Natural Disasters and Growth: The Role of Foreign Aid and Disaster Insurance

Appendix

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1 Proofs

1.1 Proposition 1

The Bellman Equation in the *normal phase* is given by

$$\rho V_N \frac{w^{1-\gamma}}{1-\gamma} = \max_{c,\alpha,\kappa,b} \left\{ c^{1-\gamma}\frac{w^{1-\gamma}}{1-\gamma} + V_N w^{1-\gamma}\left[\left(\alpha A + (1-\alpha) H\right) - (\kappa + b + c)\right] \right. \tag{1}$$
$$\left. -\gamma V_N w^{1-\gamma}\frac{\sigma^2}{2}\alpha^2 + \eta\left[V_D \frac{w^{1-\gamma}}{1-\gamma} E\left[\left(\beta(\alpha,b,\kappa,\mu^\delta,\delta,\zeta)\right)^{1-\gamma}\right] - V_N\frac{w^{1-\gamma}}{1-\gamma}\right]\right\}.$$

The FOCs are

$$\begin{aligned}
&c_N : c_N^{-\gamma} - V_N w^{1-\gamma} = 0\\
&\alpha_N : (A-H)\, V_N w^{1-\gamma} - \sigma^2 \alpha_N \gamma V_N w^{1-\gamma}\\
&\quad + \eta V_D w^{1-\gamma}\beta(\alpha,b,\kappa,\mu^\delta,\delta,\zeta)^{-\gamma}\left\{\mu_k^\delta(\kappa)\,\delta_k\,(1+_k) - \mu_h^\delta(\kappa)\,\delta_h\,(1+_h)\right\} = 0\\
&b_N : -V_N w^{1-\gamma} + \eta V_D w^{1-\gamma}\beta(\alpha,b,\kappa,\mu^\delta,\delta,\zeta)^{-\gamma}\left(\frac{\partial I(b)}{\partial b}\right) = 0
\end{aligned}$$

Re-arranging the FOCs we obtain

$$c_N = V_N^{-\left(\frac{1}{\gamma}\right)}, \tag{2}$$

and

$$\alpha_N = \frac{A-H}{\gamma\sigma^2} + \frac{\eta\Delta_k(\kappa)}{V\beta(\alpha_N,b,\kappa,\mu^\delta,\delta,\zeta)^\gamma}, \frac{1}{\gamma\sigma^2} \tag{3}$$

where we have defined $\Delta_k(\kappa)$ to be

$$\Delta_k(\kappa) = \mu_k^\delta(\kappa)(1+\zeta_k) - \mu_h^\delta(\kappa)(1+\zeta_h).$$

Under actuarially fairly priced insurance $I(b) = \frac{b}{\eta}$, $\frac{\partial I(b)}{\partial b} = \frac{1}{\eta}$, and optimal demand for disaster insurance implies

$$V\beta(\alpha_N, b, \kappa, \mu^\delta, \delta, \zeta)^\gamma = 1. \tag{4}$$

The Bellman Equation in the *disaster phase* is given by

$$\begin{aligned}
\rho V_D \frac{w^{1-\gamma}}{1-\gamma} = \max_{c,\alpha,\kappa_D,b_D} \Bigg\{ & c^{1-\gamma}\frac{w^{1-\gamma}}{1-\gamma} + V_D w^{1-\gamma}\left[\left(\alpha A_D + (1-\alpha) H_D\right) - (\kappa_D + c + b_D)\right] \\
& - \gamma V_D w^{1-\gamma}\frac{\sigma^2}{2}\alpha^2 + \upsilon(\kappa_D)\left[V_N \frac{w^{1-\gamma}}{1-\gamma} - V_D \frac{w^{1-\gamma}}{1-\gamma}\right] \\
& + \eta \left[V_D \frac{w^{1-\gamma}}{1-\gamma} E\left[\left(\beta(\alpha, b_D, \kappa, \mu^\delta, \delta, \zeta_D)\right)^{1-\gamma}\right] - V_D \frac{w^{1-\gamma}}{1-\gamma}\right] \Bigg\}
\end{aligned} \tag{5}$$

The FOCs are

$$\begin{aligned}
& c_D : c_D^{-\gamma} - V_D w^{1-\gamma} = 0 \\
& \alpha_D : \left(A_D - H_D\right) V_D w^{1-\gamma} - \sigma^2 \alpha_D \gamma V_D w^{1-\gamma} \\
& \quad + \eta V_D w^{1-\gamma} \beta(\alpha, b_D, \kappa, \mu^\delta, \delta, \zeta_D)^{-\gamma} \left\{ \mu_k^\delta(\kappa)\, \delta_k\, (1+_k) - \mu_h^\delta(\kappa)\, \delta_h\, (1+_h) \right\} = 0 \\
& b_D : -V_D w^{1-\gamma} + \eta V_D w^{1-\gamma} \beta(\alpha, b_D, \kappa, \mu^\delta, \delta, \zeta_D)^{-\gamma} \left(\frac{\partial I(b)}{\partial b}\right) = 0
\end{aligned}$$

Re-arranging the FOCs we obtain

$$c_D = V_D^{-\left(\frac{1}{\gamma}\right)}, \tag{6}$$

and

$$\alpha_D = \frac{A_D - H_D}{\gamma\sigma^2} + \frac{\eta \Delta_k(\kappa)}{\beta(\alpha_D, b_D, \kappa, \mu^\delta, \delta, \zeta_D)^\gamma} \frac{1}{\gamma\sigma^2}. \tag{7}$$

Optimal demand for disaster insurance implies

$$\beta(\alpha_D, b_D, \kappa, \mu^\delta, \delta, \zeta_D) = 1. \tag{8}$$

Replacing (2) and (3) into (1), and dividing both sides by $V_N \frac{w^{1-\gamma}}{1-\gamma}$ we obtain

$$\begin{aligned}
(\rho + \eta) = \Bigg\{ & \gamma V_N^{-\left(\frac{1}{\gamma}\right)} + (1-\gamma)\left[\alpha_N^\star \frac{(A - H)}{\gamma\sigma^2} + H - b_N^\star - \kappa_N^\star\right] - (1-\gamma)\gamma\frac{\sigma^2}{2}\left(\alpha_N^\star\right)^2 \\
& + \eta \frac{V_D}{V_N}\left(\beta(\alpha_N^\star, b_N^\star, \kappa^\star, \mu^\delta, \delta, \zeta_N^\star)\right)^{1-\gamma} \Bigg\}
\end{aligned}$$

Similarly, replacing (6) and (7) into (5), and dividing both sides by $V_D \frac{w^{1-\gamma}}{1-\gamma}$ we obtain

$$(\rho+\eta+\nu(\kappa_D)) = \left\{ \gamma V_D^{-\left(\frac{1}{\gamma}\right)} + (1-\gamma)\left[\alpha_D^\star \frac{(A_D - H_D)}{\gamma\sigma^2} + H_D - b_D^\star - \kappa_D^\star\right] - (1-\gamma)\gamma\frac{\sigma^2}{2}(\alpha_D^\star)^2 \right.$$
$$\left. + \nu(\kappa_D)\frac{V_N}{V_D} + \eta\left(\beta(\alpha_D^\star, b_D^\star, \kappa^\star, \mu^\delta, \delta, \zeta_D^\star)\right)^{1-\gamma} \right\}$$

To economize on notation we define

$$P = H + \alpha_N(A-H) - \gamma\frac{\sigma^2}{2}\alpha_N^2 - \kappa,$$

$$P_D = H_D + \alpha_D(A_D - H_D) - \gamma\frac{\sigma^2}{2}\alpha_D^2 - \kappa_D,$$

and

$$V = \frac{V_N}{V_D}.$$

V_N is pinned down by

$$\gamma V_N^{-\left(\frac{1}{\gamma}\right)} = (\rho+\eta) - (1-\gamma)P + (1-\gamma)b_N^\star - \eta V^{-1}\left(\beta(\alpha_N^\star, b_N^\star, \kappa^\star, \mu^\delta, \delta, \zeta_N^\star)\right)^{1-\gamma}, \quad (9)$$

and V_D by

$$\gamma V_D^{-\left(\frac{1}{\gamma}\right)} = (\rho+\eta+\nu(\kappa_D)) - (1-\gamma)P_D + (1-\gamma)b_D^\star - \eta\left(\beta(\alpha_D^\star, b_D^\star, \kappa^\star, \mu^\delta, \delta, \zeta_D^\star)\right)^{1-\gamma} - \nu(\kappa_D)V. \quad (10)$$

Dividing (10) by (9), we obtain

$$V^{\left(\frac{1}{\gamma}\right)} = \frac{(\rho+\eta+\nu(\kappa_D)) - (1-\gamma)P_D + (1-\gamma)b_D^\star - \eta\left(\beta(\alpha_D^\star, b_D^\star, \kappa^\star, \mu^\delta, \delta, \zeta_D^\star)\right)^{1-\gamma} - \nu(\kappa_D)V}{(\rho+\eta) - (1-\gamma)P + (1-\gamma)b_N^\star - \eta V^{-1}\left(\beta(\alpha_N^\star, b_N^\star, \kappa^\star, \mu^\delta, \delta, \zeta_N^\star)\right)^{1-\gamma}},$$

and the equilibrium relative valuation V^* is pinned down by

$$V^{\left(\frac{1}{\gamma}\right)}\left((\rho+\eta) - (1-\gamma)P + (1-\gamma)b_N^\star\right) - \eta V^{\frac{1-\gamma}{\gamma}}\left(\beta(\alpha_N^\star, b_N^\star, \kappa^\star, \mu^\delta, \delta, \zeta_N^\star)\right)^{1-\gamma} =$$
$$(\rho+\eta+\nu(\kappa_D)) - (1-\gamma)P_D + (1-\gamma)b_D^\star - \eta\left(\beta(\alpha_D^\star, b_D^\star, \kappa^\star, \mu^\delta, \delta, \zeta_D^\star)\right)^{1-\gamma} - \nu(\kappa_D)V \quad (11)$$

Finally, the growth rates in the *normal* and *disaster* phases, conditional on no strikes, are:

$$\mu^N = \mathbb{E}\left(\frac{dw}{w} \mid N_t = 0\right) = \alpha_N(A-H) + H - \kappa + b_N + c_N$$
$$= H + \alpha_N(A-H) - \kappa - b_N - V_N^{-\left(\frac{1}{\gamma}\right)},$$

and

$$\mu^D = \mathbb{E}\left(\frac{dw}{w}|N_t = 0, M_t = 0\right) = \alpha_D\left(A_D - H_D\right) + H_D - \kappa_D + b_D + c_D$$
$$= H_D + \alpha_D\left(A_D - H_D\right) - \kappa_D - b_D - V_D^{-\left(\frac{1}{\gamma}\right)}.$$

$\square$

1.2 Existence, Uniqueness, and Properties of the Equilibrium.

Consider the case where the properties of natural disasters do not affect the composition of the portfolio. To be precise, assume

$$\mu^\delta(\kappa) = \mu_k^\delta(\kappa)(1+\zeta_k) = \mu_h^\delta(\kappa)(1+\zeta_h).$$

In this case, $\Delta_k\left(\kappa\right) = 0$, and

$$\alpha_N = \frac{A-H}{\gamma\sigma^2},$$
$$\alpha_D = \frac{A_D - H_D}{\gamma\sigma^2},$$

and

$$\beta(\alpha, b, \kappa, \mu^\delta, \delta, \zeta) = \mu^\delta(\kappa) + \frac{b_N}{\eta} + \zeta_N.$$

Re-arranging (11), the relative valuation of the problem solves

$$\begin{aligned}&(\rho+\eta) - (1-\gamma)P - \eta V^{-1}\beta(\alpha, b, \kappa, \mu^\delta, \delta, \zeta)^{1-\gamma}\\ &= \left[\rho + \eta + \upsilon(\kappa_D) - (1-\gamma)P_D - \eta\beta(\alpha, b, \kappa, \mu^\delta, \delta, \zeta_D)^{1-\gamma} - \upsilon(\kappa_D)V\right]V^{-1/\gamma}.\end{aligned}$$

To study the properties of the solution, it is convenient to analyze the left and the right hand side of this equation separately. Define $L\left(V\right)$ and $R\left(V\right)$ to be

$$L\left(V\right) = (\rho+\eta) - (1-\gamma)P - \eta V^{-1}\beta(\alpha, b, \kappa, \mu^\delta, \delta, \zeta)^{1-\gamma},$$

and

$$R\left(V\right) = \left[\rho + \eta + \upsilon(\kappa_D) - (1-\gamma)P_D - \eta\beta(\alpha, b, \kappa, \mu^\delta, \delta, \zeta_D)^{1-\gamma} - \upsilon(\kappa_D)V\right]V^{-1/\gamma}.$$

$L\left(V\right)$ has the following properties:

- $L^{'}(V) > 0$
- $L^{''}(V) < 0$
- $\underset{V\to 0}{lim} L(V) \to -\infty$
- $\underset{V\to +\infty}{lim} L(V) \to (\rho+\eta) - (1-\gamma)P$
- $L(1) = (\rho+\eta) - (1-\gamma)P - \eta\beta(\alpha, b, \kappa, \mu^{\delta}, \delta, \zeta)^{1-\gamma}$

To study the properties of $R(V)$ it is convenient to express it as

$$R(V) = \left(\rho+\eta+\upsilon(\kappa_D) - (1-\gamma)P_D - \eta\beta(\alpha, b, \kappa, \mu^{\delta}, \delta, \zeta_D)^{1-\gamma}\right) V^{-1/\gamma} - \upsilon(\kappa_D)V^{\frac{\gamma-1}{\gamma}}.$$

If $\gamma > 1$ and $\left(\rho+\eta+\upsilon(\kappa_D) + (\gamma-1)P_D - \eta\beta(\alpha, b, \kappa, \mu^{\delta}, \delta, \zeta_D)^{1-\gamma}\right) > 0$:

- $R^{'}(V) < 0$
- $R^{''}(V) < 0$
- $\underset{V\to 0}{lim} R(V) \to +\infty$
- $\underset{V\to +\infty}{lim} R(V) \to -\infty$
- $R(1) = (\rho+\eta) + (\gamma-1)P_D - \eta\beta(\alpha, b, \kappa, \mu^{\delta}, \delta, \zeta_D)^{1-\gamma}$.

On the other hand, if $\gamma \in (0,1)$ and

$$\left(\rho+\eta - (1-\gamma)P_D - \eta\beta(\alpha, b, \kappa, \mu^{\delta}, \delta, \zeta_D)^{1-\gamma}\right) > 0$$

- $R(V) > 0 \Leftrightarrow \left(\rho+\eta+\upsilon(\kappa_D) - (1-\gamma)P_D - \eta\beta(\alpha, b, \kappa, \mu^{\delta}, \delta, \zeta_D)^{1-\gamma}\right) V^{-1/\gamma} > \upsilon(\kappa_D)V^{1-\frac{1}{\gamma}}$, which requires:

$$V > \left(\frac{\rho+\eta+\upsilon(\kappa_D) - (1-\gamma)P_D - \eta\beta(\alpha, b, \kappa, \mu^{\delta}, \delta, \zeta_D)^{1-\gamma}}{\upsilon(\kappa_D)}\right) \equiv \tilde{V}.$$

- $R^{'}(V) > 0 \Leftrightarrow \upsilon(\kappa_D)(1-\gamma)V^{-1} > \left(\rho+\eta+\upsilon(\kappa_D) - (1-\gamma)P_D - \eta\beta(\alpha, b, \kappa, \mu^{\delta}, \delta, \zeta_D)^{1-\gamma}\right)$, which requires:

$$V > \left(\frac{\rho+\eta+\upsilon(\kappa_D) - (1-\gamma)P_D - \eta\beta(\alpha, b, \kappa, \mu^{\delta}, \delta, \zeta_D)^{1-\gamma}}{\upsilon(\kappa_D)(1-\gamma)}\right) \equiv \hat{V}$$

- $\hat{V}$ and $\tilde{V}$ have the following properties:

- $\tilde{V} < 1$
- $\hat{V} > \tilde{V}$
- $\hat{V} > 1 \text{if} \rho + \eta + \upsilon(\kappa_D) - (1-\gamma)\left(P_D - \upsilon(\kappa_D)\right) - \eta\beta(\alpha, b, \kappa, \mu^\delta, \delta, \zeta_D)^{1-\gamma} > 0$

- $\lim_{V \to 0} R(V) \to -\infty$
- $\lim_{V \to +\infty} R(V) \to 0$
- $R(1) = (\rho + \eta) - (1-\gamma)P_D - \eta\beta(\alpha, b, \kappa, \mu^\delta, \delta, \zeta_D)^{1-\gamma}$.

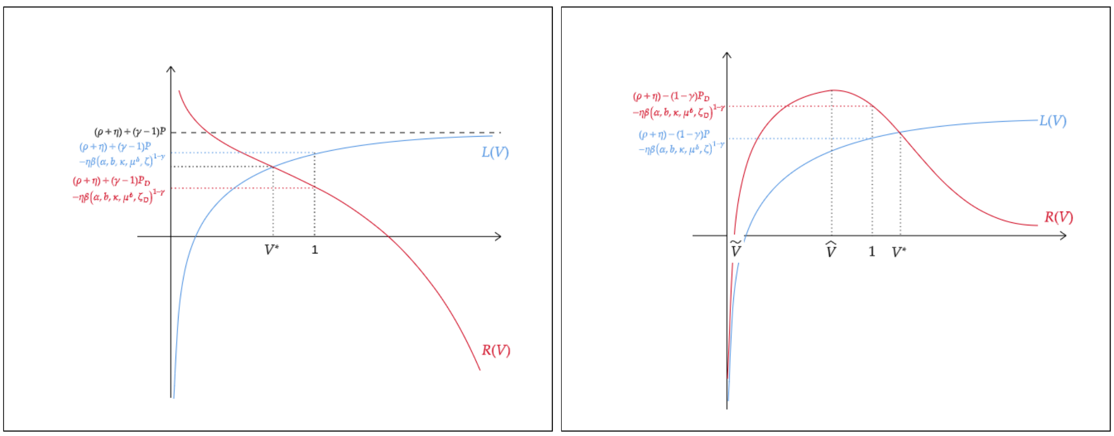
The properties of $R(V)$ and $L(V)$ above described, together with conditions imposed on the model parameters are sufficient conditions for the solution to exist and be unique. Furthermore, assuming $\zeta_D = \zeta = \bar{\zeta}$, when $\gamma > 1$ we have

$$L(1) > R(1)$$
$$(\rho + \eta) + (\gamma - 1)P - \eta\beta(\alpha, b, \kappa, \mu^\delta, \delta, \bar{\zeta})^{1-\gamma} > (\rho + \eta) + (\gamma - 1)P_D - \eta\beta(\alpha, b, \kappa, \mu^\delta, \delta, \bar{\zeta})^{1-\gamma}$$
$$(\gamma - 1)P > (\gamma - 1)P_D,$$

and since $L(V)$ and $R(V)$ are strictly increasing and strictly decreasing in V, the solution is such that $V^* < 1$.When $\gamma < 1$, $R(1) > L(1)$, $L(V)$ is strictly increasing in V. Since $\hat{V} < 1$, V is in the region of the domain where $R(V)$ it is strictly decreasing, and the solution is such that $V^* > 1$. The properties of the equation that pins down the equilibrium are summarized in the figure below:

Figure 1: Characterization of the Equilibrium for Different Values of Intertemporal Elasticity of Substitution.

(*Left Panel* $\gamma > 1$ - *Right Panel* $\gamma \in (0, 1)$)



1.3 Proposition 2.

The proof follows from equation (11), which pins down the equilibrium value of V^*.

$$L(V) = R(V)$$

$$(\rho+\eta) - (1-\gamma)P - \eta V^{-1}\beta(\alpha,b,\kappa,\mu^{\delta},\delta,\zeta)^{1-\gamma} = \\ \left[\rho+\eta+\upsilon(\kappa_D) - (1-\gamma)P_D - \eta\beta(\alpha,b,\kappa,\mu^{\delta},\delta,\zeta_D)^{1-\gamma} - \upsilon(\kappa_D)V\right]V^{-1/\gamma}$$

There are two possible cases:

High intertemporal elasticity of substitution ($\gamma \in (0,1)$).

In this case $L(V)$ is strictly increasing in V,and in a neighbourhood V^*, $R(V)$ is strictly decreasing in V.

- $\frac{\partial V^*}{\partial \zeta} > 0$
  An increase in ζ rises $\beta(\alpha,b,\kappa,\mu^{\delta},\delta,\zeta)^{1-\gamma}$, which reduces $L(V)$. Restoring the equilibrium requires an increase in V^*.

- $\frac{\partial V^*}{\partial \zeta_D} < 0$
  An increase in ζ_D rises $\beta(\alpha,b,\kappa,\mu^{\delta},\delta,\zeta_D)^{1-\gamma}$, which reduces $R(V)$. Restoring the equilibrium requires a reduction in V^* to reduce $L(V)$.

- $\frac{\partial V^*}{\partial \upsilon(\kappa_D)} < 0$
  $\frac{\partial R(V)}{\partial \upsilon(\kappa_D)} = V^{-1/\gamma}(1-V) < 0$. Restoring the equilibrium requires a reduction in V^* to reduce $L(V)$.

- $\frac{\partial V^*}{\partial \eta}|_{\zeta=\zeta_D} < 0$
  $\frac{\partial R(V)}{\partial \eta} = \left(1-\beta(\alpha,b,\kappa,\mu^{\delta},\delta,\zeta)^{1-\gamma}V^{-1}\right) > \frac{\partial L(V)}{\partial \eta} = \left(V^{-1/\gamma}\left[1-\beta(\alpha,b,\kappa,\mu^{\delta},\delta,\zeta_D)^{1-\gamma}\right]\right)$.
  Restoring the equilibrium requires a reduction in V^*.

Low intertemporal elasticity of substitution ($\gamma > 1$).

- $\frac{\partial V^*}{\partial \zeta} < 0$
  An increase in ζ reduces $\beta(\alpha,b,\kappa,\mu^{\delta},\delta,\zeta)^{1-\gamma}$, which increases $L(V)$. Restoring the equilibrium requires a reduction in V^*.

- $\frac{\partial V^*}{\partial \zeta_D} > 0$
  An increase in ζ_D leads to a decline in $\beta(\alpha,b,\kappa,\mu^{\delta},\delta,\zeta_D)^{1-\gamma}$, which increases $R(V)$.

Restoring the equilibrium requires a rise in V^* to increase $L(V)$.

- $\frac{\partial V^*}{\partial v(\kappa_D)} > 0$
  Since $V < 1^*, \frac{\partial R(V)}{\partial v(\kappa_D)} = V^{-1/\gamma}(1-V) > 0$. Restoring the equilibrium requires a rise in V^* to increase $L(V)$.

- $\frac{\partial V^*}{\partial \eta}|_{\zeta=\zeta_D} < 0$
  $V < 1^* \implies$
  $\frac{\partial R(V)}{\partial \eta} = \left(1-\beta(\alpha,b,\kappa,\mu^\delta,\delta,\zeta)^{1-\gamma}V^{-1}\right) < \frac{\partial L(V)}{\partial \eta} = \left(V^{-1/\gamma}\left[1-\beta(\alpha,b,\kappa,\mu^\delta,\delta,\zeta_D)^{1-\gamma}\right]\right)$.
  Restoring the equilibrium requires an increase in V^*.

$\square$

1.4 Proposition 4.

It is convenient to prove Proposition 4 first before continuing with Proposition 3. From Proposition 1, the growth rates in the *normal* and *disaster* phases are

$$\mu^N = H + \alpha_N (A - H) - \kappa - b_N - V_N^{-\left(\frac{1}{\gamma}\right)},$$

$$\mu^D = H_D + \alpha_D (A_D - H_D) - \kappa_D - b_D - V_D^{-\left(\frac{1}{\gamma}\right)}.$$

Assuming $\mu^\delta(\kappa) = \mu_k^\delta(\kappa)(1+\zeta_k) = \mu_h^\delta(\kappa)(1+\zeta_h), \Delta_k(\kappa) = 0$, and $\alpha_N = \frac{A-H}{\gamma\sigma^2}$,

$$\alpha_D = \frac{A_D - H_D}{\gamma\sigma^2}, \beta(\alpha,b,\kappa,\mu^\delta,\delta,\zeta) = \mu^\delta(\kappa) + \frac{b}{\eta} + \zeta.$$

P and P_D are given by

$$P = H + \frac{1}{2}\left(\frac{A-H}{\gamma\sigma^2}\right) - \kappa,$$

and

$$P_D = H_D + \frac{1}{2}\left(\frac{A_D - H_D}{\gamma\sigma^2}\right) - \kappa_D.$$

Moreover, V_N and V_D are pinned down by

$$\gamma V_N^{-\left(\frac{1}{\gamma}\right)} = (\rho+\eta) - (1-\gamma)P + (1-\gamma)b_N^\star - \eta V^{-1}\left(\mu^\delta(\kappa) + \frac{b}{\eta} + \zeta\right)^{1-\gamma},$$

and

$$\gamma V_D^{-\left(\frac{1}{\gamma}\right)} = (\rho + \eta + \nu(\kappa_D)) - (1-\gamma)P_D + (1-\gamma)b_D^\star - \eta\left(\mu^\delta(\kappa) + \frac{b_D}{\eta} + \zeta_D\right)^{1-\gamma} - \nu(\kappa_D)V.$$

And the growth rates are

$$\mu^N = H + \frac{(A-H)^2}{\gamma\sigma^2} - \kappa - b_N - \frac{(\rho+\eta)}{\gamma} + \frac{(1-\gamma)}{\gamma}P - \frac{(1-\gamma)}{\gamma}b_N^\star + \frac{\eta}{\gamma}V^{-1}\left(\mu^\delta(\kappa) + \frac{b}{\eta} + \zeta\right)^{1-\gamma}$$

$$\mu^D = H_D + \frac{(A_D-H_D)^2}{\gamma\sigma^2} - \kappa_D - b_D - \frac{(\rho+\eta+\nu(\kappa_D))}{\gamma} + \frac{(1-\gamma)}{\gamma}P_D - \frac{(1-\gamma)}{\gamma}b_D^\star$$
$$+ \frac{\eta}{\gamma}\left(\mu^\delta(\kappa) + \frac{b_D}{\eta}\zeta_D\right)^{1-\gamma} + \frac{\nu(\kappa_D)}{\gamma}V,$$

which simplify to

$$\mu^N = \frac{1}{\gamma}\left[H + \frac{(\gamma+1)}{2}\left(\frac{(A-H)^2}{\gamma\sigma^2}\right) - (\rho+\eta) - \kappa - b_N + \eta V^{-1}\left(\mu^\delta(\kappa) + \frac{b}{\eta} + \zeta\right)^{1-\gamma}\right],$$

$$\mu^D = \frac{1}{\gamma}\left[H_D + \frac{(\gamma+1)}{2}\left(\frac{(A_D-H_D)^2}{\gamma\sigma^2}\right) - (\rho+\eta+\nu(\kappa_D)) - \kappa_D - b_D\right.$$
$$\left. + \eta\left(\mu^\delta(\kappa) + \frac{b}{\eta} + \zeta_D\right)^{1-\gamma} + \nu(\kappa_D)V\right].$$

Under no insurance markets for disasters $b_N = 0$ and $b_D = 0$. Focus on the case where $V > 1$.

- $$\frac{\partial\mu^N}{\partial\zeta} = \eta(1-\gamma)V^{-1}\left(\mu^\delta(\kappa)+\zeta\right)^{-\gamma} - \left(\frac{\partial V}{\partial\zeta}\right)\eta V^{-2}\left(\mu^\delta(\kappa)+\zeta\right)^{1-\gamma}$$
$$= \eta V^{-1}\left(\mu^\delta(\kappa)+\zeta\right)^{-\gamma}\left((1-\gamma) - \left(\frac{\partial V}{\partial\zeta}\right)\eta V^{-1}\left(\mu^\delta(\kappa)+\zeta\right)\right) > 0 \text{ or } < 0.$$
- $\frac{\partial\mu^N}{\partial\zeta_D} = \eta(-1)V^{-2}\left(\frac{\partial V}{\partial\zeta_D}\right) > 0.$
- $\frac{\partial\mu^D}{\partial\zeta} = \left(\frac{1}{\gamma}\right)\nu(\kappa_D)\left(\frac{\partial V}{\partial\zeta}\right) < 0.$
- $\frac{\partial\mu^D}{\partial\zeta_D} = \left(\frac{1}{\gamma}\right)\left(\eta(1-\gamma)\left(\mu^\delta(\kappa)+\zeta_D\right)^{-\gamma} + \nu(\kappa_D)\left(\frac{\partial V}{\partial\zeta_D}\right)\right) > 0$ or $< 0.$

□

1.5 Proposition 3.

Under full insurance, in the normal phase $V\left(\mu^{\delta}(\kappa)+b_N+\zeta\right)^{\gamma}=1$ and $\left(\mu^{\delta}(\kappa)+b_D+\zeta_D\right)=1$. As a consequence, the equation that pins down the relative valuation is independent of ζ_D, and the growth rates are given by

$$\mu^N=\frac{1}{\gamma}\left[H+\frac{(\gamma+1)}{2}\left(\frac{(A-H)^2}{\gamma\sigma^2}\right)-(\rho+\eta)-\kappa-b_N+\eta\left(\mu^{\delta}(\kappa)+\frac{b_N}{\eta}+\zeta\right)\right],$$

and

$$\mu^D=\frac{1}{\gamma}\left[H_D+\frac{(\gamma+1)}{2}\left(\frac{(A_D-H_D)^2}{\gamma\sigma^2}\right)-(\rho+\nu(\kappa_D))-\kappa_D-b_D+\nu(\kappa_D)V\right].$$

- $\frac{\partial\mu^N}{\partial\zeta}=\left(\frac{1}{\gamma}\right)\eta>0$
- $\frac{\partial\mu^N}{\partial\zeta_D}=0$
- $\frac{\partial\mu^N}{\partial\bar{\zeta}}=\left(\frac{1}{\gamma}\right)\eta>0$
- $\frac{\partial\mu^D}{\partial\zeta}=\left(\frac{1}{\gamma}\right)\nu(\kappa_D)\left(\frac{\partial V}{\partial\zeta}\right)<0$
- $\frac{\partial\mu^D}{\partial\zeta_D}=0$
- $\frac{\partial\mu^D}{\partial\bar{\zeta}}=\left(\frac{1}{\gamma}\right)\nu(\kappa_D)\left(\frac{\partial V}{\partial\zeta}\right)<0$

$\square$

1.6 Proposition 5.

Assume the country has full access to disaster insurance. In this case, the equation that pins that the relative valuation in equilibrium is

$$(\rho+\eta)-(1-\gamma)P-\eta\left(\mu^{\delta}(\kappa)+\frac{b_N}{\eta}+\zeta\right)=\left[\rho+\upsilon(\kappa_D)-(1-\gamma)P_D-\upsilon(\kappa_D)V\right]V^{-1/\gamma}.$$

- Changes in disaster frequency (η)
  - $\frac{\partial\mu^N}{\partial\eta}=\left(\frac{1}{\gamma}\right)\left(-1+\left(\mu^{\delta}(\kappa)+\frac{b_N}{\eta}+\zeta\right)\right)<0$
  - $\frac{\partial\mu^D}{\partial\eta}=\left(\frac{1}{\gamma}\right)\left(\nu(\kappa_D)\left(\frac{\partial V}{\partial\eta}\right)\right)\begin{cases}<0 \text{ if } \gamma>1\\ >0 \text{ if } \gamma\in(0,1)\end{cases}$

- Changes recovery speed $\nu(\kappa_D)$
  - $\frac{\partial \mu^N}{\partial \nu(\kappa_D)} = 0$
  - $\frac{\partial \mu^D}{\partial \nu(\kappa_D)} = \left(\frac{1}{\gamma}\right)(-1+V) \begin{cases} < 0 \text{ if } \gamma > 1 \\ > 0 \text{ if } \gamma \in (0,1) \end{cases}$
- Changes in prevention technologies $\mu^\delta(\kappa)$
  - $\frac{\partial \mu^N}{\partial \mu(\kappa)} = \left(\frac{1}{\gamma}\right)\eta > 0$
  - $\frac{\partial \mu^D}{\partial \mu(\kappa)} = \left(\frac{1}{\gamma}\right)\left(\nu(\kappa_D)\left(\frac{\partial V}{\partial \mu(\kappa)}\right)\right) > 0$
- Changes in productivity loss associated to disasters ϕ. Defining $\frac{P_D}{P} = 1 - \phi$
  - $\frac{\partial \mu^N}{\partial \phi} = \left(\frac{1}{\gamma}\right)(P_D(-1)) < 0$
  - $\frac{\partial \mu^D}{\partial \phi} = \left(\frac{1}{\gamma}\right)\left(-P + \nu(\kappa_D)\left(\frac{\partial V}{\partial \phi}\right)\right) < 0$

1.7 Proposition 6.

Assume $\gamma > 1$. First, to pin down how changes in κ affect V^*, we compute

$$\frac{\partial L(V)}{\partial \kappa} = -(1-\gamma)\frac{\partial P}{\partial \kappa} - \eta(1-\gamma)\beta(\alpha, b, \kappa, \mu^\delta, \delta, \zeta_D)^{-\gamma}\mu'(\kappa) = (1-\gamma)\left(1 - \eta\mu'(\kappa)\beta(\alpha, b, \kappa, \mu^\delta, \delta, \zeta_D)^{-\gamma}\right) < 0$$

Thus, restoring the equilibrium requires an increase in V^* to reduce $R(V)$, and $\frac{\partial V}{\partial \kappa} > 0$.
Additionally for all γ:

$$\frac{\partial V}{\partial \kappa_D} = 0 \Longleftrightarrow \frac{\partial R(V)}{\partial \kappa_D} = 0 \Longleftrightarrow \nu'(\kappa_D)(1-V) - (1-\gamma)\left(\frac{\partial P_D}{\partial \kappa_D}\right) = 0$$

$$(1-V) = \frac{(1-\gamma)}{\nu'(\kappa_D)}\left(\frac{\partial P_D}{\partial \kappa_D}\right)$$