1. Write a python program to find the Forbenius norm $\|\mathsf{A}\|_{\mathrm{F}} = \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij}^2}$ of a matrix $\mathsf{A} \in \mathbb{R}^{m \times n}$. Here, a_{ij} represents the (i, j)-th entry of the matrix A. Do not use inbuilt function. Create the matrix A such that a_{ij} should follow a Gaussian distribution with zero mean and unit variance. Your code should take the size $(m > 6, n > 8)$ of the matrix as an input and should print the norm of the matrix. You should make function for finding Forbenius norm inside your code that should take the matrix as input and returns the norm.

2. Consider a matrix $\mathsf{B} = \mathsf{A} + \mathsf{A}^\top$, where $\mathsf{A} \in \mathbb{R}^{100 \times 100}$ is a matrix such that (i, j) entries a_{ij} follow a Gaussian distribution with zero mean and variance equal to σ^2.

   (a) Write a python program to find the eigenvalue-eigenvector decomposition (EVD) of $\mathsf{B} = \mathsf{V}\mathsf{D}\mathsf{V}^\top$. You can use an inbuilt function to find the EVD. Here, the matrix $\mathsf{V} = \begin{bmatrix} \mathsf{v}_1 & \mathsf{v}_2 & \dots & \mathsf{v}_n \end{bmatrix} \in \mathbb{R}^{n \times n}$ contains the eigenvectors v_i's as its columns.The matrix $\mathsf{D} = \mathtt{diag}(\begin{bmatrix} \lambda_1 & \lambda_2 & \dots & \lambda_n \end{bmatrix})$ contains the eigenvalues λ_i's as its diagonal entries. Ensure that $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$. Take $\sigma^2 = 1$.

   (b) Write a code to find the matrix $\mathsf{B}_k = \sum_{i=1}^{k} \lambda_i \mathsf{v}_i \mathsf{v}_i^\top$ and $\|\mathsf{B} - \mathsf{B}_k\|_{\mathrm{F}}$ for $k \in \{1, 2, \dots, n\}$. Take $\sigma^2 = 1$.

   (c) Plot $\|\mathsf{B} - \mathsf{B}_k\|_{\mathrm{F}}$ vs k. Write your observations from this plot regarding how many eigenvalues and eigenvectors do we need to ensure that B_k is approximately similar to B. Take $\sigma^2 = 1$. Take $\sigma^2 = 1$.

   (d) Write a code to find $\|\mathsf{V}^\top\mathsf{V} - \mathsf{I}\|_{\mathrm{F}}$, where I is the identity matrix. Take $\sigma^2 = 1$.

   (e) Plot $\|\mathsf{B} - \mathsf{B}_k\|_{\mathrm{F}}$ vs $\sigma^2 \in \{0, 0.01, 0.02, \dots, 1.00\}$ for $k = 5, 10, 20, 25, 30$.

3. Consider a dataset (attached as a .txt file) containing n data points $\mathsf{x}_i \in \mathbb{R}^d$ each of having d number of features.

   (a) Write a python code to read the data points from the .txt file and find the covariance matrix $\mathsf{C} \in \mathbb{R}^{d \times d}$ for this data.

   (b) Find the pairs of features which are most (positively/negatively) correlated and the least correlated.

   (c) Find the sample mean and variance of each feature.

   (d) Find the eigenvalues of the covariance matrix. Observe the relationship between the variance of the i-th feature and the i-th eigenvalue of the covariance matrix.