01204211 Discrete Mathematics
Lecture 8b: Finite automa[1]

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[1]Based on lecture notes of *Models of Computation* course by Jeff Erikson.

Example: syntax highlighting

HTML tokenizer

Game programming

State-transition graphs

More examples over $\Sigma = \{0, 1\}$

All strings, except 010.

Strings containing the subsequence 010.

Formal definitions

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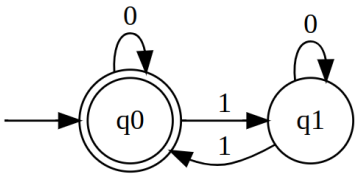
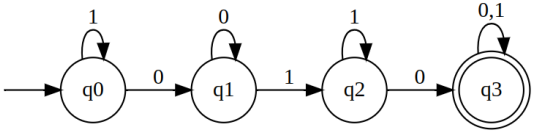
- the input alphabet Σ,
- a finite set of states Q,
- a transition function $\delta : Q \times \Sigma \longrightarrow Q$
- a start state $s \in Q$, and
- a subset $A \subseteq Q$ of accepting states.

Example 1



Example 2



Moves

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$$\delta^*(q, w) = \begin{cases} q & \text{if } w = \varepsilon, \\ \delta^*(\delta(q, a), x) & \text{if } w = ax. \end{cases}$$

The signature of δ^* is $Q \times \Sigma^* \longrightarrow Q$.

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$$\delta^*(s, w) \in A.$$

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def multiple_of_5(w):
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    for i in w:
        r = (2*r + w) % 5
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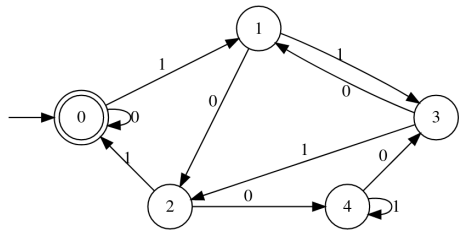

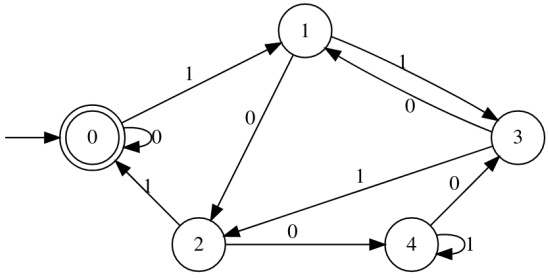
0
1
2
3
4
1
0
1
0
1
0
1
0
0
1

Digital design: Implementation

Digital design: Moore and Mealy machines

In the digital design class, you will encounter finite-state machines as well. The version we consider in this class is refered to as a **Moore machine**.
In practices, there is another variant of FSM called **Mealy machines**, whose outputs depend on input symbols as well.

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Formally, they differ in output function.

- Moore machine: $G : Q \longrightarrow [0, 1]$
- Mealy machine: $G : Q \times \Sigma \longrightarrow [0, 1]$

Example: even number of 1's

Example: strings containing 00 as a substring

Combining DFAs

What if we want to build a DFA that accepts strings with an even number of 1's and containing 00 as a substring?