Natural Disasters and Growth: The Role of Foreign Aid and Disaster Insurance (preliminary and incomplete)

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Abstract

In this paper we develop a continuous time stochastic growth model that is suitable for studying the impact of natural disasters on the short run and long run growth rate of an economy. We find that the growth effects of a natural disaster depend in complicated ways on the details of expected foreign disaster aid and the existence of catastrophe insurance markets. We show that aid can have an influence on investments in prevention and mitigation activities and can delay the recovery form a natural disaster strike.

1 Introduction

The immediate economic impact of a natural disaster strike is large and negative. The empirical literature has found mixed results about the longer run effect of natural disasters on economic activity. To the extent that global climate change will likely increase the prevalence of some forms of natural disasters it is important to develop a framework that is suitable to interpret the evidence at the same time that provides some guidance on the effect of policies.

What is the evidence? In the last few years there has been extensive empirical research on the economic impact of natural disasters on growth. A cursory reading of the literature suggests that there is a significant disagreement about the short and long run consequences of natural disaster strikes. In some cases the evidence points to a positive relationship between the risk of natural disasters and economic growth. For example, Skidmore and Toya (2002) find that the average number of natural disasters is positively correlated with growth. Kousky (2014) reviews a large number of studies and finds that natural disasters have a modest impact on economic activity. At the other end, Hsiang and Jina (2014), Berleman and Wenzel (2016) and Bakkensen and Barrage (2017) reach the opposite conclusion: a natural disaster strike —in these papers the analysis is restricted to tropical cyclones— decreases the growth rate, and the impact is relatively long lasting. We view these differences as indicating not only that the measurement of a natural disaster event is difficult and mired with error but also that it is necessary to take into account heterogeneity across countries in the activities that can influence the effect of a natural disaster.

There are two other pieces of evidence that seem relevant to motivate what a theoretical model should include. First, Berleman and Wenzel (2016) find that the growth impact of a natural disaster varies depending on the country's level of development: a natural disaster strike in a relatively rich country has very small growth effects while a similar event in a poor country results in large decreases in growth .[1] Second, von Peter et. al. (2012) find that the growth impact of a natural disaster strike depends on whether the loss was insured or not: insured losses do not appear to have a significant im-

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[1] Hsiang and Jina (2014) on the other hand find no significant differences between countries with above the median icnome and countries below. This way of categorizing rich and poor is possibly too coarse to get significant results.

pact on growth while uninsured losses have a negative impact.[2] It is not clear that these two are independent observations as it is possible that high income countries are also countries that are better insured against natural disasters. Studying the impact of hurricanes in the U.S. Deryugina (forthcoming) finds that following a strike the affected area receives transfers —emergency aid and insurance payments— in an amount close to the estimates of the damages caused by a hurricane. She finds that in her sample hurricanes have a negligible impact on income. That is, transfers that compensate for the loss result in no growth effects. We read this research as suggesting that the role of transfers and insurance should not be neglected when analyzing the economic impact of natural disasters.

In this paper we develop a continuous time stochastic growth model that is rich enough to account for the evidence. We study optimal consumption and investing under alternative market structures. We explore the role of foreign disaster aid and study its impact on saving decisions as well as the choice of sectoral allocation of investment. We also explore the impact upon endogenous decisions of the availability of actuarially fairly priced disaster insurance. Finally, we explore the effect, both in terms of growth and welfare, of delays in the provision of aid and in the payouts of insurance contracts.

Not surprisingly given the existing results on stochastic growth models, the growth impact of shocks depends on the curvature of the utility function, even though the Poisson shocks that we use to capture the large and unusual natural disaster shocks are not of the more standard variety.

Some of the less intuitive results include:

- Foreign aid received when the natural disaster impacts a country in the normal phase and aid when the country is in the disaster regime (roughly experiences two events within a short time) have potentially opposite impacts on the growth rate during the recovery period.

- Under reasonable conditions on the prevention and mitigation technologies, increases in foreign aid reduce investment in mitigation activities and, as a result, delay the recovery from the disaster (i.e. increase the expected duration of the low productivity regime).

- Depending on parameters, foreign aid can either crowd out the demand for insurance or induce a country to "over insure." It is even

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[2]The result must be interpreted with care since measured losses are, at best, a very noisy proxy for actual losses.

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theoretically possible that a country becomes a net seller of catastrophe insurance. This can happen if the country expects a large inflow of foreign aid contingent on a natural disaster strike since it can use the reverse insurance to increase consumption in normal times. Even though we do not expect this to be the outcome under a realistic calibration, the possibility shows the incentives that must be taken into account when creating a market for disaster insurance.

- Increased frequency of natural disasters has a growth effect even holding the expected loss of stocks from a strike constant. This simply illustrates the non-linearity and extensive cross equations restrictions implies by the theory. Moreover it shows that measurement matters since expected losses and frequency can have opposite growth effects.

In order to make progress quantifying the impact of natural disasters we conduct a quantitative exercise. We pick parameters to match the evidence on the effect of cyclones and we asses the effect of foreign aid, insurance and improvements in prevention and mitigation technologies (in progress).

The paper closest to ours is Bakkensen and Barrage (2017). They also analyze a growth model under normality assumptions and note the difference between natural disaster risk and strikes. The main difference is that we emphasize the role of foreign aid, insurance markets and prevention and mitigation technologies. In addition, our model allows for the possibility of higher and or lower growth rate in the post impact period while theirs implies that the growth rate is unchanged.

Our work is also related to the literature on the macro impact of large shocks which includes Barro (2009), Jones and Olken (2008), Gourio (2012) and Gabaix (2012).

In section 2 we describe the basic model and study separately the equilibrium allocations in the case in which the country does not have access to catastrophe insurance and the case in which it does. We also study the effect of delays. Section 3 (missing) contains the quantitative exercise and section 4 (missing) offers some conclusions.

2 Model

We study an economy populated by a representative dynasty. Since we abstract away from externalities the competitive equilibrium coincides with the

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solution to the planner's problem. We assume that the economy is closed except for limited access to a disaster insurance market. The model includes two types of shocks: standard TFP shocks and natural disaster shocks that are modeled as Poisson arrivals.

We view the economy as being in one of two regimes. The *normal* regime is the high productivity regime while the *disaster* regime is associated with low productivity. An economy that is in the normal regime switches to the disaster regime upon receiving a natural disaster strike. It reverts back to the normal regime with an instantaneous probability that depends on resources allocated to recover.

On the technology side we consider a standard two capital good Ak model with the following capital accumulation technologies:

$$dk = Akdt + \sigma kdZ - \left(1 - \mu_k^\delta(\kappa)\delta_k\right) kdN_t,$$

$$dh = Hhdt + \left(1 - \mu_h^\delta(\kappa)\delta_h\right) hdN_t,$$

where Z_t is a standard Brownian motion and N_t is a Poisson with parameter η. A realization of this Poisson corresponds to a natural disaster strike. The term $\mu_j^\delta(\kappa)\delta_j$ measures the amount of j-type of capital that is available after the natural disaster. We assume that $\mu_j^\delta(\kappa)\delta_j \in (0,1)$. In this context $\mu_j^\delta(\kappa)$ is the average loss rate and δ_j is random and has mean one. This component is meant to capture uncertainty about the strength of the natural disaster.

As it is standard in this Merton-type models we assume that $A > H$ but that the parameters are such that the share of both capital stocks in total wealth is strictly between zero and one.

We assume that the country can spend resources in activities that reduce the impact of a hurricane. For example, sea walls and better construction standards can significantly reduce the effect of a cyclone in a coastal area. In addition, we allow for the possibility that the country purchase insurance. In the simplest version of the model we introduce a standard insurance contract purchases from the rest of the world: the country pays a premium conditional on no natural disasters occurring and receives a payment when there is a hurricane strike.

In the simple version of the model we let total wealth be denoted by w, with $w = k + h$. Then the law of motion of wealth in the normal phase is

$$dw = \left[(\alpha A + (1 - \alpha) H) - (\kappa + b + c)\right] wdt$$
$$+\sigma \alpha wdZ_t - \left(1 - \mu_k^\delta(\kappa)\delta_k\right) \alpha wdN_t - \left(1 - \mu_h^\delta(\kappa)\delta_h\right) (1 - \alpha)wdN_t.$$

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In this specification $k = \alpha w$ and $h = (1 - \alpha)w$, κw is the total amount of resources allocated to prevention, bw is the premium corresponding to the insurance contract and cw is aggregate consumption. We assume that $\mu_j^\delta(\kappa)$ is increasing in κ.

The occurrence of a natural disaster has several effects. First, it causes a jump in the level of wealth. Let w' be the stock of wealth after the strike. Then,

$$
w' = w \left[\underbrace{\mu_k^\delta(\kappa)\delta_k(1 + \zeta_k)\alpha + \left(\mu_h^\delta(\kappa)\delta_h(1 + \zeta_h\right)(1 - \alpha) + I(b) + \zeta_w}_{\equiv \beta(\alpha, b, \kappa, \mu^\delta, \delta, \zeta)} \right]
$$

The term $\beta(\alpha, b, \mu^\delta, \delta, \zeta)$ captures the loss of wealth associated with the arrival of a natural disaster. For each capital of type j the post-hurricane level is $\mu_j^\delta(\kappa)\delta_j(1 + \zeta_j)$ of the pre-hurricane level. The term ζ_j captures the amount of capital specific foreign aid post-hurricane.[3] The term $I(b)$ is the payoff per unit of wealth of the insurance contract. Finally ζ_w stands for foreign aid that can be used at the discretion of the country. In general, we expect that the sum of all the terms except for the insurance payout will be less than one.

Second, we assume the occurrence of a natural disaster event is associated with lower the productivity of both types of capital but the loss is a function of both the resources allocated to prevention, as measured by κ as well as resources destined to mitigation which we denote by κ_D.[4] Thus, as a matter of notation we assume that post-strike the productivities are

$$
\begin{aligned}
A_D &= A(\kappa, \kappa_D) \leq A, \\
H_D &= H(\kappa, \kappa_D) \leq H.
\end{aligned}
$$

In what follows, to ease notation, we will be using (A_D, H_D) without explicitly noting their dependence on (κ, κ_D).

We assume that the duration of this low productivity phase is endogenous and depends on the amount of "mitigation resources" spent by the country. We assume that the switch back to normal times is well described by a Poisson process M_t with parameter $\upsilon(\kappa_D)$. Since the expected duration of

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[3]In a future extension we will also consider the possibility that the insurance payout is capital specific.

[4]We use the subscript D to denote the relevant values in the disaster regime.

the low productivity phase following a natural disaster strike is $1/\upsilon(\kappa_D)$ we assume that $\upsilon(\kappa_D)$ is increasing in κ_D.

The corresponding feasibility constraint during the disaster phase is given by

$$dw = \left[(\alpha_D A_D + (1 - \alpha_D) H_D) - (\kappa_D + c_D + b_D)\right] w dt$$
$$+ \sigma \alpha_D w dZ_t + w dM_t - \left(1 - \mu_k^\delta(\kappa)\delta_k\right) \alpha_D w dN_t - \left(1 - \mu_h^\delta(\kappa)\delta_h\right)(1 - \alpha_D) w dN_t,$$

where the last two terms capture the loss associated with another natural disaster strike while the country is still in the disaster phase.[5]

We assume that the utility function is given by

$$u(C) = c^{1-\gamma}\frac{w^{1-\gamma}}{1-\gamma}, \quad \text{where } C = cw.$$

Let the value function in normal (disaster) times be $V_N(w)$ $(V_D(w))$. Given the linearity in the technology and the assumption that preferences are of the CRRA variety we conjecture that

$$V_N(w) = V_N\frac{w^{1-\gamma}}{1-\gamma},$$
$$V_D(w) = V_D\frac{w^{1-\gamma}}{1-\gamma}.$$

The HJB equations of the planner's problem (which coincides with the competitive allocation) are,

$$\rho V_N\frac{w^{1-\gamma}}{1-\gamma} = \max_{c,\alpha,\kappa,b}\left\{c^{1-\gamma}\frac{w^{1-\gamma}}{1-\gamma} + V_N w^{1-\gamma}\left[(\alpha A + (1-\alpha)H) - (\kappa + b + c)\right]\right.$$

$$\left.-\gamma V_N w^{1-\gamma}\frac{\sigma^2}{2}\alpha^2 + \eta\left[V_D\frac{w^{1-\gamma}}{1-\gamma}E\left[(\beta(\alpha,b,\kappa,\mu^\delta,\delta,\zeta))^{1-\gamma}\right] - V_N\frac{w^{1-\gamma}}{1-\gamma}\right]\right\}.$$

(1)

The first three terms on the right hand side are standard. The last term captures the value loss associated with a natural disaster strike.

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[5] To keep the model stationary we assume that if there is another strike while the economy is in the disaster phase there is no further decrease in productivity. The only impact of this "second" strike is to reduce the stocks of both types of capital.

The corresponding equation for the disaster case is

$$\rho V_D \frac{w^{1-\gamma}}{1-\gamma} = \max_{c,\alpha,\kappa_D,b_D} \left\{ c^{1-\gamma}\frac{w^{1-\gamma}}{1-\gamma} + V_D w^{1-\gamma} \left[(\alpha A_D + (1-\alpha) H_D) - (\kappa_D + c + b_D) \right] \right.$$

$$\tag{2}$$

$$-\gamma V_D w^{1-\gamma}\frac{\sigma^2}{2}\alpha^2 + \upsilon(\kappa_D)\left[V_N \frac{w^{1-\gamma}}{1-\gamma} - V_D \frac{w^{1-\gamma}}{1-\gamma} \right]$$

$$\left. +\eta \left[V_D \frac{w^{1-\gamma}}{1-\gamma}E\left[(\beta(\alpha,b_D,\kappa,\mu^\delta,\delta,\zeta_D))^{1-\gamma} \right] - V_D \frac{w^{1-\gamma}}{1-\gamma} \right] \right\}.$$

In addition to the standard terms corresponding to TFP shocks the value of the problem depends on the gain associated with switching to the normal regime (and this is driven by a Poisson with parameter $\upsilon(\kappa_D)$) as well as the potential loss associated with another natural disaster hitting the economy while it is still in the disaster phase (and this is driven by an independent Poisson with parameter η). In this formulation we allow the amount of aid conditional on a natural disaster strike and the country being in the disaster phase, ζ_D, to be potentially different from ζ.[6]

2.1   No Disaster Insurance

It seems useful to understand the forces at work to consider a sequence of versions that simplify the problem by shutting down several channels. Let us first consider that case in which the loss of stocks associated with natural disaster is small. To capture this we set $\delta_k = \delta_h = 1$. We also take for now the choice of investment in prevention and mitigation as exogenous and assume no insurance.

Given that the utility function is unbounded it is clear that existence depends on parameter values. Thus, until we get to the quantitative section of the paper we will simply assume existence of an equilibrium. Put it differently the model only makes economic sense for those parameter values consistent with existence of an equilibrium.

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[6]Since on average a country that experiences another event while still in the disaster regime corresponds to a country that has been hit twice in a relatively short time by a natural disaster we allow for donors to respond differentially.

To economize on notation we define

$$P = H + \alpha_N(A - H) - \gamma\frac{\sigma^2}{2}\alpha_N^2 - \kappa,$$

$$P_D = H_D + \alpha_D(A_D - H_D) - \gamma\frac{\sigma^2}{2}\alpha_D^2 - \kappa_D.$$

$$V = \frac{V_N}{V_D}$$

It is understood that P depends on κ and the other variables that affect α_N and the same applies to P_D even though we do not make that dependence explicit.

Proposition 1 *Let (α_N, V) be the unique solution to the following equations*

$$\alpha_N = \frac{A - H}{\gamma\sigma^2} + \frac{\eta\Delta_k(\kappa)}{V\beta(\alpha_N, b, \kappa, \mu^\delta, \delta, \zeta)^\gamma}\frac{1}{\gamma\sigma^2}, \tag{3}$$

$$\alpha_D = \frac{A_D - H_D}{\gamma\sigma^2} + \frac{\eta\Delta_k(\kappa)}{\beta(\alpha_D, b_D, \kappa, \mu^\delta, \delta, \zeta_D)^\gamma}\frac{1}{\gamma\sigma^2}$$

where

$$\Delta_k(\kappa) = \mu_k^\delta(\kappa)(1 + \zeta_k) - \mu_h^\delta(\kappa)(1 + \zeta_h)$$

and

$$\rho + \eta + \upsilon(\kappa_D) - (1 - \gamma)P_D - \eta\beta(\alpha_D, b_D, \kappa, \mu^\delta, \delta, \zeta_D)^{1-\gamma} - \upsilon(\kappa_D)V\tag{4}$$
$$= [(\rho + \eta) - (1 - \gamma)P]V^{1/\gamma} - \eta\beta(\alpha_N, b, \kappa, \mu^\delta, \delta, \zeta)^{1-\gamma}V^{\frac{1-\gamma}{\gamma}}.$$

The expected growth rates in each regime conditional on no regime change are

$$\mu^N = H + \alpha_N(A - H) - \kappa - V_N^{-1/\gamma}\tag{5}$$

and

$$\mu^D = H_D + \alpha_D(A_D - H_D) - \kappa_D - V_D^{-1/\gamma},\tag{6}$$

where

$$\gamma V_D^{-1/\gamma} = \rho + \eta + \upsilon(\kappa_D) - (1-\gamma)P_D - \eta\beta(\alpha_D, b_D, \kappa, \mu^\delta, \delta, \zeta_D)^{1-\gamma} - \upsilon(\kappa_D)V, \tag{7}$$

and

$$\gamma V_N^{-1/\gamma} = \rho + \eta - (1 - \gamma)P - \eta\frac{\beta(\alpha_N, b, \kappa, \mu^\delta, \delta, \zeta)^{1-\gamma}}{V}\tag{8}$$

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Proof. (See Appendix) ∎

In a standard Merton portfolio problem the share of risky assets in the portfolio is given by

$$\frac{A - H}{\gamma \sigma^2},$$

since both α_j equal the share as prescribed by the Morton result plus a term whose sign depends on the sign of $\Delta_k(\kappa)$ it follows that when the expected capital loss associated with the k-type of capital exceeds that of the h-type of capital then the optimal α_j falls short of the Merton value and the opposite is true when the values are reversed.

This result highlights one of the channels that, in the model, can account for the difference between μ^N and μ^D. First, the fact that $A_D < A$, and $H_D < H$ implies that $\mu^D < \mu^N$. However, there are two other forces that can, potentially, reverse this. First, there is the standard saving effect captured by $V_N^{-1/\gamma}$ and $V_D^{-1/\gamma}$. In this case the reason why saving might be lower in the normal regime is that, starting from that phase, the economy will have lower returns if it switches to a disaster phase, while this is not the case if it is already in the disaster regime. Of course for this effect to dominate the income effect it is necessary that the utility function display relatively low curvature. Second, it is possible that $\alpha_D > \alpha_N$ and this increase in the share of the high return capital can increase growth. Whether this happens or not depends in complicated ways on the details of the distribution of foreign aid (ζ, ζ_D) among other effects.

2.2   Disaster Insurance and the Role of Aid

In this section we assume that the country has access to an insurance market and that insurance is fairly priced by the rest of the world. We do not need to assume that the interest rate is the same as the domestic discount rate. In this case it follows that zero profits in this activity implies that the relationship between premiums and payoffs are

$$I(b) = \frac{b}{\eta}.$$

In this case, it is possible to show that the optimal choice of insurance during the normal phase is such that[7]

$$V\beta(\alpha_N, b, \kappa, \mu^\delta, \delta, \zeta)^\gamma = 1,$$

and the share of the portfolio allocated to the risky asset is

$$\alpha_N = \frac{A - H}{\gamma\sigma^2} + \eta\Delta_k(\kappa).$$

During the disaster phase the optimal choice is

$$\beta(\alpha_D, b_D, \kappa, \mu^\delta, \delta, \zeta_D) = 1$$

which corresponds to full insurance.

Since the productivity of aggregate capital (or wealth) is given by

$$H + \alpha_N(A - H) = H + \frac{(A - H)^2}{\gamma\sigma^2} + \eta\Delta_k(\kappa)(A - H)$$

then the type of disasters that result in larger losses for physical than human capital —corresponding to $\Delta_k(\kappa) < 0$— increases in the frequency of natural disasters (i.e. an increase in η) decrease the aggregate productivity as it results in a smaller investment in the high return (and high loss in the event of a natural disaster) capital. Of course, if $\Delta_k(\kappa) > 0$ the same forces result in higher productivity.

In order to study the effects of insurance it is convenient to emphasize the version of the model in which the properties of the natural disaster do not directly affect the composition of the portfolio. To be precise, we assume that $\Delta_k(\kappa) = 0$. In this case,

$$\alpha_N = \frac{A - H}{\gamma\sigma^2},$$
$$\alpha_D = \frac{A_D - H_D}{\gamma\sigma^2}$$

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[7]This condition does not imply that the post-strike level of wealth is lower than the pre-strike. In particular, one can show that if $\gamma > 1$ then $V < 1$ and hence $\beta(\alpha_N, b, \kappa, \mu^\delta, \delta, \zeta) > 1$ which implies more post transfer wealth. In section XXXXX we deal with the case in which there are limits to the level of coverage.

are independent of natural disasters and

$$\beta(\alpha, b, \kappa, \mu^\delta, \delta, \zeta) = \mu^\delta(\kappa) + \frac{b}{\eta} + \zeta,$$

where

$$\mu^\delta(\kappa) = \mu_k^\delta(\kappa)(1 + \zeta_k) = \mu_h^\delta(\kappa)(1 + \zeta_h).$$

In our notation we distinguish between the effect of foreign aid when the natural disaster strike occurs during a normal phase, which we denoted by ζ, from the case in which the transfer follows a natural disaster strike that occurs when the country is already in the disaster phase, which we denoted by ζ_D. In some cases, it is useful to consider the case $\zeta = \zeta_D = \bar{\zeta}$ which assumes that foreign aid is not contingent on whether the country had been recently affected by another natural disaster.

It follows that

$$P = H + \frac{(A - H)^2}{2\gamma\sigma^2} - \kappa, \tag{9}$$

and

$$P_D = H_D + \frac{(A_D - H_D)^2}{2\gamma\sigma^2} - \kappa_D \tag{10}$$

are independent of properties of natural disasters and, hence, can be taken as given

The following proposition summarizes the basic implications of the model for $V = V_N/V_D$, the relative value of the problem in the normal and disaster phases in the absence of insurance.

Proposition 2 *The relative value of the problems* $V = V_N/V_D$ *solves the following equation:*

$$\left[(\rho + \eta) - (1 - \gamma)\left(P + \eta\left(\mu^\delta(\kappa) + \zeta\right)\right)\right]V^{1/\gamma} \tag{11}$$
$$= \rho + \eta + \upsilon(\kappa_D) - (1 - \gamma)\left[P_D + \eta\left(\mu^\delta(\kappa) + \zeta_D\right)\right] - \upsilon(\kappa_D)V.$$

The solution V^* has the following properties

1. *If $\gamma \in (0, 1)$*

$$\frac{\partial V^*}{\partial \zeta} > 0, \frac{\partial V^*}{\partial \zeta_D} < 0, \frac{\partial V^*}{\partial \bar{\zeta}} > 0,$$

$$\frac{\partial V^*}{\partial \upsilon(\kappa_D)} < 0, \frac{\partial V^*}{\partial \eta}\Big|_{\zeta = \zeta_D} < 0$$

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2. *if $\gamma > 1$*

$$\frac{\partial V^*}{\partial \zeta} < 0, \frac{\partial V^*}{\partial \zeta_D} > 0, \frac{\partial V^*}{\partial \bar{\zeta}} > 0,$$

$$\frac{\partial V^*}{\partial \upsilon(\kappa_D)} > 0, \frac{\partial V^*}{\partial \eta} \big|_{\zeta=\zeta_D} > 0$$

Proof. (see the Appendix) ■

The average growth rates in each regime, contingent on no strikes, are given by

$$\mu^N = \frac{1}{\gamma} \left[P + \eta \left(\mu^\delta(\kappa) + \zeta \right) - (\rho + \eta) \right]$$

$$\mu^D = \frac{1}{\gamma} \left[\left(P_D + \eta \left(\mu^\delta(\kappa) + \zeta_D \right) \right) - (\rho + \eta + \upsilon(\kappa_D)) + \upsilon(\kappa_D)V \right].$$

****Question: Do we really need/have $\left(\mu^\delta(\kappa) + \zeta_D \right)$ in μ^D? Since $\beta(\alpha_D, b_D, \kappa, \mu^\delta, \delta, \zeta_D) = 1$, it follows that c^D is

$$\gamma c^D = (\rho + \eta + \upsilon(\kappa_D)) - (1 - \gamma)P_D - \eta - \upsilon(\kappa_D)V$$

or

$$\gamma c^D = (\rho + \eta + \upsilon(\kappa_D)) - (1 - \gamma)P_D - \eta - \upsilon(\kappa_D)\beta(\alpha, b, \kappa, \mu^\delta, \delta, \zeta)^{-\gamma}.$$

It follows then that, for all γ, we have that

$$\frac{\partial c^D}{\partial \zeta} = \gamma \upsilon(\kappa_D)\beta(\alpha, b, \kappa, \mu^\delta, \delta, \zeta)^{-\gamma-1} > 0,$$

$$\frac{\partial c^D}{\partial \zeta_D} = 0.$$

This, in turn, implies that

$$\frac{\partial \mu^D}{\partial \zeta} < 0, \text{ and } \frac{\partial \mu^D}{\partial \zeta_D} = 0$$

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We can now summarize the impact of foreign aid and properties on natural disaster on the growth rate in each phase under full insurance. Not surprisingly, the qualitative implications do not depend on the curvature of the utility function.

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Proposition 3 (Full Insurance: The Effect of Foreign Aid) *The impact of aid when the country has access to full insurance is given by*

$$\frac{\partial \mu^N}{\partial \zeta} > 0, \frac{\partial \mu^N}{\partial \zeta_D} = 0, \frac{\partial \mu^N}{\partial \bar{\zeta}} > 0.$$

and

$$\frac{\partial \mu^D}{\partial \zeta} < 0, \frac{\partial \mu^D}{\partial \zeta_D} = 0, \frac{\partial \mu^D}{\partial \bar{\zeta}} < 0.$$

Proof. (see the Appendix) ∎

Proposition 4 (No Insurance: The Effect of Foreign Aid) *The impact of aid when the country has access to full insurance is given by*

$$\frac{\partial \mu^N}{\partial \zeta} < 0, \frac{\partial \mu^N}{\partial \zeta_D} > 0.$$

and

$$\frac{\partial \mu^D}{\partial \zeta} > 0, \frac{\partial \mu^D}{\partial \zeta_D} = indeterminate.$$

Proof. (see the Appendix) ∎

The model implies potentially heterogenous growth effects of foreign aid depending on the specific details of how it is awarded. Increases in regime neutral aid, as measured by $\bar{\zeta}$, unambiguously increase the growth rate in the normal phase at the same time that is decreases the growth rate in the disaster phase.

Increases in (promised) aid when the country experiences a natural disaster strike but is otherwise in a normal phase, that is, increases in ζ, have opposite effects on the growth rate on the two phases.

Next we study how the different dimensions of a natural disaster affect growth. Given the relatively simple model that we study we can summarize the relevant dimensions as

- Frequency of strikes: $1/\eta$.

- Duration of the disaster phase: $1/\upsilon(\kappa_D)$

- Loss of stocks: $\mu^\delta(\kappa)$

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- Loss of productivity: $P_D/P = 1 - \phi$

Proposition 5 (Growth and the Structure of Natural Disasters) *1.*
Changes in η

$$\frac{\partial \mu^N}{\partial \eta} < 0, \frac{\partial \mu^D}{\partial \eta} \mid_{\gamma \in (0,1)} < 0, \frac{\partial \mu^D}{\partial \eta} \mid_{\gamma > 1} = \text{ ambiguous.}$$

2. Changes in $\upsilon(\kappa_D)$

$$\frac{\partial \mu^N}{\partial \upsilon(\kappa_D)} = 0, \frac{\partial \mu^D}{\partial \upsilon(\kappa_D)} \mid_{\gamma \in (0,1)} > 0, \frac{\partial \mu^D}{\partial \upsilon(\kappa_D)} \mid_{\gamma > 1} < 0.$$

3. Changes in $\mu^\delta(\kappa)$

$$\frac{\partial \mu^N}{\partial \mu^\delta(\kappa)} > 0, \frac{\partial \mu^D}{\partial \mu^\delta(\kappa)} > 0.$$

4. Changes in ϕ

$$\frac{\partial \mu^N}{\partial \phi} < 0, \frac{\partial \mu^D}{\partial \phi} < 0.$$

Proof. (see the Appendix) ∎

Some of the results are as expected: natural disasters that result in more destruction of stocks and that are associated with lower productivity unambiguously decrease growth. However the impact of duration (or frequency) of the phenomena have less intuitive effects. Consider, for example the impact of a decrease in the duration of the low productivity phase, that is, a faster recovery. This improvement has no impact on the growth rate in the normal phase and can actually decrease the growth rate in the disaster phase. This can happen when the utility function has more curvature than the log. In this case income effects dominate and the expectation of a faster recovery (and the associated higher return to investment) does not result in higher savings. Rather, the optimal policy increases consumption.

The model is highly non-linear and it suggests that different elements of a natural disaster can have different impacts on growth. To illustrate this consider the impact of increasing the frequency of natural disasters, η, at the same time that the expected loss associated with a natural disaster is held constant. To be precise let the instantaneous return on capital be denoted

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z, then the total return taking into account that a fraction $1 - \mu^\delta(\kappa)$ is lost in the case of a natural disaster is simply

$$\frac{z}{\rho + \eta\left(1 - \mu^\delta(\kappa)\right)}.$$

Thus a measure of a natural disaster economic cost is $\eta\left(1 - \mu^\delta(\kappa)\right)$. We want to compare the growth impact of different natural disasters that are associated with exactly the same expected loss. Let $\eta\left(1 - \mu^\delta(\kappa)\right) = m$. Thus, holding m constant we want to determine the impact of more frequent (higher η) natural disasters. Thus, this captures the tradeoff between more frequent, but less destructive, events and more rare but more damaging natural disasters.

Simple algebra shows that

$$\frac{\partial \mu^N}{\partial \eta} \Big|_{m=\bar{m}} = \zeta > 0.$$

Thus, when it comes to evaluating the growth impacts of natural disasters in normal times more frequent and less severe events are growth enhancing. The result shows that empirical work that tries to ascertain the growth effects of a natural disaster and uses expected losses as its measure of impact will get biased estimates depending on the distribution of frequencies.

The effect of the expected time in between strikes $(1/\eta)$ on the growth rate in the post-strike phase is ambiguous. Formally, the impact on μ^D is

$$\frac{\partial \mu^D}{\partial \eta} \Big|_{m=\bar{m}} = \zeta_D + \upsilon(\kappa_D)\frac{\partial V}{\partial \eta} \Big|_{m=\bar{m}}.$$

The sign of the term $(\partial V/\partial \eta) \big|_{m=\bar{m}}$ depends on the elasticity of substitution. When income effects dominate ($\gamma > 1$), it is positive and more frequent natural disasters are growth enhancing. If $\gamma < 1$ the last term is negative and the whole expression would be negative if ζ_D is small.

The Demand for Disaster Insurance What is the optimal choice of insurance? One can show that the demand for insurance in the disaster phase is such that it completely offsets the capital loss, that is

$$b_D = \eta(1 - (\mu^\delta(\kappa) + \zeta_D)).$$

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In this situation there is compete crowding out of private insurance by foreign aid. This suggests that efforts to create a market for catastrophe bonds have to take into account the negative incentives associated with the expectation of foreign aid.

In the normal phase the demand for insurance is given by

$$b_N = \eta(V^{-1/\gamma} - (\mu^\delta(\kappa) + \zeta_D)).$$

Since the model imposes no restrictions on this demand it is possible for the country to "overinsure" in the sense that, in equilibrium, $\beta(\alpha_N, b, \kappa, \mu^\delta, \delta, \zeta)$, the post-strike relative wealth can be greater than one. In fact, this is the case if $\gamma > 1$. As Proposition 2 shows in this case $V < 1$ and since the optimal choice of insurance requires that

$$V\beta(\alpha_N, b, \kappa, \mu^\delta, \delta, \zeta)^\gamma = 1$$

it follows that $\beta(\alpha_N, b, \kappa, \mu^\delta, \delta, \zeta) > 1$. The reason for this is that the country is using the insurance market to insure as well against the low productivity during the disaster phase. One way of doing that is by acquiring more wealth conditional on the shock and this is exactly the type of contract that the insurance scheme offers.

In the case that $\gamma \in (0, 1)$, then the optimal choice is such that there is incomplete insurance, that is, $\beta(\alpha_N, b, \kappa, \mu^\delta, \delta, \zeta) < 1$. Finally it is possible for the country to "sell" insurance (issuer of catastrophe bonds). This corresponds to the case

$$b_N = \eta(V^{-1/\gamma} - (\mu^\delta(\kappa) + \zeta_D)) < 0$$

which can happen when V is sufficiently large. Even though this might seem paradoxical, the key driver of this role reversal is foreign aid. If the country expects a large ζ then it chooses to increase current consumption in exchange for lower future consumption. Effectively, the country is selling some of its right to the foreign aid it will receive in the case of a natural disaster strike.

2.2.1 Optimal Choice of Prevention and Mitigation

In general it is not possible theoretically to determine how changes in foreign aid will affect endogenous prevention and mitigation efforts. I this section we

make some progress and report some partial results. We take the objective function to maximize V_N. Thus function V_N satisfies

$$\gamma V_N^{-1/\gamma} = \rho + \eta + (\gamma - 1) \left[P + \eta \left(\mu^\delta(\kappa) + \zeta \right) \right] - \gamma \eta V^{-1/\gamma}. \qquad (12)$$

For an interior maximum we require that $\partial V_N/\partial \kappa$ and $\partial V_N/\partial \kappa_D$ be equal to zero. Simple algebra implies that

$$\frac{\partial V_N}{\partial \kappa} = 0 \Leftrightarrow \eta V^{-\left(\frac{1}{\gamma}+1\right)} \frac{\partial V}{\partial \kappa} = (1 - \gamma) \left[-1 + \eta \frac{d\mu^\delta}{d\kappa}(\kappa) \right], \qquad (13)$$

and

$$\frac{\partial V_N}{\partial \kappa_D} = 0 \Leftrightarrow \frac{\partial V}{\partial \kappa_D} = 0. \qquad (14)$$

In this model the relative valuation V is a complicated function of all parameters and endogenous variables. We summarize the properties of $\partial V/\partial \kappa$ and $\partial V/\partial \kappa_D$ in the following proposition

Proposition 6 *Assume that $\gamma > 1$, then*

$$\frac{\partial V}{\partial \kappa} > 0$$

and, for all γ,

$$\frac{\partial V}{\partial \kappa_D} = 0 \Leftrightarrow 1 - V = (1 - \gamma) \frac{\partial P_D/\partial \kappa_D - 1}{v'(\kappa_D)}.$$

Proof. (see the Appendix) ∎

Given that $\partial V/\partial \kappa > 0$ when $\gamma > 1$, equation (13) implies that

$$\frac{d\mu^\delta}{d\kappa}(\kappa) < \frac{1}{\eta},$$

which shows that optimal prevention in this case requires more investment than what would be required to equate the marginal cost of prevention — which is one in this case— with the marginal benefit of reducing the losses of stocks —which in this case is $\eta \times d\mu^\delta/d\kappa$. The reason is simple: In this specification investments in prevention have a positive impact on flow productivity of both forms of capital during the disaster phase. Hence this second component increases the marginal benefit.

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The optimal level of mitigation implies that

$$z(\kappa_D) \equiv \frac{\partial P_D / \partial \kappa_D - 1}{\upsilon'(\kappa_D)}$$

must satisfy

$$z(\kappa_D) = \frac{1 - V}{1 - \gamma}. \tag{15}$$

Given the results of Proposition 5 the right hand side of equation (15) is negative and this implies that, at the optimum, the marginal product of mitigation investments is less than the marginal cost. The reason is simple: mitigation also shortens the expected duration of the disaster phase and this a valuable

$$\partial P_D / \partial \kappa_D < 1.$$

Under some assumptions about the specific technologies the function $z(\kappa_D)$ is downward sloping. In that case the results in Proposition 5 imply that increases in $\bar{\zeta}$ decrease the optimal κ_D. Thus, higher expected foreign aid weakens the incentives that the country has to invest in activities that increase productivity and shorten the duration of the disaster phase. In particular, the model implies that countries that receive a higher level of foreign aid in response to a natural disaster strike will experience longer periods of low productivity.

3   Quantitative Effects: A First Pass

In this section we report the results from parameterizing the model. At this point our quantitative exercise is aimed at trying to understand the interplay between different mechanisms and forces in the model rather than trying to match any country's experience. Moreover, we were not able to find reliable data in order to estimate the relevant parameters. Instead we report the criteria that we used to select specific values.

3.1   Functional Forms

As indicated in the model the productivity in the disaster phase is lower than in the normal phase. We assume the post-strike productivities follow

$$A_D = A \times f_A(\kappa, \kappa_D)$$
$$H_D = H \times f_H(\kappa, \kappa_D)$$

with

$$f_j \in (0,1) \ j = \{A, H\}$$

and

$$\frac{\partial f_j}{\partial \kappa} > 0$$
$$\frac{\partial f_j}{\partial \kappa_D} > 0$$

More specifically, we specify that the physical and human productivities in the disaster phase are given by

$$f_A(\kappa, \kappa_D) = 1 - \phi_A \times e^{-(\lambda^A \times \kappa + \lambda_D^A \times \kappa_D)}$$
$$f_H(\kappa, \kappa_D) = 1 - \phi_H \times e^{-(\lambda^H \times \kappa + \lambda_D^H \times \kappa_D)}$$

where ϕ_A and ϕ_H are the respective productivity losses under zero investment in prevention and mitigation and λ^A and λ_D^A (λ^H and λ_D^H) are the semi-elasticities of the physical (human) capital stock loss functions (i.e. $-\phi_j \times e^{-(\lambda^j \times \kappa + \lambda_D^j \times \kappa_D)}$) with respect to investment in prevention and mitigation, respectively.

We assume that the probability of returning to the *normal* phase after the disaster hits the economy is given by

$$\upsilon(\kappa_D) = \upsilon_0(1 + \upsilon_1 \times \kappa_D)^{\upsilon_2}$$

where υ_0 is the inverse of the expected disaster duration under no investment in mitigation, and υ_1 and υ_2 are scale and curvature parameters.

The losses of the two stocks when the natural disaster hits depend on the amount of prevention resources, κ, according to

$$\mu_k^\delta(\kappa) = 1 - \mu_k^0 \times e^{-\mu_k^1 \times \kappa}$$
$$\mu_h^\delta(\kappa) = 1 - \mu_h^0 \times e^{-\mu_h^1 \times \kappa}$$

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where μ_k^0 and μ_h^0 are the physical and human capital stock losses under zero prevention investment and μ_k^1 and μ_h^1 are the semi-elasticities of physical and human capital stock losses with respect to prevention.

Our aim in choosing these functional forms was to present a fairly general approach to trying to capture the role of prevention and mitigation in reducing the impact of a natural disaster on the productive capabilities of an economy.

3.2 Calibration

Given the limited data availability on natural disasters and their impact, we take what we consider a reasonable calibration and we analyze the sensitivity of the results obtained to changes to this baseline case.

We assume that in the event of natural disaster, and under zero investment in prevention and mitigation, the physical and human capital productivities fall by 20% and 10%, respectively. Thus $\phi_A = 0.2$ and $\phi_H = 0.1$. Furthermore, we initially consider the case of equal impact of prevention and mitigation on physical and human capital productivities, which implies $\lambda^A = \lambda_D^A$ and $\lambda^H = \lambda_D^H$. To pin down λ^A and λ^H we take the approach that at relatively high levels of investment in prevention and mitigation [8] their marginal impact on productivity is almost negligible. Thus, λ^A and λ^H solve

$$\lambda^A \times 0.2 \times e^{-(\lambda^A \times \kappa + \lambda_D^A \times \kappa_D)} \Big|_{\kappa=0.05; \kappa_D=0.05} \approx 0$$

$$\lambda^H \times 0.1 \times e^{-(\lambda^H \times \kappa + \lambda_D^H \times \kappa_D)} \Big|_{\kappa=0.05; \kappa_D=0.05} \approx 0.$$

We find that $\lambda^A = \lambda_D^A \approx 140$ and $\lambda^H = \lambda_D^H \approx 110$.

In the case of the stock loss functions we suppose that without investment in prevention the physical and human capital losses are 10% and 5% of their corresponding stocks, respectively

$$\mu_k^0 = 0.10$$
$$\mu_h^0 = 0.05$$

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[8] We think of $\kappa = 0.05$ and $\kappa_D = 0.05$ as those levels, which, considering a total wealth -physical and human capital- to output ratio of six, amount to prevention and mitigation investments of around 30% of GDP.

We calibrate μ_k^1 and μ_h^1 in a similar fashion as the productivity loss functions, considering a nearly zero impact of additional investment in prevention for high investment levels. In this case μ_k^1 and μ_h^1 solve

$$0.1 \times \mu_k^1 \times e^{-\mu_k^1 \times \kappa} \quad \Big|_{\ \kappa=0.05} \approx 0$$
$$0.05 \times \mu_h^1 \times e^{-\mu_h^1 \times \kappa} \quad \Big|_{\ \kappa=0.05} \approx 0$$

which result in $\mu_k^1 \approx 145$ and $\mu_h^1 \approx 130$.

For the disaster recovery probability, we calibrate υ_0 such that the expected duration of the disaster phase is three years under zero investment in mitigation, hence $\upsilon_0 = \frac{1}{3}$. We choose υ_1 and υ_2 such that the expected recovery speed is 1.5 years and 0.5 years for prevention investments of 5% and 30% of GDP, respectively. Hence, υ_1 and υ_2 solve

$$
\begin{aligned}
1.5 &= \frac{1}{3} \times (1 + \upsilon_1 \times \frac{0.05}{6})^{\upsilon_2} \\
0.5 &= \frac{1}{3} \times (1 + \upsilon_1 \times \frac{0.30}{6})^{\upsilon_2}
\end{aligned}
$$

We obtain $\upsilon_1 \approx 510$ and $\upsilon_2 \approx 0.55$.

We take the discount rate to be $\rho = 0.04$, the relative risk aversion parameter $\gamma = 2$, and the probability of a natural disaster hitting the economy to be 0.03. The latter implies that a natural disaster hits the economy every 33 years on average, a very rare event. We take the expected return on physical capital to be 10% ($A = 0.1$), the return on human capital to be 6% ($H = 0.06$) and we calibrate the volatility of physical capital to match the historical return volatility of the S&P 500 ($\sigma = 0.16$).

3.3   Sensitivity: Preliminary Results

In this section we analyze the model's quantitative behavior under the baseline calibration, starting with the no disaster insurance case. We set the generic wealth (ζ_w) and stock-specific transfers (ζ_k and ζ_h) and find the utility maximizing levels of $(\kappa, \kappa_D, \alpha_N, \alpha_D)$.[9] We report those values as well as the growth rates on the two phases, μ_N and μ_D, the fraction of total wealth

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[9]We search over a grid of 61 equally spaced values for κ and κ_D between 0 and 0.05 to find the investment levels in prevention and mitigation that maximize utility.

left over after a disaster strikes, $\beta(\alpha, \kappa, \mu^\delta, \zeta)$, and the expected duration of the disaster phase, $1/\upsilon(\kappa_D)$.

We label our base scenario, *Case 1*. We then explore the sensitivity of the endogenous choices to changes in the basic parameterization that we label Cases 2-6.

- *Case 2*: This case displays higher semi-elasticities of investment of both prevention and mitigation for both forms of capital. We choose $\lambda_D^A = 2\lambda^A$ and $\lambda_D^H = 2\lambda^H$.

- *Case 3*: We allow for less curvature (higher marginal product) of investments in prevention and mitigation. We capture this by requiring that the marginal product be low (close to zero) when $\kappa = \kappa_D = 0.1$, instead of 0.05.

- *Case 4*: We increase the losses of the two stocks when a natural disaster increases. If no efforts in prevention and mitigation are undertaken we assume that 40% of the physical capital stock is lost and 20% of the human capital is destroyed. Thus, $\phi_A' = 0.4$ and $\phi_H' = 0.2$

- *Case 5*: We triple the volatility of the risky technology and increase σ from 0.16 to 0.48.

- *Case 6*: We assume that a natural disaster occurs, on average, every ten years. This changes η from 0.03 to 0.1.

The results are reported in Table 1

Table 1						
	Case 1	*Case 2*	*Case 3*	*Case 4*	*Case 5*	*Case 6*
κ	0.00%	0.00%	0.00%	0.00%	0.00%	0.42%
κ_D	1.20%	1.00%	1.50%	1.70%	0.60%	0.42%
μ_N	3.37%	3.37%	3.38%	3.37%	1.35%	3.20%
μ_D	1.74%	2.25%	0.86%	1.26%	0.39%	2.43%
α_N	0.746	0.746	0.745	0.745	0.08	0.726
α_D	0.701	0.735	0.638	0.706	0.07	0.651
$\frac{1}{\upsilon(\kappa_D)}$	1.03	1.11	0.92	0.87	1.40	1.60
$\beta(\alpha, \kappa, \mu^\delta, \zeta)$	0.91	0.91	0.91	0.91	0.95	0.95

The quantitative results do not vary significantly between cases *Cases 1 to 4*. In all cases it is optimal to spend no resources in prevention. Rather it is better to spend between 6% and 10% of GDP in the disaster phase increasing productivity and shortening the duration of that phase. The highest level of spending in mitigation occurs when the natural disaster is costliest in terms of stock losses (*Case 4*)

We find that the growth rate of output displays little change across all four cases in the normal phase, while the growth rate in the disaster phase depends on the impact of the disaster on productivity and the marginal returns of the mitigation technology. The intuition is straightforward: since a disaster is a very unlikely event and mitigation measures can be adopted instantaneously, it is optimal to save on prevention resources, keep a high growth rate in the normal phase and take remedy measures when the economy is hit by a disaster.

The main difference in the first four cases is the resulting growth rate in the disaster phase. In *Case 2*, the higher marginal return on mitigation allows for a reduction in mitigation investment —relative to *Case 1*—while still keeping a higher post-strike productivity $(A_D^2 = 0.099 > A_D^1 = 0.096)$. This drives up the portfolio share of the risky asset. These two elements combined result in a higher growth rate in the disaster phase. *Case 3* in, in one dimension, the opposite of *Case 2*. Given our calibration, it implies that the marginal impact of prevention and mitigation are smaller for similar investment levels than in *Case 1*.The mechanism driving the results is, therefore, the same as in *Case 2*, but acting in the opposite direction.

Case 4 requires a higher mitigation investment to keep the return of the risky asset at the same level as in *Case 1*. As a result, even if the capital portfolio shares are very close in both cases, the higher required mitigation investment drags growth down in the disaster phase.

In *Case 5* (higher volatility of the risky technology) we find, as expected, a sizable decrease in the fraction of wealth allocated to the risky asset, both in the normal and in the disaster phase. This drives down growth under both scenarios. Relative to the previous four cases it also reduces the efforts at mitigation.

As noted above, a common feature of *Cases 1* to *5* is that the optimal investment on prevention is zero in all of them. This is driven by our assumption that natural disasters are rare. When we increase the expected arrival time from 33 to 10 years we find a positive investment in prevention. We view this case as somewhere "in between" a parameterization that applies to

earthquakes —fairly rare events— and a parameterization that captures the impact of hurricanes —an almost yearly occurrence.

3.4   Aid and Welfare: Growth Effects

In this section we explore the interplay between foreign aid and the endogenous choice of investments in prevention and mitigation. We consider three different scenarios in terms of how rare natural disasters are. The three cases area natural disaster on average every 33 years ($\eta = 0.03$). every 10 years ($\eta = 0.10$), and every two years ($\eta = 0.50$).

We initially set the foreign aid (transfers) equal to zero and increase them up to 10% of the post-strike wealth. In this preliminary exercise we only study the impact of "general" (as opposed to stock specific) transfers. This corresponds to what we labeled ζ_w in the theoretical model.

In Tables 2-4 we report the same variables as in Table 1. In addition we indicate the welfare gain —relative to the no transfer case— associated with foreign aid. We follow standard practice in macro and estimate the welfare gains as the percentage increase in permanent consumption associated with the transfer.[10] Of course, higher foreign aid increases welfare in a monotonic way but transfers have less obvious effects:

1. In all cases higher transfers lower the growth rate in both phases. This decrease is driven by the country adjusting the level of overall saving, the composition of the portfolio and the investments in prevention and mitigation.

2. In the rare event scenario ($\eta = 0.03$) foreign aid and investment in mitigation are complements: the higher the transfer the higher the optimal investment in mitigation. Moreover, total saving is decreasing in the transfer in the normal phase and almost constant in the disaster phase. The portfolio effects of foreign aid are small.

3. In the intermediate and frequent case scenarios ($\eta = 0.10$ and $\eta = 0.50$) foreign aid and investment in prevention are substitutes but foreign

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[10]Label $\overline{C}(\zeta) = \overline{c}w$ the constant level of lifetime consumption that yields utility $V_N(w)$. Thus $V_N(\zeta)\frac{w^{1-\gamma}}{1-\gamma} \equiv \frac{(\overline{c}(\zeta)w)^{1-\gamma}}{1-\gamma}\frac{1}{\rho}$, which implies $\overline{c}(\zeta) = (\rho V_N(\zeta))^{\left(\frac{1}{1-\gamma}\right)}$ and $\overline{c}(0) = (\rho V_N(0))^{\left(\frac{1}{1-\gamma}\right)}$.Therefore $\frac{\overline{c}(\zeta)}{\overline{c}(0)} = \left[\frac{V_N(0)}{V_N(\zeta)}\right]^{\left(\frac{1}{1-\gamma}\right)}$.

aid and investment in mitigation are complements. The impact of aid on total investment is positive: investment in mitigation increases in a magnitude than more than compensates the fall in investment in prevention. The portfolio effects are small.

4. Growth and welfare move in opposite directions: the higher the level of foreign aid the higher the welfare and the lower the growth rate in both phases.

5. The growth impact of foreign aid is significantly smaller (although still negative) in the normal phase. Since higher transfers are associated with lower investment in prevention it must be the case that, in the normal phase, the expectation of higher transfers lowers saving in both productive assets.

6. Growth reversals. The model implies that in the case of rare natural disasters the growth rate in the normal phase is higher than in the disaster phase. However, for events that, on expectation, happen every two years the opposite is true: growth is higher in the disaster phase.

Table 2: $\eta = 0.03$					
	ζ_w				
	0.00	0.01	0.02	0.05	0.10
κ	0.000	0.000	0.000	0.000	0.000
κ_D	0.0117	0.0117	0.0117	0.0125	0.0133
μ_N	3.37%	3.36%	3.34%	3.30%	3.24%
μ_D	1.74%	1.72%	1.63%	1.63%	1.53%
α_N	0.746	0.746	0.747	0.749	0.752
α_D	0.701	0.701	0.701	0.707	0.712
$\frac{1}{\upsilon(\kappa_D)}$	1.033	1.033	1.033	1.000	0.969
c_N	0.742	0.745	0.747	0.754	0.764
c_D	0.772	0.774	0.776	0.779	0.785
$\frac{\kappa}{F_N}$	0.000	0.000	0.000	0.000	0.000
$\frac{\kappa_D}{F_D}$	0.162	0.162	0.162	0.172	0.183
$\beta(\alpha, \kappa, \mu^\delta, \zeta)$	0.91	0.92	0.93	0.99	1.01
$\Delta Welfare$	0	0.60%	1.40%	3.00%	5.80%

Table 3: $\eta = 0.10$					
	ζ_w				
	0.00	0.01	0.02	0.05	0.10
κ	0.0042	0.003	0.003	0.0025	0.0017
κ_D	0.0042	0.007	0.007	0.0100	0.0142
μ_N	3.20%	3.21%	3.16%	3.08%	2.92%
μ_D	2.43%	2.27%	2.22%	1.91%	1.40%
α_N	0.726	0.719	0.720	0.714	0.710
α_D	0.651	0.660	0.660	0.669	0.673
$\frac{1}{\upsilon(\kappa_D)}$	1.60	1.33	1.33	1.11	0.94
$\beta(\alpha, \kappa, \mu^\delta, \zeta)$	0.87	0.87	0.87	0.92	0.91
$\frac{c_N}{Y_N}$	0.700	0.706	0.713	0.732	0.762
$\frac{c_D}{Y_D}$	0.750	0.745	0.751	0.757	0.774
$\frac{\kappa}{Y_N}$	0.055	0.044	0.044	0.033	0.022
$\frac{\kappa_D}{Y_D}$	0.059	0.094	0.094	0.138	0.193
$\Delta Welfare$	0	1.85%	3.70%	9.23%	18.3%

Table 4: $\eta = 0.50$					
	ζ_w				
	0.00	0.01	0.02	0.05	0.10
κ	0.020	0.020	0.020	0.020	0.0142
κ_D	0.000	0.000	0.000	0.000	0.0175
μ_N	1.88%	1.77%	1.66%	1.35%	1.13%
μ_D	3.66%	3.57%	3.48%	3.21%	0.84%
α_N	0.765	0.765	0.766	0.766	0.741
α_D	0.753	0.753	0.753	0.753	0.733
$\frac{1}{\upsilon(\kappa_D)}$	3.00	3.00	3.00	3.00	0.849
$\beta(\alpha, \kappa, \mu^\delta, \zeta)$	0.99	1.00	1.01	1.05	1.09
$\frac{c_N}{Y_N}$	0.686	0.700	0.714	0.756	0.849
$\frac{c_D}{Y_D}$	0.703	0.716	0.728	0.765	0.839
$\frac{\kappa}{Y_N}$	0.264	0.264	0.264	0.264	0.187
$\frac{\kappa_D}{Y_D}$	0.000	0.000	0.000	0.000	0.232
$\Delta Welfare$	0	4.20%	8.50%	21.5%	53.0%

4   Conclusion

In this paper we develop a simple framework to evaluate policies that can influence the economic impact of natural disasters. We show that disaster insurance (for example in the form of cat bonds) can have a large impact on a country's choice of prevention and mitigation investments. In the current version insurance payments are instantaneous but an important extension (on going work) is to allow for delays and quantify the "value" of alternative insurance policies.

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5 Appendix 1: Catastrophe Bonds

In this appendix we show that a catastrophe bond is equivalent to an insurance contract. We assume, as before, that a natural disaster arrives as a Poison with parameter η. Let a measure of damage or intensity of the event be denoted by G. We view damage/intensity as random variable and we interpret $G(x)$ as the probability that the damage/intensity is less than or equal to x.

If a country issues a catastrophe bond —for now we look at bonds that have infinite maturity in the absence of an event— with principal B that earns a return r^c. The investors in this bond keep earning this return until a natural disaster that reaches a specific level of damage/intensity occurs. At this point the bond earns a zero return from then on. We will choose r^c so that the bond sells at par.

The funds raised by the sale of catastrophe bond are invested at the risk free rate, r, and the principal is paid to the issuer of the catastrophe bond if an event that meets the criteria occurs.

Consider a catastrophe bond that is associated with an event described by the arrival of a natural disaster with intensity greater than $\bar{x}$. The relevant pricing equation is given by

$$rB = r^c B + \eta \left[F(\bar{x}) \times B + (1 - F(\bar{x})) \times 0 - B \right].$$

This implies that the rate r^c is $r^c = r + \eta \left(1 - F(\bar{x})\right)$. Thus, the country is paying r^c every period (if there is no natural disaster) and earning r. Thus, the true cost of the bond is $r^c - r$ and this is equivalent to the insurance rate in the main body of the paper as $r^c - r = \eta \left(1 - F(\bar{x})\right)$.