Parameters

$\theta = 1/3;\ \rho = 0.04;\ \delta = 0.08;$ `(* from HRV *)`

$\eta = 0.21370396;$

$\chi = 0.22;$

$\gamma = 0.83;$ `(* Nacho's estimation *)`

`(*` η`=0.44;`χ`=0.55;`γ`=0.69;*)`

$nn = 0.0098;\ \gamma g = 0.0109;$

$\gamma s = 0.0004;$

$\gamma x = 0.0122;$

$\gamma h = 0.0041;$ `(* from HRV dataset *)`

`Ag0 = 1; As0 = 1; Ax0 = 1; (* normalization *)`

$gpg = \gamma x - \gamma g;$ `Print["g_Pg = ", gpg] (* growth rate Pg *)`

$gps = \gamma x - \gamma s;$ `Print["g_Ps = ", gps] (* growth rate Ps *)`

$gk = \dfrac{\gamma x}{1-\theta} + \gamma h;$ `Print["g_k = ", gk] (* growth rate k, e and y *)`

$\kappa = \dfrac{\rho + \delta + \chi\, gps + (1-\chi)\, gk}{\theta};$ `Print["`κ` = ", `κ`]`

$k0 = \kappa^{\frac{1}{\theta-1}}\, Ax0^{\frac{1}{1-\theta}};\ e0 = (\kappa - \delta - nn - gk)\, k0;\ y0 = \kappa\, k0;$ `(* initial values *)`

$Pg[t\_] = e^{gpg\,(t-1980)};$

$Ps[t\_] = e^{gps\,(t-1980)};$

$e[t\_] = e0\, e^{gk\,(t-1980)};$ `(* consumption expenditure per capita *)`

$k[t\_] = k0\, e^{gk\,(t-1980)};$ `(* capital per capita *)`

$y[t\_] = y0\, e^{gk\,(t-1980)};$ `(* gross income per capita *)`

$m[t\_] = y[t] - \delta\, k[t];$ `(* net income per capita *)`

$cg[t\_] = \eta \left(\dfrac{e[t]}{Ps[t]}\right)^{1-\chi} \left(\dfrac{Pg[t]}{Ps[t]}\right)^{\gamma};$ `(* equilibrium goods consumption *)`

$cs[t\_] = \dfrac{e[t]}{Ps[t]} - \eta \left(\dfrac{e[t]}{Ps[t]}\right)^{1-\chi} \left(\dfrac{Pg[t]}{Ps[t]}\right)^{\gamma};$ `(* equilibrium service consumption *)`

$laspe[t\_] = \dfrac{Pg[1980] \times cg[t] + Ps[1980] \times cs[t]}{Pg[1980] \times cg[1980] + Ps[1980] \times cs[1980]};$

`(* real consumption and deflator: Laspeyres *)`

$$\text{Plaspe[t\_]} = \frac{\text{e[t]}}{\text{laspe[t]}};$$

$$\text{paashe[t\_]} = \frac{\text{Pg[2017]} \times \text{cg[t]} + \text{Ps[2017]} \times \text{cs[t]}}{\text{Pg[2017]} \times \text{cg[1980]} + \text{Ps[2017]} \times \text{cs[1980]}};$$

```
(* real consumption and deflator: Paasche *)
```

$$\text{Ppaashe[t\_]} = \frac{\text{e[t]}}{\text{paashe[t]}};$$

```
Plot[{Log[laspe[t]] , Log[paashe[t]] } , {t, 1980, 2017},
  PlotStyle → {{GrayLevel[0.25], Null}, {GrayLevel[0.25], Dashed}},
  AxesLabel → {"time", "Real consumption_pc"}];
 =
 figGDP = Plot[{Log[laspy[t]] + nn (t - 1980), Log[paashy[t]] + nn (t - 1980),
    0.5 (Log[laspy[t]] + nn (t - 1980)) + 0.5 (Log[paashy[t]] + nn (t - 1980))},
   {t, 1980, 2017}, PlotStyle → {{Dashing[{0.05, 0.02}], GrayLevel[0.25]},
     {GrayLevel[0.25], Dashing[{0.01, 0.02}]}, {GrayLevel[0.25]}},
   AxesLabel → {"year", "Real GDP"}, PlotRange → {{1980, 2017}, {0, 1}}]

Print["Diff Laspeyres minus Passhe at 2017:", laspy[2017] - paashy[2017]]
```

$$\nu\text{[t\_]} = \frac{\text{e[t]}^{\chi-1}}{\text{Ps[t]}}; \quad \text{(* marginal value of capital *)}$$

$$\text{V[exp\_, pg\_, ps\_]} = \frac{1}{\chi}\left(\frac{\text{exp}}{\text{ps}}\right)^{\chi} - \frac{\eta}{\gamma}\left(\frac{\text{pg}}{\text{ps}}\right)^{\gamma} - \frac{1}{\chi} + \frac{\eta}{\gamma}; \quad \text{(* net income per capita *)}$$

$$\text{indu[m\_, pg\_, ps\_, nu\_]} = \text{V}\left[\left(\text{nu ps}^{\chi}\right)^{\frac{1}{\chi-1}}, \text{pg, ps}\right] + \text{nu}\left(\text{m} - \left(\text{nu ps}^{\chi}\right)^{\frac{1}{\chi-1}}\right);$$

```
(* indirect utility function *)
```

$$\text{expf[w\_, pg\_, ps\_, nu\_]} = \left(\text{nu ps}^{\chi}\right)^{\frac{1}{\chi-1}} + \frac{\text{w}}{\text{nu}} - \frac{\text{V}\left[\left(\text{nu ps}^{\chi}\right)^{\frac{1}{\chi-1}}, \text{pg, ps}\right]}{\text{nu}};$$

```
(* expenditure function *)

Mhat[t_, z_] = expf[indu[m[z], Pg[z], Ps[z], ν[t]], Pg[t], Ps[t], ν[t]];
(* hypothetical net income per capita *)

fig2017 = Plot[Log[Mhat[2017, z] + δ k[z]] - Log[Mhat[2017, 1980] + δ k[1980]] + nn (z - 1980), {z,
   1980, 2017}, PlotStyle → {GrayLevel[0.25], Dashing[{0.01, 0.02}]}]; (* 2017-base EV *)

fig1980 = Plot[Log[Mhat[1980, z] + δ k[z]] - Log[Mhat[1980, 1980] + δ k[1980]] + nn (z - 1980),
  {z, 1980, 2017}, PlotStyle → {Dashing[{0.05, 0.02}], GrayLevel[0.25]}];
```

```
(*  1980-base EV *)

figM = Plot[Log[m[z]] - Log[m[1980]] + nn (z - 1980), {z, 1980, 2017}];
(*  Net income in units of the investment good *)

fig = Show[fig2017, fig1980, PlotRange → All,
  AxesLabel → {"time", "fixed-base  Fisher-Shell"}, PlotRange → {{1980, 2017}, {0, 1.5}}]
```

Syntax: "θ = 1/3; ρ = 0.04; δ = 0.08; (*from HRV*) η = 0.21370396; χ = 0.22; γ = 0.83; (*Nacho'sestimation*)" is incomplete; more input is needed.

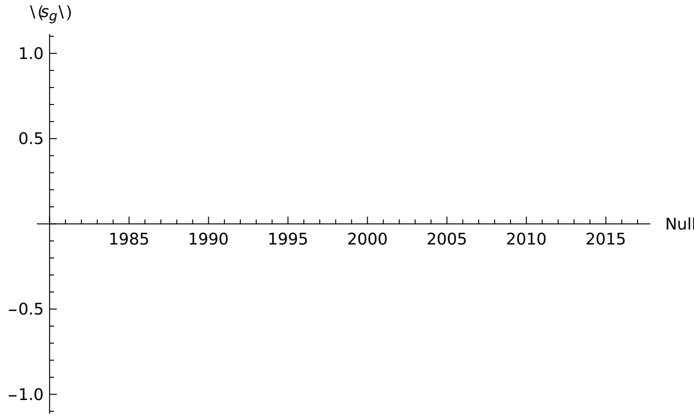
GDP

In[55]:=
```
se = e[1980]/y[1980];
Print["consumption share of net income = ", se]

gg = (1 - χ) gk + (γ - 1) gpg + (χ - γ) gps;
Print["Growth rate Cg = ", gg]

sg[t_] = η (e[t]/Ps[t])^-χ (Pg[t]/Ps[t])^γ;
Plot[sg[t], {t, 1980, 2017}, AxesLabel → {Null, "s_g"}]

gpe[t_] = sg[t] gpg + (1 - sg[t]) gps;
Plot[gpe[t], {t, 1980, 2017}, AxesLabel → {Null, "g_pe"}]

Cs[t_] = e[t]/Ps[t] (1 - sg[t]);
Plot[Cs[t], {t, 1980, 2017}];
gs[t_] = Simplify[D[Log[Cs[t]], t]];
Plot[gs[t], {t, 1980, 2017}, AxesLabel → {Null, "g_s"}]

Divisia[t_] = Simplify[se (sg[t] gg + (1 - sg[t]) gs[t]) + (1 - se) gk];
Plot[Divisia[t], {t, 1980, 2017}, AxesLabel → {Null, "Divisia"}]
Print["growth rate 1980 = ", Divisia[1980]]
Print["growth rate 2017 = ", Divisia[2017]]

Plot[e[1980] / (Mhat[1980, z] + δ k[z]), {z, 1980, 2017}, AxesLabel →
  {Null, "\!\(\*SubscriptBox[\(e\), \\(1980\)]\)/\!\(\*SubscriptBox[OverscriptBox[\(y\),
     \(^\)], \(1980, \t\)]\)"}, PlotStyle → {GrayLevel[0.25]}]

Plot[e[2017] / (Mhat[2017, z] + δ k[z]), {z, 1980, 2017}, AxesLabel →
  {Null, "\!\(\*SubscriptBox[\(e\), \\(2017\)]\)/\!\(\*SubscriptBox[OverscriptBox[\(y\),
     \(^\)], \(2017, \t\)]\)"}, PlotStyle → {GrayLevel[0.25]}]
```

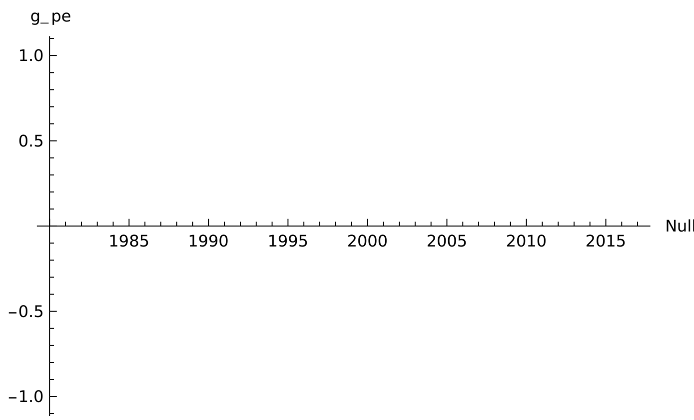
consumption share of net income = $\frac{e[1980]}{y[1980]}$

Growth rate Cg = 0.010053

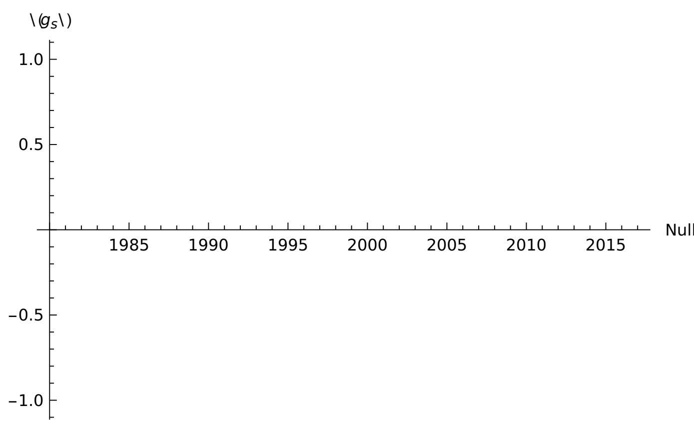
Out[60]=



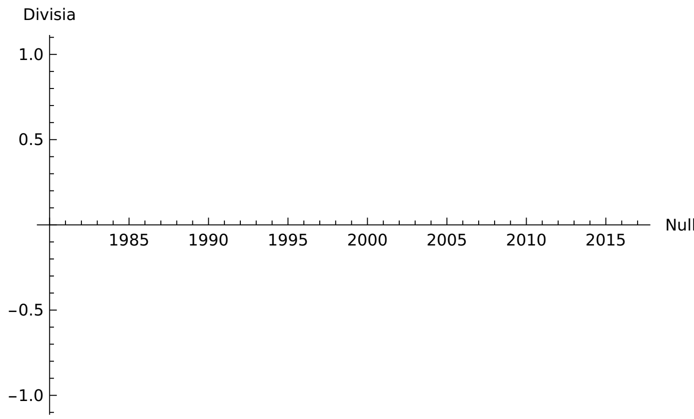
Out[62]=



Out[66]=



Out[68]=



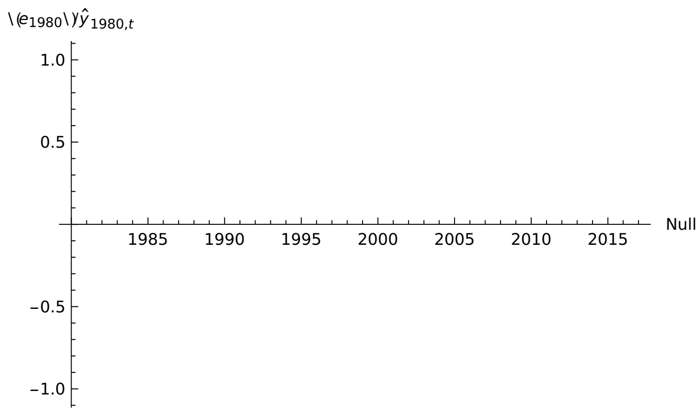
growth rate 1980 =

$$\begin{aligned}
&\Bigg(\Bigg(e[1980]\times Pg[1980]\Bigg(0.00214837\left(\frac{e[1980]}{Ps[1980]}\right)^{2.22}Ps[1980]^2-0.213704\left(\frac{e[1980]}{Ps[1980]}\right)^{1.22}Ps[1980]\,e'[1980]+\\
&\quad e[1980]\left(0.0470149\left(\frac{e[1980]}{Ps[1980]}\right)^{0.22}e'[1980]+0.344063\left(\frac{e[1980]}{Ps[1980]}\right)^{1.22}Ps'[1980]\right)\Bigg)+\\
&\quad Ps[1980]\Bigg(\left(\left(\frac{e[1980]}{Ps[1980]}\right)^{2.44}\left(\frac{Pg[1980]}{Ps[1980]}\right)^{0.17}Ps[1980]^2\left(-0.0224\,e[1980]+0.0224\,y[1980]\right)+\right.\\
&\quad 1.\,e[1980]\left(\frac{e[1980]}{Ps[1980]}\right)^{1.44}\left(\frac{Pg[1980]}{Ps[1980]}\right)^{0.17}Ps[1980]\,e'[1980]+\\
&\quad \left.e[1980]^2\left(-0.177374\left(\frac{e[1980]}{Ps[1980]}\right)^{1.22}Pg'[1980]-1.\left(\frac{e[1980]}{Ps[1980]}\right)^{1.44}\left(\frac{Pg[1980]}{Ps[1980]}\right)^{0.17}Ps'[1980]\right)\right)\Bigg)\Bigg)\Bigg/\\
&\left(\left(\frac{e[1980]}{Ps[1980]}\right)^{2.44}\left(\frac{Pg[1980]}{Ps[1980]}\right)^{0.17}Ps[1980]^3\,y[1980]\right)
\end{aligned}$$

growth rate 2017 =

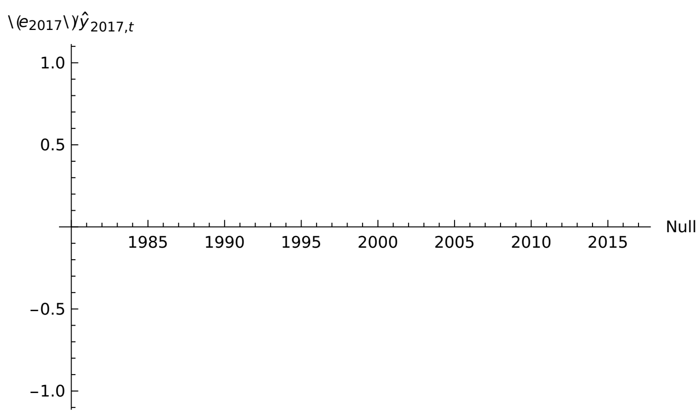
$$\begin{aligned}
&\Bigg(\Bigg(e[1980]\times Pg[2017]\Bigg(0.00214837\left(\frac{e[2017]}{Ps[2017]}\right)^{2.22}Ps[2017]^2-0.213704\left(\frac{e[2017]}{Ps[2017]}\right)^{1.22}Ps[2017]\,e'[2017]+\\
&\quad e[2017]\left(0.0470149\left(\frac{e[2017]}{Ps[2017]}\right)^{0.22}e'[2017]+0.344063\left(\frac{e[2017]}{Ps[2017]}\right)^{1.22}Ps'[2017]\right)\Bigg)+\\
&\quad Ps[2017]\Bigg(\left(\left(\frac{e[2017]}{Ps[2017]}\right)^{2.44}\left(\frac{Pg[2017]}{Ps[2017]}\right)^{0.17}Ps[2017]^2\left(-0.0224\,e[1980]+0.0224\,y[1980]\right)+\right.\\
&\quad 1.\,e[1980]\left(\frac{e[2017]}{Ps[2017]}\right)^{1.44}\left(\frac{Pg[2017]}{Ps[2017]}\right)^{0.17}Ps[2017]\,e'[2017]+e[1980]\times e[2017]\\
&\quad \left.\left(-0.177374\left(\frac{e[2017]}{Ps[2017]}\right)^{1.22}Pg'[2017]-1.\left(\frac{e[2017]}{Ps[2017]}\right)^{1.44}\left(\frac{Pg[2017]}{Ps[2017]}\right)^{0.17}Ps'[2017]\right)\right)\Bigg)\Bigg)\Bigg/\\
&\left(\left(\frac{e[2017]}{Ps[2017]}\right)^{2.44}\left(\frac{Pg[2017]}{Ps[2017]}\right)^{0.17}Ps[2017]^3\,y[1980]\right)
\end{aligned}$$

Out[71]=



\(e1980\)ŷ1980,t
1.0
0.5
−0.5
−1.0
1985
1990
1995
2000
2005
2010
2015
Null

Out[72]=



\(e2017\)ŷ2017,t
1.0
0.5
−0.5
−1.0
1985
1990
1995
2000
2005
2010
2015
Null

```
In[73]:= FS = Table[{n, NIntegrate[Divisia[t], {t, 1980, n}] + nn (n - 1980)}, {n, 1980, 2017, 0.01}];

Print["bias 1980-base = ", Log[Mhat[1980, z] + δ k[z]] -
    Log[Mhat[1980, 1980] + δ k[1980]] + nn (z - 1980) - FS[[Length[FS], 2]] /. z → 2017]
Print["bias 2017-base = ",
 -Log[Mhat[2017, 2017]] + Log[Mhat[2017, 1980]] + NIntegrate[Divisia[t], {t, 1980, 2017}]]
```

NIntegrate: The integrand
(e[1980] Pg[t] (0.00214837 (e[≪1≫] Power[≪2≫])$^{2.22}$ Ps[t]^2 - ≪1≫ + e[t] (0.0470149 Power[≪2≫] ≪1≫$^{(≪1≫)}$[≪1≫] + ≪1≫)) + ≪1≫) / $\left(\left(\frac{e[t]}{Ps[t]}\right)^{2.44} \left(\frac{Pg[t]}{Ps[t]}\right)^{0.17}$ Ps[t]^3 y[1980]$\right)$
has evaluated to non-numerical values for all sampling points in the region with boundaries {{1980, 1980.01}}.

NIntegrate: The integrand
(e[1980] Pg[t] (0.00214837 (e[≪1≫] Power[≪2≫])$^{2.22}$ Ps[t]^2 - ≪1≫ + e[t] (0.0470149 Power[≪2≫] ≪1≫$^{(≪1≫)}$[≪1≫] + ≪1≫)) + ≪1≫) / $\left(\left(\frac{e[t]}{Ps[t]}\right)^{2.44} \left(\frac{Pg[t]}{Ps[t]}\right)^{0.17}$ Ps[t]^3 y[1980]$\right)$
has evaluated to non-numerical values for all sampling points in the region with boundaries {{1980, 1980.01}}.

NIntegrate: The integrand
(e[1980] Pg[t] (0.00214837 (e[≪1≫] Power[≪2≫])$^{2.22}$ Ps[t]^2 - ≪1≫ + e[t] (0.0470149 Power[≪2≫] ≪1≫$^{(≪1≫)}$[≪1≫] + ≪1≫)) + ≪1≫) / $\left(\left(\frac{e[t]}{Ps[t]}\right)^{2.44} \left(\frac{Pg[t]}{Ps[t]}\right)^{0.17}$ Ps[t]^3 y[1980]$\right)$
has evaluated to non-numerical values for all sampling points in the region with boundaries {{1980, 1980.02}}.

General: Further output of NIntegrate::inumr will be suppressed during this calculation.

Cloud: This computation has exceeded the time limit for your plan.

```
Out[73]=
$Aborted
```

Part: Part specification FS[[0, 2]] is longer than depth of object.

```
bias 1980-base = 0.3626 - Log[0.08 k[1980] + Mhat[1980, 1980]] +
 Log[0.08 k[2017] + Mhat[1980, 2017]] - FS[[0, 2]]
```

NIntegrate: The integrand
(e[1980] Pg[t] (0.00214837 (e[≪1≫] Power[≪2≫])$^{2.22}$ Ps[t]^2 - ≪1≫ + e[t] (0.0470149 Power[≪2≫] ≪1≫$^{(≪1≫)}$[≪1≫] + ≪1≫)) + ≪1≫) / $\left(\left(\frac{e[t]}{Ps[t]}\right)^{2.44} \left(\frac{Pg[t]}{Ps[t]}\right)^{0.17}$ Ps[t]^3 y[1980]$\right)$
has evaluated to non-numerical values for all sampling points in the region with boundaries {{1980, 2017}}.

```
bias 2017-base =
 Log[Mhat[2017, 1980]] - Log[Mhat[2017, 2017]] + NIntegrate[Divisia[t], {t, 1980, 2017}]
```

In[76]:=
```
figDivisia = ListPlot[FS, Joined → True,
   AxesLabel → {"year", "GDP"}, PlotRange → {{1980, 2017}, {0, 1.5}},
   PlotStyle → {GrayLevel[0.25]}, PlotRange → {{1980, 2017}, {0, 1}}];
Show[figDivisia, fig2017, fig1980,
 PlotRange → {{1980, 2017}, {0, 1.1}}, AxesLabel → {"time", "GDP Fisher-Shell"}]
```

ListPlot: FS is not a list of numbers or pairs of numbers.

Show: Could not combine the graphics objects in Show[ListPlot[FS, Joined → True, AxesLabel → {year, GDP}, PlotRange → {{1980, 2017}, {0, 1.5}}, PlotStyle → {■}, PlotRange → {{1980, 2017}, {0, 1}}], fig2017, fig1980, PlotRange → {{1980, 2017}, {0, 1.1}}, AxesLabel → {time, GDP Fisher–Shell}].

Out[77]=
```
Show[ListPlot[FS, Joined → True, AxesLabel → {year, GDP}, PlotRange → {{1980, 2017}, {0, 1.5}},
  PlotStyle → {■}, PlotRange → {{1980, 2017}, {0, 1}}], fig2017, fig1980,
 PlotRange → {{1980, 2017}, {0, 1.1}}, AxesLabel → {time, GDP Fisher-Shell}]
```

In[78]:=
```
Show[figGDP, fig2017, fig1980, PlotRange → {{1980, 2017}, {0, 1.5}},
 AxesLabel → {"time", "GDP Fisher-Shell"}]
```

Show: Could not combine the graphics objects in Show[figGDP, fig2017, fig1980, PlotRange → {{1980, 2017}, {0, 1.5}}, AxesLabel → {time, GDP Fisher–Shell}].

Out[78]=
```
Show[figGDP, fig2017, fig1980,
 PlotRange → {{1980, 2017}, {0, 1.5}}, AxesLabel → {time, GDP Fisher-Shell}]
```

In[79]:=
```
Show[figGDP, figDivisia, PlotRange → {{1980, 2017}, {0, 1}},
 AxesLabel → {"time", "GDP Fisher-Shell"}]
```

Show: Could not combine the graphics objects in Show[figGDP, ListPlot[FS, Joined → True, AxesLabel → {year, GDP}, PlotRange → {{1980, 2017}, {0, 1.5}}, PlotStyle → {■}, PlotRange → {{1980, 2017}, {0, 1}}], PlotRange → {{1980, 2017}, {0, 1}}, AxesLabel → {time, GDP Fisher–Shell}].

Out[79]=
```
Show[figGDP, ListPlot[FS, Joined → True, AxesLabel → {year, GDP},
  PlotRange → {{1980, 2017}, {0, 1.5}}, PlotStyle → {■}, PlotRange → {{1980, 2017}, {0, 1}}],
 PlotRange → {{1980, 2017}, {0, 1}}, AxesLabel → {time, GDP Fisher-Shell}]
```

In[80]:=

$$\mathtt{Plot}\left[\frac{\mathtt{Pg[1980]}\times\mathtt{cg[t]}}{\mathtt{Pg[1980]}\times\mathtt{cg[1980]}+\mathtt{Ps[1980]}\times\mathtt{cs[1980]}},\ \{\mathtt{t},\ 1980,\ 2017\}\right]$$

$$\mathtt{Plot}\left[\frac{\mathtt{Pg[2017]}\times\mathtt{cg[t]}}{\mathtt{Pg[2017]}\times\mathtt{cg[1980]}+\mathtt{Ps[2017]}\times\mathtt{cs[1980]}},\ \{\mathtt{t},\ 1980,\ 2017\}\right]$$

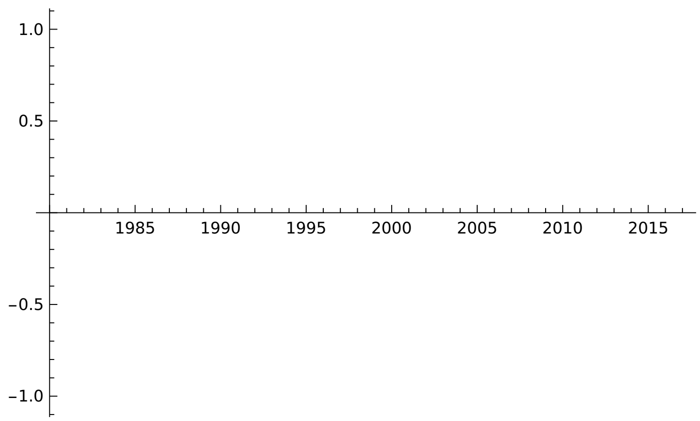
$$\mathtt{sg[1980]}$$

$$\frac{\mathtt{Pg[1980]}\times\mathtt{cg[t]}}{\mathtt{Pg[1980]}\times\mathtt{cg[1980]}+\mathtt{Ps[1980]}\times\mathtt{cs[1980]}}\ /.\ \mathtt{t}\to 1980$$

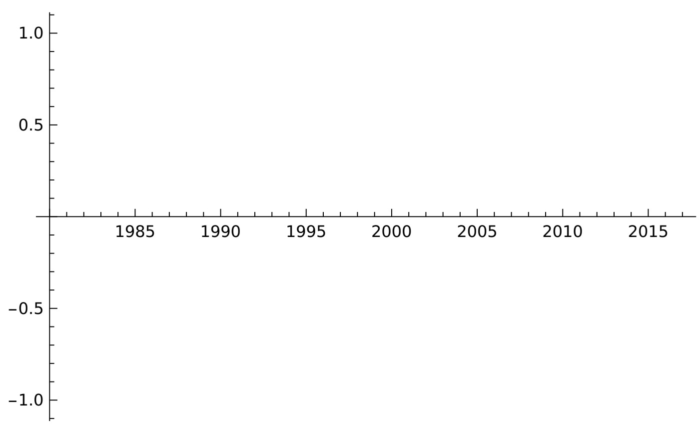
$$\mathtt{sg[2017]}$$

$$\frac{\mathtt{Pg[2017]}\times\mathtt{cg[t]}}{\mathtt{Pg[2017]}\times\mathtt{cg[1980]}+\mathtt{Ps[2017]}\times\mathtt{cs[1980]}}\ /.\ \mathtt{t}\to 2017$$

Out[80]=



Out[81]=



Out[82]=

$$\frac{0.213704\left(\frac{\mathtt{Pg[1980]}}{\mathtt{Ps[1980]}}\right)^{0.83}}{\left(\frac{\mathtt{e[1980]}}{\mathtt{Ps[1980]}}\right)^{0.22}}$$

Out[83]=

$$\frac{\mathtt{cg[1980]}\times\mathtt{Pg[1980]}}{\mathtt{cg[1980]}\times\mathtt{Pg[1980]}+\mathtt{cs[1980]}\times\mathtt{Ps[1980]}}$$

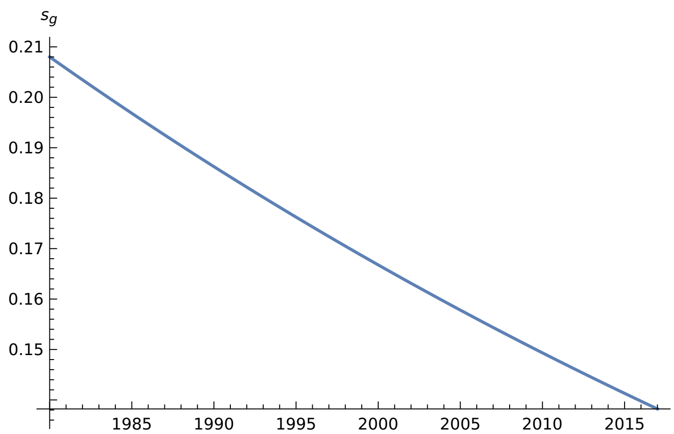
Out[84]=

$$\frac{0.213704\left(\frac{\text{Pg[2017]}}{\text{Ps[2017]}}\right)^{0.83}}{\left(\frac{\text{e[2017]}}{\text{Ps[2017]}}\right)^{0.22}}$$

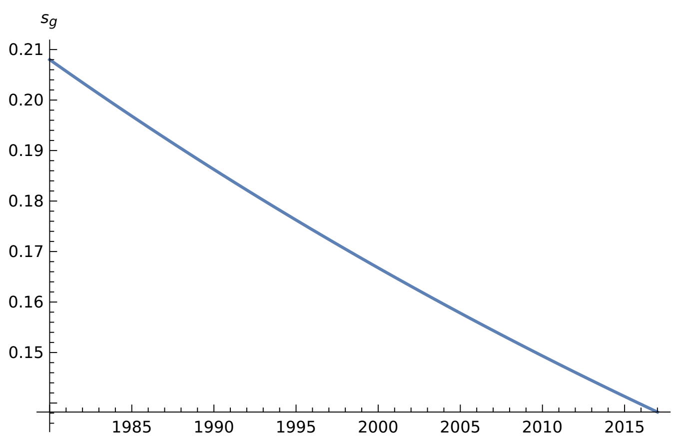
Out[85]=

$$\frac{\text{cg[2017]}\times\text{Pg[2017]}}{\text{cg[1980]}\times\text{Pg[2017]}+\text{cs[1980]}\times\text{Ps[2017]}}$$

In[86]:=



Out[86]=

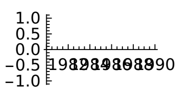
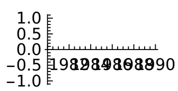
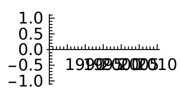
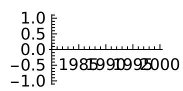


```
In[87]:= Lapeyres1980[z_] = Log[Mhat[1980, z] + δ k[z]] - Log[Mhat[1980, 1980] + δ k[1980]] + nn (z - 1980);
Paasche1990[z_] = Log[Mhat[1990, z] + δ k[z]] - Log[Mhat[1990, 1980] + δ k[1980]] + nn (z - 1980);
Paasche2000[z_] = Log[Mhat[2000, z] + δ k[z]] - Log[Mhat[2000, 1980] + δ k[1980]] + nn (z - 1980);
Paasche2010[z_] = Log[Mhat[2010, z] + δ k[z]] - Log[Mhat[2010, 1980] + δ k[1980]] + nn (z - 1980);
Paasche2017[z_] = Log[Mhat[2017, z] + δ k[z]] - Log[Mhat[2017, 1980] + δ k[1980]] + nn (z - 1980);

Print["chained 1980-1990 = ", FS⟦1001, 2⟧]
Print["chained - Laspeyres_1980 = ", FS⟦1001, 2⟧ - Lapeyres1980[1990]]
Print["chained - Paasche_1990 = ", FS⟦1001, 2⟧ - Paasche1990[1990]]
Print["chained - Paasche_2000 = ", FS⟦1001, 2⟧ - Paasche2000[1990]]
Print["chained - Paasche_2010 = ", FS⟦1001, 2⟧ - Paasche2010[1990]]
Print["chained - Paasche_2017 = ", FS⟦1001, 2⟧ - Paasche2017[1990]]

fig1980 =
  Plot[Lapeyres1980[z], {z, 1980, 1990}, PlotStyle → {Dashing[{0.05, 0.02}], GrayLevel[0.25]}];
(*  1980-base *)

fig1990 =
  Plot[Paasche1990[z], {z, 1980, 1990}, PlotStyle → {Dashing[{0.04, 0.02}], GrayLevel[0.25]}];
(*  1990-base *)

fig2000 =
  Plot[Paasche2000[z], {z, 1980, 2000}, PlotStyle → {Dashing[{0.03, 0.02}], GrayLevel[0.25]}];
(*  2000-base *)

fig2010 =
  Plot[Paasche2010[z], {z, 1980, 2010}, PlotStyle → {Dashing[{0.02, 0.02}], GrayLevel[0.25]}];
(*  2010-base *)

Show[figDivisia, fig1980, fig1990, fig2010, fig2000, fig2017,
 PlotRange → {{1980, 1990}, {0, .27}}, AxesLabel → {"time", "GDP Fisher-Shell"}, Epilog → {
   Text[Style["1980-base", GrayLevel[0.05]], {1988, 0.25}],
   Text[Style["1990-base", GrayLevel[0.05]], {1989, 0.17}],
   Text[Style["2000-base", GrayLevel[0.05]], {1989, 0.12}],
   Text[Style["2010-base", GrayLevel[0.05]], {1989, .08}],
   Text[Style["2017-base", GrayLevel[0.05]], {1989, .045}]}]

Part: Part specification FS⟦1001, 2⟧ is longer than depth of object.

chained 1980-1990 = FS⟦1001, 2⟧
```

```
Part: Part specification FS⟦1001, 2⟧ is longer than depth of object.

chained - Laspeyres_1980 =
 -0.098 + Log[0.08 k[1980] + Mhat[1980, 1980]] - Log[0.08 k[1990] + Mhat[1980, 1990]] + FS⟦1001, 2⟧

Part: Part specification FS⟦1001, 2⟧ is longer than depth of object.

chained - Paasche_1990 =
 -0.098 + Log[0.08 k[1980] + Mhat[1990, 1980]] - Log[0.08 k[1990] + Mhat[1990, 1990]] + FS⟦1001, 2⟧

Part: Part specification FS⟦1001, 2⟧ is longer than depth of object.

chained - Paasche_2000 =
 -0.098 + Log[0.08 k[1980] + Mhat[2000, 1980]] - Log[0.08 k[1990] + Mhat[2000, 1990]] + FS⟦1001, 2⟧

Part: Part specification FS⟦1001, 2⟧ is longer than depth of object.

chained - Paasche_2010 =
 -0.098 + Log[0.08 k[1980] + Mhat[2010, 1980]] - Log[0.08 k[1990] + Mhat[2010, 1990]] + FS⟦1001, 2⟧

Part: Part specification FS⟦1001, 2⟧ is longer than depth of object.

chained - Paasche_2017 =
 -0.098 + Log[0.08 k[1980] + Mhat[2017, 1980]] - Log[0.08 k[1990] + Mhat[2017, 1990]] + FS⟦1001, 2⟧

Show: Could not combine the graphics objects in Show[ListPlot[FS, Joined → True, AxesLabel → {year,
        GDP}, PlotRange → {{1980, 2017}, {0, 1.5}}, PlotStyle → {■}, PlotRange → {{1980, 2017}, {0, 1}}],
   ≪7≫, Epilog → {Text[1980–base, {1988, 0.25}], Text[1990–base, {1989, 0.17}], Text[
        2000–base, ≪1≫], Text[2010–base, {1989, 0.08}], Text[2017–base, {1989, 0.045}]}].
```

Out[106]=

```
Show[ListPlot[FS, Joined → True, AxesLabel → {year, GDP},
  PlotRange → {{1980, 2017}, {0, 1.5}}, PlotStyle → {■}, PlotRange → {{1980, 2017}, {0, 1}}],
```

```
, fig2017,
 PlotRange → {{1980, 1990}, {0, 0.27}}, AxesLabel → {time, GDP Fisher-Shell},
 Epilog → {Text[1980-base, {1988, 0.25}], Text[1990-base, {1989, 0.17}],
   Text[2000-base, {1989, 0.12}], Text[2010-base, {1989, 0.08}], Text[2017-base, {1989, 0.045}]}]
```

Checking Boppart’s Lemma 1

In[107]:=

$$\text{Plot}\left[e[t]^{\chi} - \eta \frac{1-\chi}{1-\gamma} \text{Pg}[t]^{\gamma} \text{Ps}[t]^{\chi-\gamma}, \{t, 1980, 2017\}\right]$$

Out[107]=