},$$

ไม่ขาด

we have to prove

- $L(S) \supseteq \{0^n 1^n \mid n \geq 0\}$, and $\longrightarrow$ ทุก string ที่หน้าตาแบบรูปนี้ สร้างได้
- $L(S) \subseteq \{0^n 1^n \mid n \geq 0\}$.

ทุก string ที่ generate ได้ ต้องหน้าตารูปแบบ

ไม่เกิน

Consider the grammar $S \longrightarrow 0S1 \mid \varepsilon$.

Lemma 1

$S \rightsquigarrow^* 0^n 1^n$ for every non-negative integer n.

Proof.

Consider any non-negative integer n.
Induction Hypothesis: Assume that for every non-negative integer $k < n$, $S \rightsquigarrow^* 0^k 1^k$.
There are two cases to consider.

Consider the grammar $S \longrightarrow 0S1 \mid \varepsilon$.

Lemma 1

$S \rightsquigarrow^* 0^n 1^n$ *for every non-negative integer* n.

Proof.

Consider any non-negative integer n.
Induction Hypothesis: Assume that for every non-negative integer $k < n$, $S \rightsquigarrow^* 0^k 1^k$.
There are two cases to consider.

- Case 1: $n = 0$.

$0^0 1^0 = \varepsilon$ [illegible] $S \rightsquigarrow \varepsilon$ [illegible] production rule [illegible]

$S \rightarrow \varepsilon$

Consider the grammar $S \longrightarrow 0S1 \mid \varepsilon$.

Lemma 1

$S \rightsquigarrow^* 0^n1^n$ *for every non-negative integer* n.

Proof.

Consider any non-negative integer n.
Induction Hypothesis: Assume that for every non-negative integer $k < n$, $S \rightsquigarrow^* 0^k1^k$.
There are two cases to consider.

- Case 1: $n = 0$.
- Case 2: $n > 0$. จาก induction hypothesis พบว่า $S \rightsquigarrow^* 0^k 1^k$ ที่ $k = n-1$ ได้

เนื่องจาก $S \rightsquigarrow 0S1$ และ $S \rightsquigarrow^* 0^{n-1} 1^{n-1}$

จะได้ว่า $S \rightsquigarrow^* 00^{n-1}1^{n-1}1 = 0^n1^n$ ตามต้องการ

เนื่องจากประโยคจริงในทุกกรณี, จะได้ว่า lemma จริง.

Consider the grammar $S \longrightarrow 0S1 \mid \varepsilon$.

Lemma 1

$S \rightsquigarrow^* 0^n 1^n$ *for every non-negative integer* n.

Proof.

Consider any non-negative integer n.
Induction Hypothesis: Assume that for every non-negative integer $k < n$, $S \rightsquigarrow^* 0^k 1^k$.
There are two cases to consider.

- Case 1: $n = 0$.
- Case 2: $n > 0$. From I.H., we know that

$$S \rightsquigarrow^* 0^{n-1} 1^{n-1}.$$

We can apply rule $S \longrightarrow 0S1$ to obtain $0^n 1^n$, i.e.,

$$S \longrightarrow 0S1 \rightsquigarrow^* 00^{n-1} 1^{n-1} 1 = 0^n 1^n.$$

In both cases, we conclude that $S \rightsquigarrow^* 0^n 1^n$, as required.

Consider the following grammar

$$S \longrightarrow 0S1 \mid \varepsilon$$

Lemma 2

$L(S) = \{0^n1^n \mid n \geq 0\}$ –

Proof.

แสดงไปแล้วว่า $\{0^n1^n \mid n \geq 0\} \subseteq L(S)$ เหลือแสดงว่า $L(S) \subseteq \{0^n1^n \mid n \geq 0\}$

พิสูจน์โดยแสดงว่า ถ้า $S \Rightarrow^* y$, y อยู่ในรูป 0^n1^n

Consider the following grammar

$$S \longrightarrow 0S1 \mid \varepsilon$$

Lemma 2

$L(S) = \{0^n 1^n \mid n \geq 0\}$

Proof.

Consider any string $w \in L(S)$. We show that $w = 0^n 1^n$ for some non-negative integer n.

Consider the following grammar

$$S \longrightarrow 0S1 \mid \varepsilon$$

Lemma 2

$L(S) = \{0^n 1^n \mid n \geq 0\}$

Proof.

Consider any string $w \in L(S)$. We show that $w = 0^n 1^n$ for some non-negative integer n.
I.H.: Assume that for any string $x \in L(S)$ such that $|x| < |w|$, $x = 0^k 1^k$ for some non-negative integer k.
There are

Consider the following grammar

$$S \longrightarrow 0S1 \mid \varepsilon$$

Lemma 2

$L(S) = \{0^n 1^n \mid n \geq 0\}$

Proof.

Consider any string $w \in L(S)$. We show that $w = 0^n 1^n$ for some non-negative integer n.
I.H.: Assume that for any string $x \in L(S)$ such that $|x| < |w|$, $x = 0^k 1^k$ for some non-negative integer k.
There are 2 cases:
Case 1: $w = \varepsilon$.

Consider the following grammar

$$S \longrightarrow 0S1 \mid \varepsilon$$

Lemma 2

$L(S) = \{0^n 1^n \mid n \geq 0\}$

Proof.

Consider any string $w \in L(S)$. We show that $w = 0^n 1^n$ for some non-negative integer n.
I.H.: Assume that for any string $x \in L(S)$ such that $|x| < |w|$, $x = 0^k 1^k$ for some non-negative integer k.
There are 2 cases:
Case 1: $w = \varepsilon$.
Case 2: $w = 0x1$ for some $x \in L(S)$.

Consider the following grammar

$S \to 0S1$

$$S \longrightarrow 0S1 \mid \varepsilon$$

Lemma 2

$L(S) = \{0^n 1^n \mid n \geq 0\}$

Proof.

Consider any string $w \in L(S)$. We show that $w = 0^n 1^n$ for some non-negative integer n.
I.H.: Assume that for any string $x \in L(S)$ such that $|x| < |w|$, $x = 0^k 1^k$ for some non-negative integer k.
There are 2 cases:
Case 1: $w = \varepsilon$.
Case 2: $w = 0x1$ for some $x \in L(S)$. Since $|x| = |w| - 2 < |w|$, we can apply I.H., and get that $x = 0^k 1^k$;thus $w = 00^k 1^k 1$, i.e., $w = 0^n 1^n$ where $n = k + 1$, as required. □

Careful

$\{w \mid \underline{\#(0,w)} = \underline{\#(1,w)}\} = R$

- When using inductive proof, you have to ensure that each part of the string w is shorter than w.
- Consider this grammar

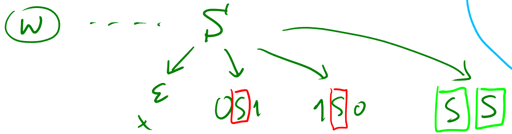
รวม strings ที่จำนวน 0 และ 1 เท่ากัน $= R$

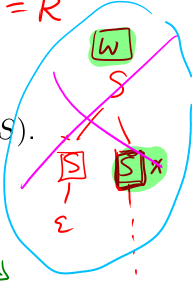
$$S \longrightarrow \varepsilon \mid SS \mid 0S1 \mid 1S0.$$

- When w is created by rule $S \to SS$, we know that $w = xy$ for $x, y \in L(S)$.
- Do we know that $|x| < |w|$ and $|y| < |w|$?

$L(S) = R$

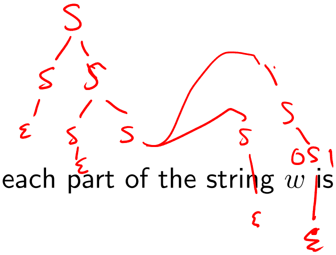
- $L(S) \subseteq R$
- $R \subseteq L(S)$





Careful

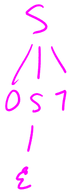
01



- When using inductive proof, you have to ensure that each part of the string w is shorter than w.
- Consider this grammar

εε εε

$$S \longrightarrow \varepsilon \mid \boxed{SS} \mid 0S1 \mid 1S0.$$



- When w is created by rule $S \to SS$, we know that $w = xy$ for $x, y \in L(S)$.
- Do we know that $|x| < |w|$ and $|y| < |w|$?
- We can consider a **minimum-length derivation** in the proof to avoid this problem.

Consider grammar $S \longrightarrow \varepsilon \mid SS \mid 0S1 \mid 1S0$. For every string $w \in L(S)$, we have $\#(0, w) = \#(1, w)$, where $\#(a, w)$ is the number of occurrences of a in w.

$L(S) \subseteq R$

Proof.

Consider $w \in L(S)$. Fix a minimum-length derivation of w.

Induction Hypothesis: Assume that for any string $x \in L(S)$ such that $|x| < |w|$, we have $\#(0, x) = \#(1, x)$.

There are four cases to consider, depending on the first production in this derivation.

- Case 1: The first production is $S \longrightarrow \varepsilon$. ✓

Consider grammar $S \longrightarrow \varepsilon \mid SS \mid 0S1 \mid 1S0$. For every string $w \in L(S)$, we have $\#(0,w) = \#(1,w)$, where $\#(a,w)$ is the number of occurrences of a in w.

Proof.

Consider $w \in L(S)$. Fix a minimum-length derivation of w.
Induction Hypothesis: Assume that for any string $x \in L(S)$ such that $|x| < |w|$, we have $\#(0,x) = \#(1,x)$.
There are four cases to consider, depending on the first production in this derivation.

- Case 1: The first production is $S \longrightarrow \varepsilon$.
- Case 2: The first production is $S \longrightarrow 0\boxed{S}1$.

ให้ $w = 0\underline{x}1$

เนื่องจาก $|x| < |w|$,

จาก I.H. $\#(0,x) = \#(1,x)$

ดังนั้น $\#(0,w) = 1 + \#(0,x)$

$\#(1,w) = 1 + \#(1,x)$

$\Downarrow$

$\#(0,w) = \#(1,w)$.

Consider grammar $S \longrightarrow \varepsilon \mid SS \mid 0S1 \mid 1S0$. For every string $w \in L(S)$, we have $\#(0, w) = \#(1, w)$, where $\#(a, w)$ is the number of occurrences of a in w.

Proof.

Consider $w \in L(S)$. Fix a minimum-length derivation of w.
Induction Hypothesis: Assume that for any string $x \in L(S)$ such that $|x| < |w|$, we have $\#(0, x) = \#(1, x)$.
There are four cases to consider, depending on the first production in this derivation.

- Case 1: The first production is $S \longrightarrow \varepsilon$.
- Case 2: The first production is $S \longrightarrow 0S1$. Case 3: The first production is $S \longrightarrow 1S0$.

Consider grammar $S \longrightarrow \varepsilon \mid SS \mid 0S1 \mid 1S0$. For every string $w \in L(S)$, we have $\#(0, w) = \#(1, w)$, where $\#(a, w)$ is the number of occurrences of a in w.

Proof.

Consider $w \in L(S)$. Fix a minimum-length derivation of w. *

Induction Hypothesis: Assume that for any string $x \in L(S)$ such that $|x| < |w|$, we have $\#(0, x) = \#(1, x)$.

There are four cases to consider, depending on the first production in this derivation.

- Case 1: The first production is $S \longrightarrow \varepsilon$.
- Case 2: The first production is $S \longrightarrow 0S1$. Case 3: The first production is $S \longrightarrow 1S0$.
- Case 4: The first production is $S \longrightarrow SS$.

$\underline{w} = \underline{xy}$ โดยที่ $x \in L(S)$, $y \in L(S)$

เนื่องจากเงื่อนไข min length derivation,

จะได้ว่า $\underline{x \neq \varepsilon}$, $\underline{y \neq \varepsilon}$

$\Rightarrow \underline{|x| < |w|}$, $\underline{|y| < |w|} \Rightarrow$ *

Consider grammar $S \longrightarrow \varepsilon \mid SS \mid 0S1 \mid 1S0$. For every string $w \in L(S)$, we have $\#(0, w) = \#(1, w)$, where $\#(a, w)$ is the number of occurrences of a in w.

Proof.

Consider $w \in L(S)$. Fix a minimum-length derivation of w.
Induction Hypothesis: Assume that for any string $x \in L(S)$ such that $|x| < |w|$, we have $\#(0, x) = \#(1, x)$.
There are four cases to consider, depending on the first production in this derivation.

- Case 1: The first production is $S \longrightarrow \varepsilon$.
- Case 2: The first production is $S \longrightarrow 0S1$. Case 3: The first production is $S \longrightarrow 1S0$.
- Case 4: The first production is $S \longrightarrow SS$. In this case $w = xy$ for some $x, y \in L(S)$.

Consider grammar $S \longrightarrow \varepsilon \mid SS \mid 0S1 \mid 1S0$. For every string $w \in L(S)$, we have $\#(0, w) = \#(1, w)$, where $\#(a, w)$ is the number of occurrences of a in w.

Proof.

Consider $w \in L(S)$. Fix a minimum-length derivation of w.
Induction Hypothesis: Assume that for any string $x \in L(S)$ such that $|x| < |w|$, we have $\#(0, x) = \#(1, x)$.
There are four cases to consider, depending on the first production in this derivation.

- Case 1: The first production is $S \longrightarrow \varepsilon$.
- Case 2: The first production is $S \longrightarrow 0S1$. Case 3: The first production is $S \longrightarrow 1S0$.
- Case 4: The first production is $S \longrightarrow SS$. In this case $w = xy$ for some $x, y \in L(S)$. Since we assume the minimum-length derivation, x and y cannot be ε because

Consider grammar $S \longrightarrow \varepsilon \mid SS \mid 0S1 \mid 1S0$. For every string $w \in L(S)$, we have $\#(0,w) = \#(1,w)$, where $\#(a,w)$ is the number of occurrences of a in w.

Proof.

Consider $w \in L(S)$. Fix a minimum-length derivation of w.
Induction Hypothesis: Assume that for any string $x \in L(S)$ such that $|x| < |w|$, we have $\#(0,x) = \#(1,x)$.
There are four cases to consider, depending on the first production in this derivation.

- Case 1: The first production is $S \longrightarrow \varepsilon$.
- Case 2: The first production is $S \longrightarrow 0S1$. Case 3: The first production is $S \longrightarrow 1S0$.
- Case 4: The first production is $S \longrightarrow SS$. In this case $w = xy$ for some $x, y \in L(S)$. Since we assume the minimum-length derivation, x and y cannot be ε because in that case we can shorten the derivation of w.

Consider grammar $S \longrightarrow \varepsilon \mid SS \mid 0S1 \mid 1S0$. For every string $w \in L(S)$, we have $\#(0, w) = \#(1, w)$, where $\#(a, w)$ is the number of occurrences of a in w.

Proof.

Consider $w \in L(S)$. Fix a minimum-length derivation of w.
Induction Hypothesis: Assume that for any string $x \in L(S)$ such that $|x| < |w|$, we have $\#(0, x) = \#(1, x)$.
There are four cases to consider, depending on the first production in this derivation.

- Case 1: The first production is $S \longrightarrow \varepsilon$.
- Case 2: The first production is $S \longrightarrow 0S1$. Case 3: The first production is $S \longrightarrow 1S0$.
- Case 4: The first production is $S \longrightarrow SS$. In this case $w = xy$ for some $x, y \in L(S)$. Since we assume the minimum-length derivation, x and y cannot be ε because in that case we can shorten the derivation of w.

  From I.H.,

Consider grammar $S \longrightarrow \varepsilon \mid SS \mid 0S1 \mid 1S0$. For every string $w \in L(S)$, we have $\#(0,w) = \#(1,w)$, where $\#(a,w)$ is the number of occurrences of a in w.

Proof.

Consider $w \in L(S)$. Fix a minimum-length derivation of w.
Induction Hypothesis: Assume that for any string $x \in L(S)$ such that $|x| < |w|$, we have $\#(0,x) = \#(1,x)$.
There are four cases to consider, depending on the first production in this derivation.

- Case 1: The first production is $S \longrightarrow \varepsilon$.
- Case 2: The first production is $S \longrightarrow 0S1$. Case 3: The first production is $S \longrightarrow 1S0$.
- Case 4: The first production is $S \longrightarrow SS$. In this case $w = xy$ for some $x, y \in L(S)$. Since we assume the minimum-length derivation, x and y cannot be ε because in that case we can shorten the derivation of w. $\Rightarrow$ $|x| < |w|$, $|y| < |w|$

  From I.H., we know that $\#(0,x) = \$(1,x)$ an $\#(0,y) = \#(1,y)$; thus,

$$\#(0,w) \quad = \quad \#(0,x) + \#(0,y)$$

Consider grammar $S \longrightarrow \varepsilon \mid SS \mid 0S1 \mid 1S0$. For every string $w \in L(S)$, we have $\#(0, w) = \#(1, w)$, where $\#(a, w)$ is the number of occurrences of a in w.

Proof.

Consider $w \in L(S)$. Fix a minimum-length derivation of w.
Induction Hypothesis: Assume that for any string $x \in L(S)$ such that $|x| < |w|$, we have $\#(0, x) = \#(1, x)$.
There are four cases to consider, depending on the first production in this derivation.

- Case 1: The first production is $S \longrightarrow \varepsilon$.
- Case 2: The first production is $S \longrightarrow 0S1$. Case 3: The first production is $S \longrightarrow 1S0$.
- Case 4: The first production is $S \longrightarrow SS$. In this case $w = xy$ for some $x, y \in L(S)$. Since we assume the minimum-length derivation, x and y cannot be ε because in that case we can shorten the derivation of w.

  From I.H., we know that $\#(0, x) = \$(1, x)$ an $\#(0, y) = \#(1, y)$; thus,

$$\begin{aligned} \#(0, w) &= \#(0, x) + \#(0, y) \\ &= \#(1, x) + \#(1, y) \end{aligned}$$

Consider grammar $S \longrightarrow \varepsilon \mid SS \mid 0S1 \mid 1S0$. For every string $w \in L(S)$, we have $\#(0,w) = \#(1,w)$, where $\#(a,w)$ is the number of occurrences of a in w. $L(S) \subseteq R$

Proof.

Consider $w \in L(S)$. Fix a minimum-length derivation of w.
Induction Hypothesis: Assume that for any string $x \in L(S)$ such that $|x| < |w|$, we have $\#(0,x) = \#(1,x)$.
There are four cases to consider, depending on the first production in this derivation.

- Case 1: The first production is $S \longrightarrow \varepsilon$.
- Case 2: The first production is $S \longrightarrow 0S1$. Case 3: The first production is $S \longrightarrow 1S0$.
- Case 4: The first production is $S \longrightarrow SS$. In this case $w = xy$ for some $x, y \in L(S)$. Since we assume the minimum-length derivation, x and y cannot be ε because in that case we can shorten the derivation of w.

  From I.H., we know that $\#(0,x) = \$(1,x)$ an $\#(0,y) = \#(1,y)$; thus,

$$
\begin{aligned}
\#(0,w) &= \#(0,x) + \#(0,y) \quad \text{— จาก I.H.} \\
&= \#(1,x) + \#(1,y) = \#(1,w)
\end{aligned}
$$

In all cases, we conclude that $\#(0,w) = \#(1,w)$.

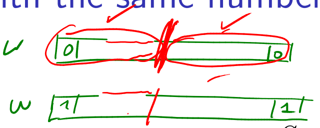
Examples: Not palindromes

Strings in $(0+1)^*$ that are not palindromes.

$$\begin{aligned} S &\longrightarrow 0S0 \mid 1S1 \mid 0Z1 \mid 1Z0 \\ Z &\longrightarrow \varepsilon \mid 0Z \mid 1Z \end{aligned}$$

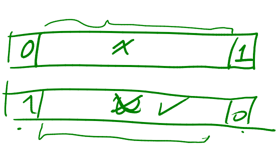
Why does this work?

Strings with the same number of 0s and 1s



Case 1

Case 2



$$S \longrightarrow \varepsilon \mid \boxed{SS} \mid \underline{0S1} \mid \underline{1S0}.$$

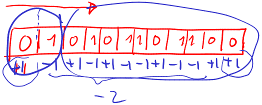
We already show that every string in $L(S)$ contains the same number of 0s and 1s. Why does it contain all possible required strings?

Thm $R \subseteq L(S)$

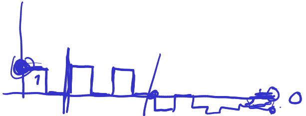
พิสูจน์ $\boxed{w}$ ที่ $\#(0,w) = \#(1,w)$

I.H. assume ทุกๆ string x ที่ $\underline{|x| < |w|}$ และ $\#(0,x) = \#(1,x)$,

จะ: $\underline{x \in L(S)}$



$\boxed{0110} \mid \boxed{1100}$



Strings in which the number of 0s is greater than or equal to the number of 1s

We can start with the previous grammar

$$S \longrightarrow \varepsilon \mid SS \mid 0S1 \mid 1S0.$$

And try to add more rules.

Strings in which the number of 0s is greater than or equal to the number of 1s

We can start with the previous grammar

$$S \longrightarrow \varepsilon \mid SS \mid 0S1 \mid 1S0.$$

And try to add more rules.

$$S \longrightarrow \varepsilon \mid SS \mid 0S1 \mid 1S0 \mid 0S \mid S0.$$

Strings with different numbers of 0s and 1s

We can start with the previous grammar E of strings with equal number of 0 and 1.

$$E \longrightarrow \varepsilon \mid EE \mid 0E1 \mid 1E0.$$

There are two cases.

Strings with different numbers of 0s and 1s

We can start with the previous grammar E of strings with equal number of 0 and 1.

$$E \longrightarrow \varepsilon \mid EE \mid 0E1 \mid 1E0.$$

There are two cases.

$$S \longrightarrow O \mid I$$

Strings with different numbers of 0s and 1s

We can start with the previous grammar E of strings with equal number of 0 and 1.

$$E \longrightarrow \varepsilon \mid EE \mid 0E1 \mid 1E0.$$

There are two cases.

$$S \longrightarrow O \mid I$$

$$O \longrightarrow E0O \mid E0E$$

How about I?

Strings with different numbers of 0s and 1s

We can start with the previous grammar E of strings with equal number of $\mathtt{0}$ and $\mathtt{1}$.

$$E \longrightarrow \varepsilon \mid EE \mid \mathtt{0}E\mathtt{1} \mid \mathtt{1}E\mathtt{0}.$$

There are two cases.

$$S \longrightarrow O \mid I$$

$$O \longrightarrow E\mathtt{0}O \mid E\mathtt{0}E$$

How about I?

$$I \longrightarrow E\mathtt{1}I \mid E\mathtt{1}E$$

Balanced parentheses

$$S \longrightarrow (S) \mid SS \mid \varepsilon$$

Balanced parentheses

$$S \longrightarrow (S) \mid SS \mid \varepsilon$$

$$S \longrightarrow (S)S \mid \varepsilon$$

Mutual induction

Consider grammar

$$S \longrightarrow \mathtt{0}A\mathtt{1} \mid \varepsilon \qquad\qquad A \longrightarrow \mathtt{1}S\mathtt{0} \mid \varepsilon$$

What is $L(S)$?

Mutual induction

Consider grammar

$$S \longrightarrow \mathtt{0}A\mathtt{1} \mid \varepsilon \qquad\qquad A \longrightarrow \mathtt{1}S\mathtt{0} \mid \varepsilon$$

What is $L(S)$?
From inspection, we may guess that $L(S) = (\mathtt{01})^*$. But how can we prove that?

Mutual induction

Consider grammar

$$S \longrightarrow \mathtt{0}A\mathtt{1} \mid \varepsilon \qquad\qquad A \longrightarrow \mathtt{1}S\mathtt{0} \mid \varepsilon$$

What is $L(S)$?
From inspection, we may guess that $L(S) = (\mathtt{01})^*$. But how can we prove that?
To prove $L(S) = (\mathtt{01})^*$, we must also prove $L(A) = (\mathtt{10})^*$ *at the same time*.