Homework 2

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Question1

Part A

As we can see that all v_i of the matrix X, are Lineally dependent, then all the rank is equal to three.

We can see that if we solve the following system of equations:

$$\begin{gathered}\alpha + \beta + \gamma = 0 \\ \alpha + \beta = 0 \\ \alpha = 0\end{gathered}$$

Then we can see that the only solution that solve the system is the trivial one then all three v_i are LI.

Part B

We can solve for w, and we get:

$$w = X^{-1}y$$

Question 2

Part A

```
import numpy as np
x=np.array([[25,0,1],[20,1,2],[40,1,6]])
y=np.array([[110],[110],[210]])
def solver2(y,x_s):
solution = np.dot(np.linalg.inv(x_s),y)
return solution
solver2(y,x)
```

Part B

As the issue is in the second column that are the new unkwows and as we know the actual value of w, we nee to solve the following system:

$$\begin{bmatrix} 25 & x & 1 \\ 20 & y & 2 \\ 40 & z & 6 \end{bmatrix} \begin{bmatrix} 4 \\ 9 \\ 4 \end{bmatrix} = \begin{bmatrix} 110 \\ 110 \\ 210 \end{bmatrix}$$

$$4 \begin{bmatrix} 25 \\ 20 \\ 40 \end{bmatrix} + 9 \begin{bmatrix} x \\ y \\ z \end{bmatrix} + 4 \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix} = \begin{bmatrix} 110 \\ 110 \\ 210 \end{bmatrix}$$

$$9 \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 110 \\ 110 \\ 210 \end{bmatrix} - 4 \begin{bmatrix} 26 \\ 22 \\ 46 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{6}{9} \\ \frac{22}{9} \\ \frac{26}{9} \end{bmatrix}$$

Part C

We cannot estimate the result because we can see that $v_2 + v_3 = v_1$, this implies that we cannot take the inverse of $X^T X$ because we have perfect collienallity, this will generate that the Least Squares Estimator or solving $w = X^{-1} y$ cannot be estimated . But we could drop $v1$ from the model, then if we try to apply Least Squares(LS) we dont have any problem as n>p, where n is the number of cereals and p the nutrients in the cereals. Then we could estimate the true w for the four of each component. If we apply LS we got:$w = [4, 4, 9, 4]$, this are the true values of w that we get in the previous part.

Question 3

A-

$$f(w) = w^T (2x)$$

$$\frac{df(w)}{dw} = (2x)$$

B-

$$f(w) = 3w^T x - 0.5x^T w$$

As we know that $a^t b = b^t a$, the we have:

$$f(w) = 2.5w^T x$$

$$\frac{df(w)}{dw} = 2.5x$$

C-

$$f(w) = w^T \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} w$$

We can use the property that $\frac{dx^T Qx}{dw} = Qx + Q^T x$

$$\frac{df(w)}{dw} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} w + \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} w$$

$$\frac{df(w)}{dw} = \begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix} w$$

D-

$$f(w) = w^T \begin{bmatrix} 1 & 2.5 \\ 2.5 & 4 \end{bmatrix} w$$

$$\frac{df(w)}{dw} = 2 \begin{bmatrix} 1 & 2.5 \\ 2.5 & 4 \end{bmatrix} w$$

$$\frac{df(w)}{dw} = \begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix} w$$

Question4

In the latter part of the document.

Question 5

A-

$$y = \sum_{i=1}^{d} w_i x^i + w_0$$

B-

We can build one equation for every sample (x_i,y_i) that we have in the data set, the for any polynomial d, the n equations are:

$$\begin{gathered}\sum_{i=0}^{d} w_i x_1^i = y_1 \\ \vdots \\ \vdots \\ \sum_{i=0}^{d} w_i x_n^i = y_n\end{gathered}$$

Then, if we transform this in vectors, in the right side we will have the y vector and in the left the p(z) for a especific n.

$$\begin{bmatrix} \sum_{i=0}^{d} w_i x_1^i \\ \vdots \\ \vdots \\ \sum_{i=0}^{d} w_i x_1^i \end{bmatrix} = \begin{bmatrix} y_1 \\ \vdots \\ \vdots \\ y_n \end{bmatrix}$$

Now we can see that every summation is multiplication of x_n and the vector w, then:

$$\begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^d \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^d \end{bmatrix} \begin{bmatrix} w_0 \\ \vdots \\ \vdots \\ w_d \end{bmatrix} = \begin{bmatrix} y_1 \\ \vdots \\ \vdots \\ y_n \end{bmatrix}$$

HW2

October 22, 2019

1 Coding part

Juan Vila

1.1 Q2.1

```
In [1]: import numpy as np
        import scipy.io as sio
        import matplotlib.pyplot as plt


        x=np.array([[25,0,1],[20,1,2],[40,1,6]])
        y=np.array([[110],[110],[210]])

In [4]: def solver2(y,x_s):
            solution = np.dot(np.linalg.inv(x_s),y)
            return solution
        w = solver2(y,x)
        print(w)
```

```
[[ 4.25]
 [17.5 ]
 [ 3.75]]
```

Comment: w=[4.25,17.5,3.74]

1.2 Q4

```
In [7]: #import data
        q4_data = sio.loadmat('face_emotion_data.mat')
        q4_x=q4_data['X']
        q4_y=q4_data['y']
        #Functions
        def beta_est(Y,x,cons=True):
            if cons:
                X = np.column_stack((x,np.ones([len(x),1])))
            else:
```

```python
        X=x.copy()
    return np.dot(np.linalg.inv(np.dot(np.transpose(X),X)),\
                np.dot(np.transpose(X),Y))

def polynomial(x,d):
    rv = x.copy()
    if d>1:
        for i in range(2,d+1):
           rv =  np.column_stack((rv,x ** i))
    else:
        return x

    return rv

def proyection(Y,x, cons=True):
    if cons:
        X = np.column_stack((x,np.ones([len(x),1])))
    else:
        X=x.copy()
    y_hat = np.dot(X,beta_est(Y,x,cons))
    return y_hat

def cross_val(y,X,n):
    data = np.column_stack((y,X))
    np.random.shuffle(data)
    data_cv = np.split(data,n)
    rv =[]
    for i in range(n):
        temp=data_cv.copy()
        test_set = temp.pop(i)
        y_test, x_test =  test_set[:,[0]], test_set[:,1:]
        train_set = np.concatenate(temp,axis=0)
        y_train, x_train = train_set[:,[0]], train_set[:,1:]
        w = beta_est(y_train, x_train)
        x_test = np.column_stack((x_test,np.ones([len(x_test),1])))
        y_hat = np.dot(x_test,w)
        count = 0
        y_label_assig = np.where(y_hat>0,1,-1)
        for i,j in enumerate(y_test):
            if y_label_assig[i]==1 and y_test[i]==-1 or\
            y_label_assig[i]==-1 and y_test[i]==+1:
                count+=1
        rv.append(count/16)
    rv=np.array(rv)

    return rv.mean()
```

1.2.1 Part A

```
In [12]: wq4a = beta_est(q4_y, q4_x)
         print(wq4a)
```

```
[[ 0.94366942]
 [ 0.21373778]
 [ 0.26641775]
 [-0.39221373]
 [-0.00538552]
 [-0.01764687]
 [-0.16632809]
 [-0.0822838 ]
 [-0.16644364]
 [ 0.078125  ]]
```

1.2.2 Part B

I would predict using the Y variable using this wheigths and then i would input the labels with the following rule: If Y_hat>0 y_labels_p=1 , 0 otherwise. The code is implemented in the command cross val as the following: y_label_assig = np.where(y_hat>0,1,-1)

1.2.3 Part C

Comment: I will select the three biggest absolute value weights, this are x1, x3 and x4. The estimator value are .94,.26 and -.39 respectively.

1.2.4 Part D

```
In [13]: q4_x_d = q4_x[:,[0]]
         q4_x_d = np.column_stack((q4_x_d,q4_x[:,[2]]))
         q4_x_d = np.column_stack((q4_x_d,q4_x[:,[3]]))
         beta_est(q4_y, q4_x_d)
```

```
Out[13]: array([[ 0.70546316],
                [ 0.8737872 ],
                [-0.78805643],
                [ 0.078125  ]])
```

Comment: If we run the regression with this only three estimates we found that the new estimates are .70,.87 and -.78, which made sence because we dropped the other variables.

1.2.5 Part E

```
In [20]: np.random.seed(1234)
         cross_val(q4_y,q4_x,8)
```

```
Out[20]: 0.0390625
```

Comment: The mean error is 3.9%

1.2.6 Part F

```
In [21]: cross_val(q4_y,q4_x_d,8)
```

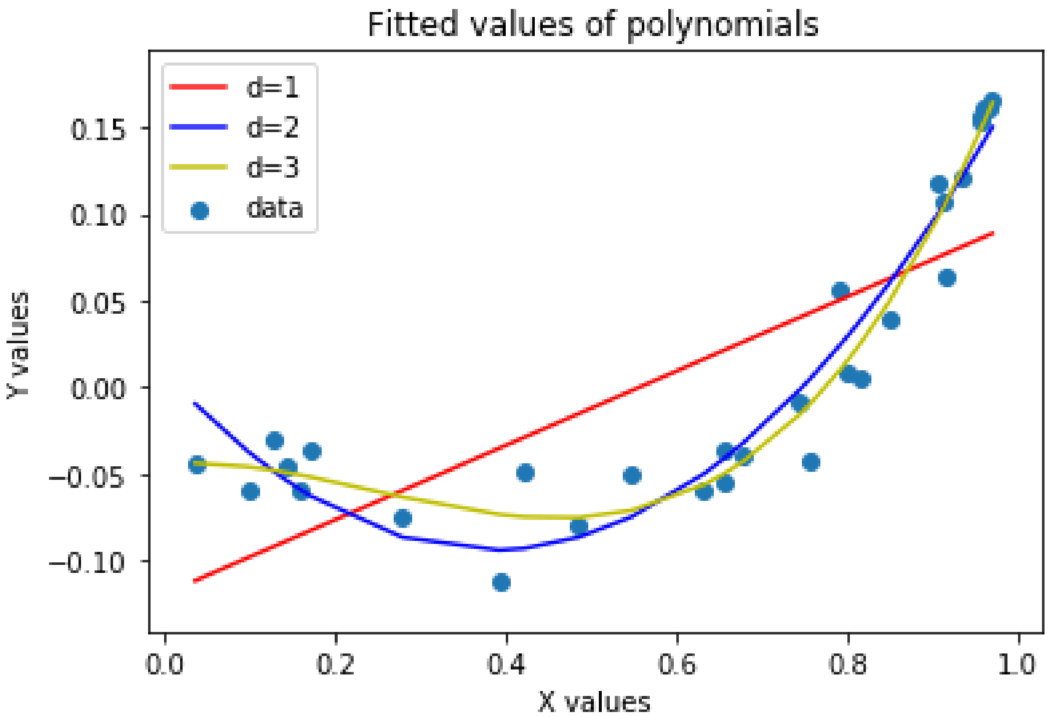
```
Out[21]: 0.0625
```

Comment: The mean error is 6.25% which made sence because LS as you use more features will improve the prediction. That dosent mean that we have to put infinite features because we can have other problems as bias.

1.3 Q5.c

```
In [22]: pol=[1,2,3]
         d = sio.loadmat('polydata.mat')
         x_5 = d['x']
         y_5 = d['y']
         y_comp = x_5.copy()
         y_comp = np.column_stack((y_comp,y_5))
         for i in pol:
             x_pol = polynomial(x_5,i)
             y_hat_temp = proyection(y_5,x_pol)
             y_comp =  np.column_stack((y_comp,y_hat_temp))
         g1 = y_comp[:,0:2]
         g2 = y_comp[:,[0]]
         g2 =  np.column_stack((g2,y_comp[:,[2]]))
         g2 = g2[g2[:, 0].argsort()]
         g4 = y_comp[:,[0]]
         g4 =  np.column_stack((g4,y_comp[:,[4]]))
         g4 = g4[g4[:, 0].argsort()]
         g3 = y_comp[:,[0]]
         g3 =  np.column_stack((g3,y_comp[:,[3]]))
         g3 = g3[g3[:, 0].argsort()]

         plt.scatter(g1[:,[0]],g1[:,[1]],label = 'data')
         plt.plot(g2[:,[0]],g2[:,[1]],color='r', label='d=1')
         plt.plot(g3[:,[0]], g3[:,[1]],color='b',label='d=2')
         plt.plot(g4[:,[0]], g4[:,[1]],color='y',label='d=3')
         plt.ylabel('Y values')
         plt.xlabel('X values')
         plt.title('Fitted values of polynomials')
         plt.legend()
         plt.show()
```


Fitted values of polynomials
d=1
d=2
d=3
data
0.15
0.10
0.05
0.00
−0.05
−0.10
Y values
0.0
0.2
0.4
0.6
0.8
1.0
X values

```
In [ ]:
```