1)

a)

$$(\hat{\mu}, \hat{\Sigma}, \hat{w}) = \arg\min - \sum_i \log \sum_{j=1}^{2} w_j \, N(x_i / \mu_j, \Sigma_j)$$

b)

$$R = \begin{bmatrix} 1 & 0 \\ 0{,}4 & 0{,}6 \\ 0 & 1 \end{bmatrix} \quad D = \begin{pmatrix} 1 \\ 10 \\ 20 \end{pmatrix}$$

$$w_1 = \frac{1{,}4}{3} \qquad w_2 = \frac{1{,}6}{3}$$

c)

$$\mu_1 = \frac{1+4}{1{,}4} = \frac{5}{1{,}4} \qquad \mu_2 = \frac{20+6+0}{1{,}6} = \frac{26}{1{,}6}$$

d)

$x - \mu_1$		$(x-\mu_1)^2$	$(x-\mu_1)^2 \cdot \gamma_1$
$1 - \frac{5}{1{,}4}$	$\approx -2{,}57$	6,6049	6.6049
$10 - 5/1{,}4$	$= 6.42$	41.216	16.4864
$20 - 5/1{,}4$	$= 16.42$	269.89	0

$$\sum_{i=1}^{3} \gamma_1(x_i)(x_i - \mu_j)(x_i - \mu_j)^T = 23{,}09$$

$$\Sigma_1 = \frac{23{,}09}{1{,}4} = 16.49$$

$(x-\mu_2)$		$(x-\mu_1)^2$	$\gamma_2 \cdot (x-\mu_2)^2$
$1 - \frac{26}{1{,}6}$	$= -15.25$	264.06	~~264.06~~ 0
$10 - \frac{26}{1{,}6}$	$= -6.25$	39.0625	23.43
$20 - \frac{26}{1{,}6}$	$= 3{,}75$	14.0625	14.0625
			37.49

$$\Sigma_2 = \frac{37.49}{1{,}6} = 23.43$$

2a

$$\sum_i \log p(x^{(i)}/\theta) = \sum_i \log \sum_{z^{(i)}} P(x^{(i)}, z^{(i)}; \theta)$$

$$=$$

$$\log p(x/\theta) = \log \sum_{z} p(z/\theta) \cdot p(x/z,\theta) \frac{q(z)}{Q(z)}$$

$$= \log$$

$$\log E_q\left[\frac{p(z/\theta) \cdot p(x/z,\theta)}{q(z)} \right]$$ Jensen Inequality

$$l(\theta) \geq E_q\left[\log \frac{p(z/\theta)\, p(x/z,\theta)}{q(z)} \right]$$

$$\geq E_q\left[\log(p(z/\theta)\, p(x/z,\theta))\right] - E_q\left[Q(z)\right]$$

$$\geq E_q\left[\log P(x,z)\right] - \sum_{z=i}^{m} \log Q(z) \cdot Q(z)$$

2b.

2b)

Since we know that.

$$l(\theta) \geq E_q\left[\log\left(\frac{P(z/\theta)\cdot P(X/z,\theta)}{q(z)}\right)\right]$$

$$\geq \sum_z Q(z)\cdot\log\left(\quad \right)$$

$$\geq \sum_z q(z)\log\left(\frac{P(z/\theta)}{Q(z)}\right) + \sum_z q(z)\cdot\log P(X/\theta)$$

$$\geq -KL\left[Q(z);P(z/X,\theta)\right] + \log p(X/\theta)$$

Then if $Q^*(z) = \underline{P(z/\theta)}$

$$l(\theta) \geq \log p(X/\theta) \Rightarrow l(\theta) = l(\theta)$$

$\Rightarrow$ we maximize for every fix θ if $P(z,X,\theta) = Q^*(z)$ we can find a lower Bound close to the ML equation

2c lets suppose that we start with $\theta^{(\tau)}$ fix.

If we try to maximize the lower bound we have to choose a
$Q^*(z) = \arg\max L(q^*, \theta^{(\tau)})$

To choose a θ^{new} we need
$\theta^{(\tau+1)} = \arg\max L(q^*, \theta^{(\tau)})$

Then $\xrightarrow{\text{lower bound}}$

$$L(q^*, \theta^{\tau+1}) \geq L(q^*, \theta^{\tau})$$

By definition of θ^{new}, we get that

$$\log P(x/\theta^{\tau+1}) \geq L(q^*/\theta^{\tau+1}) \geq L(q^*/\theta^{\tau})$$

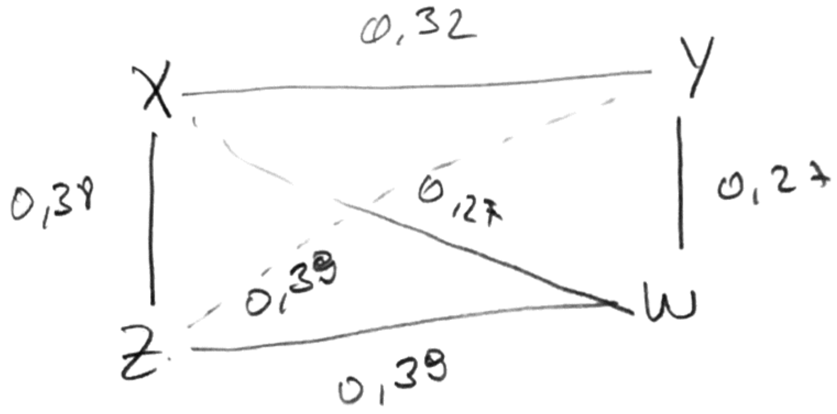
Since we re-optimize we expect that the value of optimized likelyhood function to be optimized

But, for KL we know that for every fix θ, we could find a $Q = P(X, Z)$, then

$$\log P(X/\theta^{\tau+1}) = \log P(X/\theta^{\tau})$$

Then This ensure a bounded from part B, and we show that is monotonically, then the Two properties ensure convergence

3a) We know that a fully conected Bayes network maximize the MLE estimation. Then



3b) Since we start by the edges, we observe that Z, have the largest values for every node; then.