Chained Indices Unchained:
Structural Transformation and the Welfare Foundations of Income Growth Measurement

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Preliminary Draft: August 2025

Abstract

This paper studies how to accurately measure welfare-relevant growth in the presence of structural transformation, focusing on the role of chained quantity indices. Using a structural transformation model calibrated to the U.S. economy since 1980, we compare two approaches grounded in the Fisher–Shell principle: a current-base equivalent variation measure and a chained Fisher–Shell index. We show that the chained index aligns with the Divisia index and consistently tracks welfare gains along the balanced growth path, while fixed-base indices misrepresent historical growth. These distortions arise from evaluating income using a single base preference order in a context of evolving preferences and relative prices. Our findings reinforce the theoretical justification for using chained indices in national accounts, not only for statistical accuracy but also for consistency with welfare measurement in dynamic economies. We conclude that structural transformation motivates and legitimises the use of chained indices to measure real income growth.

Keywords: Structural transformation, Non-homothetic preferences, Investment specific technical change, Chained quantity indexes, GDP measurement, Equivalent variation, Divisia index, Fisher-ideal index, and Fisher–Shell index.

JEL classification numbers: C43, E01, E13, O11, O14, O41, O47.

*The author expresses gratitude to Akos Valentiniy for sharing the data used in Herrendorf et al. (2021), and Ariel Burstein for a highly beneficial discussion held during a visit to UCLA in April 2024.

1 Introduction

Modern macroeconomics was built upon the well-established **?** facts. These empirical regularities describe the stability of key aggregate variables, such as the growth rates of output and capital and the constancy of factor income shares. They motivated the development of the Neoclassical growth model, in which the economy converges to a balanced growth path (BGP) along which all aggregates grow at constant rates and relative prices remain stable. However, two strands of empirical research challenge the universality of this framework by documenting systematic departures from balanced growth: the literature on structural transformation and the literature on embodied technical progress. These departures raise fundamental measurement challenges, as standard national accounting practices may fail to accurately capture welfare-relevant output in the presence of persistent changes in relative prices and shifting expenditure patterns.

Structural transformation, the progressive reallocation of economic activity from agriculture to industry and then to services as economies develop, has profound implications for understanding long-run growth. First systematically documented by **?**, this process is reflected in sustained changes in sectoral labour shares, consumption patterns, and the composition of value added. Because services typically exhibit slower productivity growth and rising relative prices, their increasing weight in aggregate consumption alters the composition of output in ways that standard measurement frameworks may misrepresent.[1]

To explain these unbalanced properties of the data, a rich theoretical literature has emerged since the early 2000s. **?** propose a unified growth model in which non-homothetic preferences drive sectoral reallocation even when all sectors experience identical productivity growth. Their model reconciles Kuznets's observations with the stylised facts of balanced growth previously documented by **?**. **?** show that differential sectoral productivity growth leads to systematic changes in labour and consumption shares across sectors even when preferences are homothetic. **?** further highlight the role of capital deepening, demonstrating how differences in capital intensity can generate un-

[1]This observation is closely related to **?**, who highlighted the rising cost of services due to slower productivity improvements, a phenomenon known as Baumol's cost disease.

balanced sectoral growth within an economy experiencing balanced aggregate growth. Together, these models formalise the interaction between income effects, relative price dynamics, and technological heterogeneity in shaping the evolution of the sectoral structure. **?** unify earlier mechanisms by jointly allowing for non-homothetic preferences and sector-specific productivity growth, showing how both income and substitution effects shape the reallocation of economic activity. By closely matching key empirical trends in expenditure shares, labour allocations, and relative prices, their model provides a benchmark for interpreting the long-run patterns of structural change.[2]

Embodied technical progress refers to improvements in technology that are incorporated into new capital goods, making newer vintages of capital more productive than older ones. The literature on embodied technical progress builds on one of the most persistent empirical regularities in postwar U.S. data: the secular decline in the relative price of investment goods, particularly equipment and durable goods, compared to non-durable consumption and services. This trend reflects the fact that new capital goods embody increasingly advanced technologies, a phenomenon known as investment-specific technological change (ISTC). In a perfectly competitive economy, if consumption and investment are distinct goods and embodied technical progress is strictly positive, then the relative price of investment must decline over time, since the same nominal expenditure purchases increasingly efficient capital. At equilibrium, as shown by Greenwood et al. (1997), the relative price of capital goods in terms of consumption goods is inversely proportional to investment efficiency and therefore declines at the rate of embodied technical progress.

Empirically, this decline has been well documented beginning with the seminal work of **?**, who showed that quality-adjusted price indexes for durable goods fell significantly in the postwar period. His findings led to major revisions in U.S. national accounting practices. In 1996, the Bureau of Economic Analysis (BEA) adopted chained Fisher-ideal in-

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[2] For a comprehensive exposition of their approach, its empirical implementation and the relation with the previous literature, see **?**. For alternative assumptions on non-homothetic preferences in this context, see also **?**, Boppart (2014), **?**, **?**, **?** and **?**. A large body of empirical research confirms the systematic nature of structural transformation across countries. In a cross-country analysis of 29 economies over the postwar period, **?** document common patterns of sectoral reallocation, particularly the movement of labour from agriculture to industry and then to services, accompanied by rising productivity gaps across sectors.

dices to replace fixed-base Laspeyres indices in GDP and investment price measurement, recognising that fixed-base methods could not accurately track the evolving expenditure patterns resulting from persistent changes in relative prices. These revisions followed the sharp price declines in equipment investment and durable goods consumption observed in the 1980s, which exposed the limitations of fixed-base indices and the distortions they introduced in growth measurement.[3] The chained Fisher-ideal index improves the accuracy and stability of GDP measurement by continuously updating expenditure weights in line with consumption and investment dynamics. However, its welfare implications remain less well understood, especially in the context of structural transformation. This paper evaluates whether chained indices reliably track welfare-relevant output growth, grounding the analysis in the economic theory of index numbers.[4]

Building on these insights, Herrendorf et al. (2021) propose a unified framework that integrates structural transformation and embodied technical progress into a single general equilibrium model. Their approach allows for both sector-specific productivity trends and investment-specific technological change, capturing the dual sources of unbalanced growth documented in the data. By embedding vintage capital and non-homothetic preferences within a three-sector model, they show how the simultaneous evolution of preferences, productivity, and relative prices jointly determine the sectoral composition of output and investment over time. This framework enables a richer understanding of long-run growth dynamics and highlights the importance of using consistent measurement tools, such as chained indices and quality-adjusted prices—to accurately track welfare-relevant changes in economic activity..

It is important to notice that all evidence supporting the more than 20 years of research on both structural transformation and embodied technical progress are founded in the chained indices used in the national accounts methodology. This paper contributes to the structural transformation and the embodied technical progress literature by investigating how and why the measurement of real income, especially through chained Divisia indices, can more effectively capture welfare-relevant growth. We analyse this question within a general equilibrium model that reflects the key features of structural

[3]See Parker and Triplett (1996); Whelan (2002), among many others.

[4]For a revision of this literature, see Fisher (1922); Fisher and Shell (1968); Diewert (1976, 1978); Caves et al. (1982), among many others.

change. The analysis highlights the importance of index number theory in tracking the evolution of welfare over time.

In the framework of the structural transformation model proposed by Herrendorf et al. (2021), this paper examines how real income measurement affects the evaluation of welfare gains. The analysis focuses on the Divisia index, which is well approximated in practice by a Fisher-ideal quantity index and is increasingly used in national accounts. We contrast two recent theoretical frameworks that provide differing welfare interpretations for such chained indices: Baqaee and Burstein (2023) and Durán and Licandro (2025). Both approaches build on the Fisher–Shell principle, originally proposed by Fisher and Shell (1968), which asserts that in a world of changing preferences, welfare comparisons should adopt a consistent preference order. This principle is especially relevant when preferences are non-homothetic and evolve over time, as is typical in structural transformation models. According to this view, comparing welfare between two different points in time requires selecting a common preference order, or *base preference order*, to isolate the welfare consequences of income and expenditure changes. While both frameworks invoke the Fisher–Shell principle, they diverge in how they apply it to dynamic settings with structural transformation and changing price structures. This divergence reflects the broader theoretical and methodological challenge of applying index number theory, which traditionally assumes homothetic and stable preferences in a static framework, to contexts with persistent shifts in preferences and relative prices in a dynamic model.[5]

Baqaee and Burstein (2023) propose an equivalent variation measure of welfare gains, using current preferences as a base preference order to evaluate the entire history of income growth. Their approach requires stability and homotheticity of preferences to ensure that the chained Divisia index accurately reflects welfare gains. Any deviation from these conditions makes the chained Divisia index inconsistent with their welfare measure.

Durán and Licandro (2025), by contrast, present an alternative approach that applies the Fisher–Shell principle in a dynamic context by chaining welfare gains between

[5]See also the seminal paper by Licandro et al. (2002) and the application to discrete time by Duernecker et al. (2021).

contiguous periods. They refer to it as the chained Fisher–Shell quantity index. A salient advantage of this method is that it does not need the stability and homotheticity assumptions required by Baqaee and Burstein (2023). Instead, in the framework of continuous-time dynamic general equilibrium models, Durán and Licandro (2025) show that a chained Fisher–Shell true quantity index aligns with a chained Divisia index. Then, chained Divisia indices continuously adapt to changes in preferences, income, and prices, offering a flexible welfare-based measure of income growth. Under this approach, at any moment in time, welfare gains are measured using the preferences relevant to that specific time and then chained.

The result in Durán and Licandro (2025) does not invalidate the measure proposed by Baqaee and Burstein (2023); rather, it highlights that chained Divisia indices are also welfare-based. Thus, it illustrates the broader principle that multiple true quantity indices can capture welfare gains in the same context. In light of this result, Proposition 1 in Baqaee and Burstein (2023) can be understood to demonstrate that the chained Divisia index and the Baqaee-Burstein equivalent variation measure, being alternative measures of welfare gains, both grounded on the Fisher-Shell principle, converge to each other only under homothetic and stable preferences.

To understand the implications of these contrasting approaches, we quantitatively evaluate a slightly modified version of the structural transformation model proposed by Herrendorf et al. (2021). The model captures key features of the structural transformation observed in the U.S. economy since 1980. Within this framework, we compare two welfare-based measures of real income: a current-base Fisher–Shell equivalent variation measure, following the approach of Baqaee and Burstein (2023), and a chained Fisher–Shell index, as developed in Durán and Licandro (2025).

The analysis shows that a chained Fisher–Shell index, which continuously updates preferences and prices, yields a welfare-consistent measure of real GDP growth, precisely equal to a chained Divisia index. This measure is invariant to the choice of reference time and accurately reflects the balanced growth path of the economy. By contrast, a current-base index systematically underestimates welfare gains along the structural transformation path when compared to a Divisia index, while a past-base Fisher–Shell index overestimates them. As time passes and the history of growth is evaluated through successive current-base Fisher–Shell indices, past economic progress appears to shrink repeatedly. These distortions arise from evaluating welfare using static, time-specific preferences and prices, which fail to capture the evolving nature of consumption patterns. Chapters 2 and 3 formalise the theoretical link between equivalent variation measures and index number formulas, while Chapter 4 quantifies the resulting biases in real income measurement through a calibrated version of the model.

This result underscores the central role of measurement in the analysis of structural transformation. Although chained indices such as the Fisher–ideal index are already widely used in statistical practice, largely for pragmatic reasons such as avoiding large revisions when base years change, their deeper welfare-theoretic foundation lies in their capacity to accommodate changing preferences and relative prices. From this perspective, the contribution is not simply a technical refinement in growth accounting, but a broader assertion of the conceptual coherence of welfare-based measurement in dynamic settings. Rather than treating GDP measurement and structural transformation as distinct domains, the paper argues that the latter provides a compelling rationale for the former: accurate welfare-based measurement is essential to understand the true nature of economic progress.

In addition to its theoretical implications, the approach offers a direct and transparent connection between model and data. The chained Divisia index can be constructed from national accounts variables, specifically, consumption value-added shares and the investment share in aggregate expenditure, without requiring auxiliary assumptions or data transformations. These same expenditure shares, which play a central role in the empirical structural transformation literature, thus also serve as sufficient statistics for measuring welfare gains. This dual function enhances the empirical applicability of the framework and reinforces the interpretability of welfare-based growth accounting.

2 Structural Transformation Model

To compare the behavior of fixed-base Fisher–Shell indices with that of the chained Fisher–Shell index, this section adopts the structural transformation model developed by Herrendorf et al. (2021). Hereafter, we refer to it as the HRV structural transformation model. In a close economy, the model features three sectors: goods, services, and investment, with non-homothetic preferences defined over consumption of goods and services. Both goods and services are also used in the production of investment, which is governed by a CES technology. This framework provides a rich environment for evaluating how fixed-base and chained true quantity indices capture welfare-relevant income growth along a balanced growth path with non-homothetic preferences and time-varying income elasticities and elasticities of substitution.

Description of technology. We assume that goods and services are produced by distinct sectors. Value added in each sector is generated using Cobb–Douglas production technologies

$$Y_{j,t} = A_{j,t} K_{j,t}^{\theta} L_{j,t}^{1-\theta}, \tag{1}$$

where j, $j \in \{g, s\}$, indexes the goods and services sectors, respectively. These production functions share a common capital intensity parameter θ, $\theta \in (0,1)$, but differ in total factor productivity, denoted $A_{j,t}$. Each sector employs the same homogeneous production factors, capital $K_{j,t}$ and labor $L_{j,t}$, which are freely mobile across sectors. It follows that

$$K_{g,t} + K_{s,t} = K_t \quad \text{and} \quad L_{g,t} + L_{s,t} = L_t,$$

where K_t and L_t denote the economy's total capital and labor endowments, respectively.

Investment is produced using a CES technology

$$I_t = A_{x,t} \left(\omega^{\frac{1}{\varepsilon}} X_{g,t}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\omega)^{\frac{1}{\varepsilon}} X_{s,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2}$$

where $X_{g,t}$ and $X_{s,t}$ denote inputs from the goods and services sectors, respectively. The parameter ε, $\varepsilon > 0$, governs the elasticity of substitution between these inputs, and ω, $\omega \in (0,1)$, captures their relative weight in the production of investment. The term $A_{x,t}$ represents investment-specific productivity, which is neutral with respect to input composition.

Capital depreciates at a constant rate $\delta > 0$ and evolves according to the standard law of motion

$$\dot{K}_t = I_t - \delta K_t. \tag{3}$$

In equilibrium, efficiency requires that output in each sector be allocated between consumption and investment inputs, s.t.,

$$C_{g,t} + X_{g,t} = Y_{g,t} \quad \text{and} \quad C_{s,t} + X_{s,t} = Y_{s,t},$$

where $C_{g,t}$ and $C_{s,t}$ denote consumption of goods and services, respectively.

Equilibrium prices. Under constant returns to scale in the investment sector and perfectly competitive markets, the equilibrium path of relative prices depends solely on differences in sectoral technical progress. This property ensures that the evolution of prices is independent of the interaction between income and substitution effects in consumption decisions. Moreover, to better map the evolution of relative prices, we assume that the price of consumption goods is distorted by the factor $\mathrm{e}^{-\zeta t}$, $\zeta > 0$. [6] It is straightforward to show that the relative price of goods, $P_{g,t}$, to services, $P_{s,t}$, satisfies

$$\frac{P_{g,t}}{P_{s,t}} = \frac{A_{s,t}}{A_{g,t}\,\mathrm{e}^{\zeta t}},$$

that is, it equals the inverse of the relative sectoral TFPs, adjusted by the price distortion.

We adopt the investment good as the numeraire. It is easy to see that, at equilibrium, the prices of goods and services, relative to the prices of investment, are given by

$$P_{g,t} = \frac{\mathcal{A}_t}{A_{g,t}\,\mathrm{e}^{\zeta t}}, \qquad \text{and} \qquad P_{s,t} = \frac{\mathcal{A}_t}{A_{s,t}}, \tag{4}$$

where

$$\mathcal{A}_t = A_{x,t}\left(\omega A_{g,t}^{\varepsilon-1} + (1-\omega)A_{s,t}^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}. \tag{5}$$

As shown below, $\mathcal{A}_t$ corresponds to total factor productivity in the investment sector. Notice that the implicit assumption is that the prices of goods and services are not distorted in the market for investment goods. The price distortion is then restricted to the market for consumption goods. At equilibrium in the investment sector, the ratios of expenditure shares and input quantities on goods and services are given by

$$\frac{P_{g,t}X_{g,t}}{P_{s,t}X_{s,t}} = \frac{\omega}{1-\omega}\left(\frac{A_{s,t}}{A_{g,t}\mathrm{e}^{\zeta t}}\right)^{1-\varepsilon} \qquad \text{and} \qquad \frac{X_{g,t}}{X_{s,t}} = \frac{\omega}{1-\omega}\left(\frac{A_{g,t}\mathrm{e}^{\zeta t}}{A_{s,t}}\right)^{\varepsilon}.$$

In what follows, and consistently with the data, we impose the following assumption on sectoral TFP:

Assumption 1. *Total factor productivity evolves according to* $A_{g,t} = A_{g,0}\,e^{g_g t}$, $A_{s,t} = A_{s,0}\,e^{g_s t}$, *and* $\mathcal{A}_t = \mathcal{A}_0\,e^{g_{\mathcal{A}} t}$, *where the growth rates satisfy* $g_{\mathcal{A}} > g_g > g_s$. [7]

[6] As will become clear later, this distortion does not affect the allocation of resources between consumption expenditure and investment, but rather the allocation between the consumption of goods and the consumption of services. Because the distortion does not affect the dynamics of the model, it may be transitory to the period under study.

[7] In equation (5), $A_{x,t}$ adjusts to be consistent with this assumption.

As a consequence, the growth rates of $P_{g,t}$ and $P_{s,t}$, denoted respectively π_g and π_s, are given by

$$\pi_g = g_{\mathcal{A}} - g_g - \zeta < g_{\mathcal{A}} - g_s = \pi_s.$$

Under Assumption 1, the model delivers the prediction, consistent with the data, that consumption service prices grow faster than consumption goods prices, and that the overall consumption price index increases more rapidly than the price of investment goods.

Aggregate production technology. Like in National Accounts, we define aggregate final output, measured in units of the investment good, as

$$Y_t = P_{g,t}C_{g,t} + P_{s,t}C_{s,t} + I_t. \tag{6}$$

Herrendorf et al. (2021) show that at equilibrium the aggregate production function is

$$Y_t = \mathcal{A}_t K_t^{\theta} L_t^{1-\theta}, \tag{7}$$

where $\mathcal{A}_t$ is the investment-sector productivity index defined in equation (5). Notice, however, that Y_t is arbitrarily measured in units of the investment good.

Aggregate dynamics. Combining equations (3), (6), and (7), the law of motion for capital, for $t \geqslant 0$, given the exogenous path of $\mathcal{A}_t$ and an initial capital stock $K_0 > 0$, can be written as

$$\dot{K}_t = \underbrace{\mathcal{A}_t K_t^{\theta} L_t^{1-\theta} - E_t}_{I_t} - \delta K_t,$$

where

$$E_t = P_{g,t}C_{g,t} + P_{s,t}C_{s,t} \tag{8}$$

denotes total consumption expenditure, measured in units of the investment good.

Non-homothetic preferences. Let population be denoted by N_t, growing at a constant rate n, $n > 0$. At each point in time t, every individual supplies h_t units of human capital, which grows exogenously at rate g_h, $g_h > 0$. Total labor supplied is thus $L_t = h_t N_t$, and is offered inelastically.

The economy features an infinitely lived representative household whose preferences are represented by the intertemporal utility function

$$\int_0^{\infty} U(c_{g,t}, c_{s,t})\, \mathrm{e}^{(n-\rho)t}\, \mathrm{d}t, \tag{9}$$

where $\rho > n$ is the subjective discount rate and $U(\cdot)$ is per capita utility. The instantaneous utility function $U(\cdot)$ is assumed to belong to the *price-independent generalised linear* (PIGL) class –see **?**. It depends on per capita consumption of goods and services, defined as $c_{g,t} = C_{g,t}/N_t$ and $c_{s,t} = C_{s,t}/N_t$, respectively.

Since the PIGL class generally lacks a closed-form direct utility representation, we work with its indirect utility form $V(e_t, P_{g,t}, P_{s,t})$, where $e_t = E_t/N_t$ denotes per capita consumption expenditure. Following Boppart (2014), we make the following assumption

Assumption 2. *The instantaneous utility function $U(c_{g,t}, c_{s,t}; e_t)$ belongs to the PIGL class and has the following indirect utility representation:*

$$V(e_t, P_{g,t}, P_{s,t}) = \frac{1}{\chi}\left(\left(\frac{e_t}{P_{s,t}}\right)^{\chi} - 1\right) + \frac{\eta}{\gamma}\left(1 - \left(\frac{P_{g,t}}{P_{s,t}}\right)^{\gamma}\right), \tag{10}$$

where $\eta > 0$ and $1 > \gamma > \chi > 0$.

The functional form of the indirect utility function in (10) implies non-homothetic preferences whenever $\chi > 0$, and it captures the two classical channels behind structural transformation: income and price effects. The two terms on the right-hand side of (10) admit a natural interpretation. The first term represents the utility obtained from allocating all consumption expenditure to services, while the second term measures the additional utility gain from reallocating part of that expenditure to the consumption of goods. The first term depends positively on total income e, reflecting the *income effect*, while the second term depends negatively on the relative price of goods to services, P_g/P_s, reflecting the *substitution effect*.

The expenditure share on goods, derived using Roy's identity, is given by

$$s_{g,t} := \frac{P_{g,t}c_{g,t}}{e_t} = \eta\left(\frac{e_t}{P_{s,t}}\right)^{-\chi}\left(\frac{P_{g,t}}{P_{s,t}}\right)^{\gamma}. \tag{11}$$

The first term on the right-hand side of (11) captures the income effect: as total expenditure e_t rises, the consumption bundle shifts from goods to services when $\chi > 0$, consistent with the interpretation of services as luxuries and goods as necessities. The second term captures the substitution effect: under $\gamma > 0$, goods and services are complements, and the magnitude of the substitution effect increases with γ.

Assumption 3 below states the necessary and sufficient conditions for the indirect utility function in (10) to satisfy standard regularity conditions –Lemma 1 in Boppart (2014).

Assumption 3. *The following necessary and sufficient condition, for the indirect utility function in (10) to satisfy standard regularity conditions, holds*

$$e_t^{\chi} \geqslant \frac{1-\chi}{1-\gamma}\,\eta\, P_{g,t}^{\gamma} P_{s,t}^{\chi-\gamma} \qquad \textit{or equivalently} \qquad s_{g,t} \leqslant \frac{1-\gamma}{1-\chi}.$$

The second inequality follows directly from applying equation (11). In the next section, we restrict the set of calibrated parameters to those that satisfy this condition.

By substituting the expenditure share (11) into the indirect utility function (10), we obtain an alternative representation:

$$V(e_t, P_{g,t}, P_{s,t}) = \left(\frac{1}{\chi} - \frac{s_{g,t}}{\gamma}\right)\left(\frac{e_t}{P_{s,t}}\right)^{\chi} - \frac{1}{\chi} + \frac{\eta}{\gamma}. \tag{12}$$

At the optimal allocation, utility increases with total consumption expenditure and decreases with the share of goods in consumption, reinforcing the role of non-homothetic preferences and relative prices in shaping structural transformation.

Intertemporal problem and intratemporal allocation. Under Assumption 2, the representative household chooses a path $\{e_t, k_t\}$ for per capita consumption expenditure and capital that solves the following dynamic program

$$v(k_t) = \max_{\{e_t, k_t\}} \int_0^{\infty} V(e_t, P_{g,t}, P_{s,t})\, \mathrm{e}^{(n-\rho)t} \mathrm{d}t, \tag{13}$$

subject to the law of motion for capital per capita

$$\dot{k}_t = \widehat{\mathcal{A}}_t k_t^{\theta} - e_t - (\delta + n)k_t, \tag{14}$$

where $k_t = K_t/N_t$ and $\widehat{\mathcal{A}}_t = \mathcal{A}_t h_t^{1-\theta}$ and $\rho > n > 0$. Notice that the only relevant term in the indirect utility function is $\frac{1}{\chi}\left(\frac{e_t}{P_{s,t}}\right)^{\chi}$. Consequently, preferences are constant intertemporal elasticity of substitution (CIES) with respect to consumption expenditure, with the discount factor given by $\Gamma_t = P_{s,t}^{-\chi} \mathrm{e}^{(n-\rho)t}$, which from Assumption 1 declines over time since $P_{s,t}$ increases, as in the data.[8]

[8]In this framework, the intertemporal elasticity of substitution is $1/(1-\chi)$.

The Euler equation characterising the solution to the household's problem is[9]

$$\frac{\dot{e}_t}{e_t} = \frac{1}{1-\chi}\left(\theta\widehat{\mathcal{A}}_t k_t^{\theta-1} - \rho - \delta - \chi\,\pi_s\right). \tag{15}$$

An equilibrium path for $\{e_t, k_t\}$ is given by the system formed by equations (14) and (15), given the initial condition k_0, $k_0 > 0$, and a standard transversality condition.[10] The intratemporal allocation of expenditure between goods and services can then be characterized by equation (11). Given this expression, $c_{g,t}$ can be solved for directly, and $c_{s,t}$ follows by inverting the identity $e_t = P_{g,t}c_{g,t} + P_{s,t}c_{s,t}$.

Aggregate Balanced Growth Path (ABGP). Along the aggregate balanced growth path (ABGP), the per capita variables $\{k_t, e_t, y_t\}$ grow at the constant rate

$$g_k = \frac{g_{\mathcal{A}}}{1-\theta} + g_h.$$

From the Euler equation (15), capital per capita evolves according to

$$k_t^* = \kappa^{\frac{1}{\theta-1}}\widehat{\mathcal{A}}_t^{\frac{1}{1-\theta}}, \qquad \text{where} \qquad \kappa := \frac{\rho+\delta+\chi\,\pi_s+(1-\chi)g_k}{\theta}. \tag{16}$$

Substituting into equation (14), the path of per capita consumption expenditure satisfies

$$e_t^* = (\kappa-\delta-n-g_k)k_t^*. \tag{17}$$

From the aggregate production function (7), gross and net nominal income per capita are given by

$$y_t^* = \widehat{\mathcal{A}}_t {k_t^*}^{\theta} = \kappa k_t^* \quad \text{and} \quad m_t^* = (\kappa-\delta)k_t^*, \tag{18}$$

where the second equality in the first equation follows directly from equation (16). The consumption shares of gross and net income, respectively, are

$$\frac{e_t^*}{y_t^*} = \frac{\kappa-\delta-n-g_k}{\kappa} \quad \text{and} \quad \frac{e_t^*}{m_t^*} = \frac{\kappa-\delta-n-g_k}{\kappa-\delta}.$$

We impose Assumption 4, for Proposition 1 in Herrendorf et al. (2021) to hold, implying that a ABGP exists.

[9]See Lemma 4 in Boppart (2014).

[10]In the particular case of homothetic preferences, $\chi = 0$ and the Euler equation corresponds to the case of unit intertemporal elasticity of substitution. The model becomes the two-sector growth model in Greenwood et al. (1997) without leisure.

Assumption 4. *For a ABGP to exist, let us assume*

$$\rho > \chi \left(\frac{\theta}{1-\theta} g_{\mathcal{A}} + g_h + g_s \right) + n.$$

Assumption 4 is the standard condition ensuring that the objective in (13) is bounded.

On the Divisia index. In the following section, we discuss the role of the Divisia index in measuring welfare gains. Before that, we compute the Divisia index in the ABGP of the structural transformation economy and study its main components.

From (11), the growth rate of consumption goods along a ABGP is

$$g_{c_g} = (1-\chi)\frac{\theta}{1-\theta} g_{\mathcal{A}} + (1-\chi) g_h + (1-\gamma)(g_g + \zeta) + (\gamma - \chi) g_s > 0.$$

It is constant and directly benefits from sectoral technical progress, g_g and g_s, human capital accumulation, g_h, and embodied technical progress, $\frac{\theta}{1-\theta} g_{\mathcal{A}}$, but suffers from the indirect effect of the shift in expenditure towards services, as represented by all terms related to χ and γ.

The share of goods in total consumption expenditure is, from equation (11),

$$s_{g,t} = \eta \left(\frac{e_t}{P_{s,t}} \right)^{-\chi} \left(\frac{P_{g,t}}{P_{s,t}} \right)^{\gamma}.$$

It is easy to see that along the ABGP $s_{g,t}$ is decreasing, a well-known property of the consumption expenditure share on goods observed in the data, and converging to zero as time goes to infinity. Consequently, at an ABGP, if Assumption (3) holds at the initial time, it holds forever.

From equation (11), the real consumption of services satisfies

$$c_{s,t} = \frac{e_t}{P_{s,t}} - \eta \left(\frac{e_t}{P_{s,t}} \right)^{1-\chi} \left(\frac{P_{g,t}}{P_{s,t}} \right)^{\gamma} = (1 - s_{g,t}) \frac{e_t}{P_{s,t}}. \tag{19}$$

The growth rate $g_{c_s,t} := \frac{\dot{c}_{s,t}}{c_{s,t}}$ can be computed from this expression. As time goes to infinity, $g_{c_s,t}$ converges from above to $g_k - \pi_s = \frac{\theta}{1-\theta} g_{\mathcal{A}} + g_h + g_s$. The consumption of services benefits not only from direct gains in sectoral TFP, as given by g_s, but also from gains in human capital, g_h, and investment-specific technical progress, as given by $\frac{\theta}{1-\theta} g_{\mathcal{A}}$.

Finally, along the ABGP, the Divisia index for gross and net domestic product per capita is defined as follows:

$$g_t^D = s_e \underbrace{\left(s_{g,t} g_{c_g} + (1 - s_{g,t}) g_{c_s,t}\right)}_{\widehat{g}_{e,t}} + (1 - s_e) g_x, \tag{20}$$

where $\widehat{g}_{e,t}$ is the growth rate of real consumption expenditure per capita, and $g_x = g_k$ is the growth rate of per capita investment. The shares of consumption expenditure in gross and net income are given by $s_e = \left\{\frac{e_t^*}{y_t^*}, \frac{e_t^*}{m_t^*}\right\}$, respectively. Note that $\widehat{g}_{e,t}$ differs from the growth rate of consumption expenditure measured in units of the investment good, $g_e = g_k$.

The effect of structural transformation along a ABGP depends on two factors related exclusively to consumption expenditure. On one side, consumption shifts from goods to services. Since services grow in real terms faster than consumption, this has a positive effect on growth. Moreover, the growth rate of services is reducing along the ABGP, which has a negative effect on the Divisia index.

However, at the ABGP, output in units of the investment good is growing at a constant rate. Since the price of consumption expenditure grows faster than the price of investment, when output is deflated by the true price index, its growth rate should be decreasing.

Proposition 1. *Under Assumptions 1 to 4, at an ABGP of the structural transformation economy, the growth rate of real GDP, as measured by the Divisia index, is permanently declining.*

PROOF: See Appendix A.1.

It is important to notice that the Divisia price index for consumption expenditure is given by

$$\pi_{e,t} = s_{g,t}\pi_g + (1 - s_{g,t})\pi_s.$$

Since both π_g and π_s are strictly positive and $\pi_s > \pi_g$, as observed in the data, the price of investment declines permanently relative to the price of consumption. Moreover, this relative decline accelerates as the share of goods in total consumption expenditure, $s_{g,t}$, decreases over time. As a result, the growth rate of real GDP, and thus the growth rate of TFP, when measured by a Divisia index exhibit a persistent downward trend, providing a theoretical explanation of the observed productivity slowdown.

3 Index Number Theory and GDP Growth

3.1 Bellman Representation and the Fisher–Shell Principle

Bellman representation. Following Durán and Licandro (2025), the Bellman representation of the preferences of the household at time t is given by

$$W(c_{g,t}, c_{s,t}, x_t; \nu_t) = U(c_{g,t}, c_{s,t}) + \nu_t x_t, \tag{21}$$

where $x_t = \dot{k}_t$ denotes net investment per capita and $\nu_t = v'(k_t)$ is the marginal value of capital at time t, which is taken as given when consumption and investment decisions are made. In this representation, preferences at time t are non-homothetic and time-varying, indexed by the marginal value of capital ν_t. The quasi-linearity of the Bellman representation is an artifact of the additive separability of intertemporal preferences, which implies that the marginal value of capital is independent of current consumption decisions.

To assess the welfare implications of output growth in a structural transformation economy, it is necessary to represent household preferences over per capita current consumption in goods and services and per capita current investment in a way that is compatible with intertemporal optimization. The Bellman representation offers such a framework by mapping the recursive structure of preferences into a static formulation that depends only on current choices, net investment being valued at the marginal value of capital. This representation captures the trade-off between present consumption and future utility derived from investment, enabling meaningful welfare comparisons over time. Crucially, because it summarizes the value of postponed consumption through the marginal value of capital, the Bellman representation provides a consistent basis for applying index number theory —such as the Fisher–Shell true quantity index— in dynamic settings. This allows the construction of output growth indices that accurately reflect welfare changes without requiring knowledge of the entire flow of consumption utility.[11]

At any time t, the representative household maximizes the Bellman representation of preferences (21) with respect to $\{c_{g,t}, c_{s,t}, x_t\}$, subject to the per capita budget constraint

$$P_{g,t}c_{g,t} + P_{s,t}c_{s,t} + x_t = m_t, \tag{22}$$

where m_t denotes current net per capita income. Note that in equilibrium $m_t = y_t - \delta k_t$. It is important to emphasize that y_t and m_t refer to nominal income per capita, gross, and net, as they are expressed in units of the numeraire. They do not represent real expenditure, although they are measured in units of the investment good. Consequently, aggregate nominal income is given by $Y_t = y_t N_t$ and $M_t = m_t N_t$, which we take as our measures of nominal GDP and NDP, respectively. All the arguments that follow are independent of the choice of the numeraire.

Before applying index number theory to the Bellman representation in (21), Proposition 2 below derives the corresponding indirect utility function and the associated expenditure function in the particular case of PIGL preferences.

Proposition 2. *Under Assumptions 1 to 4, the indirect utility and expenditure functions associated with the Bellman representation of preferences in equation (21) are, respectively, given by*

$$u(m, P_g, P_s; \nu) = V\left((\nu P_s^{\chi})^{\frac{1}{\chi-1}}, P_g, P_s\right) + \nu\left(m - (\nu P_s^{\chi})^{\frac{1}{\chi-1}}\right), \tag{23}$$

and

$$f(w, P_g, P_s; \nu) = (\nu P_s^{\chi})^{\frac{1}{\chi-1}} + \frac{w}{\nu} - \frac{V\left((\nu P_s^{\chi})^{\frac{1}{\chi-1}}, P_g, P_s\right)}{\nu}, \tag{24}$$

where w is an arbitrary level of utility as in (21).

Proof: See Appendix A.2.

Thanks to the quasi-linearity of the Bellman representation, which as explained above is a direct implication of intertemporal separable preferences, the indirect utility function is linear in net income m, and the expenditure function is linear in the contribution of current consumption and current net investment to welfare w.

[11]See Durán and Licandro (2025) for a more detailed discussion.

An important property of the HRV structural transformation model is that the marginal value of consumption expenditure measured in units of the investment good, $\frac{\partial V(\cdot)}{\partial e}$, must be equal to the marginal value of capital, ν, which implies that consumption expenditure in equilibrium is[12]

$$e = (\nu P_s^{\chi})^{\frac{1}{\chi-1}} . \tag{25}$$

The higher the value of capital, ν, the greater the opportunity cost of consumption, and therefore the lower the consumption expenditure.

Along the ABGP, $P_{s,t}$ and e_t grow at the constant rates $\pi_s = g_{\mathcal{A}} - g_s > 0$, and $g_k = \frac{g_{\mathcal{A}}}{1-\theta} + g_h$, respectively. Consequently, the marginal value of capital ν_t grows at rate $g_\nu = -\big((1-\chi)g_k + \chi\pi_s\big) < 0$. Since $g_k > \pi_s$, we have $|g_\nu| > \pi_s$. Notice that the income effect in equation (11) depends on expenditure measured in units of services, $\frac{e_t}{P_{s,t}} = \big(\nu_t P_{s,t}\big)^{\frac{1}{\chi-1}}$. In an ABGP, the income effect is operative for $\chi > 0$, since ν declines at a faster rate than P_s increases. In a world where preferences are homothetic, $\chi = 0$, the intertemporal elasticity of substitution is one, the income effect vanishes and the marginal value of capital decreases at the same rate as capital grows.

Notice that the indirect utility function (23) can be written as

$$u(m, P_g, P_s; \nu) = \nu\, m + \left(V\left((\nu P_s^{\chi})^{\frac{1}{\chi-1}}, P_g, P_s \right) - \nu\, (\nu P_s^{\chi})^{\frac{1}{\chi-1}} \right), \tag{26}$$

where $\nu\, m$ represents the utility of allocating all income to investment, and the second term captures the welfare gain from reallocating part of it to consumption –measured as the difference between the utility of consumption and its opportunity cost. From now on, we will refer to the second term as the *consumption reallocation gain*. Similarly, we can write the expenditure function (24) as

$$f(w, P_g, P_s; \nu) = \frac{w}{\nu} - \left(\frac{V\left((\nu P_s^{\chi})^{\frac{1}{\chi-1}}, P_g, P_s \right) - \nu\, (\nu P_s^{\chi})^{\frac{1}{\chi-1}}}{\nu} \right), \tag{27}$$

which represents the income required to attain utility level w by investing all income, minus the saving from reallocating part of it to consumption. The only difference between the second terms in the two expressions is the unit of measurement: in (23), it is expressed in consumption utility units; in (24), in investment units, with ν representing the marginal value of capital.

[12] A formal derivation is in the proof of Proposition 2 in Appendix A.2.

When preferences belong to the PIGL class, from (11) and (10), the consumption reallocation gain becomes

$$V\left(e, P_g, P_s\right) - \nu e = \left(\frac{1}{\chi} - \frac{s_g}{\gamma} - 1\right)\nu e - \frac{1}{\chi} + \frac{\eta}{\gamma} \quad \text{where} \quad e = \left(\nu P_s^{\chi}\right)^{\frac{1}{\chi - 1}}. \tag{28}$$

Along the ABGP, the parentheses in the right-hand side must be positive; otherwise the consumption of goods consumption to the consumption reallocation gain will be negative, all income will be allocated to investment, and the economy will be on a dynamically inefficient equilibrium. It is easy to show that Assumption 3 implies that $\frac{1}{\chi} - \frac{s_g}{\gamma} - 1 > 0$. More important, the consumption reallocation gain depends positively on the income effect, as given by $\frac{e}{\chi}$, and negatively on the substitution effect, as given by $\frac{s_g e}{\gamma}$.

Fisher–Shell principle. Based on the Fisher–Shell principle, which states that intertemporal welfare comparisons must be made using a consistent preference ordering, and following the logic of the equivalent variation measure introduced by Licandro et al. (2002), and refined by Durán and Licandro (2025), we adopt time t as the base preference order to compare welfare at time τ with welfare at time z by means of the equivalent variation measure[13]

$$\widehat{m}_{t,z,\tau} = f\Big(u\big(m_\tau, P_{g,\tau}, P_{s,\tau}; \nu_t\big), P_{g,z}, P_{s,z}; \nu_t\Big). \tag{29}$$

We use preferences ν_t to measure the income needed in τ to generate time-τ utility at z prices. In other words, the quantity $\widehat{m}_{t,z,\tau}$ represents the hypothetical level of income per capita, valued at time z prices, that the representative household would have needed at time τ to attain the utility achievable under historical income and prices at τ, when evaluated using the Bellman representation of preferences at time t. In (29), the time-t Bellman representation of preferences is called the *base preference order*, and the time-s prices are called the *base prices*. They are captured by the first and second subindices in $\widehat{m}_{t,z,\tau}$, respectively. Substituting (26) evaluated at τ and (27) evaluated at z into (29), and reorganising terms, we get

$$\widehat{m}_{t,z,\tau} = m_\tau + \left(\frac{V\left(\widehat{e}_{t,\tau}, P_{g,\tau}, P_{s,\tau}\right) - \nu_t \widehat{e}_{t,\tau}}{\nu_t}\right) - \left(\frac{V\left(\widehat{e}_{t,z}, P_{g,z}, P_{s,z}\right) - \nu_t \widehat{e}_{t,z}}{\nu_t}\right), \tag{30}$$

where $\widehat{e}_{t,x} = \left(\nu_t P_{s,x}^{\chi}\right)^{\frac{1}{\chi-1}}$, for $x = \{z, \tau\}$. The equivalent variation measure at τ, $\widehat{m}_{t,z,\tau}$, is equal to current income, m_τ, plus the difference between consumption reallocation

[13]This equivalent variation measure is consistent with the equivalent variation measure suggested by Baqaee and Burstein (2023) (Definition 3).

gains at time-τ prices and at base prices, both evaluated using time-t preferences, the base preference order. In what follows, all variables without a hat represent equilibrium objects. In turn, hat variables represent equivalent variation measures.[14]

The following lemma shows that the gap between the equivalent variation income measure and observed (equilibrium) income fundamentally depends on the share of consumption goods in total consumption expenditure, as evaluated using the base preference order. It fundamentally collapses to differences in prices between the time under evaluation and the base prices.

Proposition 3. *Under the conditions of Proposition 2*

$$\widehat{m}_{t,z,\tau} = m_\tau + B_{t,z,\tau}\, \widehat{e}_{t,z},$$

where

$$B_{t,z,\tau} := \left(\frac{1}{\chi} - 1 - \frac{\widehat{s}_{g,t,z}}{\gamma}\right) - \left(\frac{1}{\chi} - 1 - \frac{\widehat{s}_{g,t,\tau}}{\gamma}\right)\left(\frac{P_{s,z}}{P_{s,\tau}}\right)^{\frac{\chi}{1-\chi}}.$$

and, for $x \in \{z, \tau\}$,

$$\widehat{s}_{g,t,x} = \eta\left(\frac{\widehat{e}_{t,x}}{P_{s,x}}\right)^{-\chi}\left(\frac{P_{g,x}}{P_{s,x}}\right)^{\gamma} \qquad \textit{and} \qquad \widehat{e}_{t,x} = \left(\nu_t P_{s,x}^{\chi}\right)^{\frac{1}{\chi-1}}.$$

Proof: See Appendix A.3.

In Proposition 3, the gap between the income equivalent variation measure $\widehat{m}_{t,z,\tau}$ and current income m_τ in (30) simplifies to a factor $B_{t,z,\tau}$ that multiplies the equivalent variation measure of consumption expenditure in z, $\widehat{e}_{t,z}$. This factor critically depends on the difference between the equivalent variation measures of the consumption reallocation gains in τ and z, the last one corrected by the change in the price of services, the price used in (10) to measure real income gains. This deviation depends critically on the fixed-base preference order, as represented by ν_t. However, when measured around z, that is, when evaluated at $\tau = z$, it completely vanishes since consumption reallocation gains tend to be equal. Then $\widehat{m}_{t,z,z} = m_z$ irrespective of the fixed-base preference order t.

When measuring consumption expenditure at z using the time-t Bellman representation of preferences, the equivalent variation measure values investment by ν_t instead

[14] Bopart's Lema 1 requires $e^{\chi} \geqslant \eta \frac{1-\chi}{1-\gamma} P_g^{\gamma} P_s^{\chi-\gamma}$ for the underline preferences to exist; otherwise the elasticity of substitution between goods and services is negative. Implicit in (30), .

of ν_z. Consequently, since the marginal value of capital decreases over the course of development, the equivalent variation measure of consumption expenditure, $\widehat{e}_{t,z}$, exceeds the observed expenditure e_z if $z < t$. At a lower investment value, it is preferable to consume more. As a result, the equivalent variation measure of the share of consumption expenditure in goods, $\widehat{s}_{g,t,z}$, is lower than $s_{g,z}$ due to a stronger income effect.

It is interesting to notice that

$$\frac{m_\tau}{\widehat{m}_{t,z,\tau}} = 1 - \frac{B_{t,z,\tau}\,\widehat{e}_{t,z}}{\widehat{m}_{t,z,\tau}}$$

is the NDP deflator associated to the per-capita real NDP measure $\widehat{m}_{t,z,\tau}$ when nominal NDP is measured by units of the investment good.

Moreover, when the equivalent variation measure is evaluated using current preferences, the growth rate of $\widehat{s}_{g,t,z}$ with respect to z may even be positive, completely reversing the well-established empirical fact that the share of consumption expenditure on goods decreases in the development process. For this reason, as we show below, moving from a chained Divisia index to a fixed-base Fisher–Shell index may be counterfactual. Under the equilibrium condition that $\pi_s > \pi_g$, reversal occurs if and only if

$$1 > \gamma > \chi > \frac{\gamma(\pi_s - \pi_g)}{\pi_s + \gamma(\pi_s - \pi_g)} > 0.$$

Proposition 3 is instrumental in showing the following proposition:

Proposition 4. *Under the conditions of Proposition 2*

$$\frac{d\log \widehat{m}_{t,z,\tau}}{d\tau} = \frac{m_\tau}{\widehat{m}_{t,z,\tau}} \cdot \left(g^D_\tau + dev_{t,z,\tau}\right) := \widehat{g}^D_{t,z,\tau},$$

where

$$g^D_z = s_{e,z}\left(s_{c_g,z}g_{c_g,z} + (1 - s_{g,z})g_{c_s,z}\right) + (1 - s_{e,z})g_{x,z},$$

and

$$dev_{t,z,\tau} = \left(s_{e,z}s_{g,z} - v_{z,\tau}\widehat{s}_{e,t,z}\widehat{s}_{g,t,\tau}\right)\pi_g + \left(s_{e,z}(1 - s_{g,z}) - v_{z,\tau}\widehat{s}_{e,t,z}(1 - \widehat{s}_{g,t,\tau})\right)\pi_s,$$

with

$$\widehat{s}_{e,t,z} = \frac{\widehat{e}_{t,z}}{m_z}.$$

PROOF: See Appendix A.4.

At any time $\tau \in (t_0, t_1)$, the growth rate of real income, as measured by the equivalent variation measure $\widehat{m}_{t,z,\tau}$ in (29), is equal to the Divisia index g^D_τ plus an additive deviation $\mathrm{dev}_{t,z,\tau}$, the sum multiplied by the scaling factor $\frac{m_\tau}{\widehat{m}_{t,z,\tau}}$, which captures the discrepancy between observed income and its equivalent variation measure. Both $\mathrm{dev}_{t,z,\tau}$ and $\frac{m_\tau}{\widehat{m}_{t,z,\tau}}$ are critically influenced by the choice of base preference order and base prices. These choices significantly affect the income deflator by altering the equivalent variation measures of the expenditure share of consumption in total income, as well as the share of goods and services within total consumption expenditure. In what follows, we analyse these effects for the fixed-base Fisher–Shell (FS) and chained FS indices, along with two hybrid indices: the price-chained FS index (which uses a fixed preference order) and the preference-chained FS index (which uses fixed base prices). Each of these indices corresponds to a different specification of the equivalent variation measures of the expenditure shares $\widehat{s}_{e,t,z}$ and $\widehat{s}_{g,t,z}$.

3.2 Fisher–Shell Indices

From now on, suppose that a National Statistical Agency (NSA) has recorded National-Accounts data from an initial date t_0 to the current date t_1. The NSA seeks to measure welfare gains in a manner consistent with the Fisher–Shell principle. A key question is: for any past date $z \in (t_0, t_1)$, which *base preference order* and which *base prices* should be adopted?

To shed light on the consequences of the different approaches available, we introduce four alternative indices.

(i) **Base-time Fisher–Shell index**. For every $z \in (t_0, t_1)$, both the base preference order and the base prices are taken from the current date t_1; the resulting measure is an equivalent-variation index relative to t_1. This index is similar to the one suggested by Baqaee and Burstein (2023). For comparability, we also define the polar index, which adopts the initial time t_0 as the base. We refer to the former as the current-base FS index and to the latter as the reference-base FS index. Since we are fixing not only preferences but also prices, the current-base index will behave like a Paasche index and the reference-base index like a Laspeyres index.

(ii) **Chained Fisher–Shell index**. At any date $z \in (t_0, t_1)$, the base preference order and base prices are those prevailing at z; we compare z with the next instant $\tau = z - \mathrm{d}z$ and then chain the resulting infinitesimal indices forward to obtain a chained index. This is equivalent to the chained Fisher–Shell index suggested by Durán and Licandro (2025).

To isolate the roles of prices and preferences, we define two hybrid measures.

(iii) **Price-chained Fisher–Shell index**. We fix the base preference order at t_1 or t_0 for all comparisons, but allow base prices to update continuously and chain over time. This is a chained index of prices with a given preference order. If the preference order is time independent, $\nu_t = \nu \ \forall t$, the indices become a chained Fisher–Shell index.

(iv) **Preference-chained Fisher–Shell index**. We fix base prices at t_1 or t_0, while the preference order updates at each date z; the resulting infinitesimal indices are again chained. These indices should behave, in spirit, similarly to Paasche and Laspeyres indices, respectively.

Comparing these four indices reveals how either dimension (preferences or prices) drives the difference between fixed-base and chained Fisher–Shell measures.

It is important to note that all Fisher–Shell indices studied in this section are welfare-based and grounded in the Fisher–Shell principle, though they rely on different base preference orders and prices. Although they yield different quantitative measures of real income, all are derived from the same underlying preference representation. The fixed-base indices apply the Fisher–Shell principle globally by evaluating welfare gains over time using a fixed preference structure and prices —either current or historical— while the chained index applies the principle locally, assessing instantaneous welfare gains at each point in time and chaining them to construct a consistent intertemporal measure. Something similar applies to the price-chained and the preference-chained indices. As a result, they may exhibit different properties in dynamic settings. In the following, we quantitatively examine the behaviour of these indices in the context of the HRV structural transformation model discussed above and evaluate their implications for the study of structural transformation.

3.2.1 Chained Fisher–Shell Index

In line with Durán and Licandro (2025), let us first study the particular case when, for all $z \in (t_0, t_1)$, income around z is evaluated using the time-z preference order and prices. The equivalent variation measure becomes $\widehat{m}_{z,z,\tau}$. From Proposition 4, when its growth rate is evaluated at $\tau = z$, $\mathrm{dev}_{z,z} = 0$, implying that the growth rate at z is the Divisia index, i.e., $\frac{\mathrm{d}\log \widehat{m}_{z,z,\tau}}{\mathrm{d}\tau}\Big|_{\tau=z} = g_z^D$. When normalized to zero at $z = t_0$, the chained Fisher–Shell index becomes the chained Divisia index

$$\mathcal{D}_z = \int_{t_0}^{z} g_h^D \mathrm{d}h.$$

Throughout this paper time t_0 is said to be the *reference time*, i.e., the time at which all indices are equal to zero. The Divisia index is then, in the framework of the HRV structural transformation model, a welfare-based measured.

From the perspective of the chained Fisher–Shell index, the output growth rate at any point in time is computed using the preferences and prices prevailing at that moment. It captures cumulative welfare gains, as perceived by the representative agent over time, based on their evolving preferences and the changing relative prices. It is from this perspective that the chained Divisia index is welfare-based.

3.2.2 Fixed-Base Fisher–Shell Indices

Following the strategy suggested by Baqaee and Burstein (2023), in this context, since the economy is currently at time t_1, we adopt t_1 as the base preference order and the base prices. In their own words, "the asymmetry of time makes current preferences more relevant than preferences in the past." Consequently, we define what we call a *current-base Fisher–Shell index*, for all $z \in (t_0, t_1)$, as

$$\mathcal{P}_{t_1,z} = \log(\widehat{m}_{t_1,t_1,z}) - \log(\widehat{m}_{t_1,t_1,t_0}). \tag{31}$$

The first subindex of $\mathcal{P}$ correspond to both the base preference order and base prices. By construction, the index is normalised at the reference time t_0, so that $\mathcal{P}_{t_1,t_0} = 0$, $\mathcal{P}_{t_1,z}$

measuring the cumulative growth between t_0 and z.

The value at time t_1 then satisfies

$$\mathcal{P}_{t_1,t_1} = \log(m_{t_1}) - \log(\widehat{m}_{t_1,t_1,t_0}),$$

which is an equivalent variation measure of welfare gains from the initial time t_0 to the current time t_1, as evaluated using current preferences and prices. For any intermediate time $z \in (t_0, t_1)$, the difference $\mathcal{P}_{t_1,t_1} - \mathcal{P}_{t_1,z} = \log(m_{t_1}) - \log(\widehat{m}_{t_1,t_1,z})$ captures the welfare gains from time z to t_1.

The term $\widehat{m}_{t_1,t_1,z}$ can be interpreted as the level of income per capita at time z, after correcting for changes in the price level, corrections being made from the perspective of the current Bellman representation of preferences and current prices. As such, the ratio $m_{t_1}/\widehat{m}_{t_1,t_1,z}$ provides a money-metric measure of the welfare gains between z and t_1 when evaluated from today's point of view. This approach mirrors the logic of a Paasche index in that it values past allocations using current prices, in addition to current preferences.

The current-base Fisher–Shell index implicitly defines a notion of instantaneous, welfare-based growth. For any $z \in (t_0, t_1)$, the growth rate is given by

$$\frac{\mathrm{d}\mathcal{P}_{t_1,z}}{\mathrm{d}z} = \frac{\mathrm{d}\log \widehat{m}_{t_1,t_1,z}}{\mathrm{d}z} = \frac{1}{\widehat{m}_{t_1,t_1,z}} \frac{\mathrm{d}\widehat{m}_{t_1,t_1,z}}{\mathrm{d}z}. \tag{32}$$

As we demonstrate in Proposition 5 below, the instantaneous growth rate at the base time t_1, i.e. $\left.\frac{\mathrm{d}\mathcal{P}_{t_1,z}}{dz}\right|_{z=t_1}$, coincides with the Divisia index at time t_1.

We also adopt the polar view and measure welfare gains from the perspective of the initial time t_0. To do so, we can rely on (29) to measure welfare gains of moving from t_0 to z, for $z \in (t_0, t_1)$. This alternative approach is more closely aligned with a Laspeyres index. To fix ideas, let us adopt t_0 as base time. In the following, we will represent the reference-base Fisher–Shell index, for $z \in (t_0, t_1)$, as

$$\mathcal{L}_{t_0,z} = \log \widehat{m}_{t_0,t_0,z} - \log m_{t_0}. \tag{33}$$

The index is normalised to $\mathcal{L}_{t_0,t_0} = 0$ and $\mathcal{L}_{t_0,t_1} = \log \widehat{m}_{t_0,t_1} - \log m_{t_0}$. Implicit in this index, the instantaneous equilibrium growth rate of the economy at z is measured by

$$\frac{\mathrm{d}\mathcal{L}_{t_0,z}}{\mathrm{d}z} = \frac{\mathrm{d}\log \widehat{m}_{t_0,t_0,z}}{\mathrm{d}z} = \frac{1}{\widehat{m}_{t_0,t_0,z}} \frac{\mathrm{d}\widehat{m}_{t_0,t_0,z}}{\mathrm{d}z}. \tag{34}$$

As shown in Proposition 5, the instantaneous growth rate at t_0, as measured by the reference-base Fisher–Shell index, i.e. $\frac{\mathrm{d}\mathcal{L}_{t_0,z}}{dz}|_{z=t_0}$, is also equal to the Divisia index at t_0.

Following Durán and Licandro (2025), we now establish a fundamental result linking the Fisher–Shell indices to the Divisia index. Specifically, we show that the growth rate of a base-time Fisher–Shell index coincides with that of the Divisia index when evaluated at the base time. More importantly, we demonstrate that for any $z \in (t_0, t_1)$, the chained Fisher–Shell index lies between the current-base and the reference-base Fisher–Shell indices: the former systematically understates growth, while the latter overstates it.

Proposition 5. *The instantaneous growth rate of the current-base and reference-base Fisher–Shell indices, when evaluated at the base time, is equal to the corresponding Divisia index:*

$$\left.\frac{\mathrm{d}\mathcal{P}_{t_1,z}}{\mathrm{d}z}\right|_{z=t_1} = g^D_{t_1} \quad \text{and} \quad \left.\frac{\mathrm{d}\mathcal{L}_{t_0,z}}{\mathrm{d}z}\right|_{z=t_0} = g^D_{t_0}. \tag{35}$$

Moreover, for all $z \in (t_0, t_1)$, $t_0 < t_1$, *under* $\left(\frac{\chi}{1-\chi} - \gamma\right)\pi_s + \gamma\pi_g > 0$,

$$\frac{\mathrm{d}\mathcal{P}_{t_1,z}}{\mathrm{d}z} < g^D_z < \frac{\mathrm{d}\mathcal{L}_{t_0,z}}{\mathrm{d}z} \quad \Rightarrow \quad \mathcal{L}_{t_0,z} > \mathcal{D}_z > \mathcal{P}_{t_1,z}. \tag{36}$$

Proof: See Appendix A.5.

This proposition formalises the result that, at the margin, both the current-base and reference-base Fisher–Shell indices match the Divisia index when evaluated at the base time. However, as time passes, the current-base index increasingly underestimates growth, while the reference-base index increasingly overestimates it, relative to the Divisia benchmark. Consequently, if real income growth is continually reevaluated following a current-base Fisher–Shell index by shifting the base, past growth rates will be systematically revised downward over time.

3.2.3 Price-chained Fisher–Shell index

A fixed-base Fisher–Shell index evaluates income at any time $z \in (t_0, t_1)$ using base-year preferences and prices. A natural question is: how much of the difference between a chained Divisia index and a fixed-base Fisher–Shell index arises from holding prices fixed at the base year, in addition to preferences? Importantly, the principle that "current preferences are more relevant than preferences in the past" does not imply that current prices must be used when comparing two different points in the past.

To isolate the role of preferences, let us construct price-chained Fisher–Shell indices that adopt either current or reference preferences as a benchmark. More formally, for all $z \in (t_0, t_1)$, define the price-chained Fisher–Shell indices as

$$\widehat{\mathcal{P}}_{t_1,z} = \int_{t_0}^{z} \widehat{g}^D_{t_1,h,h}\,\mathrm{d}h, \qquad \widehat{\mathcal{L}}_{t_0,z} = \int_{t_0}^{z} \widehat{g}^D_{t_0,h,h}\,\mathrm{d}h,$$

where $\widehat{g}^D_{t,h,h}$ is defined in Proposition 4. Any deviation from the chained Divisia index is due to the distortion term $\mathrm{dev}_{t,z,z}$ and the ratio $m_z/\widehat{m}_{t,z,z}$. The distortion reads:

$$\mathrm{dev}_{t,z,z} = \left(s_{e,z}s_{g,z} - \widehat{s}_{e,t,z}\widehat{s}_{g,t,z}\right)\pi_g + \left(s_{e,z}(1-s_{g,z}) - \widehat{s}_{e,t,z}(1-\widehat{s}_{g,t,z})\right)\pi_s,$$

with

$$\widehat{s}_{e,t,z} = \frac{\widehat{e}_{t,z}}{m_z}.$$

3.2.4 Preference-chained Fisher–Shell index

For completeness, let us adopt current or reference prices as a benchmark in an otherwise preference-chained Fisher–Shell index. Formally,

$$\widetilde{\mathcal{P}}_{t_1,z} = \int_{t_0}^{z} \widehat{g}^D_{h,t_1,h}\mathrm{d}h, \quad \text{and} \quad \widetilde{\mathcal{L}}_{t_0,z} = \int_{t_0}^{z} \widehat{g}^D_{h,t_0,h}\mathrm{d}h,$$

respectively, where $\widehat{g}^D_{h,t_1,h}$ and $\widehat{g}^D_{h,t_0,h}$ are defined in Proposition 4. Any deviation from the chained Divisia index is due to the gaps $\mathrm{dev}_{z,t_1,z}$ and $\mathrm{dev}_{z,t_0,z}$, and the ratios $\frac{m_z}{\widehat{m}_{z,t,z}}$. The deviation reads

$$\mathrm{dev}_{z,t,z} = \Big(s_{e,t}s_{g,t} - v_{t,z}\widehat{s}_{e,z,t}s_{g,z}\Big)\pi_g + \Big(s_{e,t}(1-s_{g,t}) - v_{t,z}\widehat{s}_{e,z,t}(1-s_{g,z})\Big)\pi_s,$$

with

$$\widehat{s}_{e,z,t} = \frac{\widehat{e}_{z,t}}{m_t}.$$

4 Mapping U.S. Data

In this section, we quantitatively examine the aggregation of final expenditures on consumption goods, consumption of services, and investment within the structural transformation framework introduced in Section 2, with a focus on measuring welfare gains using the alternative indices discussed in Section 3.

To align the data from the U.S. National Accounts with the structure of the model, we follow the methodology proposed by Herrendorf et al. (2021). Specifically, we define goods consumption as the sum of personal consumption expenditures on goods and net exports of goods. Services consumption includes personal consumption expenditures on services, government consumption, and net exports of services. Investment is defined as the sum of private domestic investment and government investment. In the following, all references to US data use these measures.

Our analysis proceeds in two steps. First, following Herrendorf et al. (2021), we extend their dataset to 2023 to document the key empirical patterns of structural transformation and evaluate the model's ability to replicate them.[15] Second, we calibrate the model and use the calibrated model to examine the properties of alternative real income measures, focusing on how measurement affects the assessment of welfare gains and the characterization of structural transformation.

4.1 Structural Transformation Facts

Figure 1 displays the evolution for the U.S. of the relative price of investment with respect to the price of the consumption aggregate, left panel, and the effective investment-specific TFP, right panel.[16] The relative price of investment exhibits a clear downward trend beginning in 1980, falling at an average annual rate of 1.1%. This decline is mirrored by a strong upward trend in effective investment-specific TFP, which rises at an average annual rate of 1.2% over the same period.

[15] This extension uses data from the BEA National Income and Product Accounts for expenditure aggregates, the Input–Output Accounts to decompose value added into consumption and investment, the Industry Economic Accounts, and the BEA–BLS Integrated Industry-Level Production Accounts to update the March 2017 release of USA KLEMS.

[16] We compute effective investment-specific TFP following the top-down approach in Herrendorf et al. (2021). To be precise, we apply growth accounting methods to the equilibrium aggregate production in equation (6) and build residually a Törnqvist index for $\mathcal{A}_t$ using data on aggregate value added expressed in units of investment, aggregate capital and labor services, and average aggregate factor shares.

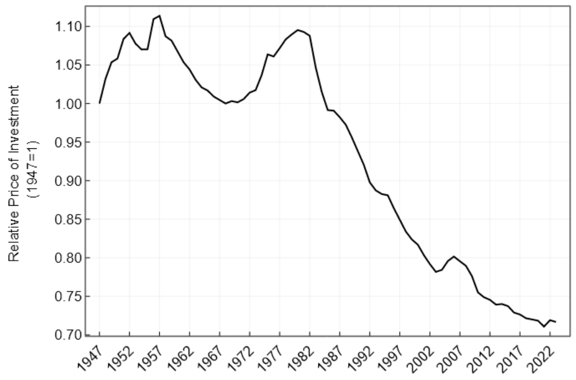


Figure 1: Relative Investment Prices and Effective Investment-Specific TFP.

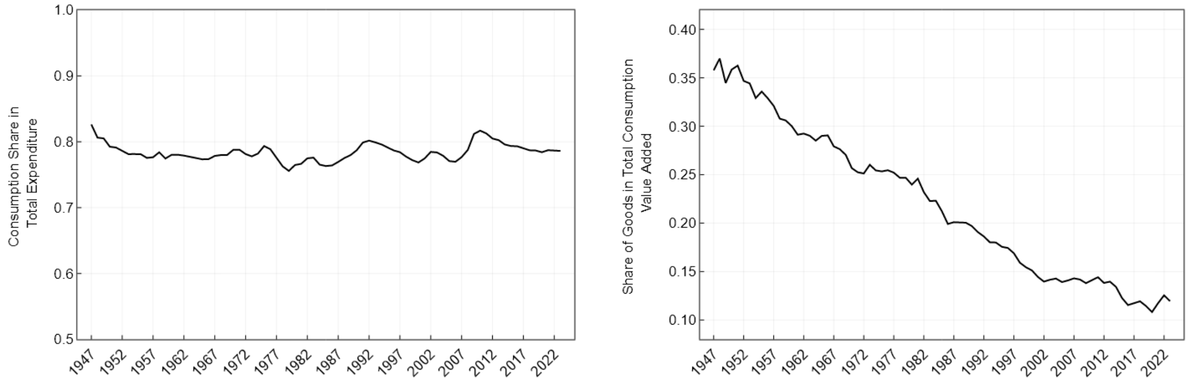


Figure 2: Consumption Expenditure and Goods Consumption Shares.

Figure 2 complements this evidence by showing that the share of total consumption expenditure in nominal GDP fluctuates around 78% throughout the sample period. At the same time, the share of goods consumption in total nominal consumption exhibits a sustained decline, falling from around 35% in 1947 to approximately 22.5% in 1980, and to below 12.5% in 2023.

4.2 Measuring GDP Growth in Practice

In this section, we assume that the theoretical economy studied in Section 2 has been on an ABGP since 1980. We choose this starting point because, as shown in Figure 1, the relative price of investment and the effective investment-specific TFP exhibit a clear linear trend only from 1980 onward, thus satisfying the requirement that the relative price

Table 1: Calibrated Parameter Values

θ	ρ	δ	n	g_h	$g_{\mathcal{A}}$	g_s	g_g	ζ
0.333	0.04	0.08	0.0091	0.0046	0.0130	0.0024	0.0097	0.0137

Note: θ, γ, δ are taken from HRV. TFP growth rates are computed using the indices from HRV. Population growth is taken from the U.S. Census Bureau. g_h is computed using the labor services index calculated by HRV. We estimate ζ by fitting an exponential trend to the ratio between relative prices of goods and services in the data versus their undistorted counterparts in the model.

of investment declines and that $\mathcal{A}$ grows at a constant rate along the ABGP.[17] We also assume that a National Statistical Agency (NSA) observes past and current equilibrium quantities and prices for the consumption of goods, the consumption of services, and investment, and can use this information to measure output growth.

Calibration. Our strategy involves a combination of calibration and structural estimation. Parameter values are reported in Tables 1 and 2. We take the capital share parameter θ, the depreciation rate δ, and the subjective discount rate ρ directly from Herrendorf et al. (2021).

For sectoral TFPs, we assume that Assumption 1 holds and productivity grows at constant exponential rates, such that $A_{j,t} = A_{j,1980}\, \mathrm{e}^{g_j t}$, for $j \in \{g, s\}$, and $\mathcal{A}_t = \mathcal{A}_{1980}\, \mathrm{e}^{g_{\mathcal{A}} t}$. Using the same growth accounting methodology as in Herrendorf et al. (2021), we extend their series of sectoral TFPs, to then normalise $A_{g,1980} = A_{s,1980} = \mathcal{A}_{1980} = 1$, and set g_j, for $j \in \{g, s, \mathcal{A}\}$, to match the normalised TFP values at $t = 2023$. The investment-specific TFP, $A_{x,t}$, is computed residually using equation (5) to guarantee that $\mathcal{A}_t$ grows at a constant rate.[18]

To estimate ζ, we proceed in three steps. First, we compute the relative price of goods to services in the data. Second, we construct the corresponding relative price implied by the model without distortions, given by $A_{s,t}/A_{g,t}$. Third, we take the ratio of the data to the model-implied relative price and fit an exponential trend to this series.

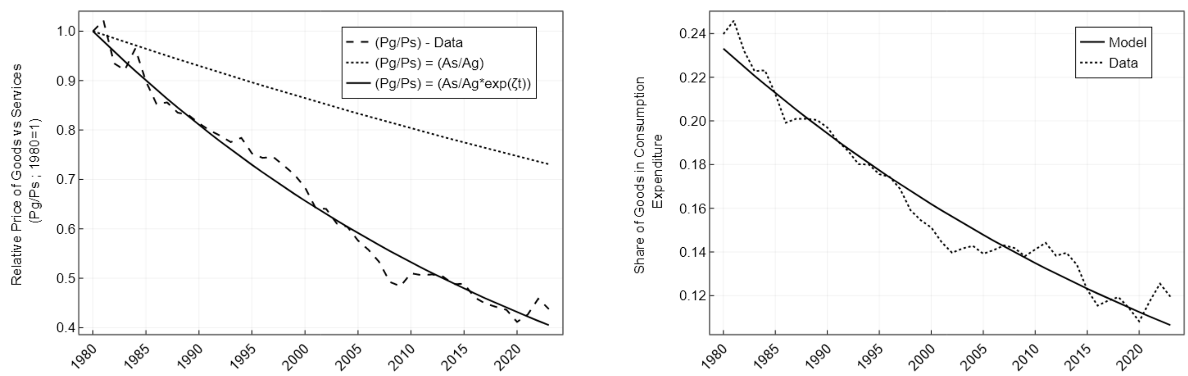
[17] See Herrendorf et al. (2021). In their paper the bottom panel of Figure 6 presents the measured investment TFP, while Proposition 1 discusses the conditions for the existence of the ABGP.

[18] To this end, we take the parameters that discipline the sectoral weights ω and the elasticity of substitution ε in the investment aggregator (2) from Herrendorf et al. (2021).

Table 2: Preference Parameters Estimated via SMM

χ	η	γ
0.305	0.242	0.672

Note: η, χ and γ are estimated via SMM, targeting the share of consumption expenditure on goods in the data for the period of 1980–2023.



(a) Consumption Goods Relative Price (b) Consumption Goods Share

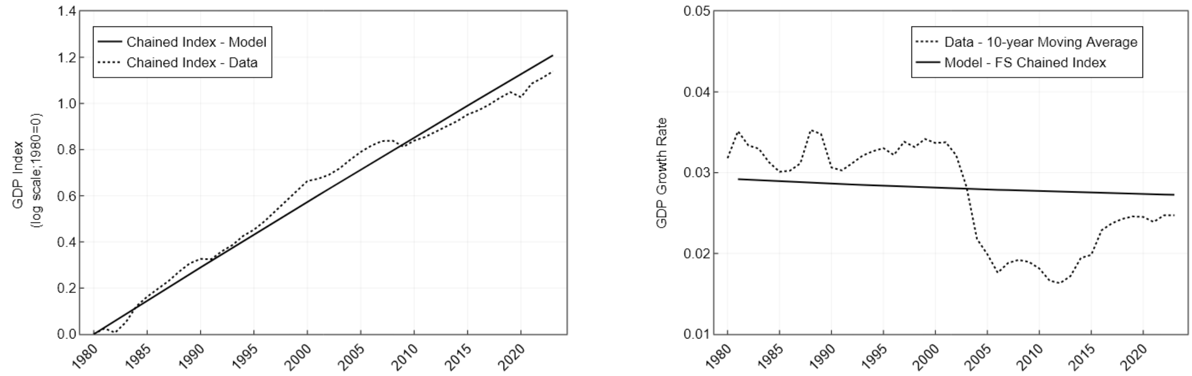
Figure 3: Consumption Goods Prices and Share

This procedure yields $\zeta = 0.0137$. Figure 3(a) shows the fit of the model together with the relative prices implied by the productivity ratio.

With these parameters in hand, we simulate the economy along the ABGP and estimate the preference parameters η, χ, γ structurally via a simulated moment method (SMM), using as a data target the observed share of consumption expenditure on goods from 1980 to 2023.[19] Figure 3(b) shows the share of goods in the total consumption expenditure in the data (dotted line) and the equilibrium share in the ABGP of the model economy (solid line).

In the following , we discuss the use of the different Fisher–Shell indices discussed in the previous section, and its main implications for the characterisation of structural transformation.

[19]The SMM minimizes the sum of squared residuals of the distance between the share of consumption on goods in the model and the data. We estimate η, χ and γ under the restrictions that $\eta > 0$ and $1 > \gamma > \chi > 0$.



(a) U.S. vs model GDP

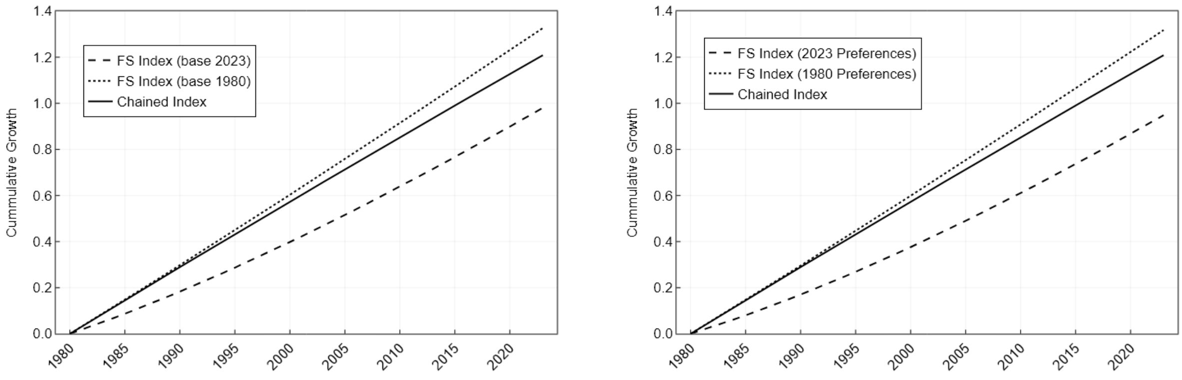
(b) U.S. vs model GDP growth

Figure 4: GDP and GDP growth

Chained Fisher-Ideal. Using current and past equilibrium quantities and prices, let the NSA measure a chained Fisher-ideal index of GDP growth, which in continuous time equals the corresponding Divisia index.

Figure 4(a) shows the evolution of U.S. GDP from 1980 to 2023, with the dotted line representing the data and the solid line depicting GDP as measured by a Fisher-ideal index along the Aggregate Balanced Growth Path (ABGP) of the structural transformation model. The model successfully captures the underlying trend of the data. Figure 4(b) plots a 10-year moving average annual GDP growth rate in the data (dotted line) alongside the model-implied GDP growth rate over the sample period (solid line). Both series show a decline over time, consistent with the findings of Herrendorf et al. (2021). The slump in observed GDP growth is a direct effect of the Great Recession, which also moves the growth rate to a lower declining trend.

Fixed-base vs chained Fisher–Shell indices. As part of our main discussion on welfare-based measurement of output growth, panel (a) of Figure 5 displays the evolution of real GDP along the ABGP of the calibrated structural transformation economy. We report three alternative measures: the chained Fisher–Shell index (solid line), corresponding to the Divisia index $\mathcal{D}_t$; the 1980-base Fisher–Shell index $\mathcal{L}_{80,t}$ (dashed line); and the 2023-base Fisher–Shell index $\mathcal{P}_{23,t}$ (dotted line). The x-axis covers the period from 1980 to 2023, and the y-axis plots the logarithm of real GDP, with all series normalised to zero in 1980.



(a) Fixed-Base FS vs. Chained Divisia (b) Price-Chained FS vs. Chained Divisia

Figure 5: Welfare-Based Real GDP Measures.

Panel (a): Chained FS (solid), 1980-base FS (dotted), and 2023-base FS (dashed). Panel (b): Chained FS (solid), 1980-base price-chained FS (dotted), and 2023-base price-chained FS (dashed).

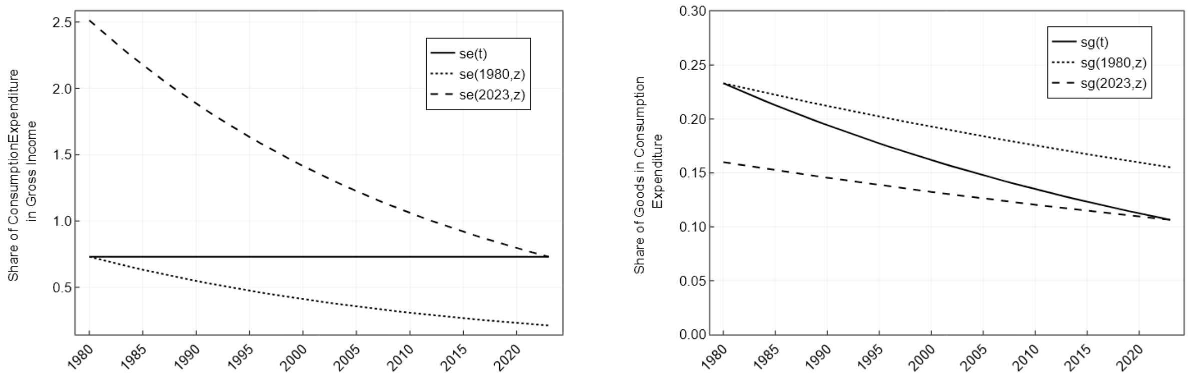
As shown in Proposition 5, relative to the chained Fisher–Shell index, the reference-base FS index $\mathcal{L}_{80,t}$ overstates growth and the current-base FS index $\mathcal{P}_{23,t}$ understates it, the chained FS lying between the two. The differences between the three indices are quantitatively large.

Let us first examine the behaviour of the reference-base Fisher–Shell index. When the representative agent adopts 1980 as the base time, the 1980-base Fisher–Shell index, shown by the dashed line in panel (a) of Figure 5, results in an index that is 11.7 p.p. higher than the chained index in 2023, overstating income growth during this period.

The fundamental reason lies in the relative weights that each index assigns to consumption expenditure and investment when measuring the growth rate of real income. For the reference-base Fisher–Shell index, the equivalent variation measure of the consumption expenditure share is given by

$$\frac{\widehat{e}_{80,z}}{\widehat{m}_{80,80,z}}, \qquad \text{for all } z \in (1980, 2023).$$

It is straightforward to verify that this share is decreasing in z. On the one hand, since $\widehat{e}_{80,z} = \left(\nu_{80} P_{s,z}^{\chi}\right)^{\frac{1}{\chi-1}}$, and the price of services $P_{s,z}$ increases over time, the equivalent variation measure of consumption expenditure declines with z. On the other hand, equilibrium income m_z, and therefore the equivalent variation measure $\widehat{m}_{80,80,z}$, increases along the ABGP. In other words, because ν_z decreases over time, adopting 1980-preferences



(a) Consumption Expenditure Share

(b) Goods Consumption Share

Figure 6: Equivalent Variation Measures for Past- and Current-base Fisher–Shell Indices.

Equilibrium (solid line), 1980-base FS (dotted) and 2023-base FS (dashed).

leads the equivalent variation measure to increasingly overvalue investment relative to consumption as z rises. Using past preferences reduces the willingness to consume and amplifies the incentive to invest as the economy evolves away from the base year. As a result, the reference-base Fisher–Shell index assigns a growing weight to investment, which grows faster than consumption expenditure in equilibrium. Consequently, the reference-base Fisher–Shell index grows faster than the Divisia index.

Figure 6(a) illustrates the behaviour of the consumption expenditure share. Under equilibrium and when measured using the Divisia index, the consumption expenditure share remains constant at around 78%. In contrast, the equivalent variation measure using 1980 as the base shows a declining pattern, starting at 78% in 1980 and decreasing thereafter.

It is also interesting to note that when income at time z is evaluated using 1980 preferences, the income effect reverses. Specifically, the equivalent variation measure of real income is given by

$$\frac{\widehat{e}_{80,z}}{P_{s,z}} = (\nu_{80} P_{s,z})^{\frac{1}{\chi-1}},$$

which decreases as z increases. This implies a decline in real income when evaluated with 1980 preferences, leading to a shift in consumption from services to goods. This shift at least partially offsets the effect of rising service prices, which otherwise reduce the share of goods in total consumption expenditure.

Figure 6(b) illustrates this pattern: the dotted line shows the 1980-base equivalent variation measure of the goods consumption share, which is systematically higher than the corresponding share under the Divisia index. This reflects the substitution towards goods caused by the declining real income when evaluated at past preferences.[20]

We now turn to the 2023-base Fisher–Shell index, which evaluates real income growth using 2023 preferences and prices. As shown by the dashed line in panel (b) of Figure 5, this index substantially understates income growth relative to the chained Divisia index, leading to an index value in 1980 that is 22.8 p.p. lower than the chained index.

This understatement arises for reasons symmetrical to those discussed for the 1980-base index. Specifically, for the current-base Fisher–Shell index with 2023 as the base, the equivalent variation measure of the consumption expenditure share is given by

$$\frac{\widehat{e}_{23,z}}{\widehat{m}_{23,23,z}}, \qquad \text{for all } z \in (1980, 2023),$$

which is increasing in z. In this case, the equivalent variation measure of consumption expenditure,

$$\widehat{e}_{23,z} = \left(\nu_{23} P_{s,z}^{\chi}\right)^{\frac{1}{\chi-1}},$$

rises with z due to the decline in the price of services $P_{s,z}$ when the evaluation moves further in the past. Meanwhile, equilibrium income m_z, and hence the equivalent variation measure $\widehat{m}_{23,23,z}$, decreases as we move backward in time along the ABGP. Taken together, this implies an increasing equivalent variation consumption share, and a decreasing investment share. Thus, as we go back in time, the 2023-base Fisher–Shell index assigns a rising weight to consumption and a shrinking weight to investment as, the slowest-growing component all along the ABGP.

As a result, the 2023-base Fisher–Shell index grows more slowly than the Divisia index and understates past economic growth and welfare gains. This behaviour mirrors the distortion described for the 1980-base index but with opposite sign, as expected from the time-reversal symmetry implied by using current-base preferences. Figure 6(a) confirms this pattern: the equivalent variation measure of the consumption expenditure

[20]Under the calibration in Table 2, $\frac{1-\gamma}{1-\chi} = 0.472$. As shown in Figure 5, the share of goods in total consumption expenditure is systematically smaller than 0.25, not only at equilibrium but also for all equivalent variation measures in both the current-base and reference-base Fisher–Shell indices. It is therefore easy to verify that Assumption 3 holds and preferences are well-defined.

share is well above one at 1980, meaning negative investment, declining over time and converging to 78% in 2023. Figure 6(b) further illustrates that the goods consumption share is systematically below the corresponding share under the Divisia index. This is explained by the reversed income effect: using future preferences to evaluate income at earlier dates generates an upward bias in real income and shifts consumption from goods towards services, reducing the measured share of goods in total consumption expenditure. Again, this is the mirror image of the substitution effect observed in the 1980-base case.

Finally, as a robustness check, Panel (b) of Figure 5 shows the evolution of real income when measured using the price-chained Fisher Shell index at current preferences (dashed line) and 1980 preferences (dotted line). In both indices, base preferences are set at the current or reference time, but base prices are allowed to update continuously and the chained over time. As it can be observed, most of the difference between the fixed-base Fisher-Shell indices and the Divisia index are due to fixing preferences.

Current-time upgrade and GDP history revision. If a welfare-based index of income growth is evaluated from the perspective of current preferences, a suggested by Baqaee and Burstein (2023), as time passes the full history of economic growth will be revised again and again. What then will be the effect of these revisions on the measurement of economic growth in the framework of the structural transformation model discussed in this paper?

Figure 7 reports the annual growth rate of the calibrated structural transformation model, as measured by the current-base Fisher–Shell index, updated repeatedly as the base year moves forward. The chained Divisia index, shown by the solid line, indicates a declining annual growth rate of GDP between 1980 and 2023, by approximately a fifth of a percentage point, consistent with U.S. data. This decline reflects the slowdown in economic growth driven by structural change: as consumption shifts from goods to services, the economy increasingly relies on the sector with the lowest rate of technical progress. In contrast, all fixed-base Fisher–Shell indices display an accelerating growth pattern: they overstate growth after the base year and understate it before. As discussed above, this distortion arises from the counterfactual equivalent variation expenditure shares generated by fixing the preference order and prices at the base year. These shares fail to track the actual evolution of consumption patterns, resulting in a systematic mismeasurement of real income growth.

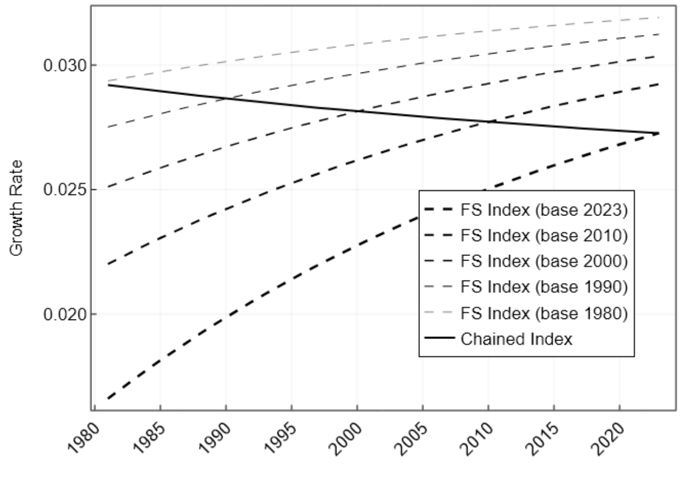


Figure 7: GDP Revisions

Summary. We show that, in line with the main contribution of the paper, the chained Fisher–Shell index provides a consistent measure of welfare-based income growth. As formally established in Proposition 5, this index coincides with the Divisia index, which measures instantaneous welfare gains along the equilibrium path of the economy. This property aligns the chained Fisher–Shell index with the growth rates reported in the National Accounts, which are also constructed using a chained Fisher-ideal index.

In contrast, current-base Fisher–Shell indices systematically underestimate past growth. This underestimation becomes more severe the further back in time, as the evaluation is conducted using today's prices and preferences. Moreover, current-base indices require permanent revisions: as time passes, the base is updated, past growth is re-evaluated, and typically revised downward. This feature underscores a key limitation of fixed-base indices and highlights the advantages of chaining to capture welfare-relevant growth dynamics.

5 Conclusions

This paper has examined the role of real income measurement in the context of structural transformation, focusing on the welfare properties of chained indices. Within a general equilibrium model that captures key features of the U.S. structural transformation since 1980, we compared two welfare-based approaches, the current-base Fisher–Shell equivalent variation measure inspired on Baqaee and Burstein (2023) and the chained Fisher–Shell index proposed by Durán and Licandro (2025), .

Our main result is that the chained Fisher–Shell index, which updates both preferences and prices continuously, aligns with the Divisia index and provides a time-invariant, welfare-consistent measure of real GDP growth. By contrast, current-base indices distort the measurement of past growth, understating welfare gains. These distortions are particularly severe when preferences income and substitution effects are large, in a world of large changes in relative prices evolve over time, as it is the case in models of structural change.

From a policy and statistical perspective, our findings reinforce the theoretical foundation for the use of chained indices in national accounts. While the move to chained indices was initially motivated by empirical considerations—such as reducing revisions in measured growth—their adoption also finds justification in the economic theory of index numbers when applied to dynamic, non-homothetic settings.

More broadly, this paper reaffirms the deep link between structural transformation and measurement. Accurately capturing welfare-relevant growth in the face of evolving consumption patterns requires moving beyond fixed-base approaches. Chained Fisher–Shell and Divisia indices offer flexible, theoretically grounded tools for this purpose, allowing for more accurate assessments of economic progress in transforming economies.

Appendices

A Proof of Propositions

A.1 Proof of Proposition 1

Proposition 1. *At an ABGP of the structural transformation economy, the growth rate of real GDP, as measured by the Divisia index, is permanently declining.*

PROOF: The Divisia quantity index at the ABGP of the HRV structural transformation economy is

$$g_t^D = s_e\big(s_{g,t}\, g_{c_g} + (1 - s_{g,t})g_{c_s,t}\big) + (1 - s_e)g_x \,.$$

The corresponding Divisia price index is

$$\pi_t^D = s_e\big(s_{g,t}\, \pi_g + (1 - s_{g,t})\pi_s\big) \,.$$

Since the investment good is the numeraire, $\pi_x = 0$. It is easy to show that $g_t^D = g_k - \pi_t^D$, where g_k is the growth rate of nominal income, as measured in units of the investment good. Since $s_{g,t}$ is declining along the ABGP and $\pi_s > \pi_g$, consumption inflation is increasing converging to service inflation when time goes to infinity. The growth rate of real GDP is then decreasing. □

Quantitative Evaluation. In the quantitative exercise, the difference in growth rates between 1980 and 2023 is

$$g_{23}^D - g_{80}^D = \pi_{80}^D - \pi_{23}^D = s_e\big(s_{g,80} - s_{g,23}\big)\big(\pi_g - \pi_s\big) < 0.$$

Since in the calibration $s_e \simeq .78$, $s_{g,23} \simeq 0.12$, $s_{g,80} \simeq 0.22$ and $\pi_s - \pi_g \simeq 0.012$, the annual growth rate of real GDP should have declined in 2023 by 0.1 p.p. relative to its value in 1980.

New proof:

$$s_{g,t} := \frac{P_{g,t}c_{g,t}}{e_t} = \eta \left(\frac{e_t}{P_{s,t}}\right)^{-\chi} \left(\frac{P_{g,t}}{P_{s,t}}\right)^{\gamma} . \tag{11}$$

A.2 Proof of Proposition 2

Proposition 2. *The indirect utility and expenditure functions associated with the Bellman representation of preferences in equation (21) are, respectively, given by*

$$u(m, P_g, P_s; \nu) = V\left((\nu P_s^{\chi})^{\frac{1}{\chi-1}}, P_g, P_s\right) + \nu\left(m - (\nu P_s^{\chi})^{\frac{1}{\chi-1}}\right), \tag{23}$$

and

$$f(w, P_g, P_s; \nu) = (\nu P_s^{\chi})^{\frac{1}{\chi-1}} + \frac{w}{\nu} - \frac{V\left((\nu P_s^{\chi})^{\frac{1}{\chi-1}}, P_g, P_s\right)}{\nu}, \tag{24}$$

where w is an arbitrary level of utility as in (21).

Proof: The household's primal problem of maximizing (21), subject to the budget constraint (22), can be solved in two stages. We are omitting index t to simplify notation. In the first stage, by the definition of the indirect utility function,

$$V(e, P_g, P_s) = \max_{\{c_g, c_s\}: P_g c_g + P_s c_s = e} U(c_g, c_s).$$

In the second stage, the household chooses x to solve

$$\max_x \; V(m - x, P_g, P_s) + \nu x.$$

The first-order condition is

$$\frac{\partial V}{\partial e}(e, P_g, P_s) = \nu.$$

Using the PIGL functional form in (10), $\frac{\partial V}{\partial e} = e^{\chi-1} P_s^{-\chi}$, we obtain

$$e^{\chi-1} P_s^{-\chi} = \nu \quad \Rightarrow \quad e = (\nu P_s^{\chi})^{\frac{1}{\chi-1}}.$$

It follows that

$$x = m - e = m - (\nu P_s^{\chi})^{\frac{1}{\chi-1}}.$$

Substituting back, the indirect utility function is

$$u(m, P_g, P_s; \nu) = V\left((\nu P_s^{\chi})^{\frac{1}{\chi-1}}, P_g, P_s\right) + \nu\left(m - (\nu P_s^{\chi})^{\frac{1}{\chi-1}}\right).$$

The expenditure function is derived from the dual problem

$$\min_{e,x} \; e + x \quad \text{s.t.} \quad V(e, P_g, P_s) + \nu x = w,$$

where w is an arbitrary level of utility. The F.O.C.s with respect to e and x are

$$1 = \mu \frac{\partial V}{\partial e} = \lambda e^{\chi - 1} P_s^{-\chi} \quad \text{and} \quad 1 = \mu \nu,$$

where μ is the Lagrangian multiplier associated to the constraint. Equating multipliers yields

$$e^{\chi-1} P_s^{-\chi} = \nu \quad \Rightarrow \quad e = \left(\nu P_s^{\chi}\right)^{\frac{1}{\chi-1}}.$$

The same condition as in the primal problem. Solving for x

$$x = \frac{w - V\left(\left(\nu P_s^{\chi}\right)^{\frac{1}{\chi-1}}, P_g, P_s\right)}{\nu}.$$

Thus, the expenditure function is

$$f(w, P_g, P_s; \nu) = \left(\nu P_s^{\chi}\right)^{\frac{1}{\chi-1}} + \frac{w - V\left(\left(\nu P_s^{\chi}\right)^{\frac{1}{\chi-1}}, P_g, P_s\right)}{\nu},$$

which completes the proof. $\square$

A.3 Proof of Proposition 3

Proposition 3. *Under the conditions of Proposition 2*

$$\widehat{m}_{t,z,\tau} = m_\tau + B_{t,z,\tau}\, \widehat{e}_{t,z},$$

where

$$B_{t,z,\tau} := \left(\frac{1}{\chi} - 1 - \frac{\widehat{s}_{g,t,z}}{\gamma}\right) - \left(\frac{1}{\chi} - 1 - \frac{\widehat{s}_{g,t,\tau}}{\gamma}\right)\left(\frac{P_{s,z}}{P_{s,\tau}}\right)^{\frac{\chi}{1-\chi}},$$

and, for $x \in \{z, \tau\}$,

$$\widehat{s}_{g,t,x} = \eta \left(\frac{\widehat{e}_{t,x}}{P_{s,x}}\right)^{-\chi} \left(\frac{P_{g,x}}{P_{s,x}}\right)^{\gamma} \quad \text{and} \quad \widehat{e}_{t,x} = \left(\nu_t P_{s,x}^{\chi}\right)^{\frac{1}{\chi-1}}.$$

Proof: From definition in (29),

$$\widehat{m}_{t,z,\tau} = f\Big(u\big(m_\tau, P_{g,\tau}, P_{s,\tau}; \nu_t\big), P_{g,z}, P_{s,z}; \nu_t\Big). \tag{29}$$

We use preferences ν_t to measure the income needed in τ to generate τ utility at z prices.

From Proposition 2,

$$u(m, P_g, P_s; \nu) = V\left(\left(\nu P_s^{\chi}\right)^{\frac{1}{\chi-1}}, P_g, P_s\right) + \nu\left(m - \left(\nu P_s^{\chi}\right)^{\frac{1}{\chi-1}}\right), \tag{23}$$

and

$$f(w, P_g, P_s; \nu) = (\nu P_s^{\chi})^{\frac{1}{\chi-1}} + \frac{w}{\nu} - \frac{V\left((\nu P_s^{\chi})^{\frac{1}{\chi-1}}, P_g, P_s\right)}{\nu}. \tag{24}$$

Moreover, from Assumption 2,

$$V(e, P_g, P_s) = \frac{1}{\chi}\left(\frac{e}{P_s}\right)^{\chi} - \frac{\eta}{\gamma}\left(\frac{P_g}{P_s}\right)^{\gamma} - \frac{1}{\chi} + \frac{\eta}{\gamma}, \tag{10}$$

where $\eta > 0$ and $1 > \gamma \geqslant \chi \geqslant 0$.

Substituting (23) into (24), then into (29), we get

$$\begin{aligned}
\widehat{m}_{t,z,\tau} &= f\Big(V(\widehat{e}_{t,\tau}, P_{g,\tau}, P_{s,\tau}) + \nu_t(m_\tau - \widehat{e}_{t,\tau}), P_{g,z}, P_{s,z}; \nu_t\Big) \\
&= \widehat{e}_{t,z} + \frac{1}{\nu_t}\Big(V(\widehat{e}_{t,\tau}, P_{g,\tau}, P_{s,\tau}) + \nu_t(m_\tau - \widehat{e}_{t,\tau})\Big) - \frac{1}{\nu_t}V(\widehat{e}_{t,z}, P_{g,z}, P_{s,z}),
\end{aligned}$$

where $\widehat{e}_{t,x} = \left(\nu_t P_{s,x}^{\chi}\right)^{\frac{1}{\chi-1}}$ for $x \in \{z, \tau\}$.

Reordering terms, we get (30)

$$\widehat{m}_{t,z,\tau} = m_\tau + \left(\frac{V(\widehat{e}_{t,\tau}, P_{g,\tau}, P_{s,\tau}) - \nu_t \widehat{e}_{t,\tau}}{\nu_t}\right) - \left(\frac{V(\widehat{e}_{t,z}, P_{g,\tau}, P_{s,z}) - \nu_t \widehat{e}_{t,z}}{\nu_t}\right). \tag{30}$$

Using (28),

$$\widehat{m}_{t,z,\tau} = m_\tau + B_{t,z,\tau}\, \widehat{e}_{t,z},$$

where

$$B_{t,z,\tau} := \left(\frac{1}{\chi} - 1 - \frac{\widehat{s}_{g,t,\tau}}{\gamma}\right)\left(\frac{P_{s,z}}{P_{s,\tau}}\right)^{\frac{\chi}{1-\chi}} - \left(\frac{1}{\chi} - 1 - \frac{\widehat{s}_{g,t,z}}{\gamma}\right),$$

which completes the proof. □

A.4 Proof of Proposition 4

Proposition 4. *Under the conditions of Proposition 2*

$$\frac{\mathrm{d}\log \widehat{m}_{t,z,\tau}}{\mathrm{d}\tau} = \frac{m_\tau}{\widehat{m}_{t,z,\tau}} \cdot \left(g_\tau^D + \mathrm{dev}_{t,z,\tau}\right) := \widehat{g}_{t,z,\tau}^D,$$

where

$$g_z^D = s_{e,z}\left(s_{c_g,z} g_{c_g,z} + (1 - s_{g,z}) g_{c_s,z}\right) + (1 - s_{e,z}) g_{x,z},$$

and

$$\mathrm{dev}_{t,z,\tau} = \Big(s_{e,z} s_{g,z} - v_{z,\tau} \widehat{s}_{e,t,z} \widehat{s}_{g,t,\tau}\Big)\pi_g + \Big(s_{e,z}(1 - s_{g,z}) - v_{z,\tau} \widehat{s}_{e,t,z}(1 - \widehat{s}_{g,t,\tau})\Big)\pi_s,$$

with

$$\widehat{s}_{e,t,z} = \frac{\widehat{e}_{t,z}}{m_z}.$$

Proof: From Proposition 3,

$$\widehat{m}_{t,z,\tau} = m_\tau + B_{t,z,\tau}\,\widehat{e}_{t,z},$$

where

$$B_{t,z,\tau} := \left(\frac{1}{\chi} - 1 - \frac{\widehat{s}_{g,t,\tau}}{\gamma}\right)\left(\frac{P_{s,z}}{P_{s,\tau}}\right)^{\frac{\chi}{1-\chi}} - \left(\frac{1}{\chi} - 1 - \frac{\widehat{s}_{g,t,z}}{\gamma}\right),$$

Differentiating $B_{t,z,\tau}$ with respect to τ,

$$\frac{\mathrm{d}B_{t,z,\tau}}{\mathrm{d}\tau} = -\frac{v_{z,\tau}}{\gamma}\frac{\mathrm{d}\,\widehat{s}_{g,t,\tau}}{\mathrm{d}\tau} + \left(\frac{1}{\chi} - 1 - \frac{\widehat{s}_{g,t,\tau}}{\gamma}\right)\frac{\mathrm{d}v_{z,\tau}}{\mathrm{d}\tau},$$

where $v_{z,\tau} = \left(\frac{P_{s,z}}{P_{s,\tau}}\right)^{\frac{\chi}{1-\chi}}$. From the definition of $v_{z,\tau}$ above,

$$\frac{\mathrm{d}v_{z,\tau}}{\mathrm{d}\tau} = -\frac{\chi}{1-\chi}\pi_s v_{z,\tau}.$$

From the definition of $\widehat{s}_{g,t,x}$ and $\widehat{e}_{t,x}$, we get

$$\widehat{s}_{g,t,\tau} = \eta\left(\frac{\widehat{e}_{t,\tau}}{P_{s,\tau}}\right)^{-\chi}\left(\frac{P_{g,\tau}}{P_{s,\tau}}\right)^{\gamma} = \eta\,\nu_t^{\frac{\chi}{1-\chi}}\,P_{s,\tau}^{\frac{\chi}{1-\chi}-\gamma}\,P_{g,\tau}^{\gamma}.$$

Differentiating with respect to τ,

$$\frac{\mathrm{d}\,\widehat{s}_{g,t,\tau}}{\mathrm{d}\tau} = \widehat{s}_{g,t,\tau}\left[\left(\frac{\chi}{1-\chi} - \gamma\right)\pi_s + \gamma\,\pi_g\right].$$

Plugging both derivatives into $\frac{\mathrm{d}B_{t,z,\tau}}{\mathrm{d}\tau}$, we get

$$\frac{\mathrm{d}B_{t,z,\tau}}{\mathrm{d}\tau} = -v_{z,\tau}\left(\frac{\widehat{s}_{g,t,\tau}}{\gamma}\left[\left(\frac{\chi}{1-\chi} - \gamma\right)\pi_s + \gamma\,\pi_g\right] + \left(\frac{1}{\chi} - 1 - \frac{\widehat{s}_{g,t,\tau}}{\gamma}\right)\frac{\chi}{1-\chi}\pi_s\right),$$

which collapses to

$$\frac{\mathrm{d}B_{t,z,\tau}}{\mathrm{d}\tau} = -v_{z,\tau}\Big(\widehat{s}_{g,t,\tau}\,\pi_g + \big(1 - \widehat{s}_{g,t,\tau}\big)\pi_s\Big).$$

The derivative of $\log\widehat{m}_{t,z,\tau}$ with respect to τ is

$$\frac{\mathrm{d}\log\widehat{m}_{t,z,\tau}}{\mathrm{d}\tau} = \frac{\dot{m}_\tau}{\widehat{m}_{t,z,\tau}} + \frac{\mathrm{d}B_{t,z,\tau}}{\mathrm{d}\tau}\cdot\frac{\widehat{e}_{t,z}}{\widehat{m}_{t,z,\tau}}.$$

Since

$$\frac{\dot{m}_z}{m_z} = s_{e,z}g_{e,z} + (1 - s_{e,z})g_{x,z},$$

and

$$g_{e,z} = s_{g,z}(g_{c_g,z} + \pi_g) + (1 - s_{g,z})(g_{c_s,z} + \pi_s),$$

Then,
$$\frac{\mathrm{d}\log \widehat{m}_{t,z,\tau}}{\mathrm{d}\tau} = \frac{m_\tau}{\widehat{m}_{t,z,\tau}} \cdot \left(g_\tau^D + \mathrm{dev}_{t,z,\tau}\right) := \widehat{g}_{t,z,\tau}^D,$$
where
$$g_z^D = s_{e,z}\left(s_{c_g,z}g_{g,z} + (1-s_{g,z})g_{c_s,z}\right) + (1-s_{e,z})g_{x,z},$$
and
$$\mathrm{dev}_{t,z,\tau} = \left(s_{e,z}s_{g,z} - v_{z,\tau}\widehat{s}_{e,t,z}\widehat{s}_{g,t,\tau}\right)\pi_g + \left(s_{e,z}(1-s_{g,z}) - v_{z,\tau}\widehat{s}_{e,t,z}(1-\widehat{s}_{g,t,\tau})\right)\pi_s. \quad \square$$

A.5 Proof of Proposition 5

(TO BE REVISED)

Proposition 5. *The instantaneous growth rate of the current-base and reference-base Fisher–Shell indices evaluated at the base time are equal to the corresponding Divisia index*
$$\left.\frac{\mathrm{d}\mathcal{P}_{t_1,z}}{\mathrm{d}z}\right|_{z=t_1} = g_{t_1}^D \quad \text{and} \quad \left.\frac{\mathrm{d}\mathcal{L}_{t_0,z}}{\mathrm{d}z}\right|_{z=t_0} = g_{t_0}^D. \tag{35}$$
Moreover, for all $z \in (t_0, t_1)$, $t_0 < t_1$, under $\left(\frac{\chi}{1-\chi} - \gamma\right)\pi_s + \gamma\pi_g > 0$
$$\frac{\mathrm{d}\mathcal{P}_{t_1,z}}{\mathrm{d}z} < g_z^D < \frac{\mathrm{d}\mathcal{L}_{t_0,z}}{\mathrm{d}z} \quad \Rightarrow \quad \mathcal{P}_{t_1,z} < \mathcal{D}_z < \mathcal{L}_{t_0,z}. \tag{36}$$

Proof: We will prove it in two stages.

Stage 1: Let us first prove (35). From the definition of the current-base and reference-base Fisher–Shell indices in (31) and (33), respectively,
$$\mathcal{P}_{t_1,z} = \log \widehat{m}_{t_1,t_1,z} - \log \widehat{m}_{t_1,t_1,t_0} \quad \text{and} \quad \mathcal{L}_{t_0,z} = \log \widehat{m}_{t_0,t_0,z} - \log m_{t_0}.$$
Then, the growth rate of the current-base and reference-base Fisher–Shell indices are
$$\left.\frac{\mathrm{d}\mathcal{P}_{t_1,z}}{\mathrm{d}z}\right|_{z=t_1} = \frac{1}{\widehat{m}_{t_1,t_1,z}} \cdot \left.\frac{\mathrm{d}\widehat{m}_{t_1,t_1,z}}{\mathrm{d}z}\right|_{z=t_1} \quad \text{and} \quad \left.\frac{\mathrm{d}\mathcal{L}_{t_0,z}}{\mathrm{d}z}\right|_{z=t_0} = \frac{1}{\widehat{m}_{t_0,t_0,z}} \cdot \left.\frac{\mathrm{d}\widehat{m}_{t_0,t_0,z}}{\mathrm{d}z}\right|_{z=t_0}.$$
From Proposition 4,
$$\left.\frac{\mathrm{d}\log \widehat{m}_{t,t,z}}{\mathrm{d}z}\right|_{z=t} = g_t^D$$
since
$$\widehat{m}_{t,t,t} = m_t \quad \text{and} \quad \mathrm{dev}_{t,t,t} = 0.$$

The previous result is true, because $v_{t,t} = 1$, $\widehat{s}_{e,t,t} = s_{e,t}$ and $\widehat{s}_{g,t,t} = s_{g,t}$. We have then shown that

$$\left.\frac{\mathrm{d}\mathcal{P}_{t_1,z}}{\mathrm{d}z}\right|_{z=t_1} = g^D_{t_1} \quad \text{and} \quad \left.\frac{\mathrm{d}\mathcal{L}_{t_0,z}}{\mathrm{d}z}\right|_{z=t_0} = g^D_{t_0}.$$

Stage 2: Let us now prove (36). From Proposition 4,

$$\frac{\mathrm{d}\log \widehat{m}_{t,t,z}}{\mathrm{d}z} = \frac{m_z}{\widehat{m}_{t,t,z}} \cdot \left(g^D_z + \mathrm{dev}_{t,t,z}\right),$$

where

$$\mathrm{dev}_{t,t,z} = s_{e,t}\Big(\big(1 - v_{t,z}\big)\pi_s + \big(s_{g,t} - v_{t,z}\widehat{s}_{g,t,z}\big)\big(\pi_g - \pi_s\big)\Big).$$

From Proposition 3

$$\widehat{m}_{t,t,z} = m_z + B_{t,t,z}\, e_t,$$

where

$$B_{t,t,z} := \left(\frac{1}{\chi} - 1 - \frac{s_{g,t}}{\gamma}\right) - \left(\frac{1}{\chi} - 1 - \frac{\widehat{s}_{g,t,z}}{\gamma}\right)\left(\frac{P_{s,t}}{P_{s,z}}\right)^{\frac{\chi}{1-\chi}}.$$

Since

$$\widehat{s}_{g,t,x} = \eta\left(\frac{\widehat{e}_{t,x}}{P_{s,x}}\right)^{-\chi}\left(\frac{P_{g,x}}{P_{s,x}}\right)^{\gamma} \quad \textit{and} \quad \widehat{e}_{t,x} = \left(\nu_t P^{\chi}_{s,x}\right)^{\frac{1}{\chi-1}}.$$

From the definition of $v_{t,z}$ and the Roy's identity,

$$v_{t,z} = \left(\frac{P_{s,t}}{P_{s,z}}\right)^{\frac{\chi}{1-\chi}} \quad \text{and} \quad \widehat{s}_{g,t,z} = \eta\left(\frac{\widehat{e}_{t,z}}{P_{s,z}}\right)^{-\chi}\left(\frac{P_{g,z}}{P_{s,\tau}}\right)^{\gamma} = \eta\, \nu_t^{\frac{\chi}{1-\chi}}\, P_{s,z}^{\frac{\chi}{1-\chi}-\gamma}\, P^{\gamma}_{g,z}.$$

Then,

$$\mathrm{dev}_{t,t,z} = s_{e,t}\Big(\big(1 - v_{t,z}\big)\pi_s + \big(s_{g,t} - v_{t,z}\widehat{s}_{g,t,z}\big)\big(\pi_g - \pi_s\big)\Big).$$

Let's first show that $\mathrm{dev}_{t_1,t_1,z} < 0$ by showing that the factors multiplying π_g and π_s are both negative. Let's start by

$$s_{g,t} - v_{t,z}\widehat{s}_{g,t,z} = \eta\left(\frac{\nu_t}{P_{s,t}}\right)^{\frac{\chi}{1-\chi}}\left(\left(\frac{P_{g,t}}{P_{s,t}}\right)^{\gamma} - \left(\frac{P_{g,z}}{P_{s,z}}\right)^{\gamma}\right) \gtrless 0, \quad \text{iff} \quad z \lessgtr t,$$

which is true since $\pi_s > \pi_g$. Moreover, under $\left(\frac{\chi}{1-\chi} - \gamma\right)\pi_s + \gamma\pi_g > 0$,

$$\widehat{s}_{g,t_1,z} = \eta\, \nu_{t_1}^{\frac{\chi}{1-\chi}}\, P_{s,z}^{\frac{\chi}{1-\chi}-\gamma}\, P^{\gamma}_{g,z} \leqslant \eta\, \nu_{t_1}^{\frac{\chi}{1-\chi}}\, P_{s,t_1}^{\frac{\chi}{1-\chi}-\gamma}\, P^{\gamma}_{g,t_1} = s_{g,t_1}.$$

Moreover, since $\pi_s > 0$ and $z < t_1$,

$$v_{t_1,z} := \left(\frac{P_{s,t_1}}{P_{s,z}}\right)^{\frac{\chi}{1-\chi}} > 1.$$

Implying

$$(1 - s_{g,t_1}) - v_{t_1,z}(1 - \widehat{s}_{g,t_1,z}) < 0.$$

Consequently, $\text{dev}_{t_1,t_1,z} < 0$.

Finally, let us show that

$$\frac{m_z}{\widehat{m}_{t_1,t_1,z}} < 1.$$

From Proposition 3.

$$\widehat{m}_{t_1,t_1,z} = m_z + B_{t_1,t_1,z}\, e_{t_1},$$

where

$$B_{t_1,t_1,z} := \left(\frac{1}{\chi} - 1 - \frac{s_{g,t_1}}{\gamma}\right) - \left(\frac{1}{\chi} - 1 - \frac{\widehat{s}_{g,t_1,z}}{\gamma}\right)\left(\frac{P_{s,t_1}}{P_{s,z}}\right)^{\frac{\chi}{1-\chi}},$$

Since, as shown above $\widehat{s}_{g,t_1,z} \leqslant s_{g,t_1}$ and $\frac{P_{s,t_1}}{P_{s,z}} > 1$, $B_{t_1,t_1,z} < 0$.

It is easy to check that the opposite is true for $\mathcal{L}_{t_0,z}$, which completes the proof. □

B Bopart's Lemma 1

In order for the preference ordering underlying the indirect utility function $V(e, P_g, P_s)$ to be well-defined, Boppart's Lemma 1 requires that, for all $t \in (t_0, t_1)$,

$$e_t^{\chi} \geqslant \frac{1-\chi}{1-\gamma}\,\eta\, P_{g,t}^{\gamma} P_{s,t}^{\chi-\gamma}. \tag{37}$$

Under the calibration presented in Section 4, this condition holds along the equilibrium path of the structural transformation economy. Note that inequality (37) is equivalent to

$$s_{g,t} \leqslant \frac{1-\gamma}{1-\chi}.$$

Given the calibrated values of $\gamma = 0.9$ and $\chi = 0.55$, this bound implies that $s_{g,t} \leqslant 0.\overline{2}$ for all $t \geqslant 1980$, as shown in Figure 6.

For the preference order behind a current-base Fisher–Shell index $\mathcal{P}_{t_0,z}$, $z \in (t_0, t_1)$, to be well-defined,

$$\widehat{e}_{t_1,z}^{\chi} \geqslant \frac{1-\chi}{1-\gamma}\,\eta\, P_{g,z}^{\gamma} P_{s,z}^{\chi-\gamma} \quad \text{or equivalently} \quad \widehat{s}_{g,t_1,z} \leqslant \frac{1-\gamma}{1-\chi}.$$

and

$$e_{t_1}^{\chi} \geqslant \frac{1-\chi}{1-\gamma}\,\eta\, P_{g,t_1}^{\gamma} P_{s,t_1}^{\chi-\gamma}. \quad \text{or equivalently} \quad s_{g,t_1} \leqslant \frac{1-\gamma}{1-\chi}.$$

The first condition corresponds to the application of Bopart's Lemma 1 to the indirect utility $u(\widehat{e}_{t_1,z}, P_{g,z}, P_{s,z}; \nu_{t_1})$ and the second to expenditure function $f(w, P_{g,t_1}, P_{s,t_1}; \nu_{t_1})$. The second condition holds, since Bopart's Lemma 1 hold at any point in the equilibrium path. Since ν_z is decreasing along the equilibrium path,

$$\widehat{e}_{t_1,z} = \left(\frac{\nu_{t_1}}{\nu_z}\right)^{\frac{1}{\chi-1}} e_z > e_z > \frac{1-\chi}{1-\gamma}\,\eta\, P_{g,z}^{\gamma} P_{s,z}^{\chi-\gamma},$$

for all $z \in (t_0, t_1)$ at the ABGP equilibrium of the structural transformation economy. This can be observed in the behavior of $\widehat{s}_{g,t_1,z}$ in Figure 6, which is always below the critical value $0.\overline{2}$.

For the preference order behind a reference-base Fisher–Shell index $\mathcal{L}_{t_0,z}$, $z \in (t_0, t_1)$, to be well-defined,

$$\widehat{e}_{t_0,z}^{\chi} \geqslant \frac{1-\chi}{1-\gamma}\,\eta\, P_{g,z}^{\gamma} P_{s,z}^{\chi-\gamma}.$$

and

$$e_{t_0}^{\chi} \geqslant \frac{1-\chi}{1-\gamma}\,\eta\, P_{g,t_0}^{\gamma} P_{s,t_0}^{\chi-\gamma}.$$

As for the current-base Fisher–Shell index, the second condition holds, since Bopart's Lemma 1 hold at any point in the equilibrium path. However, the first condition may not hold, as it can be observed in Figure 6.

Replication Files

Supplementary replication materials can be found at https://github.com/jivizcaino/Chained-Indices-Unchained .

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