▷ มีบางปัญหา ที่คอมพิวเตอร์ ไม่สามารถ แก้ได้

language.

01204211 Discrete Mathematics
Lecture 12a: Undecidability (1)

Turing machine

▶ มี language L

ที่ไม่มีโปรแกรม ภาษาไพธอน P ใดๆ

ที่ (1) รับ input แล้วจบการทำงานเสมอ

(2) $\forall w \in L$, $P(w) =$ "yes"

(3) $\forall w \notin L$, $P(w) =$ "no"

Jittat Fakcharoenphol

September 18, 2025

มีโปรแกรม ภาษาไพธอน ที่ $\boxed{\text{แก้ได้}}$ decides

Non-context-free languages

$S \to 0S1 \mid \varepsilon$

While

$$\{0^n 1^n \mid n \geq 0\}$$

is context free, the language

$$\{0^n 1^n 0^n \mid n \geq 0\}$$

is not.

Non-context-free languages

While

$$\{0^n1^n \mid n \geq 0\}$$

is context free, the language

$$\{0^n1^n0^n \mid n \geq 0\}$$

is not.
Can we write a python program to check if a string w belongs to the language $\{0^n1^n0^n \mid n \geq 0\}$?

Big question

Is there a python program that “solves” any possible problem?

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Is there an algorithm that solves every problem?
What is the limit of computation?

Answer by a counting argument

If there are “more” problems than any possible algorithms, then there should be some problem that algorithms cannot solve.
(Think of an algorithm as “a python program.)

Answer by a counting argument

If there are "more" problems than any possible algorithms, then there should be some problem that algorithms cannot solve.
(Think of an algorithm as "a python program.)
However, there are infinitely many python programs and there are infinitely many problems. It is not obvious how to much such an argument formally.

Bijections

Definition

- A function $f : A \longrightarrow B$ from domain A to range B is **one-to-one** if for any $x \neq y \in A$, $f(x) \neq f(y)$.
- A function $f : A \longrightarrow B$ from domain A to range B is **onto** if for any $x' \in B$, there exists $x \in A$ such that $f(x) = x'$.
- A function $f : A \longrightarrow B$ is a **bijection** (or bijective) if it is one-to-one and onto.

Bijection: examples

Lemma 1

For any set A, there is no bijective function $f : A \longrightarrow 2^A$.

Proof.

We prove by contradiction. Assume that there exists a bijective function f from A to 2^A. We construct a set $B \subseteq A$ such that there is no $x \in A$ such that $f(x) = B$.

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We define B as follows.

$$B = \{x \in A \mid x \notin f(x)\}.$$

Now suppose that there exists $x \in A$ such that $f(x) = B$. There are two cases to consider:
Case 1: $x \in B$.

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Now suppose that there exists $x \in A$ such that $f(x) = B$. There are two cases to consider:
Case 1: $x \in B$.
Case 2: $x \notin B$.
In both case, we have a contradiction; therefore, our assumption is false. Thus, there is no bijection between A and 2^A. $\square$

Example: finite set

Let $A = 1, 2, 3, 4, 5, 6, 7$. Consider function $f : A \longrightarrow 2^A$ defined as

$$
\begin{aligned}
f(1) &= \{\} \\
f(2) &= \{1, 2, 3\} \\
f(3) &= \{1, 2, 3, 4, 5, 6, 7\} \\
f(4) &= \{1, 3, 5, 7\} \\
f(5) &= \{2, 4, 6\} \\
f(6) &= \{7\} \\
f(7) &= \{1, 2, 3\}
\end{aligned}
$$

$B =$

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$$

	1	2	3	4	5	6	7
1							
2							
3							
4							
5							
6							
7							

$B =$

Example: infinite set

Let $A = \mathbb{N} = \{1, 2, 3, 4, \ldots\}$. Consider function $f : A \longrightarrow 2^A$ defined as

$$
\begin{array}{rcl}
f(1) & = & \{\} \\
f(2) & = & \{1, 2, 3\} \\
f(3) & = & \{1, 2, 3, 4, 5, 6, 7, \ldots\} \\
f(4) & = & \{1, 3, 5, 7, \ldots\} \\
f(5) & = & \{2, 4, 6, \ldots\} \\
f(6) & = & \{7\} \\
f(7) & = & \{1, 2, 3, 11, 12, 13, 21, 22, 23, \ldots\} \\
\vdots & & \vdots
\end{array}
$$

	1	2	3	4	5	6	7	...
1								
2								
3								
4								
5								
6								
7								
$\vdots$								

$B =$

The previous lemma informally states that there are "more" subsets than the number of elements in the set.
Let's think about:

- A set of all python programs, and
- A set of all languages.

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Let's think about:

- A set of all python programs, and
- A set of all languages.

Since each python program "solves" at most one language, there are not "enough" python programs to solve all possible language.
But what exactly is a problem that cannot be "solved"?

Decision problems

- Given an integer x, is x odd?
- Given a string w, is w palindrome?
- Given a string w, is $w \in \{0^n 1^n \mid n \geq 0\}$?
- Given a map, a starting position s, a destination t, and an integer k, does there exist a path from s to t with distance at most k?
- Given a program P and input string w, when running P with w as an input, does P terminate?

Decision problems and languages

For this problem:

Given an integer x, is x odd?

we can define a corresponding language

$$L_E = \{, \ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}.$$

To solve this problem, given x, we can ask if $x \in L_E$.

Languages and programs

We will talk about languages of particular programs. For example, let $\mathbb{P}$ be the set of all python programs. In this case, $\mathbb{P}$ is a language.

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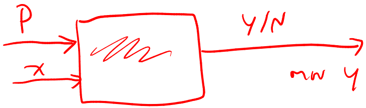
$$L = \{(P, x) \mid P \in \mathbb{P}, P(x) \text{ terminates}\}$$



ans Y iff P(x) terminates.

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$$\{(P, x) \mid P \in \mathbb{P}, P(x) \text{ terminates}\}$$

$$\{(P, Q, x) \mid P, Q \in \mathbb{P}, P(x) \text{ and } Q(x) \text{ terminate with the same output.}\}$$

Programs and inputs

```
x = int(input())
if x % 2 == 0:
    print('yes')
else:
    print('no')
```

Programs and inputs

```
$ python le.py
10
yes
$ python le.py
7
no
```

```
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```
$ python le.py
fjdsklfjsdf
Traceback (most recent call last):
  File "le.py", line 1, in <module>
    x = int(input())
ValueError: invalid literal for int()
with base 10: 'fjdsklfjsdf'
```

Programs and inputs

x

./a.out < x

```
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Nice programs

We can systematically modify any python program P so that

- P contains a main function that works with the input as a string.
- P never crashes. (If the original P crashes, the modified P outputs `no`.)

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```
x = int(input())
if x % 2 == 0:
    print('yes')
else:
    print('no')
```

```
import sys
def main(w):
    try:
        x = int(w)
        if x % 2 == 0:
            print('yes')
        else:
            print('no')
    except:
        print('no')

if __name__ == '__main__':
    w = sys.stdin.read()
    main(w)
```

When running a program

When you run a program P with input x, there are three possible outcomes:

- P terminates and outputs **yes**,
- P terminates and outputs **no**, and
- P does not terminate. (It runs forever.)

When running a program

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- P terminates and outputs **yes**,
- P terminates and outputs **no**, and
- P does not terminate. (It runs forever.)

Remarks: if P crashes (even after modification), we treat it as if it terminates and outputs **no**.

Proving impossibility

ทราบ
ปัญหา R แก้ไม่ได้

$\neg Q$ ①
(Turing)

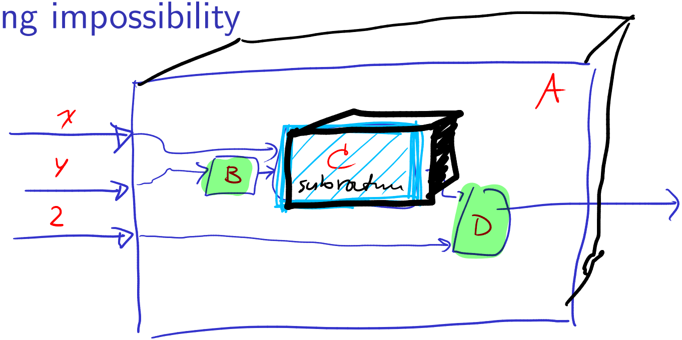
$\boxed{P \Rightarrow Q}$ ②

$\neg P$

Goal: ปัญหา (language) L แก้ไม่ได้
$(\neg P)$

Strategy: จะพิสูจน์ว่า ถ้า L แก้ได้ จะสามารถแก้ R ได้

Proving impossibility



Ⓐ ~~solves~~ decides language R

Ⓒ ~~solves~~ decides language Ⓛ

$$A(x,y,z) = D(C(x, B(y)), z)$$

- B ทำได้
- D ทำได้
- C ไม่รู้

} สร้าง A

① ถ้าปัญหา L แก้ได้

⇒ ปัญหา R แก้ได้ด้วย

② R แก้ไม่ได้ ☹

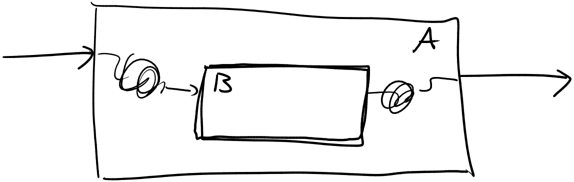
$P \Rightarrow Q$ ✓

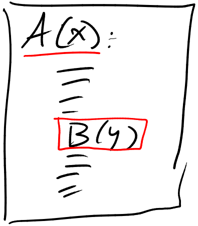
$\neg Q$ ✓

Reduction: rough idea

A แก้ปัญหา R

B แก้ปัญหา L





→ ถ้าแก้ปัญหา L ได้
⇒ แก้ปัญหา R ได้

ลดรูปจาก R ไป L

ทำ reduction จาก R ไป L

Language A

x.py

python x.py < x.py

Let $\mathbb{P}$ be the set of all python programs. Let the language A be

$$\{P \in \mathbb{P} \mid \text{when running } P \text{ with } P \text{ as an input, } P \text{ terminates}\}$$

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We use a function call notation $P(x)$ when refering to the execution of program P with input x.
We restate the definition of A as

$$\{P \in \mathbb{P} \mid P(P) \text{ terminates}\}.$$

Deciders

We say that a python program P **decides** the language L if for any input string x, P when running with x as an input,

- P always terminates,
- P outputs **yes**, if $x \in L$, and
- P outputs **no**, if $x \notin L$.

Deciders: more examples

Language A

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Language A

program Q(P):
//run P with P as an input
result ← P(P)
string
return True.

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We use a function call notation $P(x)$ when refering to the execution of program P with input x.
We restate the definition of A as

$$A = \{P \in \mathbb{P} \mid P(P) \text{ terminates}\}.$$

Case 1: พิจาร input $P \in A$.
- $Q(P)$ จะตอบ .. True

Case 2: input $P \notin A$
- $Q(P)$ .. ~~ตอบ~~ .... ทำงานไม่รู้จบ.

Q เป็น decider ของ A?

Not a decider for A

```
Input: python program P (as a string)

1. Load module P as Pmod
2. Call Pmod.main(P)
3. print('yes')                # we reach this line,
                               #  only if M.main(P) terminates
```

Lemma 2

There is no python program that decides A.

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We will see the proof at the end of class.

Undecidability

If we believe that anything that a computer can do can be written as a python program,

Undecidability

If we believe that anything that a computer can do can be written as a python program, and there is no python program that decides A, when we say that

A is undecidable.

Language A will be very important later on, we give it a proper name as $\textsc{SelfHalt}$.

The proof as a table

List all python programs in $\mathbb{P}$ as $P_1, P_2, P_3, \ldots$.

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	$\ldots$
P_1								
P_2								
P_3								
P_4								
P_5								
P_6								
$\vdots$								
(B)								

What does B do on each input program P_i?

Another language HALT

ทราบ SelfHalt undecidable

ถ้า SELFHALT แก้ได้ → Halt แก้ได้

1 สร้างโปรแกรมที่ ~~solve~~ ได้ Halt จาก SELFHALT

2 สร้างโปรแกรมที่ solve SelfHalt จาก Halt

Let

$$\textsc{Halt} = \{(P, w) \mid P \text{ is a python program such that } P(w) \text{ terminates}\}$$

We shall prove that HALT is also undecidable (if we believe that python programs represent all possible computation).

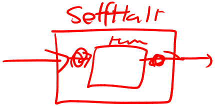
program Q(P)

R(. . .)

แก้ SelfHalt

R แก้ Halt

* ถ้า Halt แก้ได้ จะแก้ SELFHALT ได้



Lemma 3

HALT *is undecidable.*

Proof.

We prove the lemma by contradiction. Assume that there is a python program P that decides HALT.

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Proof.

We prove the lemma by contradiction. Assume that there is a python program P that decides HALT.
We construct a program C as follows

```
Program C
Input Q
1.    Load P as module Pmod
2.    if Pmod.main(Q,Q) == 'yes':
3.        print('yes')
4.    else
5.        print('no')
```

□

Proof.

We prove the lemma by contradiction. Assume that there is a python program P that decides $\textsc{Halt}$.
We construct a program C as follows

```
Program C
Input Q
1.    if P(Q,Q) == 'yes':
2.        print('yes')
3.    else
4.        print('no')
```

Proof.

We prove the lemma by contradiction. Assume that there is a python program P that decides HALT.

We construct a program C as follows

```
Program C
Input Q
if P(Q,Q) == 'yes':
    print('yes')
else
    print('no')
```

Case 1: $Q \in$ SelfHalt.

- $P(Q,Q)$ returns yes

$\rightarrow C$ returns yes

Case 2: $Q \notin$ SelfHalt

then $Q(Q)$ runs forever

- $P(Q,Q)$ returns no, C returns no

Given program P, we can construct a program C that decides SELFHALT.

Proof.

We prove the lemma by contradiction. Assume that there is a python program P that decides HALT.
We construct a program C as follows

```
Program C
Input Q
1.    if P(Q,Q) == 'yes':
2.        print('yes')
3.    else
4.        print('no')
```

Given program P, we can construct a program C that decides SELFHALT. However, we know that SELFHALT is undecidable; thus, we reach a contradiction.
We conclude that there does not exist a python program P that decides HALT. ∎

Reduction

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- We show that if HALT is decidable, then SELFHALT is also decidable.

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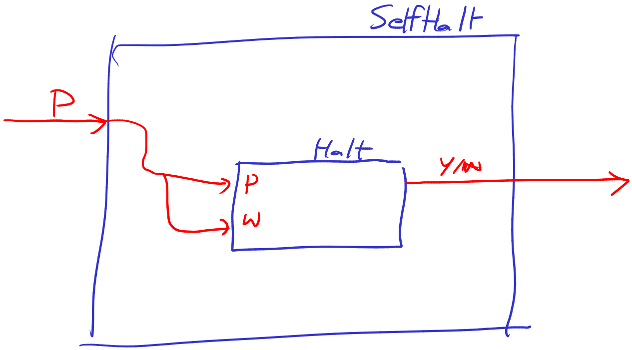
- However, SELFHALT IS UNDECIDABLE.

Reduction

- We show that if HALT is decidable, then SELFHALT is also decidable.
- However, SELFHALT IS UNDECIDABLE.
- We conclude that HALT is also undecidable.

Reduction in picture



Halt = {(P, w) | P ∈ ℙ, P(w) halts}

ตอบ "yes"

Halt undecidable

¬Q

Let ACCEPT $= \{(P, w) \mid P \in \mathbb{P}$ and $P(w)$ terminates with acceptance$\}$.

Lemma 4

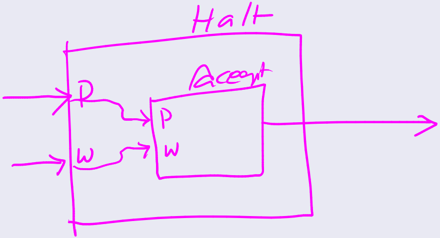
ACCEPT *is undecidable.* ¬P

Proof.

We prove the lemma by contradiction. Assume that there is a python program Q that decides ACCEPT.

program C
input P, w
- result ← $\boxed{Q(P, w)}$
- return result.

* ทำงาน (P, w) reject



▷ Case 1: (P, w) ∈ Halt

ถ้า (P, w) halts แล้ว reject
Q(P, w) . return "no" ✗

▷ Case 2: (P, w) ∉ Halt // P(w) ติดลูป

→ Q(P, w) return "no"
C return "no" ✓

Let $\textsc{Accept} = \{(P, w) \mid P \in \mathbb{P}$ and $P(w)$ terminates with acceptance$\}$.

Lemma 4

$\textsc{Accept}$ *is undecidable.*

Proof.

We prove the lemma by contradiction. Assume that there is a python program Q that decides $\textsc{Accept}$.

1. Assume ACCEPT decidable
2. มี program Q ที่ decides ACCEPT (จาก 1.)
3. แสดงว่ามีโปรแกรม C ที่ decides HALT.
   (ใช้ Q เป็น subroutine ได้)
4. HALT is decidable. ✓

ขัด contradiction
แย้ง HALT is undecidable

D [illegible] P
[illegible] yes
[illegible]

Let $\textsc{Accept} = \{(P, w) \mid P \in \mathbb{P}$ and $P(w)$ terminates with acceptance$\}$.

Lemma 4

$\textsc{Accept}$ is undecidable.

Proof.

We prove the lemma by contradiction. Assume that there is a python program Q that decides $\textsc{Accept}$.

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Program C
Input P,w
1.   Replace every "print('no')" statement in P with "print('yes')"
1.   if Q(P,w) == 'yes':
2.      print('yes')
3.   else
4.      print('no')
```

Proof (cont.) C decides HALT

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Program C
Input P,w
    Replace every "print('no')" statement in P with "print('yes')"
    if Q(P,w) == 'yes':
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We have to make sure that our reduction is correct by considering two cases.
Case 1: when $P(w)$ halts. $(P, w) \in HALT$

- โปรแกรม P ที่แก้แล้ว จะ accepts w

$\rightarrow$ Q(P, w) returns "yes"

$\rightarrow$ C returns "yes" ✓

Proof (cont.)

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Input P,w
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We have to make sure that our reduction is correct by considering two cases.
Case 1: when $P(w)$ halts.
Case 2: when $P(w)$ does not halt.

Q(P,w) returns "no"

C returns "no"

Proof (cont.)

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Program C
Input P,w
    Replace every "print('no')" statement in P with "print('yes')"
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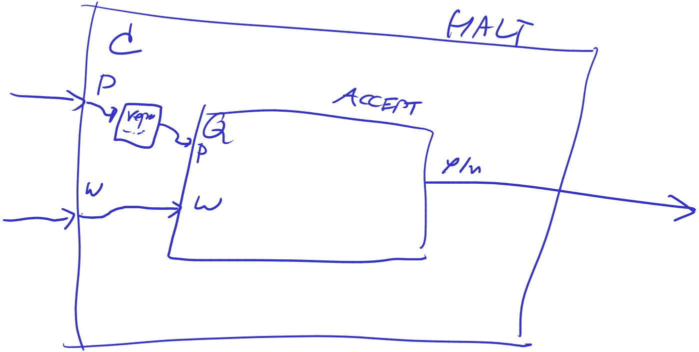
We have to make sure that our reduction is correct by considering two cases.
Case 1: when $P(w)$ halts.
Case 2: when $P(w)$ does not halt.
Since in both cases, C answers correctly, we know that given program Q deciding $\textsc{Accept}$, we can construct a program C that decides $\textsc{Halt}$. However, we know that $\textsc{Halt}$ is undecidable; thus, we reach a contradiction. We conclude that $\textsc{Accept}$ is also undecidable. $\square$

Reduction from Halt to Accept in picture



How about $\textsc{Reject}$?

Let

$$\textsc{Reject} = \{(P, w) \mid P \in \mathbb{P} \text{ and } P \text{ rejects } w\}.$$

Lemma 5

There is no python program that decides $\textsc{SelfHalt}$*.*

Proof.

We prove by contradiction. Assume that there is a python program P that decides $\textsc{SelfHalt}$.

Lemma 5

There is no python program that decides SELFHALT.

Proof.

We prove by contradiction. Assume that there is a python program P that decides SELFHALT.

We describe a python program B that reads a string Q as an input as follows:

```
Program B
Input Q
1.    Load P as module Pmod
2.    if Pmod.main(Q) == 'yes':    # when Pmod outputs yes
3.       while True: pass          #   loop forever
4.    else:                        # when Pmod outputs no
5.       quit()                    #   halt
```

Given program Q as an input, B loops forever when

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Input Q
1.    Load P as module Pmod
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5.       quit()                    #   halt
```

Given program Q as an input, B loops forever when

It terminates when

□

Proof.

We know that

- $B(Q)$ loops when $Q(Q)$ terminates, and
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Does running B using B as an input terminate?

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Let's try to plug in $Q = B$. We have

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- $B(B)$ terminates when $B(B)$ loops.

Since either $B(B)$ loops or terminates, and we cannot be in any of the cases, we obtain a contradiction.
Therefore, we conclude that program P does not exist. $\square$

Python as computation

Do you believe in this assumption:

Anything that a computer can do can be written as a python program.

Turing machines

Anything that a computer can do can be carried out using Turing machines.

Turing machines

Anything that a computer can do can be carried out using Turing machines.

Any possible computation can be performed by Turing machines.