# 01204211 Discrete Mathematics
## Lecture 10a: Applications

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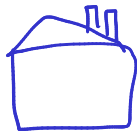
September 6, 2022

# Factory Production

A food factory can produce 2 products: a sleeping candy bar ($S$) and an energy bar ($E$).

- ▶ There are two ingredients: $A$ and $B$. The factory has $120$ units of $A$ and $100$ units of $B$.
- ▶ To produce $1$ unit of candy bar $S$, you need $15$ units of $A$ and $10$ units of $B$.
- ▶ To produce $1$ unit of energy bar $E$, you need $10$ units of $A$ and $20$ units of $B$.

# Factory Production

A food factory can produce 2 products: a sleeping candy bar ($S$) and an energy bar ($E$).

- There are two ingredients: $A$ and $B$. The factory has 120 units of $A$ and 100 units of $B$.
- To produce 1 unit of candy bar $S$, you need 15 units of $A$ and 10 units of $B$.
- To produce 1 unit of energy bar $E$, you need 10 units of $A$ and 20 units of $B$.

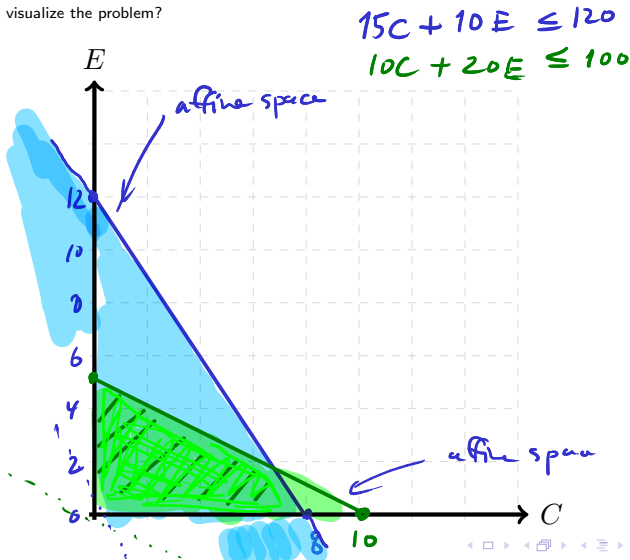How can we visualize the problem?

$$15 \cdot C + 10E \leq 120$$
$$10C + 20E \leq 100$$

# Factory Production

A food factory can produce 2 products: a sleeping candy bar ($C$) and an energy bar ($E$).

▶ There are two ingredients: $A$ and $B$. The factory has 120 units of $A$ and 100 units of $B$.

▶ To produce 1 unit of candy bar $C$, you need 15 units of $A$ and 10 units of $B$.

▶ To produce 1 unit of energy bar $E$, you need 10 units of $A$ and 20 units of $B$.

How can we visualize the problem?

$$15C + 10E \leq 120$$
$$10C + 20E \leq 100$$



affine space

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$E$

$C$

# Factory Production

How can we choose the amount of $C$ and $E$ to produce?

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It depends on the prices per unit of $C$ and $E$.
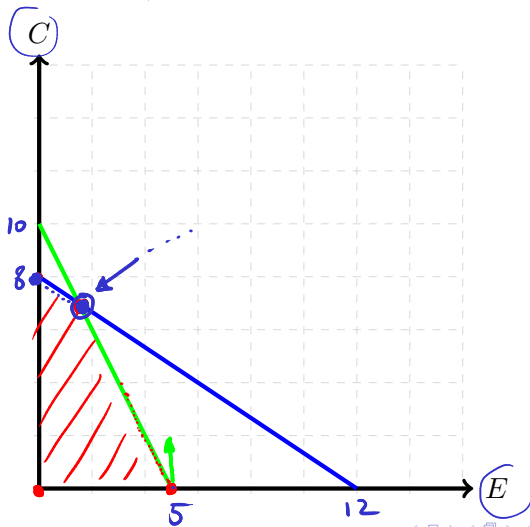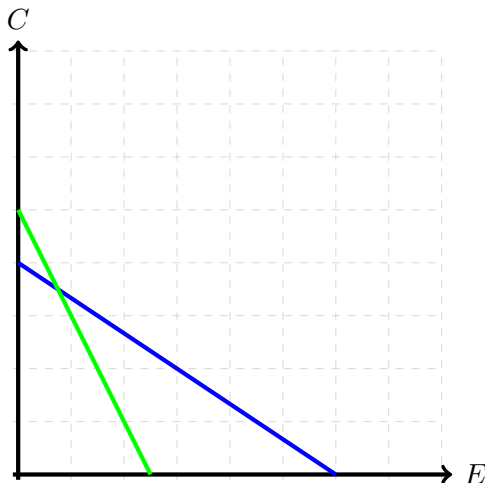
# Factory Production

How can we choose the amount of $C$ and $E$ to produce?

It depends on the prices per unit of $C$ and $E$.

What if 1 unit of $C$ is 1 baht and 1 unit of $E$ is also 1 baht?

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What if 1 unit of $C$ is 1 baht and 1 unit of $E$ is also 1 baht?

# Factory Production

What if $1$ unit of $C$ is 10 baht and $1$ unit of $E$ is also $1$ baht?
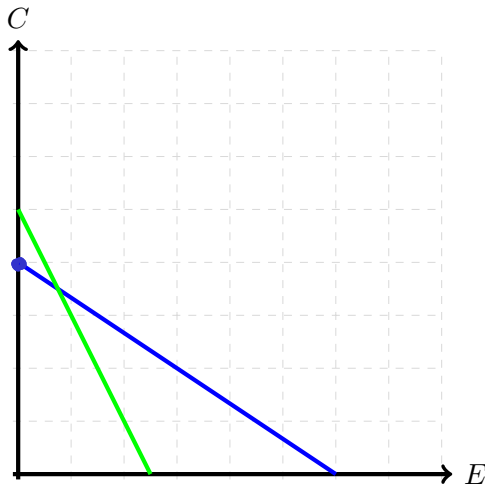
# Factory Production

What if 1 unit of $C$ is 1 baht and 1 unit of $E$ is also 10 baht?
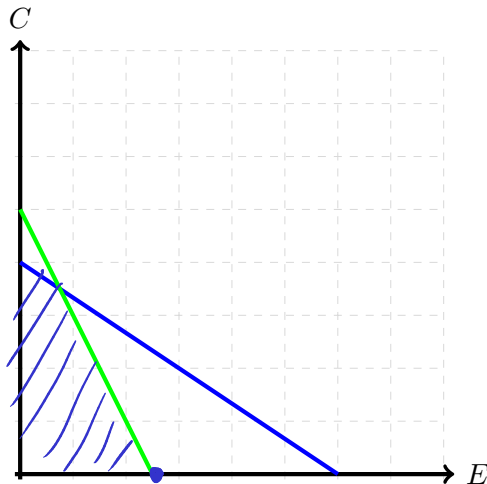
# Objective functions

If we produce $x_1$ units of $C$ and $x_2$ units of $E$, we would make

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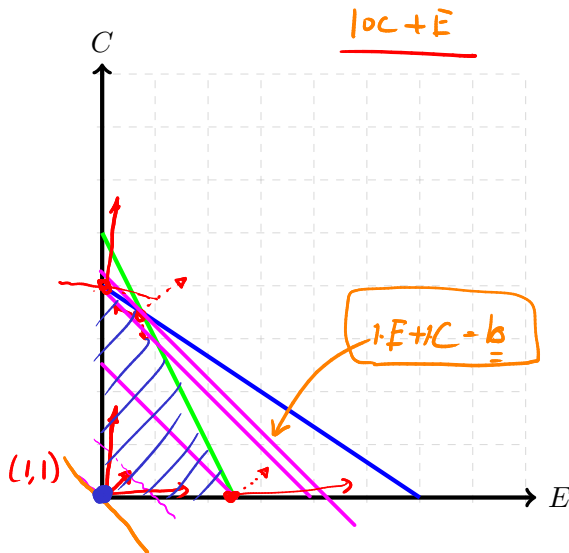
$$p_C \cdot x_1 + p_E \cdot x_2,$$

where $p_C$ and $p_E$ are unit prices for $C$ and $E$.

## Objective functions

If we produce $x_1$ units of $C$ and $x_2$ units of $E$, we would make

$$p_C \cdot x_1 + p_E \cdot x_2,$$

where $p_C$ and $p_E$ are unit prices for $C$ and $E$.
This is called an objective function. So far, we tried 3 objective
functions:

- $x_1 + x_2$
- $10 \cdot x_1 + x_2$
- $x_1 + 10 \cdot x_2$

# Objective functions
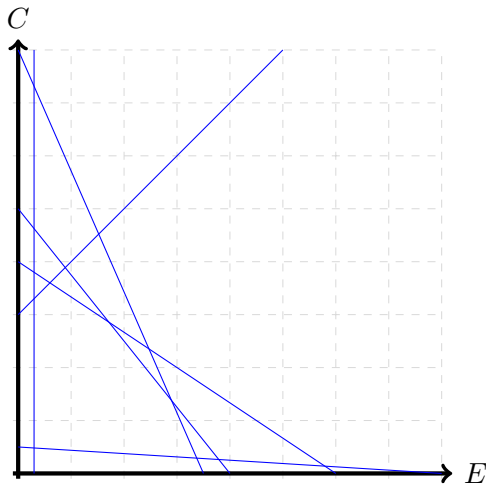
Let's see what they look like again.

# Vertices

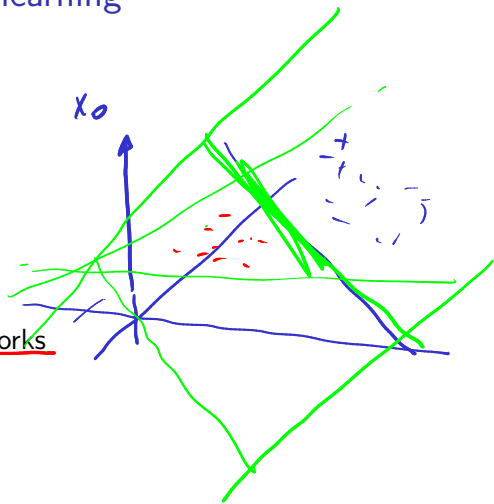Interesting things happen at the intersections. What are they?

# Simplex algorithm
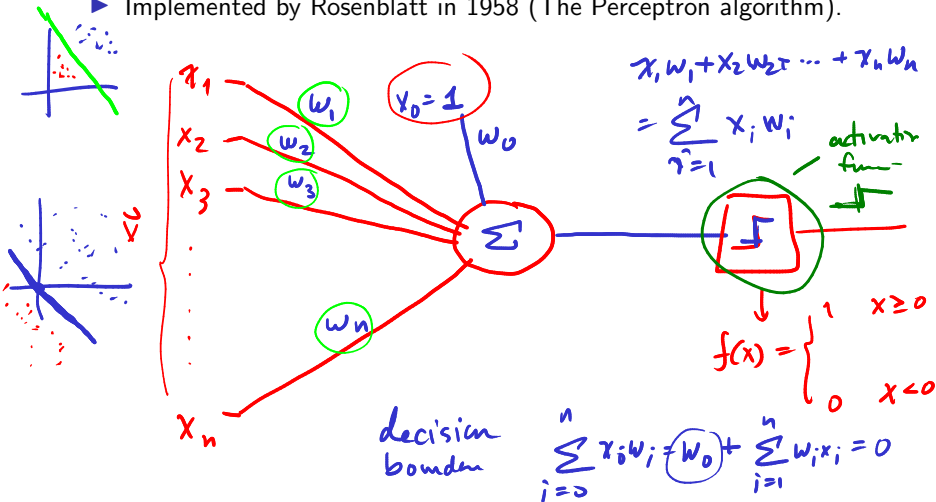
linear programming
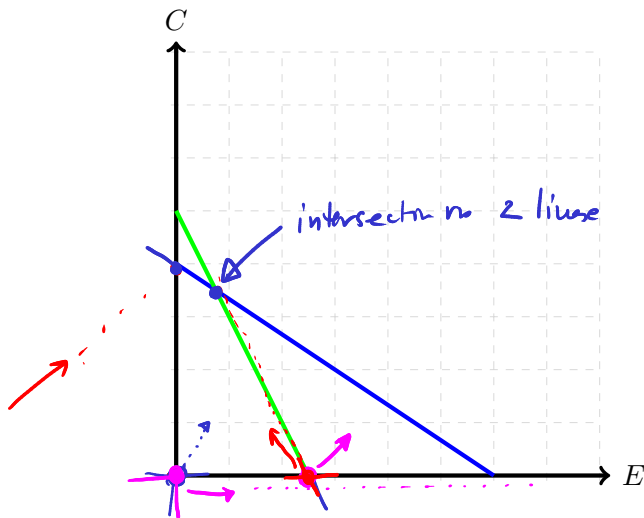
# Simplex algorithm - How do you move?

# A quick history of machine learning

- Perceptrons
- Neural networks
- Convolutional neural networks

# Perceptrons

- Invented in 1943 by McCulloch and Pitts.
- Implemented by Rosenblatt in 1958 (The Perceptron algorithm).
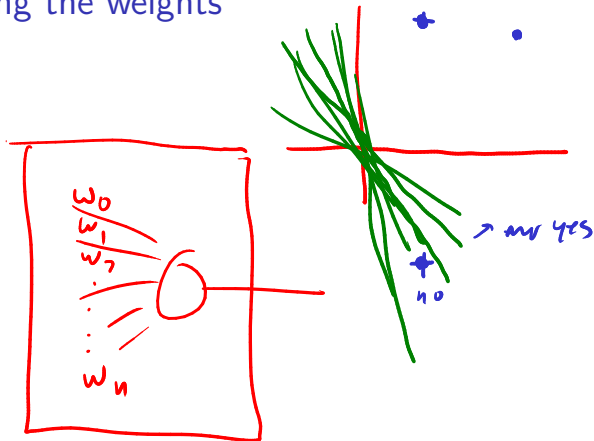
# Perceptrons: Training the weights
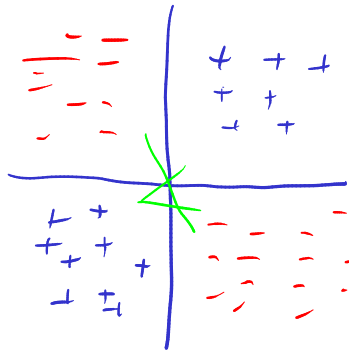
data

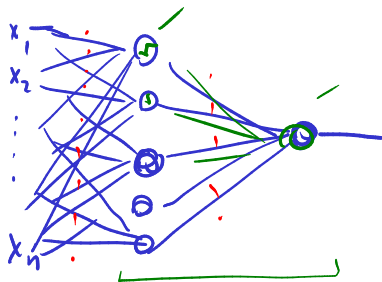$\vec{x}_1, y_1$

$\vec{x}_2, y_2$

$\vec{x}_3, y_3$

⋮

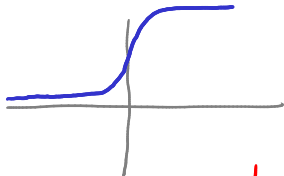$\vec{x}_m, y_m$

$w_0$
$w_1$
$w_7$
⋯
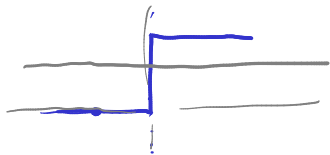$w_n$

→ say yes

no

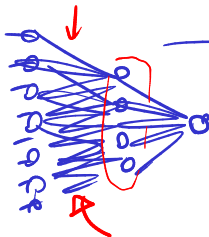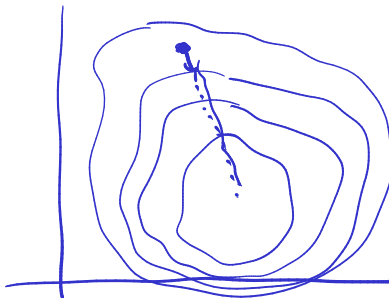# Perceptrons: XOR limit and multilayer perceptrons



XOR

# Neural networks



objective ERROR ...

# Inspiration from visual cortex study

# Convolutional neural networks



1 pattern

convolution filter | linear