Occupational Shifts and Human Capital Accumulation: The Role of Cognitive-Intensive Work in Economic Development

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Abstract

This paper examines how occupational structure shapes cross-country differences in human capital stocks. Using harmonized microdata, I document that cognitive occupations are associated with higher schooling and higher returns to education and experience, underscoring their greater skill intensity. I then develop a dynamic model of schooling, on-the-job training, and occupational choice, calibrated and structurally estimated with data from Brazil and the United States. The results show that while average human capital in cognitive occupations in the U.S. is only about 10 percent higher than in Brazil, the gap widens to nearly threefold in routine jobs. A decomposition attributes 25 percent of the U.S.–Brazil gap to differences in the occupational structure, 30 percent to the higher human capital embodied in U.S. routine work, and the remainder to relative occupational efficiency and cognitive skill differences.

Keywords: Economic Development, Human Capital, Occupational Structure, Cognitive Skill, Labor Market Transformation, Skill-Biased Technological Change, Education and Training, Labor Productivity.

JEL Codes: 015, 040, J24, O15, O33, O41, D91

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1 Introduction

Cross-country differences in standards of living and labor force attributes are noticeable. Understanding what shapes the observed disparities in workers' qualifications and how they translate into different prosperity paths is a central element in the analysis of economic development.

The traditional view suggests that there are significant disparities in productivity across countries, with labor-augmenting technological progress playing a major role in economic development (Uzawa (1961),Acemoglu (2003),Jones (1995)). A challenge is determining to what extent these productivity gaps are solely due to access to superior technologies in rich countries versus a more skilled workforce.

More recent studies, based on newly available data and improved measuring techniques, tackle this issue by performing more detailed analyses of countries' workforces as they develop. They document that technological progress has heterogeneous effects on workers, favouring disproportionately more skilled workers (Caselli and Coleman (2006),Jones (2014), Caselli (2005a), Rossi (2017), Malmberg (2018)).

In this paper, I take a different approach. Motivated by the central empirical fact that as countries develop, their occupational structures shift systematically, with employment moving away from routine-intensive jobs and toward occupations that rely more heavily on cognitive tasks, I study the implications of this shift for human capital accumulation and, ultimately, for cross-country income differences.

Using harmonized microdata from IPUMS International (see Ruggles et al. (2024)) on occupations, educational attainment, labor income, and returns to education and experience, I document two robust patterns. First, workers in cognitive occupations attain higher levels of education at all stages of development. Second, these occupations also yield higher returns to both education and experience, pointing to greater skill intensity. Taken together, these facts suggest that the reallocation of employment across occupations is an important channel through which development shapes the level and composition of human capital.

To do so, I build a model of endogenous human capital accumulation and occupational choice, where workers optimally choose their human capital investment profiles and career paths, conditional on their skill endowment and time constraints.

In the model, workers make forward-looking career and occupational choices that go beyond static trade-offs. Early in life, they can accumulate human capital through formal schooling, which requires foregoing current earnings and paying fixed educational costs. The benefit is a higher stock of human capital that raises future wages over the remaining lifetime. These dynamic gains from schooling can be further enhanced by investing in human capital on the job, as workers allocate part of their time to training during their working phase.

An immediate implication is that the developmental shift toward cognitive occupations amplifies cross-country differences in human capital. Since these jobs combine higher schooling, steeper experience-earnings profiles, and greater scope for on-the-job training, economies where a larger share of workers are employed in cognitive occupations accumulate human capital more rapidly over the life cycle. Conversely, when employment is concentrated in routine jobs, the dynamic returns to education and experience are muted, limiting the contribution of human capital to aggregate income.

To shed light on the quantitative importance of the model mechanisms described, I perform a combination of calibration and structural estimation to match salient data moments on labor earnings for countries at two different development stages: Brazil and the United States (US). These two countries provide an ideal environment for studying the main model mechanisms quantitatively for a few reasons. Firstly, the availability of harmonized microdata allows for the computation of data moments that are key to disciplining the model. Second, there is a clear shift in the occupational structure between these two countries. In 2010, 15.1 percent of workers in Brazil were in cognitive occupations, while the US had a share that was twice as high, at 36.2 percent. Third, these countries show a clear gap in educational attainment. In 2010, the average years of schooling were approximately 8.5 years in Brazil and 12.6 years in the US.

Having calibrated the model to match the salient features of the data, I perform a series of quantitative exercises that allow me to understand the importance of the shift in the occupational structure in shaping human capital accumulation patterns and labor market outcomes.

In the first exercise, I use the model to compute human capital stocks by occupation and in the aggregate. The results show that, while average human capital in the United States is only about 10 percent higher than in Brazil, the gap widens substantially within occupations: in routine jobs, U.S. workers hold nearly three times as much human capital as their Brazilian counterparts.

This suggests that while the occupational shift compositionally raises aggregate human capital, an equally important factor lies in the transformation of routine work itself: routine occupations in the United States embody substantially more human capital than their counterparts in Brazil. To quantify the contribution of these channels to cross-country gaps, I perform a simple decomposition. The results indicate that about 25 percent of the human capital gap between Brazil and the United States is attributable to differences in occupational structure. A further 30 percent reflects the higher measured human capital stock in U.S. routine occupations, while the remaining gap is accounted for by differences in the relative efficiency of cognitive jobs (24 percent) and in cognitive skills themselves (20 percent).

These findings suggest that biased technological shifts across countries, by reshaping occupational structures, are a major driver of cross-country human capital gaps.

Related Literature. From a broad point of view, this paper is related to the literature that measures the contribution of human capital to development. Earlier contributors are Mankiw et al. (1992), Klenow and Rodriguez-Clare (1997), and Hall and Jones (1999). More recently, Erosa et al. (2010), Manuelli and Seshadri (2014), Jones (2014), and Hanushek and Woessmann (2015) find human capital to be an important factor in explaining disparities in wealth levels between countries.

My paper is also related to the literature studying skill acquisition over the life-cycle. The seminal work of Becker (1964) on human capital theory laid the foundation for understanding the role of education and training in determining wage profiles and career progression. Building on this, Heckman et al. (2006) discuss the importance of both formal schooling and non-cognitive skills in shaping long-term labor market outcomes. Acemoglu and Autor (2011b) further contribute to this literature by analysing how technological change has increased the returns to cognitive skills, particularly in advanced economies.

This paper contributes to the strand of the economic development literature that studies how technological differences have heterogeneous impacts on different groups of workers as countries develop. An early contribution is by Caselli and Coleman (2006), who propose an aggregate technology framework to unveil cross-country skilled-biased gaps in technological efficiency. Caselli (2016) updates and expands this study to a broader set of countries and other factors besides labor, but still based on an aggregate technology approach. More recently, Malmberg (2018) proposes a novel approach to estimate the relative efficiency of skilled and unskilled labor based on disaggregated trade and industry data. Another recent contribution is given by Rossi (2017), who compares labor market outcomes of immigrants with different levels of educational attainment to identify differences in the relative efficiency and the relative quality of skilled and unskilled labor. The main difference between my work and these studies is that I propose a model-based method to estimate human capital stocks and technological efficiencies, allowing them to flexibly vary across countries. In addition, and except for the case of Malmberg (2018), I use occupational attainment instead of educational attainment data to identify qualitative differences between workers.

My paper is also related to a large body of literature that finds evidence of skilled-biased technical change across time and within countries. Katz and Autor (1999) provide a comprehensive survey of this literature, that includes Katz and Murphy (1992), Acemoglu (1998), Autor et al. (1998)}, Acemoglu (2002).

The remainder of the paper is organized as follows. Section 2 presents descriptive evidence on shifts in the occupational structure of employment and their relationship with human capital accumulation. Section 3 presents the theoretical framework and characterizes the equilibrium. Section 4 reports the quantitative analysis. Section 5 concludes.

2 Empirical Evidence

In this section I document a series of motivating facts that are useful to understand how the occupational structure shapes labor market and human capital outcomes. To that end, I use microdata from IPUMS International (IPUMS-I; see Ruggles et al. (2024)) to compute a series of summary statistics characterizing labor market and human capital accumulation outcomes by workers' occupations. To capture how these outcomes evolve along the development process, I merge these data with a GDP per capita [1]from Penn World Table 10.1 (PWT; see Feenstra et al. (2015)). The result is a series of summary statistics by country and year, coupled with the corresponding GDP per capita for each country.

A central element of my analysis is that the task intensity of occupations provides a stylized but general characterization of how the occupational structure, labor market, and human capital outcomes vary along the process of development.[2] To that end, I use harmonized data on worker's main occupation[3] and group them into two broad groups: cognitive-intensive and routine-intensive occupations (cognitive and routine, from now on).[4]

Cognitive occupations are those in which worker's more intensively perform abstract, non-routine cognitive tasks, while in routine jobs usually entail routine cognitive, routine manual, and non-routine manual tasks.

[1]GDP per capita is computed by calculating the ratio of *cgdpo* (Output-side real GDP at current PPPs (in mil. 2017US$)) and *pop* (Population (in millions)) in PWT 10.1.

[2]This approach is also taken by other studies focusing on cross-country human capital differences. For example, Caselli and Coleman (2006) split workers into high- and low-skill according to their level of educational attainment, with the underlying idea that higher levels of education allow workers to perform different tasks. I, instead, focus on the tasks that workers perform more intensively *directly,* showing that skill acquisition serves as a vehicle for augmenting worker's productivity when performing tasks.

[3]I use variable *OCCISCO*, which records the person's primary occupation coded according to the major, one-digit categories, in the International Standard Classification of Occupations (ISCO) scheme for 1998. Cognitive occupations include *OCCISCO* codes 01-Legislators,senior officials and managers, 02-Professionals, and 03-Technicians and associate professionals. I discard workers in *OCCISCO* codes 10-Armed forces, 11-Other occupations, unspecified or n.e.c.,and 97-Response suppressed.

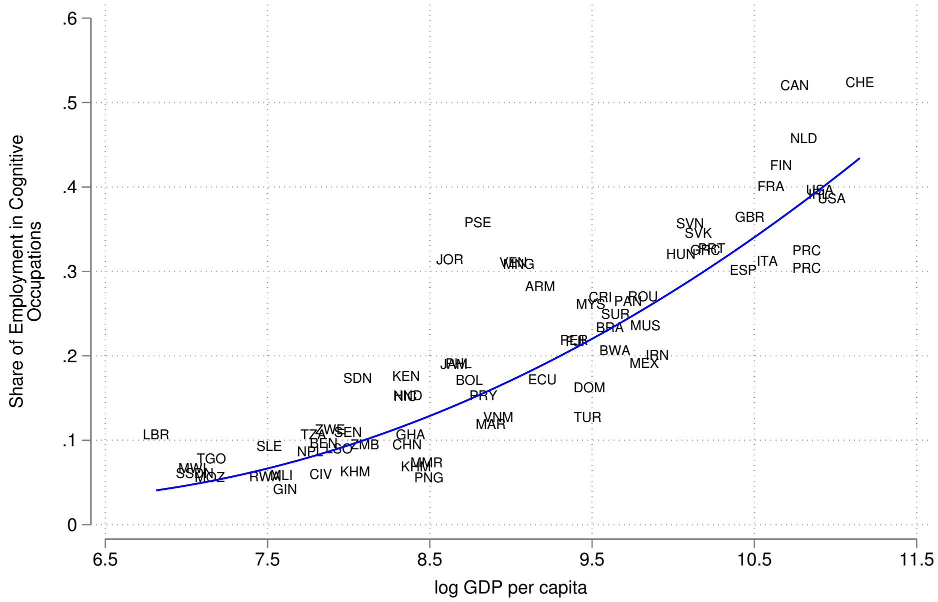
[4]Acemoglu and Autor (2011b) provide a clear characterization of the occupational task intensity.

With the summary statistics on labor market outcomes by broad occupation in hand, I document a series of facts that show the implications of the impact that the shift in the occupational structure has on labor market and human capital accumulation outcomes. I first show that as countries develop, a larger share of their workforce is employed in cognitive intensive occupations, with routine employment loosing importance. Additionally, I provide evidence that workers attain, on average, higher levels of education in cognitive occupations all along the development spectrum. Finally, focusing on a subset of countries that have available data, I show that returns to education are higher in cognitive occupations at three different levels of development. At the same time, cognitive intensive occupations also exhibit steeper experience-earnings profiles. These last two facts together suggest that the higher levels of educational attainment observed in cognitive occupations do not come at the cost of lower human capital accumulation returns.

2.1 The Occupational Structure Across Development.

I start by documenting that there is a substantial shift in the occupational structure alongside the development process. To do so, I compute the share of employment in cognitive occupations by country and plot them against their corresponding level of GDP per capita.The results are plotted in Figure A.1 below.

Figure 1: Share of Employment in Cognitive-Intensive Occupations Across Development



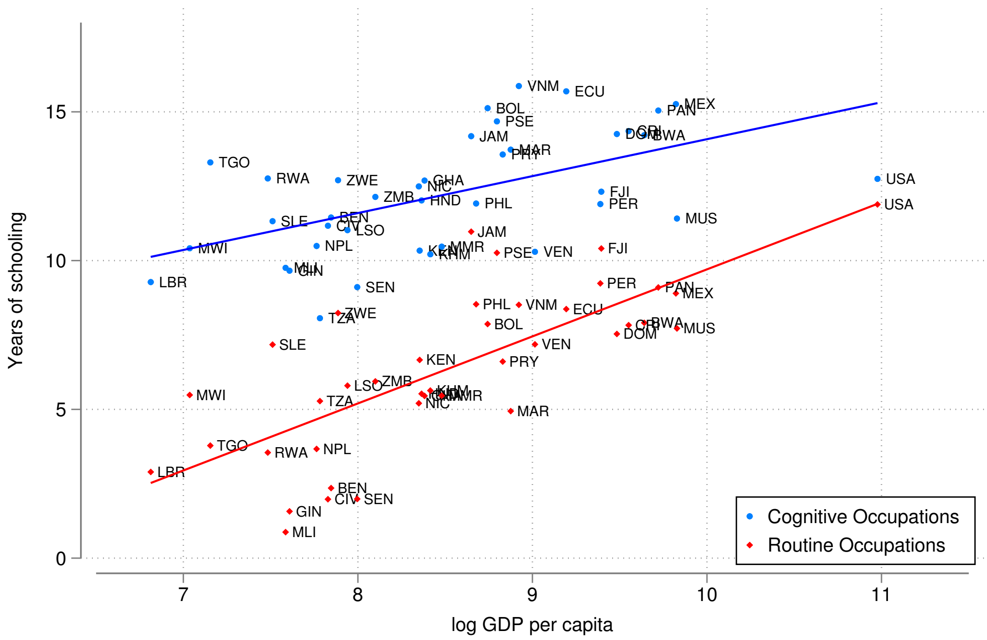
Note: *Cognitive occupations include one digit ISCO codes 01-Legislators, senior officials and managers, 02-Professionals, and 03-Technicians and associate professionals. Workers are weighted using IPUMS-I perwt. See Appendix D for a description of the countries in the sample.*

The message in Figure A.1 is clear: as countries develop and grow richer, there is a major change in the occupational structure. To be precise, while in the poorest countries of my sample only around ten percent of workers are in cognitive occupations, the share of employment in such jobs increases to over forty percent at the top of the GDP per capita distribution in my sample. In what follows, I show that this occupational shift is also associated with substantial differences in human capital accumulation.

2.2 The Occupational Structure and Educational Attainment.

I now proceed to show that cognitive and routine occupations differ in their educational intensity at all levels of development. To do so, I compute average years of schooling [5] by country and for each broad occupational group. The results are presented on Figure 1 below.

Figure 2: Educational Attainment by Occupation Across Development



Note: *Average years of schooling by broad occupational group are computed using the harmonized IPUMS-I variable* ***yrschool****. See Appendix D for further details.*

A few comments are in order. First, years of schooling increase with GDP per capita, independently of the occupation workers are in. This is a well-known fact in the literature. A somewhat less well-known fact is that this educational deepening holds for both occupations, exhibiting an even more pronounced trend in routine jobs. For instance,

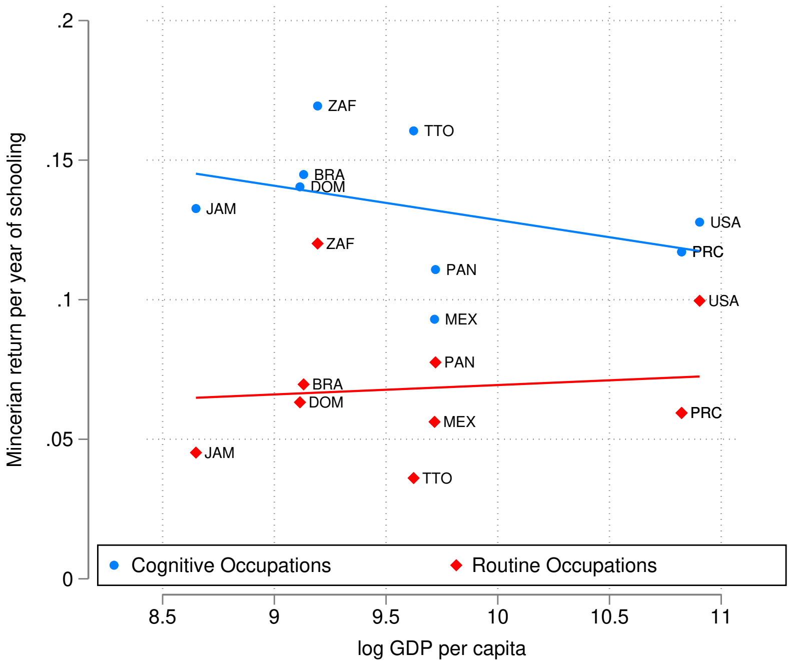
[5] I calculate average years of schooling using IPUMS-I variable *yrschool*. This variable indicates the highest grade/level of formal schooling the person had completed, in years, ranging from 0 to18.

while average years of schooling in cognitive occupations increase from 10.6 to 13.5 as we move from the first to the fourth quartile of GDP per capita, in routine occupations we see a rise from 3.6 to 8.9 years on average, respectively. Second, workers attain higher levels of education in cognitive-intensive occupations at all levels of development. The gap is stark. While average years of schooling in routine-intensive occupations is 6.4, this figure almost doubles to 12.4 in cognitive-intensive occupations. In what follows I provide evidence that this higher human capital intensity in cognitive work, at least in terms of years of schooling, does not come at the cost of lower returns to human capital accumulation.

2.3 Mincerian Returns to Education by Occupation.

I begin the analysis of human capital accumulation by examining returns to education. Specifically, I estimate Mincerian returns per additional year of schooling for the subset of countries with data of sufficient quality to allow such calculations.[6]

Figure 3: Returns to Education



Note: *Returns to schooling are estimated from Mincer regressions of log hourly earnings on education and a quadratic in experience with interactions. The sample is smaller than in Figure A.1 because it requires observed earnings, education, and hours worked. Further details are provided in Appendix D for further details.*

[6]See Appendix D for details on data requirements and methodology.

Figure 3 reports the results. Although many low-income countries are dropped from the sample due to data requirements, the estimates reveal a consistent pattern: in every country, returns to schooling are higher in cognitive than in routine occupations, independent of the level of development.

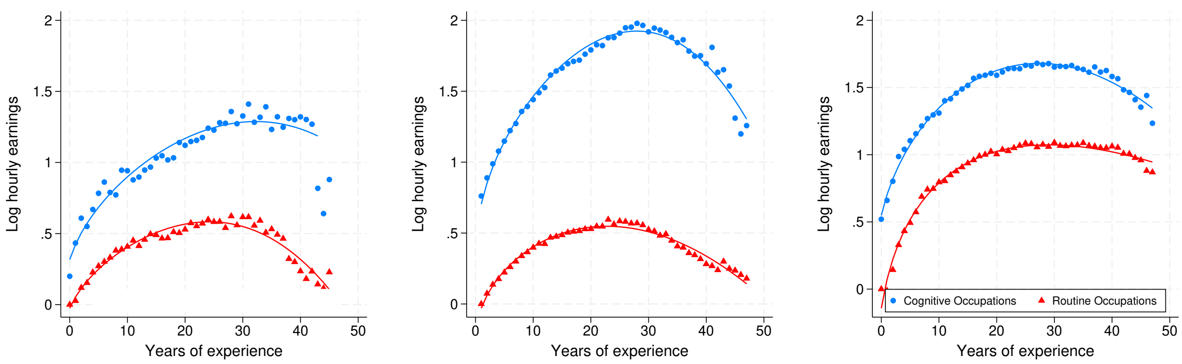
Together with Figure 1, these findings establish that higher educational attainment in cognitive occupations is accompanied by higher marginal returns to schooling.

2.4 Experience-Earnings Profiles.

To complement the evidence on returns to schooling, I now examine experience-earnings profiles across occupations. For each country and year, I compute log hourly earnings for workers at different experience levels. [7] Earnings are normalized by subtracting the log earnings for workers with no experience in routine occupations. This normalization sets a common reference point: the value at one year of experience for cognitive occupations reflects the earnings gap relative to routine occupations upon entry, while the slope of each profile captures differences in earnings growth over the life cycle.

Figure 4: Experience Earnings Profiles for a Selected Group of Countries

(a) Indonesia **(b)** Brazil **(c)** USA



Note: *Experience–earnings profiles are synthetic cohorts, obtained as weighted means of log hourly earnings by potential experience and occupation. U.S. profiles use the 2010 IPUMS-I cross-section, Brazil 2000, and Indonesia 1995. Blue circles denote cognitive occupations, red triangles routine. A fractional polynomial fit is overlaid. See Appendix D for data handling and profile construction.*

Figure 4 presents results for three countries representing low-, middle-, and high-income economies: Indonesia, Brazil, and the United States. In addition, the profiles are steeper

[7] Potential experience is calculated following the methodology in Lagakos et al. (2018): for workers with secondary complete or more, as $age - yearsofschooling - 6$; for workers with less than secondary complete, as $age - 18$. Because age varies in the cross section, these profiles correspond to synthetic cohorts.

in cognitive occupations in the three countries. Between entry and the peak of the age–earning profile, earnings in cognitive occupations grow by approximately 96 log points in Indonesia, 154 log points in Brazil, and 114 log points in the United States, compared to 61, 73, and 100 log points, respectively, in routine occupations.

These results show that workers in cognitive-intensive occupations not only start at higher earnings levels but also experience faster earnings growth over the life cycle, consistent across countries at different stages of development.

Taken together, these patterns highlight systematic differences in educational attainment, returns to schooling, and experience-earnings profiles across occupations. They point to the central role of cognitive-intensive work in shaping human capital accumulation along the development path. These empirical regularities provide the motivation for the model developed in the next section, which formalizes the interaction between occupational choice, human capital investment, and aggregate outcomes.

3 Model

Environment. Time is continuous and denoted by t. The economy is populated by a continuum of agents of mass one, indexed by i, who vary in their age $a(i)$ and innate ability $\boldsymbol{z}(i)$. The active life cycle of individuals begins at age a_0 and ends at age A, when they retire from the labor market. In each period, agents are endowed with one unit of time, which they decide how to allocate between work and human capital accumulation. If they work, agents must choose between one of two occupations, which can be cognitive or routine intensive. Before turning a_0, agents draw a pair of occupation-specific innate abilities $\boldsymbol{z}(i) = \{z_c(i), z_r(i)\}$ from the joint skill distribution $g(z_c, z_r)$, where $z_c(i)$ and $z_r(i)$ denote the endowment of skills of the agent i in cognitive and routine occupations, respectively.[8] Let $f_t(a)$ represent the measure of workers of age a in period t, such that $\int_{a=a_0}^{A} f_t(a)da = 1$.

Let $w_j(t)$ denote the wage per unit of human capital in calendar period t, and assume that wages grow exponentially at rate g_j. For simplicity, we express wages as a function of **worker age** rather than calendar time. Then, at any age $a \geq a_0$, the wage per unit of human capital is given by $w_j(a) = w_j(a_0)\, e^{g_j(a-a_0)}$. However, given the exponential nature of wages $w_j(a_0)$ can be pinned down by knowing the age of the worker in period t. For instance, for a worker who is of age $\tilde{a}$ in period t, the initial wage is $w_j(a_0) = w_j(t)\, e^{-g_j(\tilde{a}-a_0)}$.

There is no uncertainty, and agents fully know at age a_0 all the prices and technologies they face during their life cycles. This allows them to make an optimal career decision

[8] Notice that at any given time t, the pair $\{t, a\}$ is enough to characterize a cohort.

at age a_0, which entails choosing an occupation together with path for time allocation between work and human capital accumulation. There is only one final good in the economy, which is assumed to be the numeraire. All markets are competitive and capital markets are perfect.

Representative Firm. There is a single representative firm in the economy, which produces the final good and operates in competitive output and input markets. The firm has access to the following technology

$$Y(t) = A(t)K(t)^{\alpha}H(t)^{(1-\alpha)}, \tag{1}$$

where $Y(t)$ is gross output per capita, $A(t)$ captures total factor productivity, $K(t)$ is the capital stock per capita, α is the elasticity of output with respect to capital, and $H(t)$ is the following CES human capital aggregator, also measured in per capita terms

$$H(t) = \left[(A_c(t)H_c(t))^{\left(\frac{\sigma-1}{\sigma}\right)} + (A_r(t)H_r(t))^{\left(\frac{\sigma-1}{\sigma}\right)} \right]^{\left(\frac{\sigma}{\sigma-1}\right)}. \tag{2}$$

In Equation 2, $H_c(t)$,$H_r(t)$ are aggregate human capital stocks in the cognitive and the routine occupation, respectively, both measured in per capita terms.

Worker's Life Cycle. Workers can accumulate human capital through two distinct channels: formal education and on-the-job training. Human capital accumulation follows a two-stage process. In the first stage, individuals may invest in formal education, allocating a fraction $i^s(a) \in [0,1]$ of their time at age a to schooling. The second stage begins once work becomes feasible, at which point workers can invest a fraction $i^o(a) \in [0,1]$ of their working time in on-the-job training, with the remainder $l(a)$ devoted to production.

The working stage can only start after age a_w. Schooling, however, may be extended beyond a_w if workers find it optimal to do so. Within the schooling stage, workers choose how much time to allocate to human capital accumulation through $i^s(a)$. Beforea_w, neither work nor on-the-job training are possible ($l(a) = 0$ and $i^o(a) = 0$ for all $a < a_w$).[9]

This makes the termination of the schooling stage an endogenous decision. Let a^s denote the age at which schooling ends. The effective entry age into the labor market is then $\hat{a} = \max\{a_w, a^s\}$. If workers choose to leave school before a_w, they cannot accumulate additional human capital, and their existing stock depreciates until they reach the minimum working age.

Workers face a fundamental trade-off between current and future earnings. Time spent

[9]This assumption aligns the model with the standard approach to computing experience in the literature on age–earnings profiles; see, for example, Lagakos et al. (2018).

in education or on-the-job training reduces contemporaneous income and, in the case of schooling, also entails a direct cost. However, these investments increase skill levels and thus raise future productivity. In making such decisions, workers choose not only how much to invest at each age, but also when to complete formal schooling and enter the labor market. The timing and intensity of investment reflect a balance between the immediate costs of human capital accumulation and the long-term gains in earnings.

Human Capital Accumulation Technologies. I assume that human capital accumulation technologies vary by occupation and stage, allowing for enough flexibility so that returns to human capital accumulation are occupation and stage-specific. Dropping worker's indices to ease notation, during the schooling stage the human capital accumulation technologies available are

$$h_j(\dot{a}, z_j) = \eta_j^s q^s i_j^s(a, h_j) h_j(a, z_j) - \delta h_j(a, z_j), \tag{3}$$

where $j \in \{c, r\}$ is and occupation index, $h_j(a, z_j)$ is the human capital for a worker of age a and innate ability in occupation j given by z_j, $i_j^s(a, h_j) \in [0, 1]$ the fraction of time allocated to human capital accumulation via schooling for such worker. In addition, η_j^s measures the gross return per period of schooling, δ the human capital depreciation rate, and $q^s > 0$ is a schooling quality parameter. [10]

Workers can also accumulate human capital through on-the-job training in their working stage. In this case, the technologies available are

$$h_j(a, \dot{y}_j^s, z_j) = \eta_j^o h_j(a, y_j^s, z_j)^{\gamma_j} i_j^o(a, h_j)^{\alpha_j} - \delta h_j(a, y_j^s, z_j) \tag{4}$$

for $j \in \{c, r\}$. In this case, $h_j(a, y_j^s, z_j)$ is the occupation-j specific human capital for a worker of age a, innate ability z_j, and years of schooling y_j^s. In turn, $i_j^o(a, h_j) \in [0, 1]$ is the share of time allocated to on-the-job training, with α_j regulating its returns, and γ_j being a dynamic complementarity parameter that captures the extent to which current human capital affects investment via on-the-job training, and η_j^o measures the gross return on the job of training per unit of effective time spent.

Note that human capital accumulation technologies are rich enough to accommodate differential returns to investment in schooling η_j^s and on-the-job training (η_j^o, α_j) between occupations. I also allow for different degrees of dynamic complementarity (γ_j), capturing the potential for past investment decisions to have differential effects on current human capital accumulation across occupations.

[10] Notice that the gross returns to schooling are allowed to vary by occupation, while the quality of schooling is common across occupations.

Earnings. Let $w_j(t)$ represent the wage per unit of human capital in occupation $j \in \{c, r\}$ and period t. Assume that wages grow exponentially at rate g_j, such that $w_j(t) = w_j(0)e^{g_j t}$. Let $a_j^s(z_j)$ be the optimal age at which a worker with ability z_j finishes schooling in occupation j. Net earnings for a worker of age a and innate ability z_j in occupation j are stage dependent and given by

$$E_j\left(t, a, h_j(a, z_j)\right) = \begin{cases} w_j(t)h_j(a, z_j)\left(1 - i_j^s(a, h_j)\right) - \mathbb{1}\left\{i_j^s(a, h_j) > 0\right\}c & \text{if } a \in \left[a_0, a_j^s(z_j)\right] \\ \\ w_j(t)h_j(a, z_j)\left(1 - i_j^o(a, h_j)\right) & \text{if } a \in \left(a_j^s(z_j), A\right]. \end{cases} \quad (5)$$

Earnings are stage-dependent. During the schooling phase, for $a \in \left[a_0, a_j^s(z_j)\right]$, labor income is increasing in the wage per unit of human capita and the worker's human capital, and decreasing in the fraction of time allocated to human capital accumulation $i_j^s(a, h_j)$. In addition, individuals face a fixed cost c per period of specialization, but only if they spend time at school (i.e. $i_j^s(a, h_j) > 0$). After school completion and until retirement, for$a \in \left(a_j^*(z_j), A\right]$, workers allocate time between productive work and on-the-job training. Net earnings in this stage equal $w_j(t)h_j(a, z_j)\left(1 - i_j^o(a, h_j)\right)$,reflecting labor income adjusted for the time spent in on-the-job training.

This formulation highlights the core trade-off that workers face: human capital accumulation, either via education or on-the-job training, lowers current earnings via foregone earnings and the costs associated to schooling, but increases future earnings through higher human capital.

Occupational Choice. Workers can only work in one of two occupations: cognitive or routine. Human capital accumulation is occupation-specific and cannot be transferred across occupations. On-the-job training occurs only within the chosen occupation. I assume that switching occupations is not allowed, so that each worker commits to a single occupation for the entire working life. [11]

Worker's Problem.

Having laid out the model's environment, I now proceed to describe the worker's problem in further detail.

[11] The model abstracts from occupational switching. This is consistent with the empirical literature estimating experience–earnings profiles by occupation using cross-sectional data, where experience is inferred and job tenure is typically unobserved.

General Workers' Problem.

Consider the problem of a worker endowed with a vector of innate abilities $\boldsymbol{z} = \{z_c(i), z_r(i)\}$ and human capital $\boldsymbol{h}(\boldsymbol{z}(i), a) = \{h_c(z_c(i), a), h_r(z_r(i), a)\}$. Dropping individual subscripts to ease notation, her goal is

$$\underset{\{c(a,\boldsymbol{h}),b(a,\boldsymbol{h})\}_{a=a_0}^{A}}{max} \left\{ \int_{a=a_0}^{A} e^{-\rho a} u\left(c(a, \boldsymbol{h}(z, a))\right) da \right\} \tag{6}$$

subject to

$$c(a, \boldsymbol{h}) + \dot{b}(a, \boldsymbol{h}) = E(a, \boldsymbol{h}) + rb(a, \boldsymbol{h})$$

where $c(a, \boldsymbol{h})$ is consumption, $b(a, \boldsymbol{h})$ assets, r is the real interest rate, and $E(a, \boldsymbol{h})$ represent earnings at age a for an individual with human capital $\boldsymbol{h}$.

Solution to the Worker's Problem. Given perfect capital markets, and since human capital accumulation decisions do not affect the worker's utility, problem 3 can be separated into three sub-problems: an inter-temporal utility maximization problem, where the individual chooses an optimal sequence for consumption and assets taken (optimal) income as given, an occupational choice problem, where the worker picks the career that yields maximum lifetime income, and a lifetime income maximization problem, where the optimal human capital accumulation path is chosen in order to maximize lifetime earnings for a given occupation.

I start by describing and characterizing the worker's optimal human capital investment problem, to then, with the optimal earnings by occupation in hand, present the occupational choice problem.

Optimal Human Capital Investment. I start by describing the worker's optimal human capital accumulation path, conditional on working in occupation j. Notice that this entails optimally choosing a sequence of time spent in human capital investment during schooling $\left\{i_j^{s\star}(a, h_j)\right\}_{a=a_0}^{a_j^{s\star}(a,h_j)}$, a sequence of time spent accumulating human capital via on-the-job training $\left\{i_j^{o\star}(a, h_j)\right\}_{a_j^{s\star}(a,h_j)}^{A}$, and an optimal schooling stopping age $a_j^{s\star}(h_j)$. In the following, I first describe how the optimal sequences for investment in on-the-job training and schooling are chosen, fixing $a_j^{s\star}(h_j)$. Having characterized these choices, I then proceed to show how the optimal schooling period is chosen.

Worker's Problem During the Working Period. Consider a worker with innate ability z_j who finished school at age $a_j^s(h_j)$,having accumulated $y_j^s(h_j) = a_j^s(h_j) - a_0$ years of education. Her problem is to choose a sequence of time spent acquiring human capital via on-the-job training (OJT), given by $i^o_j(a, h_j)$,and time allocated to work, given by $1 - i^o_j(a, h_j)$, to maximize the continuation value of earnings during her working phase. Formally, denoting by $V^o_j(h_j, y^s_j)$ the maximum net present discounted value of earnings for a worker with human capital h_j and y^s_j years of schooling, the worker's problem is

$$V^o_j(h_j, y^s_j) = \max_{\{i^o_j(a,z_j)\}^A_{a=a^s_j(h_j)}} \left\{ \int_{a=a_0+y^s_j}^{A} e^{-\rho(a-a_0)} w_j(a) h_j(a, z_j, y^s_j) \left(1 - i^o_j(a, h_j)\right) da \right\}$$

subject to

$$\begin{aligned} \dot{h_j}(a, y^s_j, z_j) &= \eta^o_j h_j(a, y^s_j, z_j)^{\gamma_j} i^o_j(a, h_j)^{\alpha_j} - \delta h_j(a, y^s_j, z_j), \\ i^o_j(a, h_j) &\in [0, 1]. \end{aligned}$$

Worker's Problem During the Schooling Phase. Keeping the focus on occupation j, consider the problem of a worker with innate ability $z_j$during the formal education phase. She chooses an optimal human capital investment policy $i^s_j(a, h_j)$ and a terminal age $a^{s*}_j(h_j)$ to maximize the net present discounted value of lifetime earnings. To be precise, her problem is

$$\begin{aligned} V_j(z_j) = \max_{\{i^s_j(a,h_j)\}^{a^{s*}_j(h_j)}_{a=a_0}, a^{s*}_j(h_j)} \Bigg\{ & \int_{a=a_0}^{a^*_j(z_j)} e^{-\rho(a-a_0)} \Big(w_j(a) h_j(a, z_j)(1 - i^s_j(a, h_j)) - c_s \Big) da \\ & + e^{-\rho(a^*_j(z_j)-a_0)} V^o_j\left(z_j, a^*_j(z_j)\right) \Bigg\} \end{aligned} \tag{7}$$

subject to the human capital accumulation technology,

$$\dot{h_j}(a, z_j) = \eta^s_j q^s h_j(a, z_j) i^s_j(a, h_j) - \delta h_j(a, z_j),$$

the time use constraint,

$$i^s_j(a, z_j) \in [0, 1],$$

and a^s_j given by

$$a^s_j(z_j) = \sup \left\{ a : i^s_j(a, z_j) > 0 \right\}.$$

Notice that ending schooling later in life is associated with higher forgone earnings and the payment of the fixed schooling cost, with the potential benefit of increasing the labor market entry level of earnings, which in turn happen later in life and are discounted more heavily.

Proposition 1. *Suppose that $\eta_j^s q^s > (\rho+\delta) > g_j$, and , and that individuals cannot engage in formal work and on-the-job training until age a_w. The solution to the worker's lifetime income maximization problem for a given occupation j is given by*

1. *During the schooling period, for $a \in \left[a_0, a_j^*(z_j)\right]$,a human capital investment policy such that*

$$i_j^s(a) = \begin{cases} 1 & \text{if} \quad \mu_j^s(a)\eta_j^s q^s \geq e^{-(\rho-g_j)(a-a_0)} w_j(a_0), \\ 0 & \text{otherwise,} \end{cases}$$

The shadow value of human capital $\mu_j^s(a)$ during this stage is

$$\mu_j^s(a) = \mu_j^o(a_j^*) e^{\left(\eta_j^s q^s - (\delta+\rho)\right)(a_j^* - a)},$$

and human capital evolves according to

$$h_j^s(a, z_j) = z_j e^{\left(\eta_j^s q^s - \delta\right)(a-a_0)}.$$

$$h^s(y^s, z) = z e^{(\eta^s q^s - \delta) y^s}$$

2. *During the working stage, for $a \in \left(a_j^*(z_j), A\right]$,the human capital accumulation policy given is*

$$i_j^o(a) = \begin{cases} \left(\frac{1}{h_j(a)}\right)\left(\frac{(\alpha_j \eta_j^o)\left(1-e^{-(\rho+\delta-g_j)(A-a)}\right)}{(\rho+\delta-g_j)}\right)^{\left(\frac{1}{1-\alpha_j}\right)} & \text{if } \alpha_j = \beta_j \\ h_j(a)^{-\left(\frac{1-\beta_j}{1-\alpha_j}\right)}\left(\frac{\mu_j^o(a)(\alpha_j \eta_j^o) e^{(\rho+\delta-g_j)(a-a_0)}}{w_j(a_0)}\right)^{\left(\frac{1}{1-\alpha_j}\right)} & \text{if } \alpha_j \neq \beta_j \end{cases},$$

In the general case of $\alpha_j \neq \beta_j$, the shadow value of human capital $\mu_j^o(a)$ solves the following non-linear ODE

$$\dot{\mu_j^o}(a) = (\rho+\delta)\mu_j^o(a) - e^{-(\rho-g_j)(a-a_0)} w_j(a_0) + \mu_j^o(a)\left((\alpha_j - \gamma_j)\eta_j^o h_j(a)^{(\gamma_j - 1)} i_j^o(a)^{\alpha_j}\right),$$

while in the particular case of $\alpha_j = \beta_j$, it is given by

$$\mu_j^o(a) = \frac{w_j(a_0) e^{-(\rho-g_j)(a-a_0)}\left(1 - e^{-(\rho+\delta-g_j)(A-a)}\right)}{(\rho+\delta-g_j)}$$

$$\mu_j^s(a) = \mu_j^o(a_j^*)e^{\left(\eta_j^s q^s-(\delta+\rho)\right)(a_j^*-a)}.$$

3. A school completion age, $a_j^{s\star}(z_j)$*,that is pinned down by the following smooth-pasting condition*

$$w_j(a_j^{s\star})h_j(a_j^{s\star})\left(1-i_j^o(a_j^{s\star},h_j)\right)+c = \mu_j^o(a_j^{s\star})h_j(a_j^{s\star})\left(\eta_j^s q^s - \eta_j^o h_j(a_j^{s\star})^{(\gamma_j-1)} i_j^o(a_j^{s\star},h_j)^{\alpha_j}\right).$$

4. An initial working age, given by

$$a_j^{w\star}(z_j) = max\left\{a_w, a_j^{s\star}\right\}.$$

Proof. See Appendix C. □

Occupational Choice.

At the beginning of their career, workers must choose an occupation $j \in \{c, r\}$. This decision is made once and is irreversible. The choice is determined by comparing the net present discounted value (NPDV) of lifetime earnings in each occupation, as defined in Equation 7. Formally, a worker of age $\tilde{a}$ observed in period t, with ability pair$\boldsymbol{z} = (z_c, z_r)$, made her occupational decision when she was age a_0 in period $\tau = t - (\tilde{a} - a_0)$. At that time she solved

$$d_c^\star(\boldsymbol{z}, \tau) \;=\; \underset{d_c\in\{0,1\}}{\arg\max}\left\{d_c(\boldsymbol{z},\tau)V_c(z_c,\tau) + \left(1-d_c(z,\tau)\right)V_r(z_r,\tau)\right\}, \tag{8}$$

where $V_j(z_j;\tau)$ denotes the net present discounted value of earnings in occupation j at wage schedule prevailing from period τ onward. Because wages grow exponentially, we can equivalently express the decision in terms of the worker's current age $\tilde{a}$ and the wage per efficiency unit at time t:

$$\begin{aligned} d_c^\star(\boldsymbol{z},t,\tilde{a}) = \underset{d_c\in\{0,1\}}{\arg\max}\Big\{ & \, d_c(\boldsymbol{z},t,\tilde{a})\, w_c(t)\, e^{-g_c(\tilde{a}-a_0)}\, \hat{V}_c(z_c,t,\tilde{a}) \\ & + \left(1-d_c(\boldsymbol{z},t,\tilde{a})\right) w_r(t)\, e^{-g_r(\tilde{a}-a_0)}\, \hat{V}_r(z_r,t,\tilde{a})\Big\}. \end{aligned} \tag{9}$$

where $\hat{V}_j(z_j,t,\tilde{a})$ collects the human-capital-specific component of lifetime earnings.

Proposition 2. ***Occupational Choice.*** *Suppose that the wage per unit of human capital in occupation j grows exponentially at rate g_j. A worker of age $\tilde{a}$in period t chooses occupation c (i.e. $d_c^\star(\boldsymbol{z},t,\tilde{a}) = 1$) if and only if*

$$\frac{w_c(t)}{w_r(t)} > e^{(g_c-g_r)(\tilde{a}-a_0)} \frac{\hat{V}_r(z_r,\tilde{a})}{\hat{V}_c(z_c,\tilde{a})}.$$

Proof. See Appendix C.3. □

This result highlights how the dynamic setting modifies the standard Roy framework. First, the exponential term $e^{(g_c-g_r)(\tilde{a}-a_0)}$ adjusts for the fact that the wages prevailing at the time of occupational choice were lower, the higher the growth rate g_j. Thus, a higher relative growth rate g_c raises the required wage ratio $\frac{w_c(t)}{w_r(t)}$ to make cognitive occupations attractive. Second, the relative importance of human capital accumulation is explicit: occupations with steeper life-cycle skill growth ($\hat{V}_j$) may be chosen even under relatively lower prevailing wages.

Conditional on this choice, current earnings at age a are

$$E(a,\boldsymbol{h}) \;=\; d_c^\star(\boldsymbol{z},t,\tilde{a})\, E_c^\star(a,h_c) \;+\; \big(1-d_c^\star(\boldsymbol{z},t,\tilde{a})\big)\, E_r^\star(a,h_r), \tag{10}$$

where $E_j^\star(a,h_j)$ is the optimal age-a earnings in occupation $j \in \{c,r\}$ along the optimal occupation-specific human capital accumulation path.

Employment Measures. For each occupation $j \in \{c,r\}$, let $\ell_j(t,\tilde{a})$denote the fraction of individuals of age a in period t who are effectively at work in occupation j. Given individuals' optimal choices, this age-specific employment rate is

$$\ell_j(t,a) = \int_{z_c}\int_{z_r} d_c^\star\left(z_c,z_r;t,a\right)\, \mathbb{1}\left\{a \geq a_j^{w\star}(z_j)\right\}\, g(z_c,z_r)\, dz_r dz_c,$$

where $d_c^\star\left(z_c,z_r;t,a\right)$is the cohort (t,a) occupational choice rule in Equation 9 and $a_j^{w\star}(z_j)$ is the effective work-start age for ability z_j. Aggregate employment in occupation j at date t is then the population-weighted integral of the age-specific employment rate,

$$L_j(t) = \int_{a_0}^{A} f_t(a)\ell_j(t,a)da, \tag{11}$$

and total employment is $L(t) = L_c(t) + L_r(t)$. Finally, the share of employment in occupation j and period t is $\pi_j(t) = L_j(t)/L(t)$.

Inter-temporal Utility Maximization Problem. Having characterized the occupational choice in Equation 8 and the implied earnings decomposition in equation 10, we can now link these results back to the worker's intertemporal problem defined in Problem 6. In particular, the earnings term $E(a,\boldsymbol{h})$ that enters the budget constraint is fully determined by the optimal occupational decision and the associated earnings profile. With this in place, we can move directly to the characterization of the worker's intertemporal optimization problem. Assuming that preferences are CRRA and given by $u(c) = \frac{c^{1-\theta}}{1-\theta}$, the optimality conditions for the worker's problem are consist of the following Euler Equation

$$\frac{\dot{c}(a)}{c(a)} = \frac{1}{\theta}(r - \rho),$$

together with the transversality condition

$$b(A) = 0.$$

Proof. See Appendix C.4. □

Firm's Problem.

The representative firm operates the aggregate technology in Equation 1, under human capital aggregator 2. Assuming that the capital stock per capita is fixed, the firm hires human capital in cognitive ($H_c(t)$) and routine ($H_r(t)$) occupations, taken wages per unit of human capital $w_c(t)$ and $w_r(t)$ as given, in order to maximize profits.

The solution to the firm's problem is characterized by inverse effective labor demands

$$w_c(t) = \left(\frac{H(t)}{H_c(t)}\right)^{(1/\sigma)} A_c(t)^{\left(\frac{\sigma-1}{\sigma}\right)}, \tag{12}$$

and

$$w_r(t) = \left(\frac{H(t)}{H_r(t)}\right)^{(1/\sigma)} A_r(t)^{\left(\frac{\sigma-1}{\sigma}\right)}. \tag{13}$$

Taking the ratio of both yields the relative wage

$$\frac{w_c(t)}{w_r(t)} = \left(\frac{H_c(t)}{H_r(t)}\right)^{-(1/\sigma)} \left(\frac{A_c(t)}{A_r(t)}\right)^{\left(\frac{\sigma-1}{\sigma}\right)}. \tag{14}$$

Competitive Equilibrium.

Definition. *Given the state of technology $\{A(t), A_c(t), A_r(t)\}$,the joint distribution of innate abilities $g(z_c, z_r)$, the demographic structure $f_t(a)$, wage growth rates $\{g_c, g_r\}$, and human capital accumulation costs c, and the real interest rate r, a competitive equilibrium is given by wages per unit of human capital $\{w_c(t), w_r(t)\}$ such that*

- *For a worker of age $\tilde{a}$ in period t and innate ability draw $\{z_c, z_r\}$*

– *the human capital accumulation policies via schooling and on-the-job training* $\left\{i_j^{s\star}(a,z_j),\, i_j^{o\star}(a,z_j)\right\}_{a=a_0}^{A}$ *and the optimal schooling stopping age* $a_j^{s\star}(z_j)$ *maximize* $V_j(z_j)$ *in Equation 7 taking* $\{w_c(t), w_r(t)\}$ *and* c *as given.*

– *an occupational choice rule* $d_c^\star(\boldsymbol{z}; t, \tilde{a})$ *that solves the worker's lifetime income maximization problem in Equation 9, taking* $\{w_c(t), w_r(t)\}$*and* c *as given.*

– *a resulting optimal human capital path* $\{d_c^\star(\boldsymbol{z}; t, \tilde{a}) \cdot h_c^\star(a, z_c) + (1 - d_c^\star(\boldsymbol{z}; t, \tilde{a})) \cdot h_r^\star(a, z_r)\}_{a=a_0}^{A}$.

– *the sequence for consumption and assets* $\{c(a, \boldsymbol{h}), b(a, h)\}_{a=a_0}^{A}$ *solves Problem 6 taking* r *as given, and with* $E(a, \boldsymbol{h})$ *being the solution to Equation 10.*

- *For the representative firm operating the technology in Equation 1, an optimal quantity of human capital in cognitive* $h_c(t)$ *and routine* $h_r(t)$ *occupations, such that profits are maximized, taking* $w_c(t)$ *and* $w_r(t)$ *as given.*

All markets clear:

- *Goods market:*

$$Y(t) = \int_{\tilde{a}} \int_{z_c} \int_{z_r} c\left(\tilde{a}, h_c(z_c), h_r(z_r)\right) g(z_c, z_r) f_t(\tilde{a}) dz_r dz_c d\tilde{a}.$$

- *Asset market:*

$$\int_{\tilde{a}} \int_{z_c} \int_{z_r} b\left(\tilde{a}, h_c(z_c), h_r(z_r)\right) g(z_c, z_r) f_t(\tilde{a}) dz_r dz_c d\tilde{a} = 0.$$

- *Effective labor market: for* $j \in \{c, r\}$

$$H_j(t) = \int_{a_0}^{A} f_t(a) \int_{z_c}\int_{z_r} d_j^\star(\boldsymbol{z}; t, a)\, \mathbb{1}\{a \geq a_j^{w\star}(z_j)\}\, h_j(a, z_j)\, (1 - i_j^{o\star}(a, z_j))\, g(z_c, z_r)\, dz_r\, dz_c\, da \tag{15}$$

Even though the computation of the equilibrium since complex at first, all it requires is finding a fixed-point on $w_c(t)$ and $w_r(t)$. For relative effective labor demand, which pins down $\pi_c(t)$, knowing the relative wages $\frac{w_c(t)}{w_r(t)}$ provides enough information. The same holds for the optimal decision rules $d_c^\star(\boldsymbol{z}; t, \tilde{a})$. However, in general, human capital accumulation decisions also depend on the *level* of wages. But from the ideal price index

$$p(t) = \frac{1}{A(t)} \left[\left(\frac{w_c(t)}{A_c(t)}\right)^{(1-\sigma)} + \left(\frac{w_r(t)}{A_r(t)}\right)^{(1-\sigma)} \right]^{\left(\frac{1}{1-\sigma}\right)},$$

and since $p(t) = 1$,we obtain that

$$w_r(t) = \frac{A(t)}{\left[\left(\frac{w_c(t)}{w_r(t)}\frac{1}{A_c(t)}\right)^{(1-\sigma)} + \left(\frac{1}{A_r(t)}\right)^{(1-\sigma)}\right]^{\left(\frac{1}{1-\sigma}\right)}}.$$

It follows that solving for a fixed-point in relative wages $\frac{w_c(t)}{w_r(t)}$is enough to fully compute the general equilibrium of the economy.

4 Quantitative Analysis

Quantitative Model

To better map the baseline model to the data and improve its quantitative performance, I introduce some modifications to the baseline model. First, workers' innate abilities in the cognitive and routine dimensions, z_c and z_r, are drawn from a joint lognormal distribution:

$$(z_c, z_r) \sim \text{LogNormal}\left(\boldsymbol{\mu}_z, \boldsymbol{\Sigma}_z\right),$$

where $\boldsymbol{\mu}_z$ is the vector of location parameters and $\boldsymbol{\Sigma}_z$ the covariance matrix.

Second, I discretize schooling into four stages,

$$s \in \mathscr{S} = \{ns, p, sec, u\},$$

representing *no schooling*, *primary complete*, *secondary complete*, and *university complete*. Workers choose s to maximize expected lifetime utility in Equation 7, balancing costs and future earnings.

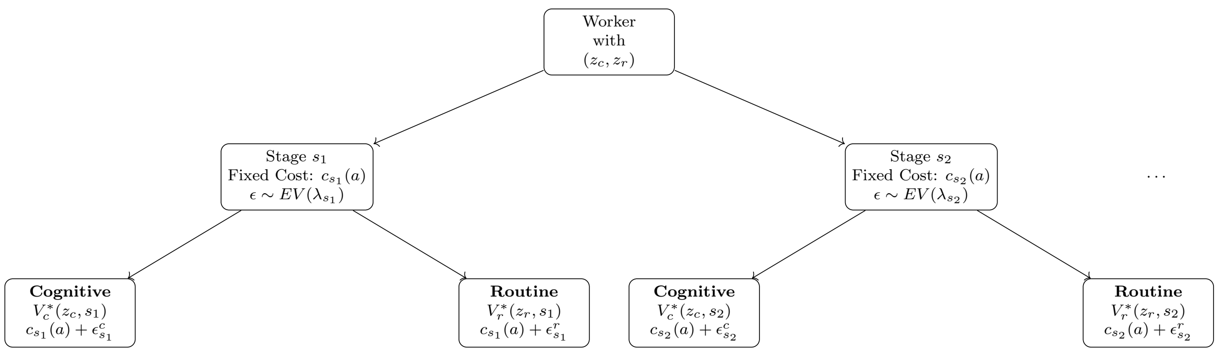
Third, I assume that the minimum starting working age a_w coincides with the age at which a worker finishes secondary education, so that beyond this stage she chooses between extending her schooling period and going to university or starting her working career.

Fourth, I assume that the per-period fixed cost of schooling c_s varies by stage. To be precise, let $c_s(a)$ denote the cost of schooling at age a, associated with each terminal age of the schooling stage. The cost structure across ages is

$$c_s(a) = \begin{cases} 0 & a = a_0 \quad \text{(no schooling)} \\ c_p & a \in (a_0, a_p] \quad \text{(primary school)} \\ c_{sec} & a \in (a_p, a_{sec}] \quad \text{(secondary school)} \\ c_u & a \in (a_{sec}, a_u] \quad \text{(university)} \end{cases}$$

In addition, I introduce an occupational mobility cost such that switching occupations entails paying a fixed cost C in units of the final good. Finally, on top of the average stage-specific fixed costs of education, each worker draws at age a_0a sequence of per-period idiosyncratic cost shocks ϵ_s^j from a generalized extreme value (GEV) distribution, with stage-specific nesting parameter λ_s (i.e. $\epsilon_s^j \sim GEV(\lambda_s)$). In this context, λ_s governs the correlation of cost shock within a nest, with $\lambda_s = 1$ implying that cost shocks are independent, while $\lambda_s \to 0$ leading to highly correlated shocks. Notice that idiosyncratic costs between nests are still independent. This cost structure generates a nested logit model for the joint education–occupation choice problem. To illustrate the decision framework, Figure 5 below provides a summary for a simplified version of the decision menu with two educational attainments.

Figure 5: Nested Logit Model for Occupational and Educational Choice.



Note: T*he diagram illustrates the nested logit structure for a worker endowed witht abilities* $\{z_c, z_r\}$. $V_j^*(z_j, s_j)$ *captures the optimal present discounted value of earnings for a workers with ability* z_j *who has computed stage* s_j; c_{s_j} *is the per-period schooling fixed cost, and* $\epsilon_{s_j}^j$ *the worker's idiosincratic cost shock in occupaiton* j *and stage* s_j.

The nested logit structure implies that education–occupation choices are no longer deterministic. Idiosyncratic cost shocks introduce randomness, so workers with identical observable characteristics may select different paths. This yields smooth aggregate choice patterns and generates probabilities of choosing each education–occupation combination, with the nested structure capturing correlation in unobserved costs within education stages. To define the choice probabilities, first notice that the net present discounted value of earnings for a worker of ability z_j conditional on finishing formal education at age a_s can be written as

$$V_j\left(z_j|\, a_s\right) = \bar{V}_j^o(z_j, a_s) + C_s(a_s) + \bar{\epsilon}_s^j,$$

where $\bar{V}_j^o(z_j|\, a_s)$ is the present discounted continuation value, $C_s(a_s)$ the present discounted sum if fixed costs until a_s, and $\bar{\epsilon}_s^j$ the present discounted sum of idiosyncratic costs until stage s for such worker. [12]

The conditional probability of choosing occupation $j \in \{c, r\}$ given stage s is

$$P\left(j \mid s, z\right) = \frac{exp\big(\big(\bar{V}_j^o(z_j, a_s) + C_s(a_s)\big) / \lambda_s\big)}{\sum_{k\in\{c,r\}} exp\big(\big(\bar{V}_k^o(z_k, a_s) + C_s(a_s)\big) / \lambda_s\big)}.$$

The inclusive value for nest (stage) s is

$$IV_s(z) = \lambda_s ln\left(\sum_{k\in\{c,r\}} exp\big(\big(\bar{V}_k^o(z_k, a_s) + C_s(a_s)\big) / \lambda_s\big)\right).$$

The probability of choosing stage s is

$$P\left(s \mid z\right) = \frac{exp\left(IV_s(z)\right)}{\sum_{s'\in\mathscr{S}} exp\left(IV_{s'}(z)\right)}.$$

With these objects in hand, we can compute the joint probability of choosing stage s and occupation j as

$$P\left(s, j \mid z\right) = P\left(s \mid z\right) \cdot P\left(j \mid s, z\right). \tag{16}$$

Calibration and Structural Estimation

With the aim of using the quantitative model to study cross-country differences in human capital and their connection to the occupational shifts documented in Section 2.1, I make the model operational by calibrating and estimating its parameters. The remainder of this section outlines the calibration and estimation strategy.

Besides a set of country-invariant and country-specific parameters—described in detail in under External Calibration below—the model can be fully mapped to a country by specifying its state of technology $\{A(t), A_c(t), A_r(t)\}$, the joint distribution of innate skills $g(z_c, z_r)$, and the demographic structure $f_t(a)$.

Mapping the model to a specific country is data intensive, as it requires estimating a thirty five. To identify these parameters, I rely on a strategy that requires at least two cross-sections of data per country covering a broad set of variables, including labor income, occupational choices, hours worked, educational attainment, and demographic

[12]See Appendix C.5 for a proof.

characteristics of workers. For this purpose, I use harmonized microdata from IPUMS-I. From the pool of countries available, I focus on Brazil as a representative middle-income economy and the United States as a high-income benchmark. Ideally, the analysis would also include a low-income country, but this is precluded by the stringent data requirements.[13]

Data. I calibrate the model to match key empirical moments related to the occupational structure, labor income, educational attainment, returns to education, and experience–earnings profiles, all disaggregated by occupation for each country. To this end, I use two cross-sections of harmonized microdata from IPUMS-I: 2010 and 2020 for the United States, and 2000 and 2010 for Brazil. The data provide information on hours worked, labor income, educational attainment and years of schooling, occupation, as well as demographic characteristics and labor force attachment. A key issue concerns the mapping of occupations in the data to the two broad categories in the model. IPUMS-I provides a harmonized variable covering eleven one-digit ISCO occupational groups, which I map into the two broad categories of the model using task-intensity measures from Acemoglu and Autor (2011a). Specifically, I assign ISCO codes 01 (Legislators, senior officials and managers), 02 (Professionals), and 03 (Technicians and associate professionals) to the cognitive group, and codes 04–09 (Clerks; Service workers and shop and market sales; Skilled agricultural and fishery workers; Crafts and related trades workers; Plant and machine operators and assemblers; and Elementary occupations) to the routine group. I discard individuals in the Armed Forces (10), unspecified occupations (11), and those with no response or unknown occupations (97–99).

Table 1 below presents the correspondence between the model's occupational categories and the ISCO one-digit occupations in the IPUMS-I data.[14]

[13] See Appendix D for details on data requirements and the list of countries that satisfy them.

[14] See the Appendix Dfor a description of the variables used to compute the data moments.

Table 1: Mapping of ISCO Occupations to Model Categories

One-Digit ISCO Code	ISCO Description	Model Category	Cognitive Intensity
01	Legislators, senior officials and managers	Cognitive	High
02	Professionals	Cognitive	Very High
03	Technicians and associate professionals	Cognitive	Medium
04	Clerks	Routine	Low
05	Service workers and shop and market sales	Routine	Low
06	Skilled agricultural and fishery workers	Routine	Low
07	Crafts and related trades workers	Routine	Low
08	Plant and machine operators and assemblers	Routine	Low
09	Elementary occupations	Routine	Low

Note: *The table maps ISCO one-digit occupational codes to the two broad categories in the model: Cognitive and Routine. Cognitive intensity is a qualitative measure (Very High, High, Medium, Low, Very Low) based on the task content of occupations, derived from Acemoglu and Autor (2011) using O*NET data. Occupations excluded from the analysis are the Armed Forces (ISCO 10), Unspecified occupations (ISCO 11), and observations with no response or unknown occupations (ISCO 97–99). The mapping uses the variable* ***OCCISCO*** *from the IPUMS-I harmonized microdata.*

Parameter Description. The model contains a total of thirty parameters, which can be divided between worker-specific and firm-specific parameters. On the worker side, I identify fifteen parameters related to skills and human capital accumulation. Five parameters govern the distribution of innate skills, summarized as

$$\boldsymbol{\mu} = \begin{bmatrix} \mu_c \\ \mu_r \end{bmatrix}, \quad \boldsymbol{\Sigma} = \begin{bmatrix} \sigma_c^2 & \sigma_{cr} \\ \sigma_{cr} & \sigma_r^2 \end{bmatrix},$$

where μ_c and μ_r are the mean cognitive and routine skills, σ_c^2 and σ_r^2 are their variances, and σ_{cr} is the covariance across skills.

Ten parameters describe the process of human capital accumulation. These include the gross returns per year of schooling, η_c^s and η_r^s, the schooling quality parameter q^s, the returns to on-the-job training η_c^o and η_r^o, the occupation-specific productivity parameters α_c and α_r, the dynamic complementarity parameters γ_c and γ_r, and the human capital depreciation parameter δ.

Given the discretization of educational attainment into four stages, I also calibrate four per-period schooling costs, $c_s = \{c_{ns}, c_p, c_{sec}, c_u\}$ and four parameters governing the correlation of idiosyncratic costs within a nest, $\lambda_s = \{\lambda_{ns}, \lambda_p, \lambda_{sec}, \lambda_u\}$.

On the firm side, five parameters must be identified: the elasticity of substitution between human capital types σ, the capital intensity parameter β, and the technology parameters A, A_c, and A_r. Finally, I need to recover the time discount factor ρ.

Under certain identification assumptions, which I describe in further detail below, the estimation procedure also allows me to recover values for the wage per unit of human capital for each country and in each occupation (w_c, w_r), together with their corresponding growth rates.

In the subsequent sections, we distinguish between parameters that are externally calibrated, either based on previous literature or auxiliary data, and those that are estimated structurally using country-specific moments.

External Calibration. I start by describing the set of parameters that are externally calibrated and are common across countries. I assume that the time discount rate ρ is 0.025. On the workers' side, I assume innate ability draws are independent and set $\sigma_{cr} = 0$. I also fix $\mu_c = \mu_r = 0$, since these parameters affect earnings levels and cannot be separately identified from wages. Furthermore, I assume that human capital depreciation is $\delta = 0.05$ for all technologies and in both countries. On the aggregate technology side, the elasticity of substitution between labor services provided by occupations is set to $\sigma = 0.60$. [15]

Structural Estimation. The remaining parameters are structurally estimated via Simulated Methods of Moments (SMM). In what follows, I describe the procedure followed together with the data moments used. I start

Returns to Education η_j^s. To estimate the parameters governing the returns to schooling in each occupation, I map the model to the data by linking the theoretical expression for human capital and earnings to empirically observable moments. In the model, human capital for a worker with y^s years of schooling and innate ability z_j in occupation j is

$$h_j^s(y^s, z_j) = z_j \, e^{(\eta_j^s q^s - \delta) y^s},$$

which implies that earnings at the end of the schooling period, assuming no work experience, are

$$E_j(t, y^s, z_j) = w_j(t) h_j^s(y^s, z_j(i)).$$

Taking the expectation conditional on the choice of occupation j yields

$$\mathbb{E}\left[\ln E_j \mid y^s, d_j^\star(\boldsymbol{z}, t, \tilde{a}) = 1\right] = \ln w_j(t) + (\eta_j^s q^s - \delta)\, y^s + \mathbb{E}\left[\ln z_j \mid y^s, d_j^\star(\boldsymbol{z}, t, \tilde{a}) = 1\right].$$

Therefore, running Mincer regressions of log earnings on years of schooling, controlling for experience and other demographic characteristics, provides an empirical counterpart

[15] See Subsection 4.1.1 for a detailed discussion about this choice and its implications for cross-country technology gaps.

to the conditional expectation in the model, with the slope identifying the *net-quality-adjusted* occupation-specific return to schooling $(\eta_j^s q^s - \delta)$. I describe in detail in Appendix D how I clean and process the IPUMS-I data for each country to construct the variables necessary to run these regressions. Notice that the intercept jointly identifies the wage per unit of human capital and the average innate ability for workers who choose occupation j $\ln w_j(t) + \mathbb{E}\left[\ln E_j \mid y^s, d_j^\star(\boldsymbol{z}, t, \tilde{a}) = 1\right]$, highlighting the need of an identification assumption for individual identification.

School Quality Parameter q^s. I normalize $q^s_{\text{US}} = 1$ and identify Brazil's q^s relative to the U.S. by relying on international standardized test scores, following the approach in Caselli (2005b).

I use international test scores as a proxy for formal skill acquisition per year, together with an external elasticity of wages with respect to test scores of $\phi = 0.20$ per 100 points (i.e., a 1-point increase raises wages by 0.002.

Let TS_c denote the average of math and science scores for country c. With $TS_{US} = 483.5$ and $TS_{BRA} = 386$, the relative quality factor that scales the gross return to schooling maps as

$$q^s_c \;\approx\; 1 + \phi \cdot \frac{TS_c - TS_{US}}{100}.$$

This gives $q^s_{\text{BRA}} \approx 1 + 0.20 \times \frac{386 - 483.5}{100} = 0.804$.

Thus, if Brazil had U.S.-level schooling quality, the gross return to schooling η_j^s would be about $1/0.804 - 1 \approx 24.2\%$ higher (roughly, one effective year is lost every four scholing years).

Wage per Unit of Human Capital in Routine Occupations $w_r(t)$. Having identified the quality adjusted gross returns per year of schooling, I proceed to estimate a series of additional moments parameters by running a simulated method of moments.

The process is sequential, and starts by identifying the wage per unit of human capital in routine occupations, $w_r(t)$. To do so, I use as data moment average log hourly earnings of workers with completed secondary education (12 years of schooling) and no experience. In the model, the conditional expectation of log earnings for such a worker is given by Equation 4. Evaluating this expression at $y^s = 12$ and imposing the occupation choice $d_r^\star(\boldsymbol{z}, t, \tilde{a}) = 1$, we obtain

$$\mathbb{E}[\ln E_r \mid y^s = 12,\, d_r^\star(\boldsymbol{z}, t, \tilde{a}) = 1] - (\eta_r^s q^s - \delta)\, 12 \;=\; \ln w_r(t) + \mathbb{E}[\ln z_r \mid y^s = 12,\, d_r^\star(\boldsymbol{z}, t, \tilde{a}) = 1]\,. \tag{17}$$

Under the identification assumption that $\mathbb{E}\big[\ln z_j \mid y^s = 12, d_j^\star(\boldsymbol{z}, t, \tilde{a}) = 1\big] = 0$,this expression allows me to identify $\ln w_j(t)$, the wage per unit of human capital in occupation j. The construction of the empirical counterpart for $\mathbb{E}\big[\ln E_j \mid y^s = 12, d_j^\star(\boldsymbol{z}, t, \tilde{a}) = 1\big]$, is outlined in Appendix D, under section average log hourly earnings.

Wage Growth Rate in Routine Occupations $g_r(t)$. To identify the wage growth rate in routine occupations g_r, I rely on the procedure described above to compute the wage per unit of human capital$w_r(t)$ in two distinct cross-sections of data: 2010 and 2020 for the United States, and 2000 and 2010 for Brazil. Assuming that wages grow exponentially at a constant rate, I recover g_r as the exponential growth rate that matches the observed change in $w_r(t)$ over the ten-year interval. This moment requires access to two harmonized cross-sections per country, thereby increasing the data demands of the whole estimation procedure.

Returns to Human Capital Accumulation via OJT $(\eta_j^o, \alpha_j, \gamma_j)$. To identify the parameters governing on-the-job human capital accumulation, I rely on experience–earnings profiles computed for synthetic cohorts in the data.[16] The returns to on-the-job training (η_c^o, η_r^o), the occupation-specific productivity parameters (α_c, α_r), and the dynamic complementarity parameters (γ_c, γ_r) are estimated using a Simulated Method of Moments (SMM). The procedure matches simulated and observed age-earnings profiles for workers between ages 18 and 65, conditional on completing a given schooling. To separately identify the effect of dynamic complementarities, I simulate profiles for two distinct levels of schooling.[17] The choice of education levels differs across occupations to ensure sufficient sample sizes: secondary and university in cognitive occupations, and primary and secondary in routine occupations.

Relative Wages $w_c(t)/w_r(t)$, Wage Growth Rate in Cognitive Occupations $g_c(t)$, Skill Distribution Variances (σ_c, σ_r) and Schooling Costs (c_s). The final

[16] Accurately measuring potential experience requires reliable data on years of schooling, measured by *yrsschool* in IPUMS-I. For that reason, I use the cross section of 2010 for the United States and 2000 for Brazil; see Appendix D for a description of how these data moments are computed.

[17] To better understand the role that human capital at the end of the schooling period has on investment in on-the-job training, see Equation C.6.

step of the estimation procedure jointly identifies relative wages $w_c(t)/w_r(t)$,wage growth rate $g_c$in cognitive occupations,the variances of the skill distribution (σ_c, σ_r), the educational attainment costs c_s, and the correlation parameters of idiosyncratic shocks within nests λ_s. The procedure is as follows. I begin by guessing values for (σ_c, σ_r) and drawing a large sample of innate ability pairs $\{z_c, z_r\}$ from the implied joint distribution. For each simulated worker, I compute the net present discounted value of lifetime earnings conditional on completing each of the four schooling levels in each occupation.

Given candidate values for relative wages, the schooling costs and correlation parameters, I then simulate individual joint choice probabilities using the nested logit structure described in Equation 16. The resulting share of workers in cognitive occupations and educational attainment shares are compared with their empirical counterparts. For identification, I normalize the cost of no schooling to zero ($c_{ns} = 0$) and set the correlation parameter in that nest to one ($\lambda_{ns} = 1$). The estimated relative wages, educational costs, and correlation parameters are those that minimize the distance between the simulated shares in the model and their corresponding data counterparts. Notice that both relative wages and costs are consistent with the equilibrium occupational and educational attainment structures observed in the data.

As in the case of the wage per unit of human capital in routine occupations, I repeat this procedure for a different cross sections of data, ten years apart. Already knowing $w_r(t)$ for these two cross sections, the resulting pair of relative wages $w_c(t)/w_r(t)$, allow me to recover $w_c(t)$ in these two periods, and its corresponding exponential growth rate $g_c(t)$.

Once the relative wages, the costs and the correlation parameters are identified, I recompute the logit choice probabilities and use them to characterize the distribution of innate abilities across educational attainment groups. Average abilities by group and occupation are scaled relative to those of workers with completed secondary education in routine occupations, in line with the wage normalization introduced earlier. I then simulate expected earnings for each attainment level, aggregate across groups using the model-implied probabilities, and compute the standard deviation of earnings in each occupation. The parameters (σ_c, σ_r) are finally adjusted until the model reproduces the observed standard deviations of log earnings by occupation. [18]

Occupational Efficiency Parameters and Elasticity (A_c, A_r) and Elasticity of Substitution Between Occupations (σ). See Subsection 4.1.1 for a description of how these parameters are recovered and the corresponding implications for cross-country technology gaps.

[18]See Appendix D for a description on how these variances are computed.

Parameter Estimates and Model Fit.

Estimated Parameters. This subsection presents the estimated parameters that underpin the quantitative results of the model. The estimates are organized into four groups for clarity. Table 2 reports the parameters related to wages and wage growth. Table 3 summarizes the parameters governing the distribution of innate ability. Third, Table 4 presents the estimated returns to schooling and the associated costs of education. Finally, Table 5 shows the parameters related to returns to on-the-job training. Together, these estimates provide the foundation for the model's calibration and subsequent analysis.

Table 2 presents the estimated wage levels and wage growth rates for cognitive and routine occupations in Brazil and the United States. A few comments are in order. First, wage levels are substantially higher in the United States: cognitive wages are about 4.5 times higher, and routine wages about 2.5 times higher than in Brazil. This already suggests that part of the observed differences in living standards between the two countries can be, at least in part, attributed to wage differentials.

Second, relative wage gaps reveal a marked cognitive bias. These shifts in the wage structure represents one of the mechanisms shaping occupational choices and the overall distribution of employment across development, as summarized by Proposition 2.

Finally, real wage growth rates are non-negligible in both countries, with wage dynamism being higher in Brazil. Although these growth rates do not directly affect the cross-sectional experience–earnings profiles—since all synthetic cohorts face the same wage per unit of human capital—they exert an important indirect influence. In particular, faster wage growth lowers the effective time discount factor, thereby encouraging workers to devote more time to human capital accumulation. [19]

Table 3 reports the estimated parameters governing the distribution of innate abilities across cognitive and routine occupations in Brazil and the United States. First, the means of the distributions are normalized to zero, since they cannot be separately identified from wage levels. Second, I assume that cognitive and routine skill draws are independent. Identifying the correlation parameter would require panel data, which is not available in this setting.[20]

Third, the reported dispersion parameters correspond to the standard deviations of the underlying normal distributions. The lognormal variances (in hat notation) are given by

$$\hat{\sigma}_j^2 = \left(e^{\sigma_j^2} - 1\right) e^{2\mu_j + \sigma_j^2},$$

[19]Equation C.10 provides a clear reference on this.
[20]See Heckman and Honoré (1990)

Table 2: Estimated Parameters
–Wages and Wage Growth–

Parameter	USA		Brazil	
	Occupation		Occupation	
	Cognitive	Routine	Cognitive	Routine
$w_c(t)/w_r(t)$	0.654		0.311	
$w_j(t)$	1.63	2.49	0.30	0.97
g_j	0.0139	0.0149	0.0262	0.0212

Note: *Wages in routine occupations are identified using log hourly earnings for 45-year-old males with completed secondary education and less than five years of experience. Relative wages are estimated via the SMM, jointly with the schooling cost parameters, in order to match the occupational and educational structure in each country. Wage growth rates are recovered following the same procedure, but using a cross-section observed ten years ahead in time. For completeness, the table also reports wage levels in cognitive occupations.*

$$\hat{\sigma}^2_{cc} = 0.477, \quad \hat{\sigma}^2_{rr} = 0.522 \quad \text{for the US,}$$

$$\hat{\sigma}^2_{cc} = 0.419, \quad \hat{\sigma}^2_{rr} = 0.502 \quad \text{for Brazil.}$$

In both countries, routine skills display higher dispersion than cognitive skills, and overall, innate abilities are more unequally distributed in the United States. Moreover, there is a clear increase in the dispersion of innate skills when moving from Brazil to the United States, consistent with the hypothesis that inequality in underlying abilities rises with development.

Finally, when comparing these distributions to the data (see Table 6 below), innate skills appear less unequally distributed than observed earnings.[21] This suggests that the mechanisms embedded in the model---such as human capital accumulation through formal schooling and on-the-job training, as well as occupational choice, and human capital accumulation frictions---amplify initial heterogeneity. This amplification is in contrast to some findings in the static, frictionless Roy model literature that states that the pursuit of comparateive advantage lowers overall wage dispersion relative to random assignment. [22]

Table 4 presents the estimated parameters for the quality-adjusted net returns to schooling and the associated schooling costs.

First, the quality-adjusted net returns to schooling---that is, the direct impact of an

[21] Table 6 reports the standard deviation of earnings. The corresponding variances for cognitive and routine occupations are 0.45 and 0.39 in the US, and 1.09 and 0.76 in Brazil.

[22] A seminal paper on the issue is Heckman and Honoré (1990), while Gould (2002) extends the result to a three-occupation Roy model.

Table 3: Estimated Parameters
-Innate Ability Distribution-

Parameter	USA		Brazil	
	Occupation		Occupation	
	Cognitive	Routine	Cognitive	Routine
σ_{cr}	0.000		0.000	
μ_j	0.000	0.000	0.000	0.000
σ_j^2	0.130	0.175	0.065	0.155

Note: *Means are normalized to zero, since they cannot be separately identified from wage levels. Cognitive and routine skill draws are assumed to be independent, as identifying their correlation would require panel data. The estimated variances correspond to the underlying Normal distribution. Their LogNormal counterparts can be obtained by applying the transformation* $\hat{\sigma}_j^2 = \left(e^{\sigma_j^2} - 1\right) e^{2\mu_j + \sigma_j^2}$*, where* $\hat{\sigma}_j^2$ *is the variance of the LogNormal distribution, and* μ_j *and* σ_j^2 *the parameters of the underlying Normal distribution.*

additional year of schooling on wages---are higher in cognitive than in routine occupations in both countries, as documented in Figure 3. Returns are also higher in Brazil than in the United States across both occupations.

Second, the quality of schooling parameter, whose estimation is described in detail in Subsection 4, implies that each year of schooling in Brazil is discounted by a factor of 1/0.805 when expressed in U.S. quality-equivalent units.

Third, schooling costs are measured on a per hour basis. To facilitate comparison, I express them as a share of earnings for a worker in the base group (routine occupation, 45-year-old male, completed secondary education and no experience). Interestingly, fixed schooling costs represent a higher share of earnings in the United States at all levels of education: primary and secondary schooling each account for more than 50 percent of hourly earnings, while university costs exceed 100 percent. Importantly, formal education requires full specialization, so forgone earnings must be added to these direct costs. An alternative way to see gauge the importance of schooling costs, is to measure relative to the earnings for the counterpart of the base group but in cognitive occupations. In this case, the picture is similar, with fixed costs representing a higher share of earnings in the US. However, the burden is somewhat smaller, with the cost of university falling to 0.93 in the US (-12 p.p.) and 0.43 in Brazil (-0.43 p.p.).

First, the estimated returns under full specialization and per unit of human capital, η_j^o, are slightly higher in the United States, as is the elasticity of time spent investing, α_j. Referring back to the optimal policy for $i_j^o(a)$ in Proposition 3, notice that the joint return to OJT investment is given by $\alpha_j \eta_j^o$, with α_j determining the curvature of earnings profiles. Thus, the higher values of $\alpha_j \eta_j^o$ estimated for the United States imply stronger incentives to invest in OJT, which translates into steeper age–earnings profiles.

Table 4: Estimated Parameters
-Returns to School and Schooling Costs-

Parameter	USA		Brazil	
	Occupation		Occupation	
	Cognitive	Routine	Cognitive	Routine
$q^s\eta_j^s - \delta$	0.118	0.067	0.146	0.074
q^s	1.000		0.805	
c_{ns}	0.00		0.00	
c_p	0.50		0.18	
c_{sec}	0.69		0.23	
c_u	1.05		0.63	

Note: *$q^s\eta_j^s - \delta$ captures net returns to schooling, which are quality-adjusted and net of human capital depreciation and measured as the effect of an additional year of schooling on wages. Schooling costs are expressed as a share of base-group earnings (routine occupation, 45-year-old males, secondary complete, no experience).*

Second, the dynamic complementarity parameter is also higher in the United States for both occupations. This finding is particularly relevant, since workers in the United States both attain higher levels of schooling and, as noted above, invest more in OJT. Routine occupations exhibit a higher degree of dynamic complementarity in both countries, making OJT outcomes more strongly dependent on previously accumulated human capital.

Table 5: Estimated Parameters
- Returns to On-the-Job Training-

Parameter	USA		Brazil	
	Occupation		Occupation	
	Cognitive	Routine	Cognitive	Routine
η_j^o	0.085	0.084	0.080	0.080
α_j	0.971	0.990	0.930	0.951
γ_j	1.016	1.031	1.000	1.016

Note: *OJT returns are estimated via the simulated method of moments (SMM), using as targets the experience–earnings profiles by occupation for two different levels of schooling. Specifically, I rely on workers with completed secondary and university education, except for routine occupations in Brazil, where I use data on workers with primary and secondary complete instead, due to the very low share of routine workers completing university.*

Targeted Data Moments. Table 6 summarizes the key data moments used in the structural estimation. These include average log hourly earnings in routine occupations, its corresponding growth rate, the standard deviation of log hourly earnings by occupation, and Mincerian returns to schooling. The share of employment in cognitive and routine occupations is also presented. To ensure comparability, all moments are computed

Table 6: Data Moments Used for Parameter Estimation

Moment	USA		Brazil	
	Occupation		Occupation	
	Cognitive	Routine	Cognitive	Routine
Avg. log Labor Income	-	1.982	-	0.853
Growth Rate in Labor Income	-	0.015	-	0.021
Employment Share	0.395	0.605	0.226	0.774
Stdv. log Labor Income	0.668	0.621	1.044	0.872
Mincerian School Return	0.118	0.067	0.146	0.074

Note: *All moments are for 45 year-old males using harmonized IPUMS-I and PWT data. Labor income measures are per hour and expressed at PPPs, measured in units of US GDP in 2010. Mincerian school return refers to the quality-adjusted net return per year of schooling. Data for the United States correspond to 2010 and for Brazil to 2000, as these are the years that contain information on all the variables required to compute the data moments. In particular, 2000 for Brazil and 2010 for the United States include information on years of schooling* ***yrschool****, which is necessary to estimate Mincerian returns and construct the experience variable. See Appendix D for a detailed explanation of how these moments are calculated.*

for 45-year-old male workers, expressed in PPP-adjusted units of U.S. 2010 GDP, and harmonized across countries using IPUMS-I and PWT data. This focus on 45-year-olds reflects a group that is well established in the labor market and thus provides a representative measure of the earnings and human capital structure at a given point in time.

A comparison of the two countries highlights several important contrasts. Average hourly labor income for workers with completed secondary education and less than five years of experience—our key moment to identify the wage per unit of human capital in routine occupations—is about three times higher in the United States. This gap is smaller than differences in broader measures of living standards, such as GDP per capita, which are roughly 5.5 to 6 times larger, reflecting that our measure captures only wage earners within a restricted group of workers. Earnings growth rates are steeper in Brazil, consistent with faster returns to labor market experience at earlier stages of development. The share of employment in cognitive occupations is nearly twice as high in the United States, mirroring the occupational shifts toward cognitive work that characterize the process of development (see Figure A.1). Earnings are also more dispersed in Brazil than in the United States across both occupational groups. Finally, Mincerian returns to schooling are higher in cognitive than in routine occupations in both countries, and their overall level is systematically higher in Brazil, in line with the evidence provided in Figure 3.

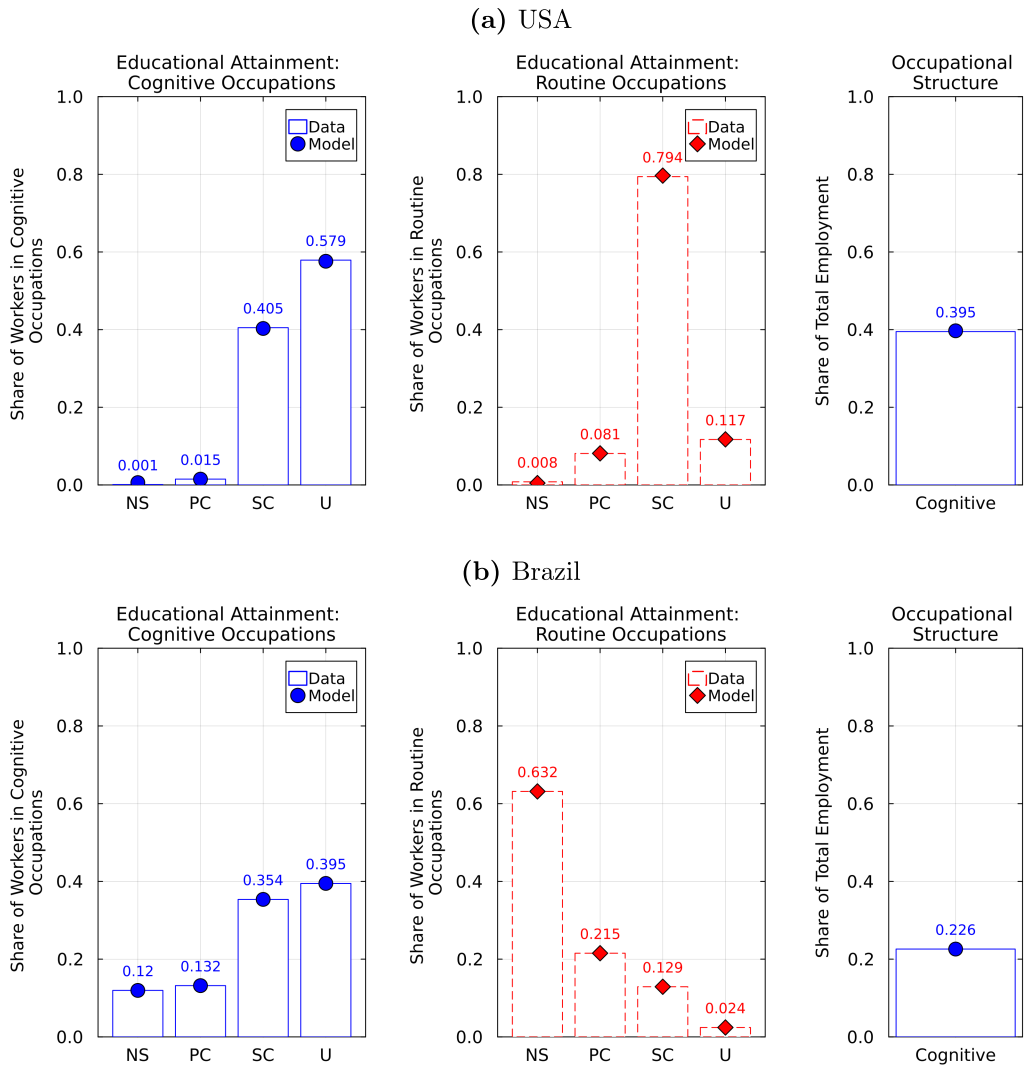
Table 6 does not exhaust the set of data moments used in the calibration. In what follows, I present the educational attainment structure by occupation and the experience–earnings profiles, which provide additional discipline for parameter identification.

Model Fit. Figure 6 presents the educational and occupational structure in Brazil and the US, comparing the data (solid bars for cognitive, dashed bars for routine occupations) and the model fit (circles for cognitive and diamonds for routine occupations).

The model perfectly reproduces both the occupational distribution and the observed educational attainment by occupation in both countries, underscoring how the inclusion of additional quantitative features described in Section 4 enables a precise match to the data.

Gaps in educational attainment are especially pronounced within routine occupations. While the US exhibits educational deepening in cognitive jobs, with the share of tertiary-educated workers rising sharply, the starkest difference lies in routine occupations. In Brazil, 63.2% of workers in routine occupations do not complete primary education (bar labeled "NS" in Brazil), whereas in the US nearly 80 percent of routine workers complete secondary schooling. This pattern aligns with the general rise in educational attainment that accompanies economic development—as shown in Figure 2—and is more pronounced in routine occupations.

Figure 6: Educational and Occupational Structure and Model Fit.



Note: *Educational attainment by occupation is computed for 45-year-old males using variable **yrschool** in IPUMS-I, for 2000 in Brazil and 2010 in the US. This cohort is chosen to ensure sufficient attachment to the labor force and to minimize sample selection issues arising from cross-country differences in women's labor force participation. Solid blue bars and red dashed bars depict the empirical data for cognitive and routine occupations, respectively, while blue circles and red diamonds denote the corresponding model fit. The educational categories are labelled as follows: **NS** (less than primary complete), **PC** (primary complete or more, but less than secondary complete), **SC** (secondary complete or more, but less than university complete), and **U** (university complete or more).*

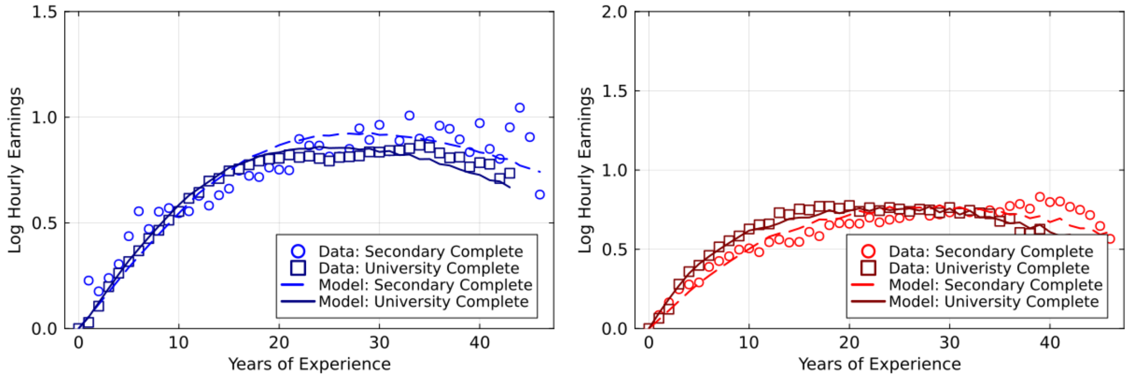
I now turn to the experience–earnings profiles that assess the model's ability to replicate life cycle human-capital accumulation. Figure 7 presents four normalized experience–earnings trajectories for synthetic cohorts in Brazil (2000) and the United States (2010), separately for cognitive and routine occupations and by two educational-attainment levels within each occupation. These dual education splits are crucial for identifying the dynamic complementarity parameter γ_j in each occupation. Earnings are normalized so that workers with zero years of experience earn zero in each country–occupation–

education cell, thereby isolating growth patterns over the life cycle.

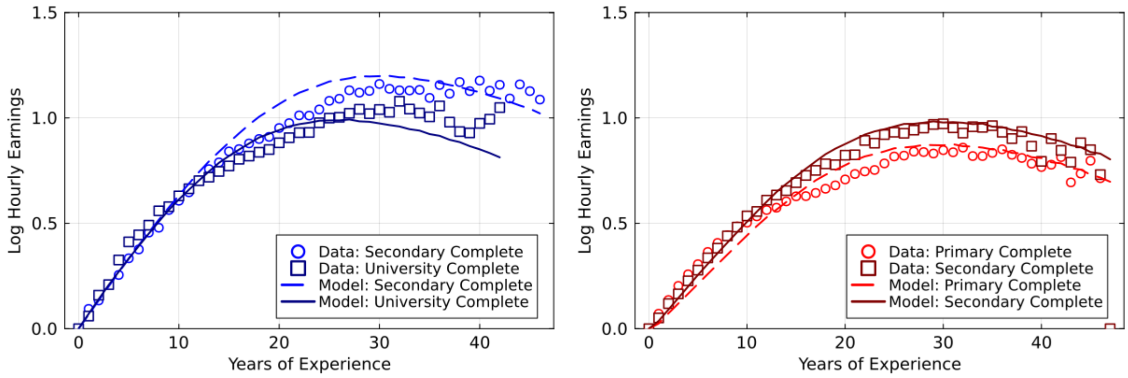
The calibrated model reproduces the observed profiles with remarkable precision across both countries and occupational categories, highlighting the importance of estimating country- and occupation-specific on-the-job training technology parameters. At the peak of the earnings curve—around twenty-five years of experience—the model attributes between 85 and 120 log-point gains in Brazil and between 75 and 100 log-point gains in the United States to human capital acquired on the job. Translated into schooling-equivalent years using each country's average return to education, these log-point increases correspond to approximately 9–13 additional years of schooling in Brazil and 7–9 additional years in the United States.

Figure 7: Experience Earnings Profiles-Data vs Model

(a) USA



(b) Brazil



Note: *Experience–earnings profiles by country, occupation, and educational attainment are computed for 45-year-old males. I use IPUMS-I data for 2000 in Brazil, and 2010 in the US beacuase information on years of schooling is available (variable* ***yrschool****). Potential experience is calculated following the methodology in Lagakos et al. (2018): for workers with secondary complete or more, as age − years of schooling − 6; for workers with less than secondary complete, as age − 18. Because age varies in the cross section, these profiles correspond to synthetic cohorts. Data are shown for workers with secondary complete and university complete, except in routine occupations in Brazil, where primary and secondary complete are used due to the low share of university-educated workers.*

Having described the calibration strategy, the data moments used, and the resulting model fit, I now turn to a series of quantitative exercises that assess how structural shifts in the occupational structure influence human capital accumulation and development.

4.1 Quantitative Exercises.

4.1.1 Occupational Choice in a Dynamic Setting.

I start by exploring quantitatively how occupational choice is determined in the context of my quantitative life cycle model. Recall from Equation 3 in Proposition C.3 that workers compare the growth opportunities associated with choosing a given occupation, summarized by $\hat{V}_j(z_j, \tilde{a})$, at the beginning of their lifetime. These relative values can be weighed against current relative wages due to the exponential nature of wage growth rates.

Recall that in the standard static Roy-type occupational choice framework, workers choose cognitive jobs if

$$\frac{w_c(t)}{w_r(t)} > \frac{z_r}{z_c},$$

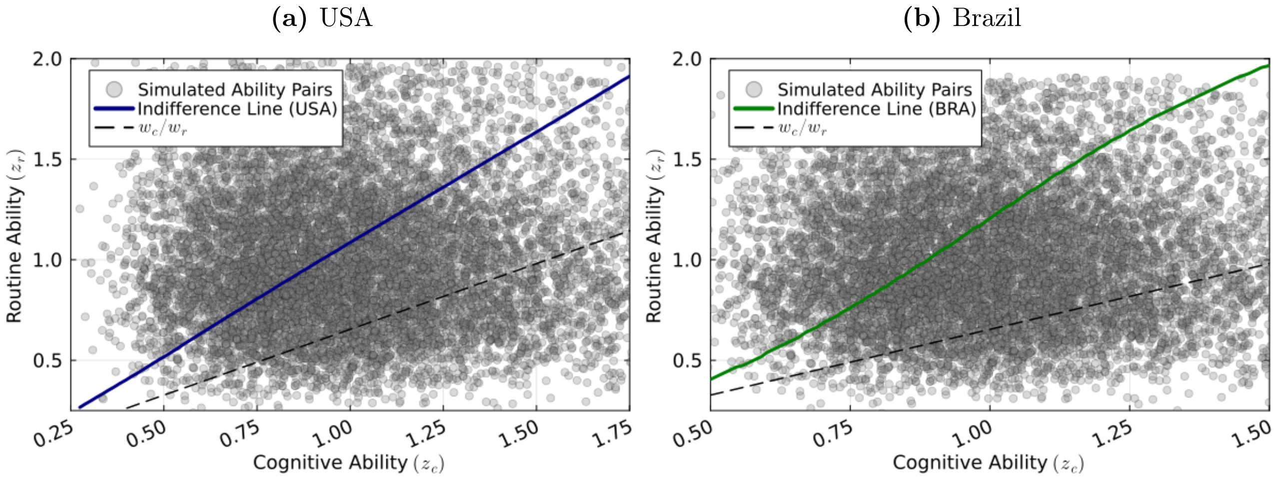
which yields the indifference mapping

$$z_c \left(\frac{w_c(t)}{w_r(t)} \right) = z_r.$$

That is, for any given z_c, one can find the corresponding z_r such that a worker endowed with this ability pair is indifferent between occupations. In this static setting, occupational sorting is entirely determined by relative wages and relative abilities. By contrast, in the dynamic model, the indifference schedule is shifted by differences in human capital accumulation opportunities across occupations, thereby incorporating not only the wage ratio but also the lifetime returns to skills.

To shed light on the implications of introducing a dynamic setting, I plot indifference schedules in the space of workers' innate abilities. The results are presented on Figure 8 below. The comparison of static and dynamic indifference schedules highlights several key insights.

Figure 8: Dynamic Occupational Choice in Brazil and the US



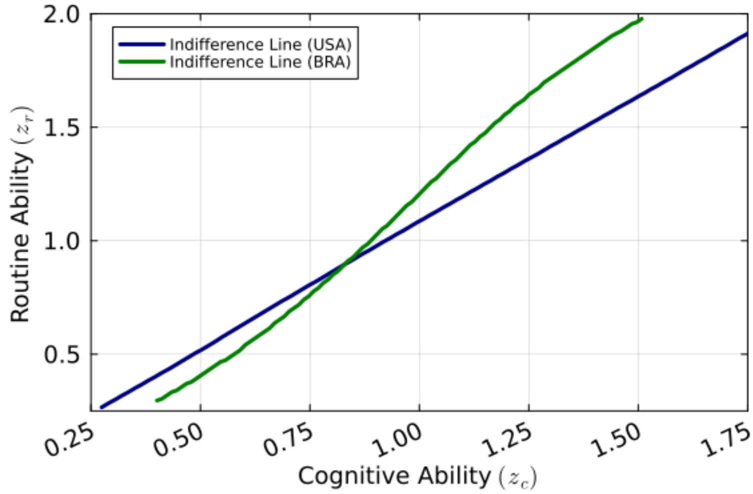
Note: *The dashed black line represents the static indifference ability schedule such that* $z_c \frac{w_c(t)}{w_r(t)} = z_r$*. The solid blue line in the US and green line in Brazil indifference ability locci such that for a worker of age* $\tilde{a}$ *in period* t*:* $\mathscr{I}(\tilde{a}, t) = \left\{ (z_c, z_r) \ : \ \hat{V}_c(z_c, \tilde{a}) \frac{w_c(t)}{w_r(t)} e^{-(g_c - g_r)(\tilde{a} - a_0)} = \hat{V}_r(z_r, \tilde{a}) \right\}$*. Dynamic schedules are for workers age 45 in both countries. The grey dots represent* $N = 5000$ *ability draws from the innate ability distribution in each country.*

First, in both countries the indifference line shifts upward once dynamic career gains are introduced, reflecting that cognitive occupations provide higher dynamic returns relative to routine work. Consequently, equilibrium requires an adjustment in relative wages, with the routine wage per unit of human capital exceeding that of cognitive occupations.

The divergence between static and dynamic indifference schedules is also stronger in Brazil than in the United States, implying that high–cognitive ability workers in Brazil are disproportionately more likely to select into cognitive occupations.

Finally, the cross-country heterogeneity in dynamic gains generates a set of workers who would optimally choose cognitive occupations in Brazil but routine occupations in the United States, those to the right of the intersection between the two indifference lines in Figure 9. This pattern relates directly to the occupational downgrading debate in the immigration literature: heterogeneous dynamic career gains provide an additional mechanism through which workers may end up in lower-skill jobs abroad, even when their innate abilities would have led them to cognitive occupations at home. Although my model abstracts from other well-known channels, such as imperfect transferability of foreign-acquired skills and post-migration reinvestment in human capital, this mechanism highlights a distinct dimension of occupational downgrading that deserves quantitative exploration.

Figure 9: Dynamic Occupational Choice in US vs Brazil



Note: *The solid blue line in the US and green line in Brazil indifference ability schedules such that for a worker of age* $\tilde{a}$ *in period* t*:* $\mathscr{I}(\tilde{a}, t) = \left\{(z_c, z_r) \ : \ \hat{V}_c(z_c, \tilde{a}) \, \frac{w_c(t)}{w_r(t)} \, e^{-(g_c - g_r)(\tilde{a} - a_0)} = \hat{V}_r(z_r, \tilde{a})\right\}$*. Dynamic schedules are for workers aged 45 in both countries.*

Human Capital Stocks.

The first quantitative exercise examines average human capital by country and occupation, along with their aggregate counterparts. The procedure entails calculating effective labor by occupation following the formula in Section 3. The procedure is as follows. A large number of workers are drawn from the joint distribution of innate skills. For a given age $\tilde{a}$, I compute, within each occupation, human capital conditional on attaining each of the four possible schooling levels: no schooling, primary, secondary, and university completion. Using the nested logit probabilities described above, I then calculate the expected human capital for each worker at age $\tilde{a}$, conditional on occupational choice. Aggregating over the age distribution of workers in each occupation yields $\bar{h}_c$ and $\bar{h}_r$, the average human capital stocks in cognitive and routine occupations, respectively.

To construct the aggregate measure, occupation-specific human capital is expressed in equivalent efficiency units of routine labor:

$$\bar{h} \ = \ \left(\frac{w_c}{w_r}\right) \pi_c \bar{h}_c \ + \ \pi_r \bar{h}_r,$$

where π_j denotes the share of employment in the occupation j. For completeness, I also report the simple weighted average, $\bar{h} \ = \ \pi_c \bar{h}_c \ + \ \pi_r \bar{h}_r$.

Table 7: Average Human Capital by Country and Occupation

Country	*Occupation*		*Aggregate*	
	Cognitive $\bar{h}_c$	Routine $\bar{h}_r$	Weighted Average	Equivalent Efficiency Units
USA	7.15	4.25	5.40	4.39
Brazil	6.54	1.43	2.58	1.73
USA/Brazil	**1.09**	**2.97**	**2.09**	**2.54**

Note: *Average human capital by occupation is computed by drawing from the skill distribution, evaluating human capital at discrete schooling levels, and weighting by nested logit choice probabilities and the age distribution. Aggregate human capital is reported both in routine-equivalent efficiency units and as a simple weighted average.*

Table 7 reports the resulting human capital stocks by country and occupation. The results show that human capital gaps vary substantially across occupations. While average human capital is only about 10% higher in cognitive occupations in the United States relative to Brazil, the gap widens to nearly threefold in routine occupations. This pattern highlights that both the occupational structure and the nature of routine work play a central role in shaping cross-country differences in human capital.

At the aggregate level, the disparity becomes even more pronounced once occupational structure is taken into account and human capital stocks are expressed in equivalent efficiency units. Because one unit of cognitive labor corresponds to $w_c/w_r = 0.65$ units of routine labor in the United States and $w_c/w_r = 0.42$ in Brazil, the aggregate gap rises to roughly a factor of 2.55. This is considerably larger than the gap implied by measures based solely on educational attainment. For example, the PWT reports human capital indices based on schooling and Mincerian returns of 3.7 for the United States in 2010 and 2.1 for Brazil in 2000, corresponding to a gap of only 1.7.

Counterfactual Stocks for Brazil.

Having computed human capital stocks by country, I now perform a series of counterfactuals to shed light on the mechanisms driving the human capital gaps between Brazil and the United States. To isolate the contribution of each channel, the exercises are implemented one at a time and are not cumulative. These counterfactuals are conducted in partial equilibrium: wages are held fixed and do not adjust to clear labor markets. Although this abstraction leaves out general equilibrium feedbacks, the exercises remain informative about the underlying sources of human capital gaps, as they allow me to identify the role of each mechanism in isolation.

Counterfactual I: Age distribution. In the first exercise, Brazil is assigned the U.S. age distribution. Since the U.S. workforce is skewed older in both occupations (the average age is 44 in cognitive and 41 in routine, compared with 36 and 34 in Brazil), the average human capital stock rises in both occupations. At the aggregate level, the human capital index increases 4.5% from 1.73 to 1.81.

Counterfactual II: Returns to schooling. In the second exercise, Brazil is given the U.S. returns to schooling. Because returns in cognitive occupations are higher in Brazil, this assignment reduces the human capital stock in cognitive jobs to about 45 percent of its original level. Conversely, the higher U.S. returns in routine occupations increase the Brazilian stock in these jobs by roughly 60 percent. Despite this gain, the routine stock remains 50 percent below its U.S. counterpart, largely due to educational attainment differences. This highlights both the importance of accounting for heterogeneity in returns across occupations and the fact that Brazil's relatively higher cognitive human capital stock is partly driven by higher returns to education, even under lower educational attainment.

Counterfactual III: Returns to On-the-Job Training. Assigning Brazil the U.S. parameters governing returns to on-the-job training has a significant impact in both occupations, though fundamentally more so in routine. The stock of human capital in cognitive occupations grows by about 40 percent (from 6.54 to 9.14), while in routine occupations it more than doubles, growing by about 136 percent (from 1.43 to 3.37). In partial equilibrium, holding the baseline occupational structure and relative wages fixed, the aggregate human capital stock grows by roughly 101 percent (from 1.73 to 3.47).

This exercise highlights two key insights. First, it reinforces that a substantial share of human capital is accumulated on the job rather than through schooling alone. Second, it underscores that the changing nature of routine occupations between Brazil and the United States is driven by the fact that, in the U.S., routine occupations offer markedly greater opportunities for human capital accumulation over the life cycle.

Counterfactual IV: Educational Attainment Costs. Next, I consider the effect of higher fixed costs of educational attainment. Specifically, Brazil is assigned schooling costs equal to the same share of average earnings faced by U.S. workers in the base group, as shown on Table 4. This not only raises the levels of all costs relative to the benchmark of no schooling, but also biases to the upside proportionately more the costs of secondary schooling (+46 percentage points) and university (+42 percentage points). As a consequence, in the counterfactual economy, 66 and 95 of workers in cognitive and routine occupations attain no schooling, reducing human capital stocks by about 40 percent in both occuupations.

This exercise highlights that educational attainment costs move in lockstep with the development process, with the incidence of the costs of higher educational levels increasing as countries face educational upgrading.

Counterfactual V: Occupational structure. Finally, I explore the effect of occupational reallocation by giving Brazil the U.S. share of employment in cognitive occupations. Interestingly, this counterfactual nearly closes the human capital gap when using the simple weighted average measure. However, under the partial equilibrium assumption that wages remain fixed, the stock in equivalent efficiency units increases by 10 percent only, remaining 55 percent below the U.S. level. This not only reflects the fact that one unit of cognitive labor is about 35 percent less valuable in Brazil, but also the predominant role of the low human capital stock in routine occupations in Brazil, which still represent a sizeable share of employment even under the US's occupational structure.

Table 8: Counterfactual Human Capital Stocks for Brazil

Country		Case	*Occupation*		*Aggregate*	
			Cognitive $\bar{h}_c$	Routine $\bar{h}_r$	Weighted Average	Equivalent Efficiency Units
Brazil		Baseline	6.54	1.43	2.58	1.73
	i.	*US's Age Distribution*	6.54	1.43	2.71	1.81
	ii.	*US's Return to Education*	3.61	2.32	2.61	2.14
	iii.	*US's Return to OJT*	9.14	3.37	4.68	3.47
	iv.	*US's Education Fixed Costs*	3.86	0.82	1.51	1.00
	v.	*US's Occupational Structure*	6.54	1.43	3.45	1.95
USA		Baseline	7.15	4.25	5.40	4.39

Note: *Counterfactuals **(i)**–**(v)** are partial equilibrium exercises designed to isolate the contribution of individual mechanisms to cross-country human capital gaps. Each counterfactual replaces one element of the Brazilian economy with its U.S. counterpart while holding all other parameters fixed: **(i)** occupation-specific age distribution, **(ii)** returns to schooling, **(iii)** returns to on-the-job training, **(iv)** fixed schooling costs (scaled by base-group earnings), and **(v)** occupational shares. These exercises are performed one at a time and are not cumulative.*

These counterfactuals highlight two central insights. First, heterogeneity in returns to schooling plays a critical role: Brazil's relatively high returns in cognitive occupations help sustain its human capital stock in those jobs despite lower average educational attainment. Second, the occupational structure strongly shapes aggregate human capital, but convergence with the U.S. requires not only a shift toward more cognitive employment but also an increase in the relative price of cognitive skills. Third, educational attainment costs evolve alongside development, with the burden of higher-level schooling becoming

Table 9: Occupational Technological Efficiency by Country

Country	$\sigma=0.60$			$\sigma=0.90$			$\sigma=1.20$			$\sigma=1.50$		
	A_c	A_r	A_c/A_r	A_c	A_r	A_c/A_r	A_c	A_r	A_c/A_r	A_c	A_r	A_c/A_r
US	1.00	0.66	1.51	1.00	0.05	20.1	1.000	8.84	0.11	1.00	3.13	0.32
Brazil	0.38	0.20	1.89	1.26	0.007	117.4	0.11	5.66	0.02	0.18	1.48	0.12
US/Brazil	*2.6*	*3.3*	*0.8*	*0.8*	*7.0*	*0.11*	*8.8*	*1.6*	*5.6*	*5.5*	*2.1*	*2.6*

Note: *Technological efficiency parameters are obtained by inverting the firm's optimality conditions, using the average human capital stocks by occupation, occupational shares, and relative wages per unit of human capital, given a value for the elasticity of substitution between occupations σ.*

more pronounced as countries undergo educational upgrading.

Recovering Technologies.

Having estimated average human capital stocks by occupation, I now proceed to recover the remaining technological parameters, which are identified conditional on a given value of the elasticity of substitution.[23]

To do so, I first express aggregate human capital by occupation as $H_j(t) = L_j(t)\bar{h}_j(t)\ avh_j(t)$,where $L_j(t)$ is total employment, $\bar{h}_j(t)$ average human capital, and $avh_j(t)$ average hours worked per year, all in occupation j. In per worker terms, we get

$$\frac{H_j(t)}{L(t)} = \pi_j(t)\bar{h}_j(t)\ avh_j(t).$$

Assuming that $avh_c(t) = avh_r(t)$, we can invert the firm's optimality condition in Equation ?? to obtain relative technologies, which are

$$\left(\frac{A_c(t)}{A_r(t)}\right) = \left[\left(\frac{w_c(t)}{w_r(t)}\right)\left(\frac{\pi_c(t)\bar{h}_c(t)}{\pi_r(t)\bar{h}_r(t)}\right)^{(1/\sigma)}\right]^{\left(\frac{\sigma}{\sigma-1}\right)}.$$

Additionally, inverting the firm's inverse demand for routine effective labor yields

$$A_r(t) = \frac{w_r(t)}{\left(\left[\left(\frac{A_c(t)}{A_r(t)}\frac{\pi_c(t)\bar{h}_c(t)}{\pi_r(t)\bar{h}_r(t)}\right)^{\left(\frac{\sigma-1}{\sigma}\right)}+1\right]^{\left(\frac{1}{\sigma-1}\right)}\right)}.$$

These two equations allow me to recover $A_c(t)$ and $A_r(t)$ up to a value for σ. The results are presented on Table 9 below, for $\sigma \in \{0.60, 0.90, 1.20, 1.50\}$.

[23]This identification strategy follows from the classic result in Diamond et al. (1978), who show that in an aggregate CES framework the elasticity of substitution and the relative technology efficiency parameters cannot be separately identified. More recent work, such as León-Ledesma et al. (2010), uses simulation methods to address this issue and provide conditions under which both sets of parameters can be identified.

I choose a benchmark value of $\sigma = 0.60$ for several reasons. First, there are relatively few empirical estimates of the elasticity of substitution between labor services provided by different occupations. To my knowledge, one of the most credible estimates is provided by Goos et al. (2014), who obtain a value of 0.90. However, their framework is based on a more disaggregated, task-based technology. Since aggregation tends to increase the degree of complementarity across inputs, it is reasonable to expect lower elasticities in my setting, where occupations are aggregated into two broad groups.

Second, selecting $\sigma < 1$ ensures that the efficiency of both cognitive and routine technologies is higher in the United States than in Brazil. This issue is discussed in detail in Caselli and Coleman (2006), but for an aggregator in which labor services vary by educational group. For my aggregate technology based on labor services provided by occupations, I prefer to err on the side of caution and adopt a value of σ that guarantees that both technologies improve as countries advance along the development path.

Finally, the choice of $\sigma = 0.60$ implies that, at least for the segment of the development process relevant for the transition from a middle-income country such as Brazil to a high-income country such as the United States, development is routine-biased. This implication is strongly supported by the detailed evidence presented in Pena and Siegel (2023), and is consistent with the evidence documented by Goos et al. (2014) for a subset of European countries.

The benchmark calibration has important implications for how the process of development is characterized, which merits some discussion. Under this parameterization, development is associated with rising technological efficiency in both occupations, but with a bias toward routine jobs. Given the complementarity between occupations, this technological upgrading raises the wage per unit of human capital in both activities. However, wages increase proportionately more in cognitive occupations, which in turn induces a reallocation of workers toward cognitive employment. This mechanism generates the occupational shifts documented in Figure A.1.

Full General Equilibrium Counterfactual Human Capital Stocks.

Having recovered the technological parameters, I now turn to a set of general equilibrium counterfactuals. Specifically, I consider a sequence of shocks to fundamentals, implemented one at a time, in which Brazil is endowed with selected features of the U.S. economy. This approach allows me to examine not only how human capital stocks respond across occupations, but also how wages adjust in equilibrium, thereby providing a clearer picture of how the returns to skills change under the counterfactual scenarios. The results are presented on Table 10 below.

Table 10: Counterfactual Average Human Capital by Occupation and Country

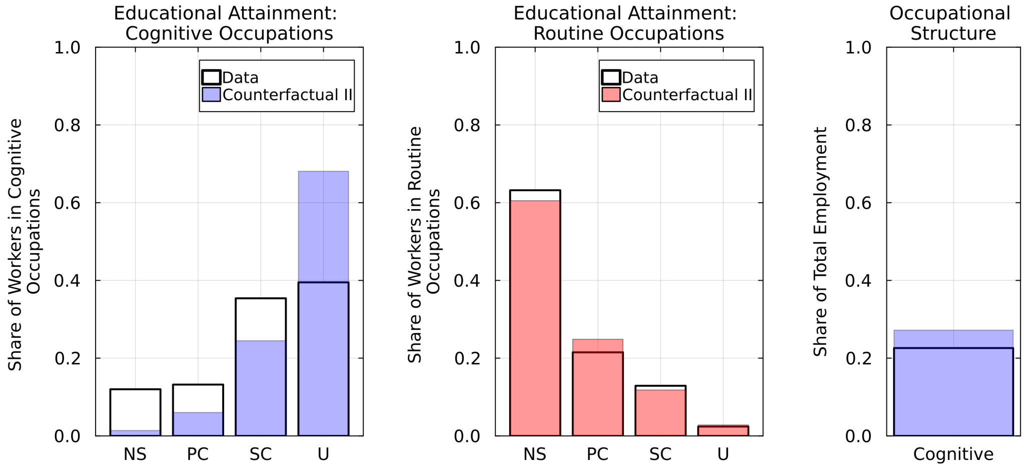
Country	Case	Equilibrium			Human Capital			
		w_c/w_r	w_r	π_c	$\bar{h}_c$	$\bar{h}_r$	Weighted Average	Equivalent Efficiency Units
Brazil	Baseline	0.42	0.97	0.226	6.54	1.43	2.58	1.73
	Counterfactual I.	0.53	1.10	0.272	8.14	1.45	3.42	2.33
	Counterfactual II.	1.06	0.67	0.362	1.51	1.15	1.28	1.32
	Counterfactual III.	0.42	0.94	0.360	5.64	2.53	3.65	2.53
USA	Baseline	0.65	2.49	0.395	7.15	4.25	5.39	4.39

Note: *Counterfactual II assigns to Brazil the U.S. technological bias by rescaling the level of efficiency in routine occupations as* $A_r^{BRA} = A_c^{BRA}/\left(A_c^{US}/A_r^{US}\right)$*.Counterfactual III assigns to Brazil the fixed educational attainment costs of the United States. Counterfactual IV assumes that idiosyncratic educational attainment costs are uncorrelated across all educational stages, i.e.* $\lambda_s = 1.000$ *for all* $s \in \{ns, p, sec, u\}$*. The weighted average stock is computed as* $\bar{h} = \pi_c\bar{h}_c + \pi_r\bar{h}_r$*,while the stock in equivalent efficiency units of routine labor is* $\bar{h} = (w_c/w_r)\pi_c\bar{h}_c + \pi_r\bar{h}_r$*.*

Counterfactual I endows Brazil with the technology bias observed in the United States. To implement this, I compute the routine-sector efficiency parameter as $A_r^{BRA} = A_c^{BRA}/\left(A_c^{US}/A_r^{US}\right)$ which guarantees that the levels of both technological efficiency parameters are higher than in the baseline. As reported in row Counterfactual I of Table 10, skill complementarity implies that this adjustment raises average human capital in cognitive occupations by about 25 percent, while leaving routine occupations largely unchanged. Figure 10 illustrates that this increase is driven by substantial educational upgrading in cognitive jobs: the share of workers attaining a university degree rises by roughly 25 percentage points.

The mechanism behind this shift operates through relative wages. The increase in routine-sector technological efficiency raises the cognitive-to-routine wage ratio, w_c/w_r which attracts more workers into cognitive occupations. This reallocation pushes up wages in both sectors, but proportionately more in cognitive jobs. The larger wage gains reduce the effective burden of fixed education costs, thereby encouraging greater educational attainment, with the effect concentrated in cognitive occupations.

Figure 10: Educational and Occupational Structure
-Counterfactual I-

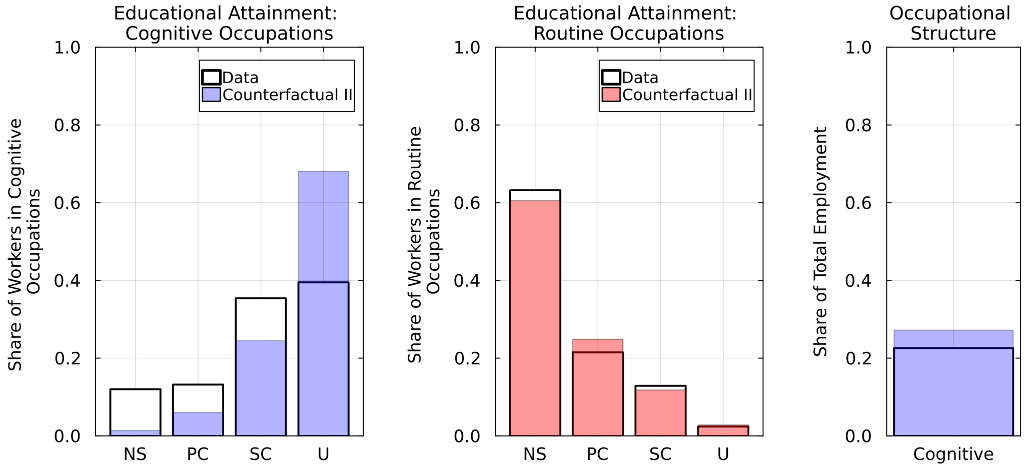


Note: *This counterfactual assigns to Brazil the U.S. technological bias by rescaling the level of efficiency in routine occupations as* $A_r^{BRA} = A_c^{BRA} / \left(A_c^{US}/A_r^{US}\right)$. *Empty black bars correspond to the data, while shaded blue and red bars represent the educational attainment shares in equilibrium in the counterfactual economy. Educational categories are defined as follows:* ***NS*** *= less than primary complete,* ***PC*** *= primary complete or more but less than secondary complete,* ***SC*** *= secondary complete or more but less than university complete, and* ***U*** *= university complete or more.*

Counterfactual II assigns to Brazil the educational fixed costs of the United States. The results, presented in row Counterfactual II of Table 10, show that the higher level of fixed costs---and the heavier burden they represent relative to Brazilian wages---induces a substantial educational downgrading, particularly in cognitive occupations (see Figure **??**). As a result, average human capital declines more than proportionately in cognitive jobs, leading to an increase in relative wages and a higher share of workers in cognitive occupations. Naturally, aggregate human capital suffers a significant reduction.

While this exercise abstracts from other institutional and policy differences, it illustrates a key mechanism: as economies develop and wages rise, educational attainment costs also increase in lock-step, partially offsetting the expansion in educational investment that accompanies development.

Figure 11: Educational and Occupational Structure
-Counterfactual II-



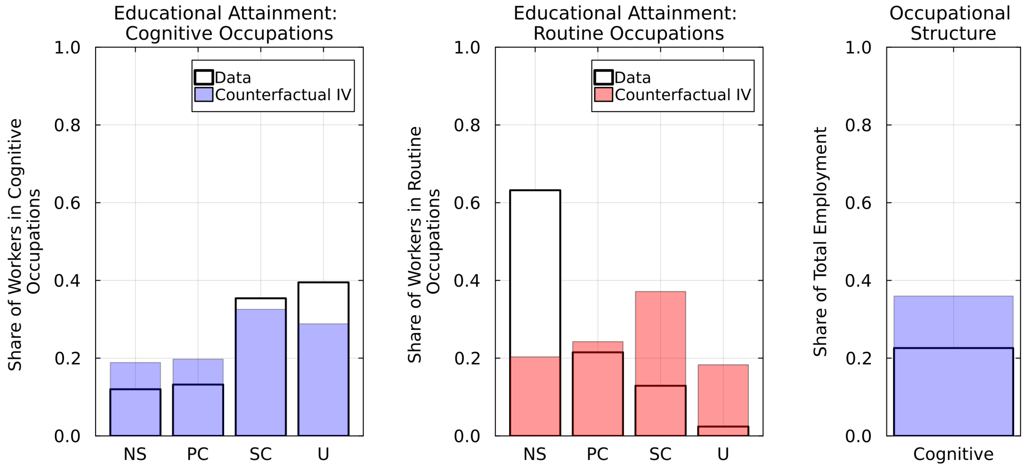
Note: *Counterfactual III assigns to Brazil the fixed educational attainment costs of the U.S.. Empty black bars correspond to the data, while shaded blue and red bars represent the educational attainment shares in equilibrium in the counterfactual economy. Educational categories are defined as follows:* ***NS*** *= less than primary complete,* ***PC*** *= primary complete or more but less than secondary complete,* ***SC*** *= secondary complete or more but less than university complete, and* ***U*** *= university complete or more.*

The previous counterfactual has shown that fixed educational attainment costs play a central role in shaping the educational structure across countries. However, to account for potential frictions that might prevent workers from choosing their optimal careers, I also incorporate idiosyncratic costs, which can disproportionately affect educational attainment at certain levels. To quantitatively gauge their importance, I assume that these costs are uncorrelated across all educational stages by setting $\lambda_s = 1.000$ for all $s \in \{ns, p, sec, u\}$.This assumption implies that workers' educational attainment and occupational choices are determined purely by the present discounted value of earnings net of education costs.

As Figure 12 shows, idiosyncratic costs play a key role in shaping the educational structure in both occupations in Brazil. With independent cost shocks, there is an educational downgrading in cognitive occupations and an upgrading in routine occupations. In particular, the share of workers not finishing schooling in routine occupations falls substantially (by more than 40 percentage points), while the share of workers completing university in cognitive jobs declines.

The overall effect is a contraction of human capital in cognitive occupations and an expansion in routine occupations, leading to a higher share of workers employed in cognitive jobs in equilibrium. Aggregate human capital rises, however, due to the shift in occupational structure and the improvement in average human capital stocks in routine occupations, given stable relative wages.

Figure 12: Educational and Occupational Structure
-Counterfactual III-



Note: *Counterfactual IV assumes that idiosincratic educational attainment costs are uncorrelated in all educational stages. That is* $\lambda_s = 1.000$ *for all* $s \in \{ns, p, sec, u\}$*. Empty black bars correspond to the data, while shaded blue and red bars represent the educational attainment shares in equilibrium in the counterfactual economy. Educational categories are defined as follows:* ***NS*** *= less than primary complete,* ***PC*** *= primary complete or more but less than secondary complete,* ***SC*** *= secondary complete or more but less than university complete, and* ***U*** *= university complete or more.*

Taken together, these counterfactuals highlight the multiple channels through which cross-country differences in fundamentals shape human capital outcomes. Fixed education costs, technology bias, and idiosyncratic frictions all leave distinct effects on the occupational and educational structure, and their relative importance varies across cognitive and routine jobs. The results underscore that both institutional and technological factors are central to understanding the human capital gaps between Brazil and the United States.

5 Conclusion

This paper has studied how shifts in occupational structure shape human capital accumulation and, in turn, cross-country income differences. Motivated by the empirical fact that development is systematically associated with a reallocation of employment from routine-intensive jobs toward cognitively intensive occupations, I documented that workers in cognitive jobs both acquire more schooling and experience steeper returns to education and experience across the development spectrum. These patterns point to occupational choice as a central channel linking structural change and human capital.

To interpret these findings, I developed a dynamic model of human capital accumulation and occupational choice, where workers optimally balance the costs of schooling and on-the-job training against future career gains. Calibrated and structurally estimated using harmonized data for Brazil and the United States, the model highlights that differences in occupational structure and the nature of routine work can substantially amplify human capital gaps. Specifically, while average human capital in the United States is only about 10 percent higher than in Brazil, the gap within routine occupations is nearly threefold. A simple decomposition attributes 25 percent of the U.S.–Brazil gap to differences in occupational structure, 30 percent to higher human capital stocks embodied in U.S. routine jobs, and the remainder to relative efficiency and cognitive skill differences.

These results underscore two main insights. First, biased technological change, by reshaping the occupational composition of employment, is a key driver of cross-country human capital gaps. Second, differences in the very nature of routine occupations across countries matter as much as compositional changes, suggesting that development involves not only occupational upgrading but also a deep transformation in the productivity of routine work.

More broadly, the analysis points to the importance of studying human capital accumulation in a framework that incorporates occupational choice and dynamic investment decisions. Future work could extend the model to allow for occupational mobility, skill transferability, and post-school reinvestment, thereby offering a richer account of phenomena such as occupational downgrading among migrants. Such extensions would further clarify how labor market structures and human capital interact to generate persistent cross-country income differences.

Appendix

A The Occupational Structure Across Development by Broad Industry.

In this Appendix, I show that the shift in the occupational structure is not purely a composition effect driven by structural transformation, or the reallocation of employment across broad industries, but a phenomenon that also occurs within broad industries across the development process. To illustrate this, I study the evolution across development of the share of cognitive occupations for three broad industries commonly used in the structural change literature (see Herrendorf et al. (2014)): *Agriculture*, *Goods*, and *Services*.

Workers are classified into industries using the IPUMS-I harmonized industry classification variable `INDGEN`. According to IPUMS-I, "`INDGEN` recodes the industrial classifications of the various samples into twelve groups that can be fairly consistently identified across all available samples. The groupings roughly conform to the International Standard Industrial Classification (`ISIC`). *Industry* refers to the activity or product of the establishment or sector in which a person worked."

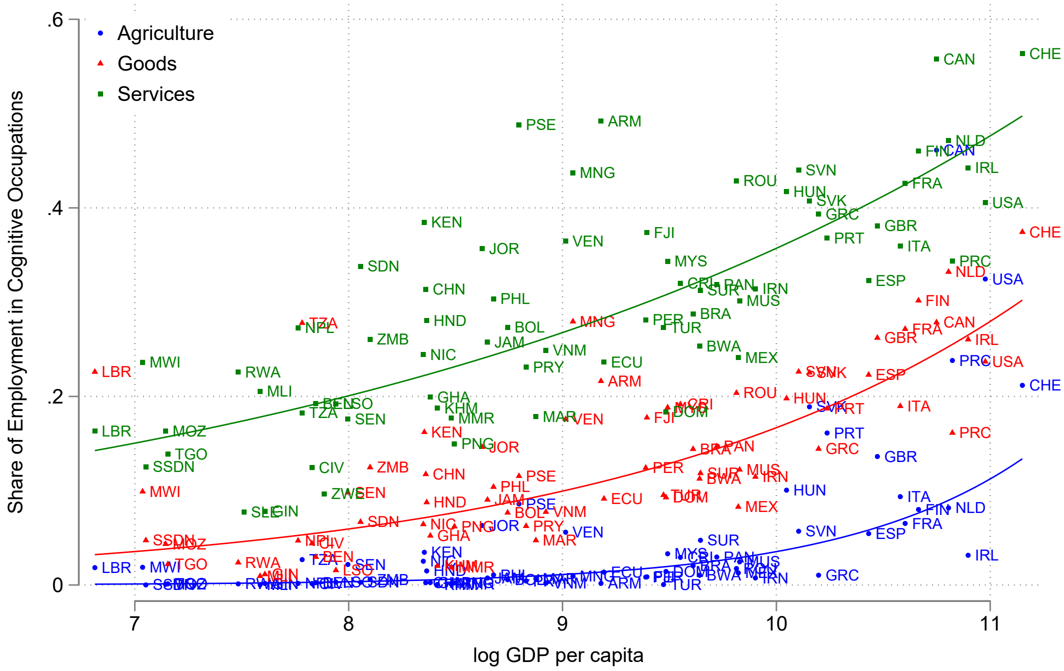
The grouping into broad sectors is done as follows, where the three digits represent the `INDGEN` industry code and the industry label is presented between parenthesis:

- **Agriculture:** 010 (Agriculture, fishing, and forestry).
- **Goods:** 020 (Mining and extraction); 030 (Manufacturing); 040 (Electricity, gas, water and waste management); 050 (Construction).
- **Services:** 060 (Wholesale and retail trade); 070 (Hotels and restaurants); 080 (Transportation, storage, and communications); 090 (Financial services and insurance); 100 (Public administration and defense); 110 (Services, not specified); 111 (Business services and real estate); 112 (Education); 114 (Other services); 120 (Private household services).

Observations belonging to industries 000 (Not in universe), 130 (Other industry, n.e.c.), 998 (Response suppressed), and 999 (Unknown) are dropped.

To demonstrate that the shift in the occupational structure toward cognitive employment is not merely a composition effect of structural change, but rather a phenomenon occurring within broad industries, I plot the share of workers in cognitive occupations by major industry groups across different levels of GDP per capita, my proxy for development. The results are shown in Figure A.1.

Figure A.1: Share of Employment in Cognitive-Intensive Occupations Across Development
-by broad Industry-



Note: *Cognitive occupations include one digit* `ISCO` *codes 01-Legislators, senior officials and managers, 02-Professionals, and 03-Technicians and associate professionals. Agriculture groups IPUMS-I* `INDGEN` *industry code 010, Goods group industry codes 020–050, and Services group codes 060–120. Industries not in universe (000), Other industry n.e.c. (130), and those with unknown industry or suppressed responses (998 and 999) are excluded. Workers are weighted using IPUMS-I perwt. See Appendix D for a description of the countries in the sample.*

A few comments are in order. First, the share of workers in cognitive occupations increases across all industries as countries grow richer. The rise is most pronounced in Agriculture, followed by Goods and then Services. Indeed, the coefficients on the exponential fits in Figure A.1 are all statistically significant at the 1 percent level, with estimated values of 1.16 for Agriculture, 0.52 for Goods, and 0.29 for Services.

Second, cognitive intensity is consistently highest in Services, followed by Goods and Agriculture. Thus, while structural transformation reallocates employment toward industries that are more cognitive-intensive, the shift toward cognitive occupations is not solely a between-industry phenomenon but also takes place within industries.

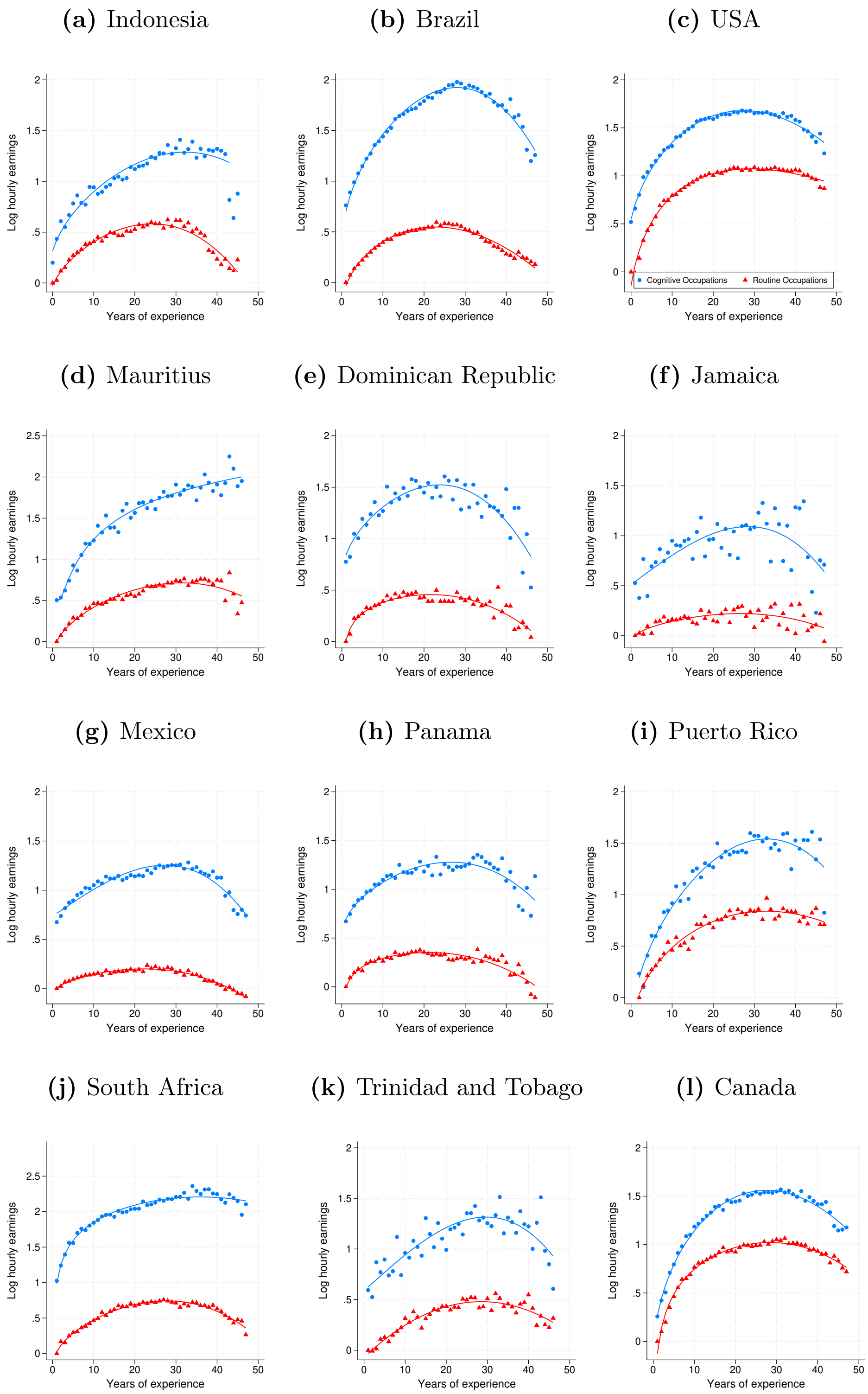
To quantitatively disentangle the role of structural change in driving the shift toward cognitive employment, I perform a simple shift–share decomposition of the change in the share of workers in cognitive occupations for the average low-, middle-, and high-income country in my sample. On average, poor, middle-, and high-income countries exhibit cognitive employment shares of 0.088, 0.191, and 0.335, respectively, with corresponding GDP per capita levels of 2,500, 8,900, and 33,600 U.S. dollars.[24] The decomposition indicates that 55 percent of the increase in cognitive employment between the average poor and middle-income country is accounted for by within-sector cognitive deepening, while the remaining 45 percent reflects structural change across sectors. Between the middle- and high-income groups, 68 percent of the increase is explained by greater cognitive intensity within sectors, with the remaining 32 percent due to structural change.

I conclude that the shift in the occupational structure associated with development is predominantly a phenomenon that occurs within broad industries and not purely the result of changes in the composition of employment toward industries that are more cognitive-intensive. Naturally, the same forces that drive cognitive deepening between industries can also engine of structural change, but that requires further investigation in the context of a multi-sector, general equilibrium model with occupation-biased technical change.

[24]These are measured at PPP and expressed in units of output of the US in 2017. See variable `cgdpo` in PWT.

B Experience Earnings Profiles.

Figure B.1: Experience Earnings Profiles for Broad Set of Countries.



Note: *Experience–earnings profiles are synthetic cohorts, obtained as weighted means of log hourly earnings by potential experience and occupation. Blue circles denote cognitive occupations, red triangles routine. A fractional polynomial fit is overlaid.*

C Proof of Proposition 3.

C.1 Working Stage.

Dropping the arguments z_j and y_j^s and the occupation indicator j to ease notation, the present-value Hamiltonian of the problem during this stage is

$$\mathscr{H}^o(a) = e^{-(\rho-g)(a-a_0)}w(a_0)h(a)(1-i^o(a)) + \mu^o(a)\left\{\eta^o h(a)^\gamma i^o(a)^\alpha - \delta h(a)\right\}. \tag{C.1}$$

The associated FOCs are

$$\begin{aligned}\frac{\partial \mathscr{H}^o(a)}{\partial i^o} = 0 \;\; &\Rightarrow \;\; -e^{-(\rho-g)(a-a_0)}w(a_0)h(a) \\ &+ \mu^o(a)\Big(\alpha\eta^o h(a)^\gamma i^o(a)^{\alpha-1}\Big) = 0, \end{aligned}\tag{C.2}$$

$$\begin{aligned}-\frac{\partial \mathscr{H}^o}{\partial h} = \dot{\mu}(a) - \rho\mu(a) \;\; &\Rightarrow \;\; -e^{-(\rho-g)(a-a_0)}w(a_0)(1-i^o(a)) \\ &- \mu^o(a)\Big(\gamma\eta^o h(a)^{\gamma-1} i^o(a)^\alpha - \delta\Big) = \dot{\mu}(a) - \rho\mu(a), \end{aligned}\tag{C.3}$$

$$\frac{\partial \mathscr{H}^o}{\partial \mu(a)} = \dot{h}(a) \;\; \Rightarrow \;\; \dot{h}(a) = \eta^o h(a)^\gamma i^o(a)^\alpha - \delta h(a). \tag{C.4}$$

And the transversality condition

$$\mu(A) = 0 \tag{C.5}$$

Re-arranging Equation C.2, we obtain

$$i^o(a) = \left(\frac{e^{(\rho-g)(a-a_0)}\mu^o(a)\left(\alpha\eta^o h(a)^{(\gamma-1)}\right)}{w(a_0)}\right)^{\left(\frac{1}{1-\alpha}\right)}. \tag{C.6}$$

Notice that from Equation C.12

$$e^{-(\rho-g)(a-a_0)}w(a_0)i^o(a) = \alpha\mu^o(a)\eta^o h(a)^{(\gamma-1)}i^o(a)^\alpha$$

which plugged in into Equation C.3, yields

$$\dot{\mu^o}(a) = -e^{-(\rho-g)(a-a_0)}w(a_0) - \mu^o(a)\left(\eta^o(\gamma-\alpha)h(a)^{(\gamma-1)}i^o(a)^\alpha - (\rho+\delta)\right). \tag{C.7}$$

In turn, Equation C.6 can be plugged in into C.7 to obtain a first-order non-linear ODE that can be solved numerically using the Equation C.5 as boundary condition.

In the particular case of $\alpha = \gamma$, Equation C.7 turns into the following first-order linear ODE

$$\dot{\mu}(a) = (\rho + \delta)\mu(a) - e^{-(\rho-g)(a-a_0)}w(a_0). \tag{C.8}$$

In this case we can obtain an analytical solution to this ODE, and the shadow value of a unit of human capital at age a is

$$\mu^o(a) = \frac{w(a_0)e^{-(\rho-g)(a-a_0)}\left(1 - e^{-(\rho+\delta-g)(A-a)}\right)}{(\rho + \delta - g)}. \tag{C.9}$$

If $\rho > g$,$\mu^o(a)$ is monotonically decreasing in a, converging to $\mu^o(A) = 0$ as the end of the worker's life cycle approaches. Notice that in this case the instantaneous return of an additional unit of human capital $\alpha\mu^o(a)\eta^o h(a)^{(\gamma-1)}i^o(a)^\alpha$ equals its instantaneous cost, given by forgone earnings $e^{-(\rho-g)(a-a_0)}w(a_0)i^o(a)$. Therefore, the benefit of investment is purely driven by the fact that the worker enjoys higher human capital for the rest of her lifetime. Consequently, as she grows older, the value of investment in human capital declines. In the general case of $\alpha \neq \gamma$ the marginal benefit and the marginal cost differ, introducing an additional channel affecting the value of human capital.

In this special case, human capital investment via OJT is

$$i^o(a) = \frac{1}{h(a)}\left(\frac{\alpha\eta^o\left(1 - e^{-(\rho+\delta-g)(A-a)}\right)}{(\rho + \delta - g)}\right)^{\left(\frac{1}{1-\alpha}\right)}, \tag{C.10}$$

which is monotonically decreasing in a, and increasing in the values of the parameters governing the human capital accumulation returns α,η^o. When $\alpha = \beta$, $i^o(a)$ is decreasing in the worker's human capital, as higher skilled workers need to invest less in order to produce a unit of human capital and therefore a given flow of earnings between the current age a and their retirement age A.

Finally, in the case of $\alpha = \gamma$, the human capital policy has a closed form representation, given by

$$h(a, y^s) = e^{-\delta(a-a_0)}\left(ze^{(\eta^s q^s-\delta)y^s} + \mathscr{I}(a_s, a)\right),$$

with

$$\mathscr{I}(a_s, a) = \int_{a_s}^{a} \eta^o\left(\frac{(\alpha\eta^o)\left(1 - e^{-(\rho+\delta-g)(A-s)}\right)}{(\rho + \delta - g)}\right)^{\left(\frac{\alpha}{1-\alpha}\right)} e^{\delta(s-a_0)}ds.$$

In the general case of $\alpha \neq \gamma$, the human capital policy does not admit a closed form solution and solves

$$h(a, y^s) = \int_{a_s}^{a} \eta^o h(s)^\gamma i^o(s) - \delta h(s)$$

subject to

$$h(a^s, y^s) = ze^{(\eta^s q^s - \delta)y^s}.$$

C.2 Schooling Stage.

Dropping the arguments z_j and y_j^s and the occupation indicator j to ease notation, the present-value Hamiltonian of the problem, conditional the worker being in the schooling stage, is

$$\mathscr{H}^s = e^{-\rho(a-a_0)}\Big(w(a)h(a)(1-i^s(a)) - c\Big)\, da + \mu^s(a)\left(\eta^s q^s h^s(a) i^s(a) - \delta h^s(a)\right) \quad \text{(C.11)}$$

The associated FOCs are

$$\frac{\partial \mathscr{H}^s}{\partial i^s(a)} = 0 \quad \Rightarrow \quad -e^{-\rho(a-a_0)} w(a)h^s(a) + \mu^s(a)\, \eta^s q^s h^s(a) = 0, \quad \text{(C.12)}$$

$$-\frac{\partial \mathscr{H}^s}{\partial h^s(a)} = \dot{\mu}^s(a) - \rho\mu^s(a) \quad \Rightarrow \quad \dot{\mu}^s(a) = \rho\mu^s(a)$$
$$- e^{-(\rho-g)(a-a_0)} w(a_0)(1-i^s(a)) - \mu^s(a)\big(\eta^s q^s i^s(a) - \delta\big), \quad \text{(C.13)}$$

$$\frac{\partial \mathscr{H}^s}{\partial \mu^s(a)} = \dot{h}^s(a) \quad \Rightarrow \quad \dot{h}^s(a) = \eta^s q^s h^s(a) i^s(a) - \delta h^s(a). \quad \text{(C.14)}$$

Together with the value-matching condition

$$\mu^s(a^{s\star}) = e^{-\rho(a_j^{s\star} - a_0)} \frac{\partial V^o(a^{s\star})}{\partial h(a^{s\star})} \equiv \mu^o(a^{s\star}) \quad \text{(C.15)}$$

and the smooth pasting condition

$$w(a^{s\star})h(a^{s\star})\left(1 - i^o(a^{s\star})\right) + c = \mu^o(a^{s\star})h(a^{s\star})\left(\eta^s q^s - \eta^o h(a^{s\star})^{(\gamma-1)} i^o(a^{s\star})^\alpha\right).$$

Starting with Equation C.12, notice that optimality condition is independent of $i^s(a)$, implying that

$$i^s(a) = \begin{cases} 1 & \text{if}\, \mu^s(a)\eta^s q^s \geq e^{-(\rho-g)(a-a_0)} w(a_0), \\ 0 & \text{otherwise.} \end{cases}$$

Intuitively, the individual fully specializes in human capital accumulation if the value of accumulating an additional unit of skills, $\mu^s(a)\eta^s q^s$, exceeds its cost, given by forgone earnings $e^{-(\rho-g)(a-a_0)} w(a_0)$.

This implies that, during the schooling period, the shadow value of human capital characterized satisfies the first-order linear ODE

$$\dot{\mu^s}(a) = -\mu^s(a) \left(\eta^s q^s - (\delta + \rho)\right),$$

with general solution

$$\mu^s(a) = K \cdot e^{-(\eta^s q^s - (\delta+\rho))(a-a_0)}.$$

We determine the constant of integration K using the boundary condition at the terminal age of the schooling period $a^{s\star}$

$$K = \mu^s(a^{s\star}) e^{(\eta^s q^s - (\delta+\rho))(a^{s\star}-a_0)},$$

and using the value matching condition $\mu^s(a^{s\star}) = \mu^o(a^{s\star})$,we get

$$\mu^s(a) = \mu^o(a^{s\star}) e^{(\eta^s q^s - (\delta+\rho))(a^{s\star}-a)}.$$

Interestingly, when the quality-adjusted return to school $(\eta^s q^s)$ exceeds the combined rate of depreciation and discounting $(\rho + \delta)$ (i.e. $\eta^s q^s > (\rho + \delta)$)the marginal value of human capital in the schooling phase is decreasing over the schooling period. This reflects the cumulative nature of early investments due to compounding.

To derive the smooth pasting condition, we go back to the definition of life time earnings. Evaluating $V(z)$ at the optimal sequences for $i^{s\star}(a)$ and $i^{o\star}(a)$ yields

$$V(a^{s\star}, z) = \int_{a=a_0}^{a^{s\star}} e^{-\rho(a-a_0)} \Big(w(t) h(a)(1 - i^{s\star}(a)) - c \Big)\, da + e^{-\rho(a_j^\star - a_0)} V^o\left(h(a^{s\star}), a^{s\star}\right),$$

where $V^o\left(h(a^{s\star}), a^{s\star}\right)$ is the continuation value measured at age $a^{s\star}$.

Totally differentiating $V(a^{s\star}, z)$ with respect to $a^{s\star}$and equating to zero yields the following optimality condition

$$\frac{dV(a^{s\star},z)}{da^{s\star}} = e^{-\rho(a^{s\star}-a_0)}\left(w(a^{s\star})h(a^{s\star})\left(1-i^{s\star}(a^{s\star})\right)-c\right)+\frac{d\left(e^{-\rho(a^{s\star}-a_0)}V^o(h(a^{s\star}),a^{s\star})\right)}{da^{s\star}} = 0,$$

with

$$\begin{aligned}\frac{d}{da^{s\star}}\Big(e^{-\rho(a^{s\star}-a_0)}V^o\big(h(a^{s\star}),a^{s\star}\big)\Big) = e^{-\rho(a^{s\star}-a_0)}\Bigg(&-\rho\, V^o(h(a^{s\star}),a^{s\star})\\ &+\frac{\partial V^o(h(a^{s\star}),a^{s\star})}{\partial a^{s\star}}+\frac{\partial V^o(h(a^{s\star}),a^{s\star})}{\partial h(a^{s\star})}\frac{\partial h(a^{s\star})}{\partial a^{s\star}}\Bigg).\end{aligned}$$

The Hamilton-Jacobi-Bellman equation for $V^o(h,a)$ is

$$\rho V^o(h,a) = w(t)h(a)\left(1-i^o(a,h)\right)+\frac{\partial V^o(h,a)}{\partial a}+\frac{\partial V^o(h,a)}{\partial h}\left(\eta^o h(a)^\gamma i^o(a)^\alpha-\delta h(a)\right),$$

which re-arranged and replaced into the equation above yields

$$\begin{aligned}\frac{d}{da^{s\star}}\Big(e^{-\rho(a^{s\star}-a_0)}V^o(h(a^{s\star}),a^{s\star})\Big) = e^{-\rho(a^{s\star}-a_0)}\Bigg[&-w(t)h(a)\big(1-i^o(a)\big)\\ &-\frac{\partial V^o(h,a^{s\star})}{\partial h(a^{s\star})}\Big(\eta^o h(a)^\gamma i^o(a)^\alpha-\delta h(a)\Big)\\ &+\frac{\partial V^o(h,a^{s\star})}{\partial h(a^{s\star})}\frac{\partial h(a^{s\star})}{\partial a^{s\star}}\Bigg].\end{aligned}$$

Using

$$e^{-\rho(a^{s\star}-a_0)}\frac{\partial V^o(h,a)}{\partial h} = \mu^o(a),$$

and

$$\frac{\partial h(a^{s\star})}{\partial a^{s\star}} = (\eta^s q^s-\delta)h(a^{s\star}),$$

we obtain

$$\begin{aligned}&e^{-\rho(a^{s\star}-a_0)}\Big(w(a^{s\star})h(a^{s\star})(1-i^{s\star}(a^{s\star}))-c\Big)\\ &\quad+e^{-\rho(a^{s\star}-a_0)}\Big(-w(t)h(a^{s\star})(1-i^o(a^{s\star}))-\mu^o(a^{s\star})\big(\eta^o h(a^{s\star})^{\gamma-1}i^o(a^{s\star})^\alpha-\delta h(a^{s\star})\big)\\ &\qquad+\mu^o(a^{s\star})(\eta^s q^s-\delta)h(a^{s\star})\Big)=0.\end{aligned}$$

Re-arranging terms, we get

$$w(a^{s\star})h(a^{s\star})\left(1-i^{s\star}(a^{s\star})\right)-c=w(a^{s\star})h(a^{s\star})\left(1-i^{o}(a^{s\star})\right)-\mu^{o}(a^{s\star})\left(\eta^{s}q^{s}-\eta^{o}h(a_j^{s\star})^{\gamma}i^{o}(a^{s\star})^{\alpha}\right).$$

Finally, given that during the schooling period $i^{s\star}(a^{s\star})=1$, we get that

$$w(a^{s\star})h(a^{s\star})\left(1-i^{o}(a^{s\star})\right)+c=\mu^{o}(a^{s\star})h(a^{s\star})\left(\eta^{s}q^{s}-\eta^{o}h(a^{s\star})^{(\gamma-1)}i^{o}(a^{s\star})^{\alpha}\right)$$

C.3 Proof of Proposition 2.

Start with the definition of the net present discount value of earnings for a worker of age $\tilde{a}$ in period t

$$\begin{aligned}V_j(z_j;t,\tilde{a})=&\int_{a=a_0}^{a_j^*(z_j)}e^{-\rho(a-a_0)}w_j(a)\left\{h_j(a,z_j)(1-i_j^s(a,h_j))-\frac{c}{w_j(a)}\right\}da\\&+e^{-\rho(a_j^*(z_j)-a_0)}V_j^o\left(z_j,a_j^*(z_j)\right).\end{aligned}$$

From the definition of

$$\begin{aligned}V_j^o(t,h_j,y_j^s)&=\int_{a=a_0+y_j^s}^{A}e^{-\rho(a-a_0)}w_j(a)h_j(a,z_j,y_j^s)\left(1-i_j^{o\star}(a,h_j)\right)da\\&=w_j(a_0)\underbrace{\int_{a=a_0+y_j^s}^{A}e^{-(\rho-g_j)(a-a_0)}h_j(a,z_j,y_j^s)\left(1-i_j^{o\star}(a,h_j)\right)da.}_{\widehat{V_j^o}\left(z_j,a_j^*(z_j)\right)}\end{aligned}$$

It follows that

$$\begin{aligned}V_j(z_j;t,\tilde{a})=w_j(a_0)\Bigg[&\int_{a=a_0}^{a_j^*(z_j)}e^{-(\rho-g_j)(a-a_0)}\left\{h_j(a,z_j)(1-i_j^s(a,h_j))-\tfrac{c}{w_j(a)}\right\}da\\&+e^{-\rho(a_j^*(z_j)-a_0)}\,\widehat{V_j^o}\left(z_j,a_j^*(z_j)\right)\Bigg].\end{aligned}$$

Moreover, since a worker of age $\tilde{a}$ was $a_0$in period $\tau=t-(\tilde{a}-a_0)$, $w_j(a_0)=w_j(\tau)$ and $w_j(\tau)=w_j(t)e^{-g_j(\tilde{a}-a_0)}$. Therefore,

$$V_j(z_j;t,\tilde{a}) = w_j(t)e^{-g_j(\tilde{a}-a_0)}\Bigg[\int_{a=a_0}^{a_j^*(z_j)} e^{-(\rho-g_j)(a-a_0)}\left\{h_j(a,z_j)(1-i_j^s(a,h_j)) - \tfrac{c}{w_j(a)}\right\}da$$
$$+ e^{-\rho(a_j^*(z_j)-a_0)}\,\widehat{V_j^o}\big(z_j,a_j^*(z_j)\big)\Bigg].$$

or

$$V_j(z_j;t,\tilde{a}) = w_j(t)e^{-g_j(\tilde{a}-a_0)}\widehat{V}_j(z_j;t,\tilde{a}).$$

Finally, $d_c^\star(\boldsymbol{z},t,\tilde{a}) = 1$ if and only if

$$V_c(z_c;t,\tilde{a}) > V_r(z_r;t,\tilde{a}),$$

which yields

$$\frac{w_c(t)}{w_r(t)} > e^{(g_c-g_r)(\tilde{a}-a_0)}\frac{\widehat{V}_r(z_r;t,\tilde{a})}{\widehat{V}_c(z_c;t,\tilde{a})}.$$

C.4 Solution to the Worker's Intertemporal Utility Maximization Problem.

Given initial assets $b(a_0)$ and earnings $E(a,\boldsymbol{h})$,the solution to Problem 6 is standard. I here present it for the sake of completeness. The worker chooses consumption $\{c(a,\boldsymbol{h}),b(a,\boldsymbol{h})\}_{a=a_0}^{A}$ to solve Problem 6, given initial assets $b(a_0)$ and earnings $E(a,\boldsymbol{h})$.

Present-value Hamiltonian. Let $\lambda(a)$ denote the present-value co-state variable. The present-value Hamiltonian is

$$\widetilde{\mathscr{H}}(a) \;=\; e^{-\rho(a-a_0)}u\big(c(a,\boldsymbol{h})\big) \;+\; \lambda(a)\big(E(a,\boldsymbol{h})+rb(a)-c(a)\big).$$

First-order conditions. Assuming that intertemporal preferences are CRRA and given by

$$u(c) = \begin{cases} \frac{c^{1-\theta}}{1-\theta}, & \text{if } \theta \neq 1 \\ log(c) & \text{if } \theta = 1 \end{cases},$$

for $\theta > 0$, the necessary conditions are

$$\widetilde{\mathscr{H}}(a) = e^{-\rho(a-a_0)}\,\frac{c^{1-\theta}}{1-\theta} + \lambda(a)\big(E(a,\boldsymbol{h})+rb(a)-c(a)\big)$$

$$\frac{\partial \widetilde{\mathscr{H}}}{\partial c} = 0 \Rightarrow \& e^{-\rho(a-a_0)} c(a)^{-\theta} = \lambda(a) \tag{C.16}$$

$$-\frac{\partial \widetilde{\mathscr{H}}}{\partial b} = \dot{\lambda(a)} \Rightarrow \qquad \& \dot{\lambda(a)} = -r\lambda(a) \tag{C.17}$$

Differentiating Equation C.16with respect to a,we get

$$\dot{\lambda(a)} = -\rho e^{-\rho(a-a_0)} c(a)^{-\theta} - \theta e^{-\rho(a-a_0)} c(a)^{-\theta} \dot{c(a)}.$$

Replacing Equation C.17 into the equation above and re-arranging terms, yields the Euler Equation

$$\frac{\dot{c(a)}}{c(a)} = \frac{1}{\theta}(r-\rho).$$

The TVC is

$$\lim_{a \to A} \lambda(a)\, b(a) = 0.$$

Replacing $\lambda(a)$ from C.16

$$\lim_{a \to A} e^{-\rho(a-a_0)} c(a)^{-\theta}\, b(a) = 0,$$

which is only satisfied if $b(A) = 0$.

C.5 Value of Choosing Schooling Stage s.

Start with the value for a worker who finishes educational stage s at age a_s. For simplicity, assume that the fixed educational cost is constant during this period. From Equation 7, we get

$$V_j^{\star}\left(z_j \middle| a_s\right) = \int_{a=a_0}^{a_s} e^{-\rho(a-a_0)} \left(w_j(a) h_j(a, z_j)(1 - i_j^{s\star}(a, h_j))) - c_s - \epsilon_s^j \right) da + e^{-\rho(a_s-a_0)} V_j^{o\star}\left(z_j, a_s\right) \Bigg\}$$

Since $i_j^{s\star}(a, h_j) = 1$during the schooling period, we get

$$V_j^{\star}\left(z_j \middle| a_s\right) = -\int_{a=a_0}^{a_s} e^{-\rho(a-a_0)} \left(c_s + \epsilon_s^j\right) da + e^{-\rho(a_s-a_0)} V_j^{o\star}\left(z_j, a_s\right).$$

Calling

$$\bar{V}_j^o(z_j, a_s) = e^{-\rho(a_s-a_0)} V_j^{o\star}(z_j, a_s),$$

$$C_s(a_s) = \int_{a=a_0}^{a_s} e^{-\rho(a-a_0)} c_s da,$$

and

$$\bar{\epsilon}_s^j = -\int_{a=a_0}^{a_s} e^{-\rho(a-a_0)} \epsilon_s^j da,$$

we obtain

$$V_j(z_j | a_s) = \bar{V}_j^o(z_j, a_s) - C_s(a_s) - \bar{\epsilon}_s^j.$$

D Data Processing and Construction of Moments.

To calibrate the model, I construct a rich set of moments capturing the occupational structure, labor income, educational attainment, returns to schooling, and experience–earnings profiles for each country. The procedure is applied to harmonized microdata from IPUMS-I for the United States (2010 and 2020) and Brazil (2000 and 2010). In addition, I use data from the Penn World Table version 10.1 (henceforth PWT) to express income in purchasing power parity (PPP)–adjusted units, comparable across countries and over time.

Sample Selection and Cleaning. For both countries, the analysis focuses on employed male workers working for wages. Females are discarded to avoid biases to differences in labor force participation over time and across the development spectrum. Observations with missing or invalid income, hours worked, or educational attainment are excluded. Individuals in the armed forces or with unspecified occupations are dropped. Only workers whose income and hours come from their main job are retained. Sample age ranges are 18–65.

Income and Hours. For both countries, nominal labor income is first converted into international dollars using purchasing power parity (PPP). Specifically, I use data from the PWT, transforming income in local currency units (LCU) into 2010 U.S. GDP dollars. The conversion uses the variable *pl_ con*, which represents the price level of consumption in each country expressed in units of U.S. GDP of 2010, together with the exchange rate variable *xr2*. Once expressed in PPP-adjusted units, hourly income is computed using usual weekly hours worked, converted to annual or monthly equivalents as appropriate.

Logarithms of hourly income are generated for use in earnings regressions and moment construction.

Occupation Classification. Occupations are classified into cognitive-intensive (ISCO codes 01, 02, 03) and routine-intensive (all remaining ISCO codes except excluded groups). This classification is consistent across countries and is based on task-intensity measures from Acemoglu and Autor (2011a).

Educational Attainment and Experience. Years of schooling are constructed from country-specific education variables (*educus* for the U.S., *educbr* for Brazil) using a mapping to equivalent schooling years. Potential labor market experience is proxied following the standard approach in the literature Lagakos et al. (2018), subtracting years of schooling and six years of pre-school education from age for high school completers and adjusting accordingly for lower educational attainment.

Moment Construction. Using the cleaned and harmonized data, I compute key moments for each country, including:

Average log hourly earnings. I compute average log hourly earnings for a restricted set of workers that serves as the reference group across countries. The procedure is as follows. First, I restrict the sample to individuals with completed secondary education (12 years of schooling). This group provides a consistent benchmark for educational attainment in both the U.S. and Brazil. Second, I limit the sample to workers with at most five years of potential experience. The idea behind these restrictions is twofold. On the one hand, fixing the level of education is essential to ensure comparability across countries. On the other, while ideally one would use workers with zero experience, this group tends to be small and potentially noisy in the data. To address this, I average across individuals with up to five years of experience, which smooths measurement noise while maintaining coverage across both samples.

For each census year, I compute the mean of log hourly earnings weighted using census weights. I retain only the census years of interest (2010 and 2020 for the U.S., 2000 and 2010 for Brazil).

Educational Attainment by Occupation. To construct the distribution of workers by educational attainment within each occupation, I proceed as follows. For each broad occupation group, I then compute the share of workers in each educational category (*edattaind*) using census weights to guarantee representativeness. This procedure is applied symmetrically to the U.S. and Brazilian samples.

In addition to the overall distribution, I also compute educational attainment shares for 45-year-olds, as this group is in their prime working age and sufficiently established in the labor force, providing an accurate representation of the prevailing educational attainment structure in a country at a given point in time.

Average and Dispersion of Log Hourly Earnings by Occupation. To capture the level and dispersion of earnings within occupations, I compute the standard deviation of log hourly earnings by occupation. I focus on 45-year-olds as the main empirical target. This group is in their prime working age and sufficiently established in the labor market, thereby offering a reliable snapshot of the earnings distribution that reflects the prevailing occupational wage structure in a given country and period.

Returns to Schooling. To estimate the returns to schooling, I run Mincer-type regressions of log hourly earnings on educational attainment and potential experience. The specification follows the approach in Acemoglu and Autor (2011), but with a slight modification: instead of including a quartic in experience, I use a quadratic in experience along with interactions between education and this quadratic term. I find that this specification provides a better fit for the IPUMS-I data while preserving the main interpretation of returns to schooling.

Formally, for individual i in occupation j and year t:

$$\begin{aligned}\log w_i = \alpha_j &+ \beta_j \,\text{SchoolYrs}_i + \gamma_{1j}\,\text{Exp}_i + \gamma_{2j}\,\text{Exp}_i^2 \\ &+ \delta_{1j}\,\text{SchoolYrs}_i \cdot \text{Exp}_i + \delta_{2j}\,\text{SchoolYrs}_i \cdot \text{Exp}_i^2 + \theta_j' \mathbf{X}_i + \varepsilon_i.\end{aligned}$$

where SchoolYrs_i is years of schooling, Exp_i is potential experience, and $\mathbf{X}_i$ includes demographic controls. Coefficients β_j measure baseline returns to schooling for occupation j, while δ_{1j} and δ_{2j} capture how returns vary with experience. Regressions are weighted using individual census weights (*perwt*).

Experience-Earnings Profiles. Experience-earnings profiles are constructed by computing the weighted mean of log-hourly earnings at each level of potential experience, separately by occupation. Since the data are cross-sectional, these profiles represent synthetic cohorts rather than true longitudinal trajectories, capturing the typical pattern of earnings as workers accumulate experience. For the United States, profiles are based on the 2010 IPUMS-I cross-section, while for Brazil, profiles are based on the 2000 cross-section.

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