

Objective. The goal of this exercise is to put forward a simple methodology to measure aggregate and sectoral TFP for an economy where total labor is normalized to unity, like in HRV2021.

Characterization of the environment. Let $L(t)$ be aggregate labor endowment and $L_g(t), L_s(t)$ the labor allocations in sector g and s , such that

$$L_g(t) + L_s(t) = L(t). \quad (1)$$

Suppose that there is an aggregate production function, given by

$$Y(t) = A(t)K(t)^\alpha L(t)^{1-\alpha}, \quad (2)$$

where $Y(t)$ is aggregate output, $K(t)$ is the aggregate capital stock, and $A(t)$ is the economy's total factor productivity.

Additionally, in each sector $j \in \{g, s\}$ output is produced according to the following technology

$$Y_j(t) = A_j(t)K_j(t)^\alpha L_j(t)^{1-\alpha} \quad \text{for } j \in \{g, s\}, \quad (3)$$

with $Y_j(t), K_j(t)$, and $A_j(t)$ representing output, the capital stock, and total factor productivity, all in sector j . Notice that α is the capital intensity parameter, which is common across sectors.

Sectoral Employment Shares. Dividing both sides of Equation (1) by $L(t)$ yields

$$l_g(t) + l_s(t) = 1,$$

where $l_g(t), l_s(t)$ are the shares of labor in goods and services in total employment.

Labor Normalization: Implications for Measurement. Suppose that we normalize the labor endowment to one, such that $L(t) = 1 \forall t$. For the aggregate economy, this implies that Equation (2) becomes

$$Y(t) = A(t)K(t)^\alpha.$$

Taking logs on both sides and re-arranging terms, we obtain that measured log TFP for the aggregate economy is

$$\ln(A(t)) = \ln(Y(t)) - \alpha \ln(K(t)).$$

For sector j , dividing both sides of Equation (3) by $L(t)$ we get

$$\begin{aligned} \frac{Y_j(t)}{L(t)} &= A_j(t) \frac{K_j(t)^\alpha L_j(t)^{1-\alpha}}{L(t)} \\ &= A_j(t) \left(\frac{K_j(t)}{L(t)} \right)^\alpha \left(\frac{L_j(t)}{L(t)} \right)^{1-\alpha} \\ &= A_j(t) \left(\frac{K_j(t)}{L(t)} \right)^\alpha (l_j(t))^{1-\alpha}. \end{aligned}$$

Since labor is normalized to unity, we obtain

$$Y_j(t) = A_j(t)K_j(t)^\alpha l_j(t)^{1-\alpha}.$$

Taking logs on both sides and re-arranging terms implies that

$$\ln(A_j(t)) = \ln(Y_j(t)) - \alpha \ln(K_j(t)) - (1 - \alpha) \ln(l_j(t)).$$

It follows that measuring sectoral TFP $A_j(t)$ when labor is normalized to 1 requires deducting from total output capital services and labor services, with the caveat that the latter are computed using the share of sector j in total employment.