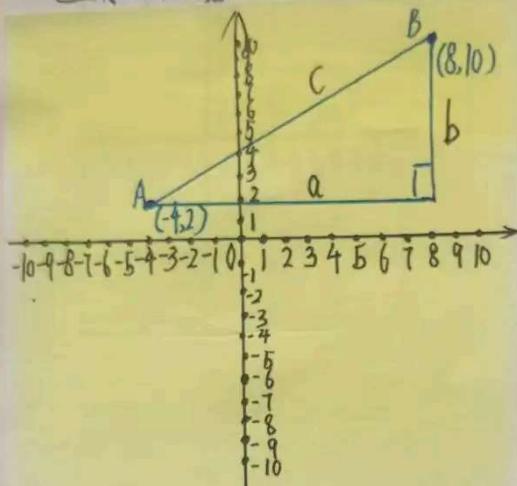


天佑城校区拓展1班笔记收集

胡浩轩

直角坐标系有点A(-4,2)、点B(8,10); AB距离为?



用勾股定理解

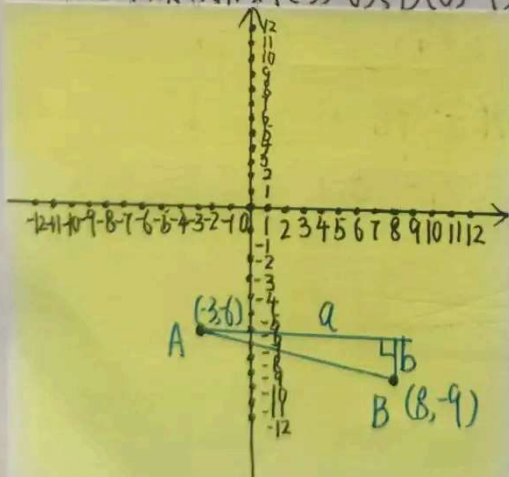
$$\because a^2 + b^2 = c^2$$

$$\therefore (8 - (-4))^2 + (10 - 2)^2$$
$$= 144 + 64$$

$$= 208 = c^2$$

$$\therefore c = \sqrt{208} = 4\sqrt{13}$$

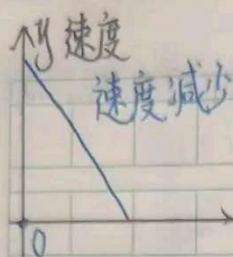
直角坐标系有点A(-3,-6)、B(8,-9); 则直线AB斜率为?



$$\text{斜率} = \frac{b}{a}$$

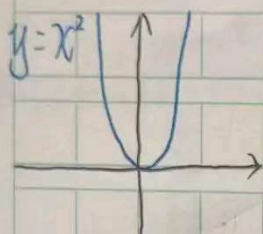
$$\therefore \frac{-9 - (-6)}{8 - (-3)} = \frac{-3}{11} = -\frac{3}{11}$$

$$\text{斜率} = -\frac{3}{11}$$



$$y = kx + b \quad (k \neq 0)$$

y 是 x 的函数

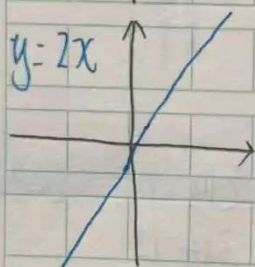


x	-2	-1	0	1	2
-----	----	----	---	---	---

y	4	1	0	1	4
-----	---	---	---	---	---

$(-2, 4) (-1, 1) (0, 0) (1, 1) (2, 4)$

对 x 每一个确定的值, y 都有唯一确定的值与其对应。

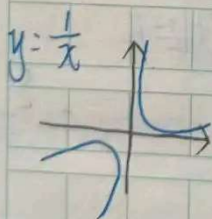


1. 列表
2. 描点
3. 连线

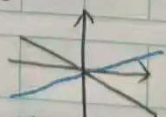
$$y = kx \quad (k \text{ 为常数}, k \neq 0)$$

正比例函数

k 为比例系数

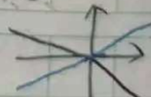


$$y = \frac{1}{2}x$$



正数方向过一三象限,
负数过二四象限

$$y = -\frac{1}{2}x$$



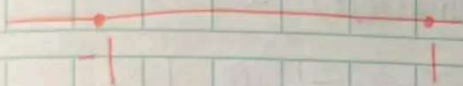
$(k > 0)$

$(k < 0)$

$k > 0: x \nearrow, y \text{ 随之} \nearrow$ $k < 0: x \nearrow, y \text{ 随之} \searrow$

$$|a-(-1)| = |b-(-1)| \quad -1 < a, b, c, d \leq 1$$

$$|c-1| = |d-1| \quad a \neq b \neq c \neq d$$



$$\sqrt{2} \approx 1.414$$

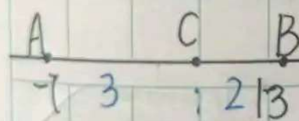
$$\sqrt{3} \approx 1.732$$

$$\sqrt{5} \approx 2.236$$

数轴上, 点 $A = -7$, $B = 13$, 点 C 在 A, B 之间, $AC:CB = 3:2$,
则点 $C = ?$

$$13 - (-7) = 20 \quad 3 \times 4 = 12$$

$$20 \div (3+2) = 4 \quad -7 + 12 = 5$$



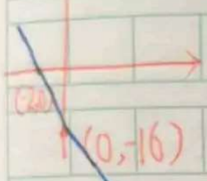
$\therefore C$ 点为 $+5$

1. 已知直线 $y = kx - 8x - 16$, 则直线与 x 轴的交点坐标为

$(-2, 0) \because y = -8x - 16 \therefore x = -2$

$\therefore 0 = -8x - 16 \therefore$ 直线与 x 轴的交点坐标

为 $(-2, 0)$



$16 = -8x$

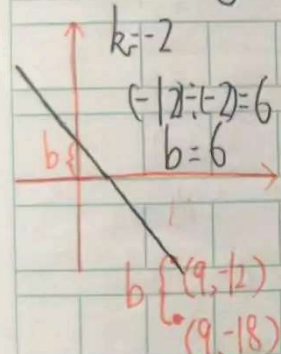
$-16 = 8x$

$x = -2$

$(-8 \times (-2) = 16 - 16 + 16 = 0)$

第0行

2. 已知直线 $y = kx + b$ 经过点 $A(9, -12)$, 且 $k = -2$, 求 $b = 6$.



$k = -2$

$(-12 = (-2) \cdot 9 + b)$

$b = 6$

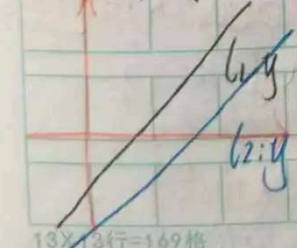
$-12 = (-2) \cdot 9 + b \Rightarrow (kx + b, x \text{ 为 } 9)$

$-12 = -18 + b$

$-12 + 18 = b$

$b = 6$

3. 已知直线 $l_1: y = 2x - 8$, 直线 $l_2: y = kx + b$ 与 l_1 平行且经过点 $(0, 12)$, 则直线 $l_2: y = 2x - 12$



\therefore 直线 $l_2: y = kx + b$ 与 直线 $l_1: y = 2x - 8$ 平行

$\therefore l_2: y = 2x + b$

$+12 = 2 \cdot 0 + b$

$-12 = b$

$\therefore l_2: y = 2x + (-12) = 2x - 12$

4

13x 13行 = 169格

正比例函数: 一次函数: (不经过原点)

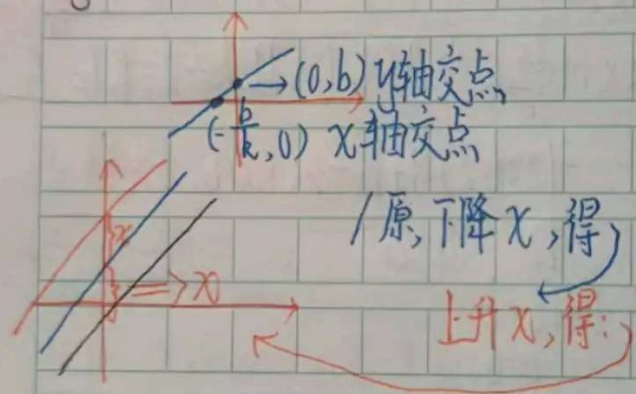
$y=kx (k \neq 0)$

$y=kx+b$

→ 常数

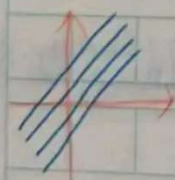
正比例函数一定是一次函数,
一次函数不一定是正比例函数

$y=kx+b (k \neq 0)$

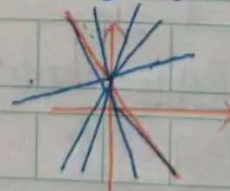


$y=b$ 表示平行于 x 轴的一条直线。

一次函数一定是一条直线, 但直线不一定是一次函数。



5 k 定 b 不定



b 定 k 不定

将点 $B(10, 0.5)$ 和 $C(20, 4.5)$

代入 $y=kx+b$:

$\begin{cases} 10k+b=0.5 \\ 20k+b=4.5 \end{cases}$

x 取值: $10 \leq x \leq 20$

有一片牧场，每天都在均匀地生长草，每头牛每天吃1份草。如果在牧场上放养15头牛，那么8天能把草吃完。如果是35头牛，那么3天能把草吃完。如果一开始放养27头牛，几天吃完？

1. 解：设原有草量为 x ，每天生长量为 y

$$\begin{cases} x+8y=15 \times 8 & ① \\ x+3y=35 \times 3 & ② \end{cases}$$

$$\begin{cases} x+8y=15 \times 8 & ① \\ x+3y=35 \times 3 & ② \end{cases}$$

①-②得：

$$5y=15 \times 8 - 35 \times 3$$

$$5y=15$$

$$y=3$$

将 $y=3$ 代入①得：

$$x+8 \times 3=120$$

$$x=120-24$$

$$x=96$$

$$\begin{cases} x=96 \\ y=3 \end{cases}$$

$$\begin{cases} x=96 \\ y=3 \end{cases}$$

$$96 \div (27-3)$$

$$=96 \div 24$$

$$=4(\text{天})$$

结果为4

$$2, (15 \times 8) - (35 \times 3) = 15(\text{份}) \quad 15 \div (8-3) = 3(\text{份/天})$$

$$(15-3) \times 8 = 96(\text{份}) \quad 96 \div (27-3) = 4(\text{天}) \quad \text{结果为4}$$

A C B 交叉相乘，得：

$$\begin{matrix} m & n \\ nA+mB \\ m+n \end{matrix}$$

$$\therefore y=kx+b$$

$$\therefore 0=kx+b$$

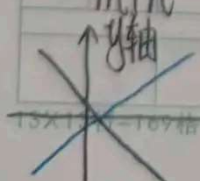
即可求出C点坐标

$$y=(>0)x$$

$$y=(<0)x$$

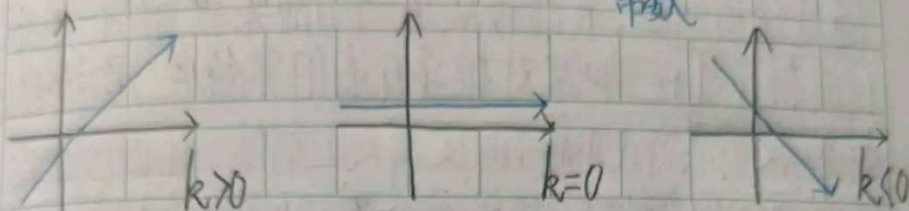
y 即为 x 的函数

6



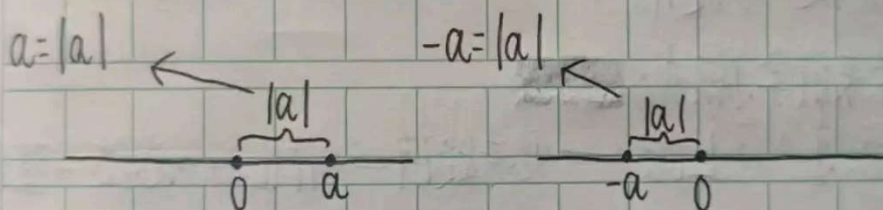
其中，正数过一三象限，负数过二四象限

正比例函数: kx 一次函数: $kx+b$ ($k \neq 0$)
常数



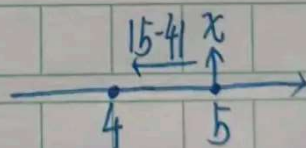
$\log_3(27) = \underline{3} \rightarrow$ 求指数 $27 = 3 \times 3 \times 3 = 3^3$ 指数为3

$a = |a|$ $-a = |a|$ $|a-b| = (-a+b) = b-a$



①求最小值。

$3|x-5| + |x-4|$



若 $x=5$, 即 $3|x-5|=0$

$|x-4|=1$

②使式子分母有理化。

$\therefore 0+1=1$, 最小值得1

$$\begin{aligned} \sqrt{a+b} &= \frac{(\sqrt{a+b})(\sqrt{a-b})}{(\sqrt{a-b})} \\ &= \frac{a-b}{\sqrt{a-b}} \\ &\downarrow \\ (a+b)(a-b) &= a^2-b^2 \end{aligned}$$

$$(x+1)^4 = ?$$

$$\textcircled{1} (x+1)(x+1)(x+1)(x+1)$$

一个扩号一个维度，
有4个扩号，即为四维

$$(x+1)(x+1)$$

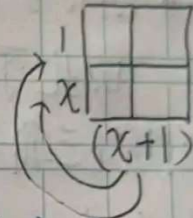
$$\text{三维: } (x+1)(x+1)(x+1)$$

$$= [(x+1)(x+1)](x+1)$$

$$= x^3 + 3x^2 + 3x + 1$$

$$\text{四维: } (x+1)(x+1)(x+1)(x+1)$$

$$= x^4 + 4x^3 + 6x^2 + 4x + 1$$



三个选2个

$$C_3^2 \Rightarrow$$

$$\frac{3 \times 2}{2 \times 1} \text{ (分子由左往右)} \text{ (分母由右往左)}$$

得3

$$C_4^3 \Rightarrow$$

$$\frac{4 \times 3 \times 2}{3 \times 2 \times 1} = 4$$

$$C_4^2 \Rightarrow$$

$$\frac{4 \times 3}{2 \times 1} = 6$$

$$\textcircled{2} (x+1)(x+1) = (x^2 + x + 1)$$

$$(x+1)(x+1)(x+1)(x+1) = (x^2 + x + 1)(x^2 + x + 1)$$

$$= (x^3 + 2x^2 + x)(x^2 + x + 1)$$

$$= x^4 + 4x^3 + 6x^2 + 4x + 1$$

依次相乘

$$\begin{array}{lcl}
 1 & 1 & \text{一维 } (x+1) \\
 ③ \quad 1 & 2 & 1 \quad \text{二维 } (x^2+2x+1) \\
 & 1 & 3 & 3 & 1 \quad \text{三维 } (x^3+3x^2+3x+1) \\
 & 1 & 4 & 6 & 4 & 1 \quad \text{四维 } (x^4+4x^3+6x^2+4x+1) \\
 & 1 & 5 & 10 & 10 & 5 & 1 \quad \text{五维 } (x^5+5x^4+10x^3+10x^2+5x+1)
 \end{array}$$

由此可以直接求出此题 $(x+1)^4 = ?$

帕斯卡三角法

$$\text{例: } (a+b)^3 = a^3 + 3a^2b + 3b^2a + b^3$$

—— $(a+b)^n$ 的展开

$$\begin{aligned}
 (x+y+z)^2 &= ? \Rightarrow (x+y)^2 \\
 &= x^2 + y^2 + 2xy
 \end{aligned}$$

$$\begin{aligned}
 \therefore (x+y+z)^2 \\
 &= x^2 + y^2 + z^2 + 2xy + 2xz + 2yz
 \end{aligned}$$

$$\begin{aligned}
 (a+b)(x+y) \\
 &= ax + ay + bx + by
 \end{aligned}$$

y	ay	by
x	ax	bx
	a	b

$$ax \div b$$

(拆解)

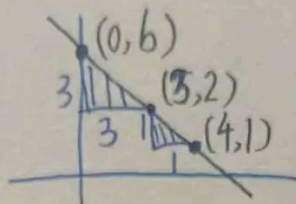
→ 实数

9

13X13行=169格

$$\begin{array}{c} \text{---} m \text{---} n \text{---} \\ | \quad | \quad | \\ A \quad C \quad B \end{array} \quad C = \frac{nA+mB}{m+n} = \frac{1B+3A}{1+3}$$

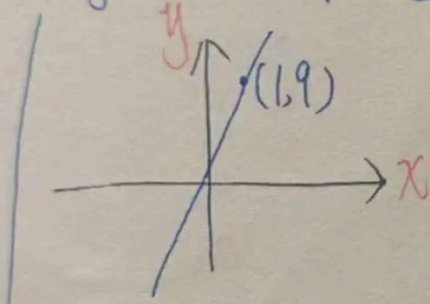
$$\frac{x - (-1)}{y - 0} = \frac{3}{1}$$



$$b = (5) \quad 2+3=5$$

line: 直线

直线 $y=kx+b$ $b=1$ 经过点 $A(1,9)$ $k=8$ slope: 斜率

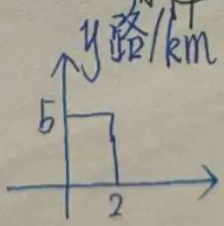
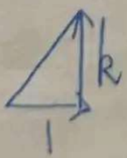


直线 $y=8x-24$, $8x-24=0$
与 x 轴交点为 3, 0 $x=3$

$\log_3(27) = 3^{\square} = 27$
求指数 (得 3)

点 $A(9, -25), B(-2, -4)$ $k=3$ $b=$
 $\frac{25-4}{9-2}=3$ 斜率

$y=kx+b$
if: $x+1$



行 2 时, 行 5 km

$\frac{1}{3} = 3^{-1}$	$\frac{1}{3} = 3^{-1}$
$\frac{1}{3} = 3^{-1}$	$3^{-1} = \frac{1}{3}$
$2^2 = \frac{1}{2^{-2}}$	$3^{-1} = \frac{1}{3}$
$\frac{1}{2} = \frac{2^0}{2^1} = 2^{(0-1)} = 2^{-1}$	$2^{-1} = \frac{1}{2}$

$\nearrow: k > 0 \rightarrow: k = 0 \searrow: k < 0$

$$(x-y)^2 = (x+y)^2 - 4xy \quad \text{完全平方公式}$$

$$\text{sgn}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases} \quad \sqrt[n]{n} = \frac{n}{n} \quad \frac{1}{a^n} = a^{-n}$$

$$(a-b)^2 + (a+b)^2 = 2a^2 + 2b^2$$

$$a^2 - b^2 = (a+b)(a-b) \quad \sqrt[n]{\cdot} = \frac{1}{n} \text{ 先} \cdot \text{后}$$

$$a^2 - 1 = (a+1)(a-1) \quad \cdot \text{减}$$

$$(-2)^2 = 4 \quad 2^2 = 4 \quad \pm 2 \text{ 是 } 4 \text{ 的平方根}$$

$$2 \text{ 是 } 4 \text{ 的算术平方根}$$

$$(a-b)(a+b) = a^2 - b^2$$

$$\sqrt{a} \cdot \sqrt{b} = a^{\frac{1}{2}} \cdot b^{\frac{1}{2}} = \sqrt{ab} \quad \sqrt[n]{a} = a^{\frac{1}{n}} \quad a^m \cdot b^m = (ab)^m$$

$$\sqrt{3} \cdot \sqrt{2} = 3^{\frac{1}{2}} \cdot 2^{\frac{1}{2}} = 6^{\frac{1}{2}} = \sqrt{6} \quad 2\sqrt{3} = 4^{\frac{1}{2}} \cdot 3^{\frac{1}{2}} = 12^{\frac{1}{2}} = \sqrt{12}$$

$$n^{-1} = \frac{1}{n} \quad 2^3 = 8 \quad 2^2 = 8 \quad a = \log_2 8 = 3 \quad \sqrt{\quad} \rightarrow \text{不是负数}$$

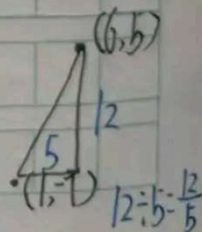
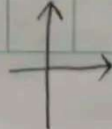
$$\sqrt{\odot} \geq 0 \quad \frac{a-b}{-2} = \frac{ba}{2} \quad \text{斜率公式: } \frac{y_2 - y_1}{x_2 - x_1}$$

$$f(n) = A \cdot n^3 + B \cdot n^2 + Cn \quad \sqrt{a^2} = |a|$$

$$\left(\frac{1}{7}\right)^{-2} = 49$$

$$15^\circ \rightarrow 3 \text{ 行 } 7 = 169$$

$$90^\circ \quad (i)^2 = -1$$



14

$$\begin{aligned}
 & \sqrt{23-6\sqrt{10+4\sqrt{3-2\sqrt{2}}}} \quad 1. \quad 1+2+3+\dots+n = \frac{n(n+1)}{2} \\
 & = \sqrt{23-6\sqrt{10+4(\sqrt{2}-1)}} \quad S = 1+2+3+\dots+n / n+(n-1)+(n-2)+\dots+1 \\
 & = \sqrt{23-6\sqrt{2}(\sqrt{2}+1)^2} \quad 2S = (n+1) \cdot n : 1+2+3+\dots+n = \frac{n(n+1)}{2} \quad \text{---} \\
 & = \sqrt{23-6\sqrt{2}(\sqrt{2}+1)^2} \quad S = \frac{n(n+1)}{2} \\
 & = \sqrt{23-6\sqrt{2}(\sqrt{2}+1)^2} \quad 2. \quad 1+3+5+\dots+(2n-1) = n^2 \quad 3-1=2 \\
 & = \sqrt{11-6\sqrt{2}} \quad 2n-1 = \frac{1}{4} \cdot [(2n+1)(2n-1) - (2n-1)(2n-3)] \\
 & = \sqrt{11-2\sqrt{18}} \quad 2n-3 = \frac{1}{4} \cdot [(2n-1)(2n-3) - (2n-3)(2n-5)] \\
 & = \sqrt{(\sqrt{9}-\sqrt{2})^2} \quad 2n-5 = \frac{1}{4} \cdot [(2n-3)(2n-5) - (2n-5)(2n-7)] \\
 & = 3-\sqrt{2}
 \end{aligned}$$

$$1 = \frac{1}{4} \cdot [(1+2) - 1(-1)]$$

$$\begin{aligned}
 \therefore S_n &= \frac{1}{4} [(2n+1)(2n-1) + 1] = \frac{1}{4} [4n^2 - 1 + 1] = \frac{1}{4} \times 4n^2 = n^2 \\
 S_n &= n^2
 \end{aligned}$$

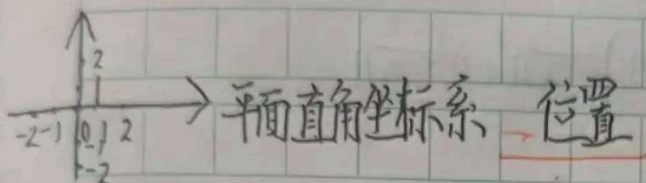
$$\begin{aligned}
 3. \quad 1^2+2^2+3^2+4^2+\dots+n^2 &= \frac{n(n+1)(2n+1)}{6} \\
 1 \times 1 + 2 \times 2 + 3 \times 3 + \dots + n \times n
 \end{aligned}$$

$$\begin{aligned}
 &= (1-1) \times 1 + 1 + (2-1) \times 2 + 2 + (3-1) \times 3 + 3 + \dots + (n-1) \times n + n \\
 &= \frac{n(n+1)}{2} + \frac{n(n+1)(2n+1)}{6}
 \end{aligned}$$

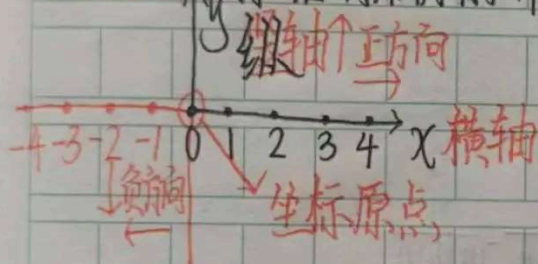
$$\begin{aligned}
 &= \frac{2(n)(n+1)(n-1) + 3n(n+1)}{6} = \frac{(n+1)n(2n-2+3)}{6} = \frac{n(n+1)(2n+1)}{6}
 \end{aligned}$$

13

双曲线：两个垂直对顶的曲面被一个不通过顶点的平面所截，截出的曲面即为双曲线。

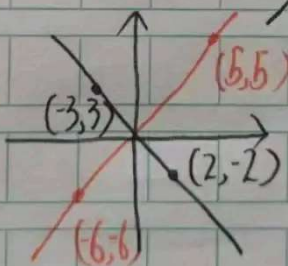
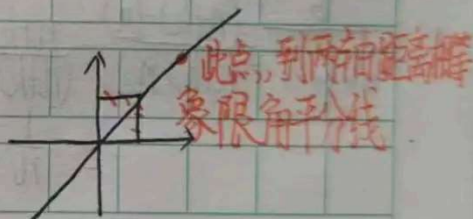


有序数对：一前一后有顺序的2个数以确定位置。

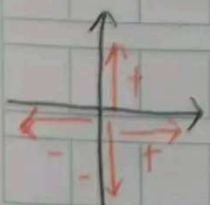


距离即为绝对值(非负)

$$|a| = |-a|$$



/ 为 - 三象限角平分线
 \ 为 - 四象限角平分线



左减右加横坐标，
 上加下减纵坐标

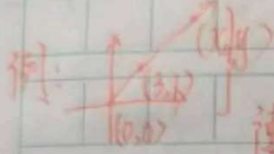
直角三角

A(-10, 0), B(-9, -3)

直线AB解析式(AB):

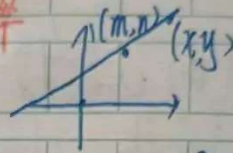
$$\frac{\Delta S}{\Delta t} = \frac{-3}{1} (0, -3)$$

$$\frac{0-y}{-10-x} = \frac{-3}{1} = \frac{\Delta S}{\Delta t}$$



速: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{1-0}{3-0} = \text{斜率}$

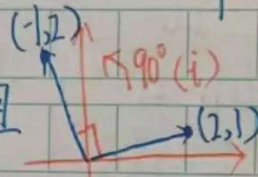
$$\frac{y}{x} = \frac{a}{b} \Rightarrow ax = by$$



$$\frac{y-n}{x-m} = \frac{1}{2}$$

$y = kx + b$ 经过点(6, -12), 且

$k = -3, b =$

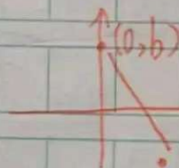


$$2y - 2n = x - m$$

$$-x + 2y + m - 2n = 0$$

$$(-1, 2)$$

$$(2, 1)$$



$$\frac{-12-b}{6-0} = -3 = \frac{-12-b}{6}$$

$$y = -3x - 12$$

$$0 = -3x - 12$$

$$-12 = (-3) \cdot 6 + b$$

$$-12 = -18 + b$$

$$b = 6$$

速度相等

距离相等

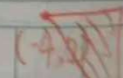
$$l_1: y = 4x + 12$$

$$l_2: y \text{ 经过 } (3, 1)$$

$$l_2: y = 4x + 3$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

x轴 y轴



$$l_1: y = -4x + 12$$

$$l_2: y = -4x + 3$$

$$l_1: y = -4x + 12$$

$$l_2: y = -4x + 3$$

把代数式 $x^2 - 2x - 3$ 化为 $(x-m)^2 + k$ 的形式, 其中 m, k 为常数, 求 m, k 的值。

1. $n^2 = -1/\dots$? $n=1$ 时, $1^2=1 \times$ $n=-1$ 时, $n=0$ 时, $0^2=0 \times$ $-1^2=1 \times$

$\therefore (x-m)^2 \neq -1$ 得 $()^2 \geq 0$

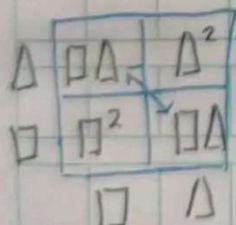
2. $(x+1)^2 + 3 \geq 3 \quad \because x \geq 0, \therefore$ 此式正确

3. $(x-1)^2 + 2 \geq 2 \quad \because x \geq 0 \therefore$ 正确

4. $x^2 - 2x + 3 \geq \frac{2}{1}$

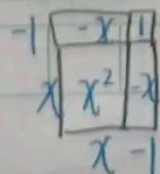
\Downarrow
 $(a+b)^2 = a^2 + 2ab + b^2$ 完全平方公式

$\square + 2\square\Delta + \Delta^2$



$x^2 - 2x - 3$

$= (x-1)^2 - 4$



$= x^2 - 2x + 1$
 $= (x-1)^2 - 4$

$(x+1)^2 = x^2 + 2x + 1$

$(x+1)^3 = x^3 + 3x^2 + 3x + 1$

$(x+1)^4 = x^4 + 4x^3 + 6x^2 + 4x + 1$ ☆☆☆

$4x+1$ ☆☆☆

$\therefore x^2 + 2x(-1) + (-1)^2 + 2$

\Downarrow
 $x^2 - 2x + 3 =$

\Downarrow
 $x^2 - 2x + 1$

$(x-1)^2 + 2 \geq 2$

10

$$\begin{array}{lcl}
 1 & 1 & \text{一维 } (x+1) \\
 ③ \quad 1 & 2 & 1 \quad \text{二维 } (x^2+2x+1) \\
 & 1 & 3 & 3 & 1 \quad \text{三维 } (x^3+3x^2+3x+1) \\
 & 1 & 4 & 6 & 4 & 1 \quad \text{四维 } (x^4+4x^3+6x^2+4x+1) \\
 & 1 & 5 & 10 & 10 & 5 & 1 \quad \text{五维 } (x^5+5x^4+10x^3+10x^2+5x+1)
 \end{array}$$

由此可以直接求出此题 $(x+1)^4 = ?$

帕斯卡三角法

$$\text{例: } (a+b)^3 = a^3 + 3a^2b + 3b^2a + b^3$$

—— $(a+b)^n$ 的展开

$$\begin{aligned}
 (x+y+z)^2 &= ? \Rightarrow (x+y)^2 \\
 &= x^2 + y^2 + 2xy
 \end{aligned}$$

$$\begin{aligned}
 \therefore (x+y+z)^2 \\
 &= x^2 + y^2 + z^2 + 2xy + 2xz + 2yz
 \end{aligned}$$

$$\begin{aligned}
 (a+b)(x+y) \\
 &= ax + ay + bx + by
 \end{aligned}$$

y	ay	by
x	ax	bx
	a	b

$$ax \div b$$

(拆解)

→ 实数

9

13X13行=169格

