Homework 2

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Question1

Part A

As we can see that all v_i of the matrix X, are Lineally dependent, then all the rank is equal to three.

We can see that if we solve the following system of equations:

$$\alpha + \beta + \gamma = 0$$
$$\alpha + \beta = 0$$
$$\alpha = 0$$

Then we can see that the only solution that solve the system is the trivial one then all three v_i are LI.

Part B

We can solve for w, and we get:

$$w = X^{-1}y$$

Question 2

Part A

```
import numpy as np
x=np.array([[25,0,1],[20,1,2],[40,1,6]])
y=np.array([[110],[110],[210]])
def solver2(y,x_s):
solution = np.dot(np.linalg.inv(x_s),y)
return solution
solver2(y,x)
```

Part B

As the issue is in the second column that are the new unkwows and as we know the actual value of w, we nee to solve the following system:

$$\begin{bmatrix} 25 & x & 1 \\ 20 & y & 2 \\ 40 & z & 6 \end{bmatrix} \begin{bmatrix} 4 \\ 9 \\ 4 \end{bmatrix} = \begin{bmatrix} 110 \\ 110 \\ 210 \end{bmatrix}$$

$$4\begin{bmatrix} 25 \\ 20 \\ 40 \end{bmatrix} + 9\begin{bmatrix} x \\ y \\ z \end{bmatrix} + 4\begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix} = \begin{bmatrix} 110 \\ 110 \\ 210 \end{bmatrix}$$

$$9\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 110 \\ 110 \\ 210 \end{bmatrix} - 4\begin{bmatrix} 26 \\ 22 \\ 46 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{6}{9} \\ \frac{22}{9} \\ \frac{26}{9} \end{bmatrix}$$

Part C

We cannot estimate the result because we can see that $v_2 + v_3 = v_1$, this implies that we cannot take the inverse of $X^T X$ because we have perfect collienallity, this will generate that the Least Squares Estimator or solving $w = X^{-1}y$ cannot be estimated . But we could drop $v1$ from the model, then if we try to apply Least Squares(LS) we dont have any problem as n>p, where n is the number of cereals and p the nutrients in the cereals. Then we could estimate the true w for the four of each component. If we apply LS we got:$w = [4, 4, 9, 4]$, this are the true values of w that we get in the previous part.

Question 3

A-

$$f(w) = w^T(2x)$$

$$\frac{df(w)}{dw} = (2x)$$

B-

$$f(w) = 3w^T x - 0.5x^T w$$

As we know that $a^t b = b^t a$, the we have:

$$f(w) = 2.5w^T x$$

$$\frac{df(w)}{dw} = 2.5x$$

C-

$$f(w) = w^T \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} w$$

We can use the property that $\frac{dx^T Qx}{dw} = Qx + Q^T x$

$$\frac{df(w)}{dw} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} w + \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} w$$

$$\frac{df(w)}{dw} = \begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix} w$$

D-

$$f(w) = w^T \begin{bmatrix} 1 & 2.5 \\ 2.5 & 4 \end{bmatrix} w$$

$$\frac{df(w)}{dw} = 2 \begin{bmatrix} 1 & 2.5 \\ 2.5 & 4 \end{bmatrix} w$$

$$\frac{df(w)}{dw} = \begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix} w$$

Question4

In the latter part of the document.

Question 5

A-

$$y = \sum_{i=1}^{d} w_i x^i + w_0$$

B-

We can build one equation for every sample (x_i,y_i) that we have in the data set, the for any polynomial d, the n equations are:

$$\begin{gathered}\sum_{i=0}^{d} w_i x_1^i = y_1 \\ \vdots \\ \vdots \\ \sum_{i=0}^{d} w_i x_n^i = y_n\end{gathered}$$

Then, if we transform this in vectors, in the right side we will have the y vector and in the left the p(z) for a especific n.

$$\begin{bmatrix} \sum_{i=0}^{d} w_i x_1^i \\ \vdots \\ \vdots \\ \sum_{i=0}^{d} w_i x_1^i \end{bmatrix} = \begin{bmatrix} y_1 \\ \vdots \\ \vdots \\ y_n \end{bmatrix}$$

Now we can see that every summation is multiplication of x_n and the vector w, then:

$$\begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^d \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^d \end{bmatrix} \begin{bmatrix} w_0 \\ \vdots \\ \vdots \\ w_d \end{bmatrix} = \begin{bmatrix} y_1 \\ \vdots \\ \vdots \\ y_n \end{bmatrix}$$