

**Objective.** The goal of this exercise is to put forward a simple methodology to measure aggregate and sectoral TFP for an economy where total labor is normalized to unity, like in HRV2021.

**Characterization of the environment.** Let  $L(t)$  be aggregate labor endowment and  $L_g(t), L_s(t)$  the labor allocations in sector  $g$  and  $s$ , such that

$$L_g(t) + L_s(t) = L(t). \quad (1)$$

Suppose that there is an aggregate production function, given by

$$Y(t) = A(t)K(t)^\alpha L(t)^{1-\alpha}, \quad (2)$$

where  $Y(t)$  is aggregate output,  $K(t)$  is the aggregate capital stock, and  $A(t)$  is the economy's total factor productivity.

Additionally, in each sector  $j \in \{g, s\}$  output is produced according to the following technology

$$Y_j(t) = A_j(t)K_j(t)^\alpha L_j(t)^{1-\alpha} \quad \text{for } j \in \{g, s\}, \quad (3)$$

with  $Y_j(t), K_j(t)$ , and  $A_j(t)$  representing output, the capital stock, and total factor productivity, all in sector  $j$ . Notice that  $\alpha$  is the capital intensity parameter, which is common across sectors.

**Sectoral Employment Shares.** Dividing both sides of Equation (1) by  $L(t)$  yields

$$l_g(t) + l_s(t) = 1,$$

where  $l_g(t), l_s(t)$  are the shares of labor in goods and services in total employment.

**Labor Normalization: Implications for Measurement.** Suppose that we normalize the labor endowment to one, such that  $L(t) = 1 \forall t$ . For the aggregate economy, this implies that Equation (2) becomes

$$Y(t) = A(t)K(t)^\alpha.$$

Taking logs on both sides and re-arranging terms, we obtain that measured log TFP for the aggregate economy is

$$\ln(A(t)) = \ln(Y(t)) - \alpha \ln(K(t)).$$

For sector  $j$ , diving both sides of Equation (3) by  $L(t)$  we get

$$\begin{aligned} \frac{Y_j(t)}{L(t)} &= A_j(t) \frac{K_j(t)^\alpha L_j(t)^{1-\alpha}}{L(t)} \\ &= A_j(t) \left( \frac{K_j(t)}{L(t)} \right)^\alpha \left( \frac{L_j(t)}{L(t)} \right)^{1-\alpha} \\ &= A_j(t) \left( \frac{K_j(t)}{L(t)} \right)^\alpha (l_j(t))^{1-\alpha}. \end{aligned}$$

Since labor is normalized to unity, we obtain

$$Y_j(t) = A_j(t)K_j(t)^\alpha l_j(t)^{1-\alpha}.$$

Taking logs on both sides and re-arranging terms implies that

$$\ln(A_j(t)) = \ln(Y_j(t)) - \alpha \ln(K_j(t)) - (1 - \alpha) \ln(l_j(t)).$$

It follows that measuring sectoral TFP  $A_j(t)$  when labor is normalized to 1 requires deducting from total output capital services and labor services, with the caveat that the latter are computed using the share of sector  $j$  in total employment.