Theory: If given a $N-length$ list of real numbers, and the elements in the array have been sorted with a specific order (in an ascend or descend sequence). Suppose each number in the array has an error bound of ε, and the exact number meets even distribution between $[x-\varepsilon, x+\varepsilon]$ where x is the current value. Then the stability of the array will be:

$$S = \frac{\sum_{1 \le i \prec j \le N} \frac{(d_{ij} - 2\varepsilon)^2}{8\varepsilon^2}}{\binom{N}{2}}$$

Whereby d_{ij} presents distance between x_i and x_j, thus $d = |x_i - x_j|$. If the array is sorted in an ascend order, then $d = x_j - x_i$, otherwise $d = x_i - x_j$.

We define S is the stability of this list, and $S = \frac{K(\tau_1, \tau_2)}{\binom{N}{2}}$, $K(\tau_1, \tau_2)$ is the Kendall tau ranking distance between two lists L_1 and L_2:

$$K(\tau_1, \tau_2) = |\{(i,j) : i < j, (\tau_1(i) < \tau_1(j) \wedge \tau_2(i) > \tau_2(j)) \vee (\tau_1(i) > \tau_1(j) \wedge \tau_2(i) < \tau_2(j))\}|$$

Where $\tau_1(i)$ and $\tau_2(i)$ are the rankings of the element i in L_1 and L_2 respectively. Therefore we could know that $K(\tau_1, \tau_2) = 0$ and $S = 0$ if the two lists are identical and $K(\tau_1, \tau_2) = \frac{n(n-1)}{2}$ and $S = 1$ (where n is the length of each list) if one list is the reverse of the other.

Proving: Assume the list is in an descend order and each element has an error bound of ε, here we arbitrarily pick up two elements i, j and $1 \le i \prec j \le N$, thus $x_i \ge x_j$. We suppose that $x_i = x_0, x_j = x_i - d$. We want to compute $P(i, j)$ which means the possibility that i and j would locate in a reverse order in the new list (remember that each element has an error bound, so the new list with exact numbers may be in a different order). Obviously, we could know $P(i, j) = 0$ if $d > 2\varepsilon$. So we only need to compute the possibility when $d \le 2\varepsilon$.

Suppose $x_j = x, x_0 - d - \varepsilon \le x \le x_0 - d + \varepsilon$, we can see that only when both i, j locate in the interval $[x_0-\varepsilon, x_0-d+\varepsilon]$ and $x_i < x_j$ that could make i, j reverse in the new list.

We could obtain the possibility function of $P(x)$:

$$P(x)=\frac{1}{2\varepsilon}\bullet\frac{x-(x_0-\varepsilon)}{2\varepsilon}=\frac{1}{4\varepsilon^2}x-\frac{x_0-\varepsilon}{4\varepsilon^2}$$

We transform the formula:

$$P = Ax+B \quad ①$$

$$A=\frac{1}{4\varepsilon^2} \quad ②$$

$$B=-\frac{x_0-\varepsilon}{4\varepsilon^2} \quad ③$$

Since x locates in the interval $[x_0-\varepsilon, x_0-d+\varepsilon]$, then calculate the integration of $P(x)$ and got $P(i,j)$:

$$P(i,j)=\int_{x_0-\varepsilon}^{x_0-d+\varepsilon} Ax+B$$

$$=\left[\frac{1}{2}Ax^2+Bx\right]_{x_0-\varepsilon}^{x_0-d+\varepsilon}=\frac{1}{2}A[(x_0-d+\varepsilon)^2-(x_0-\varepsilon)^2]+B(x_0-d+\varepsilon-x_0+\varepsilon)$$

$$=\frac{1}{2}A(d^2-2x_0d-2d\varepsilon+4x_0\varepsilon)+B(2\varepsilon-d)$$

$$=\frac{1}{8\varepsilon^2}(d^2-2x_0d-2d\varepsilon+4x_0\varepsilon)-\frac{1}{4\varepsilon^2}(x_0-\varepsilon)(2\varepsilon-d)$$

$$=\frac{1}{8\varepsilon^2}(d^2-2x_0d-2d\varepsilon+4x_0\varepsilon-4x_0\varepsilon+2x_0d+4\varepsilon^2-2\varepsilon d)$$

$$=\frac{1}{8\varepsilon^2}(d^2-4\varepsilon d+4\varepsilon^2)=\frac{(d-2\varepsilon)^2}{8\varepsilon^2}$$

Therefore:

$$P(i,j)=\frac{(d_{ij}-2\varepsilon)^2}{8\varepsilon^2}$$

Particularly, $P(i,j)=0$ when $d_{ij}=2\varepsilon$, and $P(i,j)=\frac{1}{2}$ when $d_{ij}=0$.

Finally, if we know each value in the list and the error bound ε, we could compute the stability:

$$S=\frac{\sum_{1\le i\prec j\le N}\frac{(d_{ij}-2\varepsilon)^2}{8\varepsilon^2}}{\left(\frac{N}{2}\right)}$$