State-Observer Design of a PDE-Modeled Mining Cable Elevator With Time-Varying Sensor Delays

Ji Wang, Yangjun Pi, Yumei Hu, and Zhencai Zhu

***Abstract*— A time-varying sensor delay is inevitable in a mining cable elevator operating at a maximum distance which is more than 2000-m underground, when the sensor signals are transmitted from a cage at the bottom of the cable to a control center on the ground through a set of wireless devices, including emitters and receivers. This sensor delay may degrade the estimation precision of the state observer or even destroy the stability of the closed-loop system. In this brief, we design an observer which can compensate the sensor delay with a time-varying and arbitrary length to estimate the vibration states of the mining cable elevator, including the time-varying-length mining cable and the cage. The vibration dynamics of the mining cable elevator is described by a wave partial differential equation (PDE)-ordinary differential equation (ODE) coupled system on a time-varying domain, and the sensor delay dynamics is represented by a transport PDE on a time-varying domain. A full-order infinite dimensional observer is constructed to estimate the states of such a wave PDE-ODE-transport PDE coupled system where PDEs are on time-varying domains, only using available boundary values without measurements at the connection points of the PDEs and ODE. The exponential stability of the observer error system is proved via Lyapunov analysis. The effectiveness of the proposed observer is verified via numerical simulation.**

***Index Terms*— Distributed parameter system, hybrid system, sensor delay, state observer, time-varying delay.**

I. Introduction

A. Background

A MINING cable elevator which is used to transport the cargo and miners between the ground and the working platform underground is an important equipment in depth mining exploitation [18], [28]. Although cables have the advantages of low bending and torsional stiffness, resisting relatively large axial loads, which are helpful to the heavy-load and long-distance transportation, its compliance property or stretch and contract abilities tend to cause mechanical vibrations, which leads to imprecise positioning and premature fatigue fracture [16]. For the safe manipulation, the controller is designed acting at the top of the mining cable to suppress the vibrations of the cable and the cage [29]. The controller at the top boundary of the cable always needs the vibration states distributed in the cable which are always not feasible in practical engineering. Therefore, a state observer [29] is required to provide the state estimation for the controller only using available boundary measurements obtained from the sensors placed at the boundaries. In addition to the direct feedback signals of the server actuator at the top boundary of the mining cable, some sensors are placed at the cage and the measurement signals are transmitted to the control center on the ground through a set of wireless devices, including emitters and receivers, as Fig. 1 shows. Because the maximum transmission distance from underground to the surface is more than 2000 m [18], [28], the sensing delay is inevitable. Moreover, the delay is time-varying because the delay length is related to the transmission distance which is from the bottom of the cable to its top, i.e., the length of the cable which is time-dependent. The sensor delay may degrade the estimation precision of the state observer or even destroy the stability of the closed-loop system [17]. Therefore, the time-varying delay of the sensing signals should be considered in the design of the state observer using these measurement signals, and make sure that the observer can effectively estimate the distributed vibration states in the mining cable under the time-varying sensor delay.

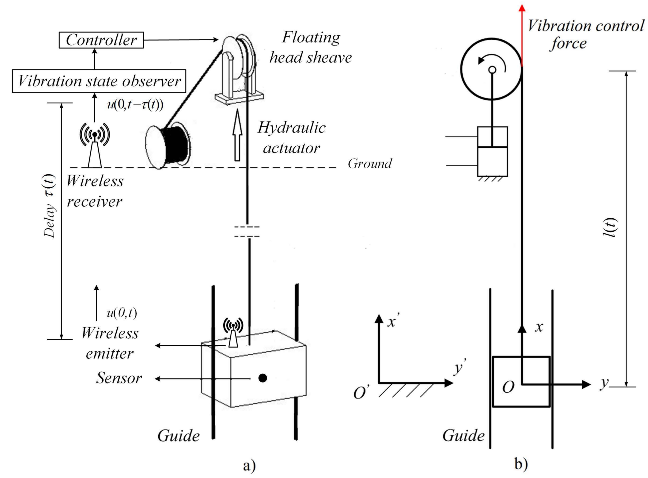

Fig. 1. Diagram of (a) mining cable elevator with a sensor delay and (b) simplified model as a time-varying-length vibrational string with a payload.

B. Observer Design for Wave PDEs

Mathematically, the vibration dynamics of the mining cable can be described by a wave partial differential equation (PDE) which represents the dynamics of a vibrational string [26],

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[27]. A state-observer design for a 1-D wave equation that contains instability at its free end is included in [23]. The first unknown input and variable structure PDE-state observer for a 1-D antistable wave equation subject to a general control matched disturbance was presented in [12]. An adaptive observer using the measured output to estimate unknown parameters of the disturbance and recover the system states for a 1-D wave equation with an anticollocated harmonic disturbance was designed in [13]. A state observer, including a disturbance estimator, was proposed to exponentially track the states of a wave PDE-ordinary differential equation (ODE) system with an anticollocated unknown disturbance in [31]. A spatial state derivative observer design was present for a wave PDE describing the oscillations of a string with some priori unknown spatially varying plant parameters in [24]. In [25], a state observer was designed for a wave PDE with collocated boundary sensing and perturbed Neumann-type actuation by using the sliding-mode technique. State-observer design for a wave PDE with in-domain viscous damping was included in [32] through an invertible pretransformation. An exponentially convergent observer was designed to reconstruct the distributed states of a wave PDE-ODE coupled system describing the vibration dynamics of a mining cable elevator in [29]. However, all the aforementioned works did not consider the measurement delay when designed the state observer for wave PDEs using boundary measurements.

C. Sensor-Delay Compensation Observer

In the past three decades, much attention has been paid to observer design in the presence of time delays. Some classical results about the design of delay compensation observers for ODE systems were proposed in [1], [8], and [10]. Compared with the above "reduced-order" observer which only estimates plant states, a full-order observer which captures the time-delay dynamics as a first-order hyperbolic PDE and estimates both plant states and sensor states was proposed in [21]. In [19], an observer for a linear time invariant (LTI) finite-dimensional system with sensor delays, which can be viewed as a coupled system consisting of an ODE and a first-order hyperbolic PDE, was designed via backstepping. The predictor-based observer design for a LTI system with a time-varying sensor delay was presented in [22]. An adaptive state observer for an ODE system subject to the sensor delay and parameter uncertainty was presented in [2]. However, the above research only considered the observer with sensor delay compensation for ODE systems. In [11], an infinite-dimensional observer was constructed to estimate the states of a wave PDE on a fixed domain under a constant sensor delay. To estimate distributed vibration states in the mining cable elevator, a state observer which can compensate a time-varying sensor delay for a wave PDE on a time-varying domain is required, and the result in this respect has not been reported yet, which motivates the research in this brief.

D. Organization

The contents in each section are briefly described as following.

1) The dynamics of a mining cable elevator with a time-varying sensor delay is presented in Section II. In detail, the vibration dynamics of the mining cable elevator simplified as a vibrating string of time-varying length with a payload at the bottom is described by a wave PDE-ODE coupled system on a time-varying domain. The dynamics of the time-varying sensor delay is represented by a transport PDE on a time-varying domain. The overall system to be estimated is a wave PDE-ODE-transport PDE coupled system where two PDEs are on time-varying domains.
2) A full-order observer for the aforementioned system is designed in Section III, only using boundary values in the input and output of the overall system without measurements at connection points of the PDEs and the ODE.
3) The stability analysis of the observer error via Lyapunov analysis is presented in Section IV.
4) The simulation is conducted in Section V. The conclusion and future work are given in Section VI.

E. Main Contribution

1) Compared with the previous sensor-delay compensation observer design only for ODE systems in [2] and [19], we develop an observer which compensates a time-varying sensor delay to estimate the states of a wave PDE on a time-varying domain.
2) Compared with our previous works where an observer is designed to estimate the states of a wave PDE-ODE coupled system in [29], in this brief, we solve a more challenging problem which is observer design for a wave PDE-ODE coupled system under a time-varying sensor delay.
3) This is the first result of state-observer design for a wave PDE-ODE coupled system on a time-varying domain with a time-varying sensor delay. That is, the full-order state-observer design for a wave PDE-ODE-transport PDE coupled system where two PDEs are on time-varying domains.
4) This is also the first design of a vibration state observer for a time-varying-length vibrating string with a payload under a sensor time delay, the length of which is time-varying and arbitrary.

II. Problem Formulation

A mining cable elevator is shown in Fig. 1(a), where a drum drives a cable through a floating sheave to lift a cage. A driving force acting at the drum regulates motion dynamics meanwhile a separate control force at the floating sheave driven by a hydraulic actuator is to suppress vibrations [29], [31]. The catenary cable (70 m) which is much shorter than the vertical cable ($\geq$2000 m) can be ignored. Due to the lateral restrict of the guides, the transverse vibrations in the vertical cable can be neglected, since they are much smaller than the axial vibrations [18]. Therefore, the model in Fig. 1(a) can be simplified as a time-varying-length axial vibrating string with a payload at the bottom and a control input at the top in Fig. 1(b).

The actual displacement $z^*(x,t)$ of each point in the cable can be considered as the sum of the translation motion $l(t)$ and the additional axial vibrations $u(x,t)$, i.e., $z^*(x,t) = l(t) + u(x,t)$. Translation motion $l(t)$ which is regulated by rigid-body dynamics, i.e., an ODE driven by a separate motion controller at the drum, is assumed as a known function in the design of the vibration controller and observer of the mining cable elevator. As Fig. 1(b) shows, the additional vibration state $u(x,t)$ is referred to a moving coordinate system associated with the translation motion $l(t)$, where the distance of the origin to the center of the cage at the initial time is $u(0,0)$, i.e., the initial elastic displacement at the connection point between the cable and the cage, caused by the stretch property of the cable and the heavy load.

Using Hamilton's principle and through the derivation in [29], the vibration dynamics-$u(x,t)$, including a sensor delay, can be obtained as

$$u_{tt}(x,t) = qu_{xx}(x,t), \quad \forall(x,t) \in [0,l(t)] \times [0,\infty), \tag{1}$$

$$u_x(l(t),t) = U(t), \tag{2}$$

$$u(0,t) = CX(t), \tag{3}$$

$$\dot{X}(t) = AX(t) + Bu_x(0,t), \tag{4}$$

$$y(t) = u(0,t-\tau(t)), \tag{5}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & \dfrac{-c_1}{M} \end{bmatrix}, \quad B = \frac{E_A}{M}\begin{bmatrix} 0 \\ -1 \end{bmatrix}, \; C = [1, 0]. \tag{6}$$

$q = E \cdot A_a/\rho$ with E being cable Young's Modulus, A_a being the cable effective steel area, and ρ being cable linear density. $E_A = E_a \times A_a$. M denotes the mass of the cage. c_1 is the cage-guide damping coefficient.

The wave PDE states $u(x,t)$ on a time-varying domain $[0,l(t)]$ describe the axial vibrations of the string with a time-varying length $l(t)$. The ODE state $X(t) = [u(0,t), u_t(0,t)]^T$ denotes the vibration displacement and vibration velocity of the cage. $U(t)$ is the vibration control force driven by a hydraulic actuator at the floating head sheave. $y(t)$ is the output signal of the system (1)–(5), i.e., the vibration displacement of the cage $u(0,t)$, with a time-varying sensor delay $\tau(t)$. As Fig. 1 shows, an acceleration sensor is placed at the cage, and through integrators, we can obtain the vibration displacement $u(0,t)$. This cage vibration displacement signal $u(0,t)$ is transmitted to the control center at ground through a set of wireless devices, including emitters and receivers. The time-varying sensor delay exists in the process of the signal transmission according to different positions of the cage, i.e., the distance from the cage to the ground.

Remark 1: We ignore the effect of the vibration states $u(x,t)$ on the motion states $l(t)$, because the vibration displacements $u(x,t)$ are much smaller than the hoisting motion $l(t)$, which is between 2000-m underground and the surface platform. Therefore, it is reasonable to consider the distance from the cage to the ground as $l(t)$. That is, the delay length $\tau(t)$ which depends on the cage position is proportional to $l(t)$.

Three assumptions on the time-varying domains $l(t)$ and $\tau(t)$ are proposed.

Assumption 1: $l(t) \in C^1(0,\infty)$. $l(t)$ is bounded: $0 < l(t) \leq L, \quad \forall t \geq 0$, where L which denotes the total length of the cable is arbitrary. $\tau(t) \in C^1(0,\infty)$. $\tau(t)$ is bounded: $0 < \tau(t) \leq \bar{\tau}$, $\forall t \geq 0$, where $\bar{\tau}$ which denotes the maximum delay is arbitrary.

Assumption 2: Velocity $\dot{l}(t)$ of the moving boundary is bounded by $|\dot{l}(t)| \leq \bar{v}_{\max} < \sqrt{q}$, where $\bar{v}_{\max}$ is the maximum velocity of the mining cable elevator.

Note: In the mining cable elevator, $\sqrt{q} \approx 775$ is much larger than the value of the maximum hoisting velocity $\bar{v}_{\max} = 16.25$ m/s, and thus $\bar{v}_{\max} < \sqrt{q}$ holds. Moreover, according to the conclusions in [14] and [15], the fact that the derivative of the moving boundary $l(t)$ is smaller than the wave speed $\sqrt{q}$ allows to prove a well-posedness result for the initial boundary value problem of (1)–(4) in the weak sense.

Assumption 3: Changing rate $\dot{\tau}(t)$ of the time-varying delay is bounded by $|\dot{\tau}(t)| \leq \bar{D} < 1$, where $\bar{D}$ is the maximum changing rate of the time-varying sensor delay.

Because $\tau(t)$ is proportional to $l(t)$ with a coefficient as the reciprocal of the wireless transmission velocity according to Remark 1, considering Assumption 2 and the large wireless transmission velocity, $|\dot{\tau}(t)|$ would be far less than 1 s. Assumption 3 is reasonable.

The real-time states of the cable-distributed vibrations which are difficult to be measured in practice are crucial for a mining cable elevator, because they not only can be used to evaluate the running status of the elevator for the supervision and real-time monitoring, but also is helpful in constructing an output-feedback vibration controller which only uses boundary values. Motivated by estimating the cable vibrations in a mining cable elevator under a time-varying sensor delay which results from the long-distance signal transmission, we develop state-observer design for a general model of (1)–(5), where $X(t) \in \mathbb{R}^n$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times 1}$, $C \in \mathbb{R}^{1\times n}$, $CB = 0$, and q is an arbitrary positive constant. There exists K to make $A - (1/\sqrt{q})BCA + ((1/\sqrt{q})BC - I)KC$ Hurwitz, where I is an identity matrix with the appropriate dimension. Note that the existence of the above K holds in the model of the mining cable elevator according to (6) and will be seen more clearly in the following. The general model also can cover some other physical problems described by the wave PDE-stable/unstable/antistable ODE coupled model with a boundary sensor delay, such as torsional vibration dynamics of drill strings with stick-slip instabilities and sensor delays at the drilling bit in deep oil drilling [6].

By defining

$$v(x,t) = u(0,t-x), \tag{7}$$

we obtain a transport PDE

$$v(0,t) = u(0,t), \tag{8}$$

$$v_t(x,t) = -v_x(x,t), \quad \forall(x,t) \in [0,\tau(t)] \times [0,\infty), \tag{9}$$

$$y(t) = v(\tau(t),t) \tag{10}$$

to describe the time-varying delay dynamics (5). Then, the plant (1)–(5) can be written as

$$u_{tt}(x,t) = qu_{xx}(x,t), \quad \forall(x,t) \in [0,l(t)] \times [0,\infty), \tag{11}$$

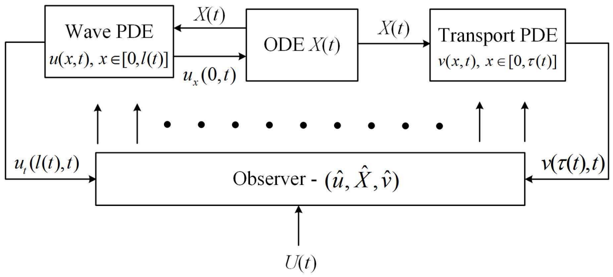

Fig. 2. Block diagram of the observer design for system (11)–(17).

$$u_x(l(t),t) = U(t), \tag{12}$$

$$u(0,t) = CX(t), \tag{13}$$

$$\dot{X}(t) = AX(t) + Bu_x(0,t), \tag{14}$$

$$v(0,t) = u(0,t), \tag{15}$$

$$v_t(x,t) = -v_x(x,t), \quad \forall (x,t) \in [0,\tau(t)] \times [0,\infty), \tag{16}$$

$$y(t) = v(\tau(t),t). \tag{17}$$

Therefore, the dynamics of axial vibrations of a time-varying-length cable with a payload and a time-varying sensor delay is represented by (11)–(17), which is a wave PDE-ODE-transport PDE coupled system, where the wave PDE and the transport PDE are on time-varying domains. The system (11)–(17) is equivalent to the original one (1)–(5). In Section III, we design an observer to estimate the states of (11)–(17) only using available boundary values. Note that the connection points of PDEs and ODE are not accessible to measure.

III. Observer Design

The available measurements of the mining cable elevator are the vibration velocity $u_t(l(t),t)$ at the head sheave and the vibration displacement of the cage under a time-varying delay, i.e., $y(t) = u(0,t-\tau(t)) = v(\tau(t),t)$. Note that $u_t(l(t),t)$ is directly obtained from the velocity feedback signal of the server actuator at the head heave, and thus the time delay can be neglected. As Fig. 2 shows, we use two boundary states $u_t(l(t),t)$ and $v(\tau(t),t)$ to estimate the states of the system (11)–(17). The observer structure consists of a copy of the plant (11)–(17) plus the boundary state error injections, described as

$$\hat{u}_{tt}(x,t) = q\hat{u}_{xx}(x,t), \tag{18}$$

$$\hat{u}_x(l(t),t) = U(t) + \frac{1}{\sqrt{q}}(u_t(l(t),t) - \hat{u}_t(l(t),t)), \tag{19}$$

$$\hat{u}(0,t) = C\hat{X}(t), \tag{20}$$

$$\begin{aligned}\dot{\hat{X}}(t) = {} & A\hat{X}(t) + B\hat{u}_x(0,t) \\ & + e^{\left(A-\frac{1}{\sqrt{q}}BCA\right)\tau(t)}K(v(\tau(t),t) - \hat{v}(\tau(t),t)),\end{aligned} \tag{21}$$

$$\hat{v}(0,t) = C\hat{X}(t), \tag{22}$$

$$\begin{aligned}\hat{v}_t(x,t) = {} & -\hat{v}_x(x,t) \\ & - Ce^{\left(A-\frac{1}{\sqrt{q}}BCA\right)(\tau(t)-x)}K(v(\tau(t),t) - \hat{v}(\tau(t),t)).\end{aligned} \tag{23}$$

Note that $v(\tau(t),t) = u(0,t-\tau(t))$ is the sensing signal with the time-varying delay. Define error states as

$$\left(\tilde{u},\tilde{v},\tilde{X}\right) = (u,v,X) - \left(\hat{u},\hat{v},\hat{X}\right). \tag{24}$$

According to (11)–(23), the observer error system is

$$\tilde{u}_{tt}(x,t) = q\tilde{u}_{xx}(x,t), \tag{25}$$

$$\tilde{u}_x(l(t),t) = -\frac{1}{\sqrt{q}}\tilde{u}_t(l(t),t), \tag{26}$$

$$\tilde{u}(0,t) = C\tilde{X}(t), \tag{27}$$

$$\begin{aligned}\dot{\tilde{X}}(t) = {} & A\tilde{X}(t) + B\tilde{u}_x(0,t) \\ & - e^{\left(A-\frac{1}{\sqrt{q}}BCA\right)\tau(t)}K\tilde{v}(\tau(t),t),\end{aligned} \tag{28}$$

$$\tilde{v}(0,t) = C\tilde{X}(t), \tag{29}$$

$$\begin{aligned}\tilde{v}_t(x,t) = {} & -\tilde{v}_x(x,t) \\ & + Ce^{\left(A-\frac{1}{\sqrt{q}}BCA\right)(\tau(t)-x)}K\tilde{v}(\tau(t),t)\end{aligned} \tag{30}$$

where K is to be determined later. In Section IV, we prove the exponential stability of the observer error system (25)–(30) via Lyapunov analysis.

IV. Stability Analysis

The main theorem of this brief is stated in the following.

Theorem 1: For any initial values $(\tilde{u}(x,0), \tilde{u}_t(x,0), \tilde{v}(x,0))$ which belong to $H^1(0,L) \times L^2(0,L) \times L^2(0,\bar{\tau})$, the observer error system (25)–(30) is exponentially stable in the sense of

$$\begin{aligned}&\int_0^{l(t)} \tilde{u}_t(x,t)^2 dx + \int_0^{l(t)} \tilde{u}_x(x,t)^2 dx \\ &\quad + \int_0^{\tau(t)} \tilde{v}(x,t)^2 dx + \left|\tilde{X}(t)\right|^2\end{aligned} \tag{31}$$

which physically illustrates that the observer (18)–(23) can effectively estimate actual vibration states $u(x,t)$ and $X(t)$ of a time-varying-length cable with a cage, and the sensor delay state $v(x,t)$, in a mining cable elevator under a time-varying sensor delay (11)–(17).

Proof: The coupled interactions between the PDEs and the ODE, and the unmatched orders between the wave PDE and the transport PDE around the ODE in (25)–(30) would pose difficulties in the stability analysis of the observer error system (25)–(30). Therefore, we apply two transformations to convert the original error system (25)–(30) into a target error system, where all PDEs are the transport type, and the coupled interaction between the delay PDE dynamics and the ODE is replaced as the cascade connection.

A. First Transformation

We introduce a transformation [5], [7] to convert the wave PDE $\tilde{u}(x,t)$ into a cascade of two transport PDEs $\alpha(x,t)$ and $\beta(x,t)$. Defining

$$\alpha(x,t) = \tilde{u}_t(x,t) + \sqrt{q}\tilde{u}_x(x,t), \tag{32}$$

$$\beta(x,t) = \tilde{u}_t(x,t) - \sqrt{q}\tilde{u}_x(x,t), \tag{33}$$

the system (25)–(30) can be written as

$$\alpha_t(x,t) = \sqrt{q}\alpha_x(x,t), \tag{34}$$

$$\beta_t(x,t) = -\sqrt{q}\beta_x(x,t), \quad (35)$$
$$\alpha(l(t),t) = 0, \quad (36)$$
$$\begin{aligned}\alpha(0,t)+\beta(0,t) = {} & 2CA\tilde{X}(t) \\ & -2Ce^{\left(A-\frac{1}{\sqrt{q}}BCA\right)\tau(t)}K\tilde{v}(\tau(t),t), \quad (37)\end{aligned}$$
$$\begin{aligned}\dot{\tilde{X}}(t) = {} & \left(A-\frac{1}{\sqrt{q}}BCA\right)\tilde{X}(t)+\frac{B}{\sqrt{q}}\alpha(0,t) \\ & +\left(\frac{1}{\sqrt{q}}BCe^{\left(A-\frac{1}{\sqrt{q}}BCA\right)\tau(t)}K \right. \\ & \left. - e^{\left(A-\frac{1}{\sqrt{q}}BCA\right)\tau(t)}K\right)\tilde{v}(\tau(t),t), \quad (38)\end{aligned}$$
$$\tilde{v}(0,t) = C\tilde{X}(t), \quad (39)$$
$$\begin{aligned}\tilde{v}_t(x,t) = {} & -\tilde{v}_x(x,t) \\ & +Ce^{\left(A-\frac{1}{\sqrt{q}}BCA\right)(\tau(t)-x)}K\tilde{v}(\tau(t),t), \quad (40)\end{aligned}$$

where

$$\begin{aligned}B\tilde{u}_x(0,t) = {} & \frac{1}{2\sqrt{q}}B(\alpha(0,t)-\beta(0,t)) \\ = {} & \frac{1}{\sqrt{q}}B\left(\alpha(0,t)-CA\tilde{X}(t)\right. \\ & \left.+Ce^{\left(A-\frac{1}{\sqrt{q}}BCA\right)\tau(t)}K\tilde{v}(\tau(t),t)\right) \quad (41)\end{aligned}$$

is used to obtain (38), and $CB = 0$ is recalled in calculating (37). Now, all PDEs are transport PDEs.

In the following transformation, we reform the bidirection interaction between the delay state $\tilde{v}$ and the ODE state $\tilde{X}(t)$ as a cascade connection and make the state matrix of the ODE (38) Hurwitz.

B. Second Transformation

Defining a transformation [19]

$$\tilde{v}(x,t) = \eta(x,t) + Ce^{-\left(A-\frac{1}{\sqrt{q}}BCA\right)x}\tilde{X}(t), \quad (42)$$

we convert the error system (34)–(40) to the target error system

$$\alpha_t(x,t) = \sqrt{q}\alpha_x(x,t), \quad (43)$$
$$\beta_t(x,t) = -\sqrt{q}\beta_x(x,t), \quad (44)$$
$$\alpha(l(t),t) = 0, \quad (45)$$
$$\begin{aligned}\alpha(0,t)+\beta(0,t) & \\ = {} & 2C\left(A-e^{\left(A-\frac{1}{\sqrt{q}}BCA\right)\tau(t)}KCe^{-\left(A-\frac{1}{\sqrt{q}}BCA\right)\tau(t)}\right) \\ & \times\tilde{X}(t)-2Ce^{\left(A-\frac{1}{\sqrt{q}}BCA\right)\tau(t)}K\eta(\tau(t),t), \quad (46)\end{aligned}$$
$$\begin{aligned}\dot{\tilde{X}}(t) = {} & \hat{A}\tilde{X}(t)+\frac{B}{\sqrt{q}}\alpha(0,t) \\ & +\left(\frac{1}{\sqrt{q}}BCe^{\left(A-\frac{1}{\sqrt{q}}BCA\right)\tau(t)}K \right. \\ & \left. -e^{\left(A-\frac{1}{\sqrt{q}}BCA\right)\tau(t)}K\right)\eta(\tau(t),t), \quad (47)\end{aligned}$$
$$\eta(0,t) = 0, \quad (48)$$
$$\eta_t(x,t) = -\eta_x(x,t), \quad (49)$$

where $\hat{A}$ is Hurwitz. Because $\alpha(x,t)$ is a transport PDE (43) with a boundary condition (45), $\alpha(0,t)\equiv 0$ for $t > (1/\sqrt{q})$. The connection between the transport PDE (48), (49) and ODE (47) is a cascade type.

The detailed transformation process is presented in the following.

Taking the time and spatial derivative of (42), we have

$$\begin{aligned}& \tilde{v}_x(x,t) \\ & = \eta_x(x,t) - C\left(A-\frac{1}{\sqrt{q}}BCA\right)e^{-\left(A-\frac{1}{\sqrt{q}}BCA\right)x}\tilde{X}(t), \quad (50)\end{aligned}$$
$$\begin{aligned}& \tilde{v}_t(x,t) \\ & = \eta_t(x,t) + Ce^{-\left(A-\frac{1}{\sqrt{q}}BCA\right)x}\left(A-\frac{1}{\sqrt{q}}BCA\right)\tilde{X}(t) \\ & \quad -Ce^{\left(A-\frac{1}{\sqrt{q}}BCA\right)(\tau(t)-x)}K\tilde{v}(\tau(t),t) \\ & \quad +Ce^{-\left(A-\frac{1}{\sqrt{q}}BCA\right)x}\frac{1}{\sqrt{q}}BCe^{\left(A-\frac{1}{\sqrt{q}}BCA\right)\tau(t)}K\tilde{v}(\tau(t),t) \\ & \quad +\frac{1}{\sqrt{q}}Ce^{-\left(A-\frac{1}{\sqrt{q}}BCA\right)x}B\alpha(0,t). \quad (51)\end{aligned}$$

Note that the last two terms in (51) are zero because $CB = 0$. Inserting (50) and (51) into (40), we have

$$\begin{aligned}& \tilde{v}_t(x,t)+\tilde{v}_x(x,t)-Ce^{\left(A-\frac{1}{\sqrt{q}}BCA\right)(\tau(t)-x)}K\tilde{v}(\tau(t),t) \\ & \qquad = \eta_t(x,t)+\eta_x(x,t) = 0, \quad (52)\end{aligned}$$

which gives (49).

Evaluating (42) at $x = 0$ and inserting the boundary condition (29) as

$$\eta(0,t) = \tilde{v}(0,t) - C\tilde{X}(t) = 0, \quad (53)$$

which gives (48). Substituting (42) into (38), we obtain (47), where

$$\begin{aligned}\hat{A} = {} & \left(A-\frac{1}{\sqrt{q}}BCA\right) \\ & +\left(\frac{1}{\sqrt{q}}BC-I\right)e^{\left(A-\frac{1}{\sqrt{q}}BCA\right)\tau(t)}KCe^{-\left(A-\frac{1}{\sqrt{q}}BCA\right)\tau(t)}\end{aligned}$$

has the same eigenvalues as $(A - (1/\sqrt{q})BCA) + ((1/\sqrt{q})BC - I)KC$. The observer gain K should be chosen to make sure $(A - (1/\sqrt{q})BCA) + ((1/\sqrt{q})BC - I)KC$ Hurwitz. Considering the plant parameters (6) of the mining cable elevator, it is easy to obtain a sufficient condition of $K = [k_1, k_2]^T$ as

$$k_1 > \frac{EA}{\sqrt{q}M} - \frac{c_1}{M}, \quad \frac{c_1}{M}k_1 + k_2 > 0 \quad (54)$$

which will be used in the simulation to determine K.

C. Stability Analysis of the Target Error System

We employ a Lyapunov function

$$\begin{aligned}V(t) = {} & \frac{a}{2}\int_0^{l(t)} e^x\alpha(x,t)^2dx + \frac{b}{2}\int_0^{l(t)} e^{-x}\beta(x,t)^2dx \\ & +\frac{c}{2}\int_0^{\tau(t)} e^{-x}\eta(x,t)^2dx + \tilde{X}^T(t)P\tilde{X}(t) \quad (55)\end{aligned}$$

where the matrix $P = P^T > 0$ is the solution to the Lyapunov equation $P\hat{A} + \hat{A}P = -Q$ for some $Q = Q^T > 0$. Defining

$$\Omega_1(t) = \|\alpha(\cdot,t)\|^2 + \|\beta(\cdot,t)\|^2 + \|\eta(\cdot,t)\|^2 + \left|\tilde{X}(t)\right|^2 \tag{56}$$

we have $\theta_1\Omega_1(t) \le V(t) \le \theta_2\Omega_1(t)$ for some positive θ_1 and θ_2.

Time derivative of $V(t)$ (55) along (43)–(49) is obtained as

$$\begin{aligned}
&\dot{V}(t)\\
&= a\int_0^{l(t)} e^x\alpha(x,t)\alpha_t(x,t)dx\\
&\quad +b\int_0^{l(t)} e^{-x}\beta(x,t)\beta_t(x,t)dx\\
&\quad +c\int_0^{\tau(t)} e^{-x}\eta(x,t)\eta_t(x,t)dx\\
&\quad +\tilde{X}(t)^T\left(\hat{A}^TP+P\hat{A}\right)\tilde{X}(t)+\frac{2}{\sqrt{q}}\tilde{X}(t)^TPB\alpha(0,t)-2\tilde{X}(t)^TP\\
&\quad \times\left(\frac{1}{\sqrt{q}}BCe^{\left(A-\frac{1}{\sqrt{q}}BCA\right)\tau(t)}K - e^{\left(A-\frac{1}{\sqrt{q}}BCA\right)\tau(t)}K\right)\\
&\quad \times\eta(\tau(t),t)+\dot{l}(t)\frac{a}{2}e^{l(t)}\alpha(l(t),t)^2\\
&\quad +\dot{l}(t)\frac{b}{2}e^{-l(t)}\beta(l(t),t)^2 + \dot{\tau}(t)\frac{c}{2}e^{-\tau(t)}\eta(\tau(t),t)^2.
\end{aligned} \tag{57}$$

Applying Young's inequality and Cauchy–Schwarz inequality to (57), the following inequality is obtained:

$$\begin{aligned}
&\dot{V}(t)\\
&\le -\left(\frac{1}{2}\lambda_{\min}(Q) - 6\sqrt{q}bM_1^2\right)\left|\tilde{X}(t)\right|^2\\
&\quad -\left(\frac{1}{2}\sqrt{q}a - \frac{4|PB|^2}{q\lambda_{\min}(Q)} - \frac{3}{2}\sqrt{q}b\right)\alpha(0,t)^2\\
&\quad -\frac{1}{2}\sqrt{q}a\int_0^{l(t)} e^x\alpha(x,t)^2dx - \frac{1}{2}\sqrt{q}b\int_0^{l(t)} e^{-x}\beta(x,t)^2dx\\
&\quad -\frac{1}{2}be^{-l(t)}(\sqrt{q}-\dot{l}(t))\beta(l(t),t)^2\\
&\quad -\left(\frac{c}{2}e^{-\bar{\tau}}(1-\dot{\tau}(t)) - \frac{4M_3^2}{\lambda_{\min}(Q)} - \frac{3}{2}\sqrt{q}bM_2^2\right)\eta(\tau(t),t)^2\\
&\quad -\frac{1}{2}c\int_0^{\tau(t)} e^{-x}\eta(x,t)^2dx,
\end{aligned} \tag{58}$$

where

$$\begin{aligned}
\frac{1}{2}\sqrt{q}b\beta(0,t)^2 \le 6\sqrt{q}bM_1^2\left|\tilde{X}(t)\right|^2 &+ \frac{3}{2}\sqrt{q}b\alpha(0,t)^2\\
&+\frac{3}{2}\sqrt{q}bM_2^2\eta(\tau(t),t)^2
\end{aligned}$$

is used by recalling (46). $\lambda_{\min}$ is the minimum eigenvalue of the corresponding matrix. Positive constants M_1, M_2, and M_3 are

$$\begin{aligned}
M_1 &= \max\left\{\left|C\left(A - e^{\left(A-\frac{1}{\sqrt{q}}BCA\right)\tau(t)}\right.\right.\right.\\
&\qquad\left.\left.\left.\times KCe^{-\left(A-\frac{1}{\sqrt{q}}BCA\right)\tau(t)}\right)\right|\right\},\\
M_2 &= \max\left\{\left|2Ce^{\left(A-\frac{1}{\sqrt{q}}BCA\right)\tau(t)}K\right|\right\},\\
M_3 &= \max\left\{\left|P\left(\frac{1}{\sqrt{q}}BCe^{\left(A-\frac{1}{\sqrt{q}}BCA\right)\tau(t)}K\right.\right.\right.\\
&\qquad\left.\left.\left.-e^{\left(A-\frac{1}{\sqrt{q}}BCA\right)\tau(t)}K\right)\right|\right\}
\end{aligned}$$

for all $\tau(t) \in [0, \bar{\tau}]$. Choose a, b, and c as

$$b < \frac{\lambda_{\min}(Q)}{12\sqrt{q}M_1^2}, \tag{59}$$

$$a > \frac{8|PB|^2}{q\sqrt{q}\lambda_{\min}(Q)} + 3b, \tag{60}$$

$$c > \frac{8e^{\bar{\tau}}M_3^2}{\lambda_{\min}(Q)(1-\dot{\tau}(t))} + \frac{3e^{\bar{\tau}}\sqrt{q}bM_2^2}{(1-\dot{\tau}(t))} \tag{61}$$

for all $|\dot{\tau}(t)| \in [0, \bar{D}]$. Note that $\bar{D} < 1$ according to Assumption 3. Together with Assumption 2 on $|\dot{l}(t)| < \sqrt{q}$, we arrive at

$$\dot{V} \le -\sigma_1 V \tag{62}$$

for some positive σ_1. Recalling (56) and (62), we obtain exponential stability in the sense of the norm $\|\alpha(\cdot,t)\| + \|\beta(\cdot,t)\| + \|\eta(\cdot,t)\| + |\tilde{X}(t)|$. Recalling the transformations (32), (33), and (42) and applying Cauchy–Schwarz inequality, we obtain

$$\begin{aligned}
\|\tilde{u}_t(\cdot,t)\|^2 &\le \frac{1}{2}\|\alpha(\cdot,t)\|^2 + \frac{1}{2}\|\beta(\cdot,t)\|^2,\\
\|\tilde{u}_x(\cdot,t)\|^2 &\le \frac{1}{2q}\|\alpha(\cdot,t)\|^2 + \frac{1}{2q}\|\beta(\cdot,t)\|^2,\\
\|\tilde{v}(\cdot,t)\| &\le 2\|\eta(\cdot,t)\|^2 + 2M_4\bar{\tau}\left|\tilde{X}(t)\right|^2,
\end{aligned}$$

where $M_4 = \max_{0\le x\le\bar{\tau}}\{|Ce^{-(A-(1/\sqrt{q})BCA)x}|\} > 0$.

Applying the exponential stability in the sense of the norm $\|\alpha(\cdot,t)\| + \|\beta(\cdot,t)\| + |\|\eta(\cdot,t)\| + |\tilde{X}(t)|$, we obtain the exponential stability in the norm $\|\tilde{u}_t(\cdot,t)\| + \|\tilde{u}_x(\cdot,t)\| + \|\tilde{v}(\cdot,t)\| + |\tilde{X}(t)|$.

Therefore, the proof of Theorem 1 is completed. ■

V. SIMULATION

In this section, we perform the numerical simulation to test the performance of the proposed state observer (18)–(23). The observer is built based on the nominal plant (11)–(17) and tested in an actual plant considering model parameter errors and additive unknown measurement noise. The model parameter errors between the actual plant and the nominal plant often appear in practice. The nominal physical parameters of the mining cable elevator in the simulation are shown in Table I. We change some plant parameters ρ, A_a, and M with respect to their nominal values, as $\rho = 7.6$, $A_a = 0.44 \times 10^{-3}$, and $M = 16\,500$, which are considered as actual plant parameters. Besides, there always exists measurement noise in the signal acquisition process of sensors, which may degrade the observer performance. In order to test the robustness of the proposed observer to the measurement noise, additive measurement noise $d(t)$ is added to (15) in the actual plant as $\upsilon(0,t) = u(0,t) + d(t)$, where $d(t)$ is the scaled white Gaussian noise with amplitude about 1×10^{-3}.

TABLE I
NOMINAL PHYSICAL PARAMETERS OF THE MINING ELEVATOR

Parameters (units)	values
Initial length L (m)	2000
Final length (m)	80
Cable effective steel area A_a (m^2)	0.47×10^{-3}
Cable effective Youngs Modulus E (N/m^2)	1.03×10^{10}
Cable linear density ρ (kg/m)	8. 1
Total hoisted mass M (kg)	15000
Gravitational acceleration g (m/s^2)	9.8
Maximum hoisting velocities $\bar{v}_{\max}$ (m/s)	16.25
Cage-guide damping coefficient c_1	0.04
Total hoisting time t_f(s)	150

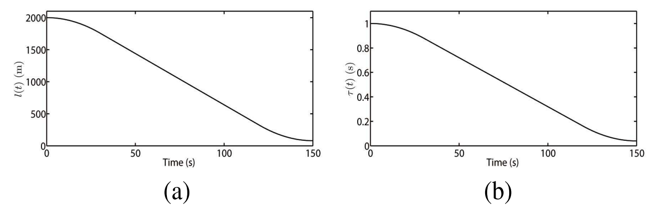

Fig. 3. (a) Time-varying length of the cable $l(t)$. (b) Time-varying sensor delay $\tau(t)$.

The initial profile of the vibration displacement in the actual plant is obtained by the force balance equation at the static state, which is written as $u(x,0)=(\rho xg+Mg)/E_A$, and the initial velocity is defined as $u_t(x,0)=0$ because the initial velocity of the each point in the cable is zero. The initial values of the transport PDE v are zeros. The initial estimation errors of u are considered as 0.02 m, i.e., the initial values of $\hat{u}$ or the nominal plant being $\hat{u}(x,0)=u(x,0)+0.02$ and $\hat{u}_t(0,t)=0$. The initial values $\hat{v}(x,0)$ are zero. Considering (54), the observer parameter K is chosen as $K=[k_1,k_2]^T=[380,5]^T$.

Because the sensor delay is related to the wireless distance between the emitter and the receiver, i.e., the length of the cable $l(t)$, we consider the time-varying delay as $\tau(t)=\lambda l(t)$ with $\lambda=0.0005$, which reflects the wireless transmission velocity of the sensor signal in the simulation. According to the maximum length of the cable $L=2000$ m, we have the maximum delay $\bar{\tau}=1$ s, which is an extremely serious case. $l(t)$ and $\tau(t)$ are priori-known functions shown in Fig. 3. The simulation is conducted by the finite-difference method with the time step and space step being 0.001 and 0.1, respectively, after introducing $\xi=x/l(t)$ and $\xi_1=x/\tau(t)$ to convert the PDEs on time-varying domains $[0,l(t)]$ and $[0,\tau(t)]$ to the PDEs on the fixed domain $[0,1]$ but with time-varying coefficients [28].

The simulation results are shown in the following. Note that the horizontal coordinates of all the figures are time. The vertical coordinates of Figs. 4, 5, and 8 are axial vibration displacements and those of Figs. 6, 7, and 9 are estimation errors of axial vibration displacements.

From Fig. 4, we can see that the vibration estimation from the proposed observer can track the actual vibration response of the midpoint at the cable effectively, even though there exist the initial estimation error, the time-varying sensor delay, the model parameter errors, and the additive measurement noise. The similar result in Fig. 5 shows that the vibration estimation from the proposed observer also recovers the actual vibration of the cage with a fast convergence. Accordingly, the estimation errors are shown in Figs. 6 and 7, which show that the estimation errors of the vibrations at the midpoint of the cable and the cage are convergent to zero very fast.

In addition to the vibration states, the proposed full-order observer also can estimate the sensor delay state, i.e., the future value of the system output $y(t+x),\forall x\in[0,\tau(t)]$. According to Fig. 8, the estimation $\hat{v}(\tau(t)/2,t)$ converges to the sensor delay state $v(\tau(t)/2,t)=u(0,t-\tau(t)/2)$ fast, i.e., effectively predicting the future value $y(t+\tau(t)/2)$ of the

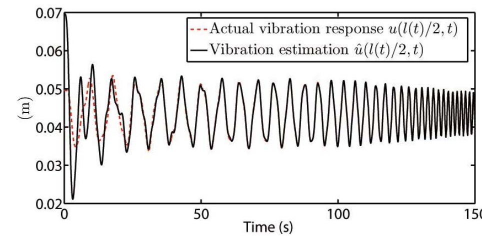

Fig. 4. Vibration response $u(l(t)/2,t)$ (red dashed line) and the vibration estimation $\hat{u}(l(t)/2,t)$ (black line) at the midpoint of the cable.

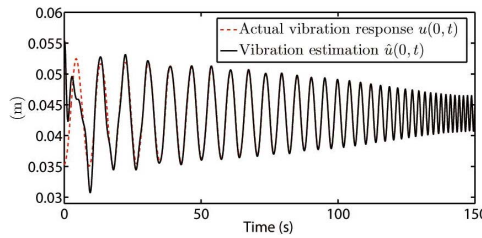

Fig. 5. Vibration response $u(0,t)$ (red dashed line) and vibration estimation $\hat{u}(0,t)$ (black line) at the cage.

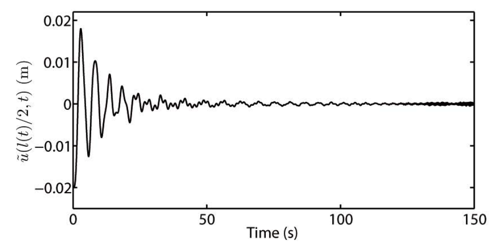

Fig. 6. Estimation error $\tilde{u}(l(t)/2,t)=u(l(t)/2,t)-\hat{u}(l(t)/2,t)$ of the vibration state at the midpoint of the cable.

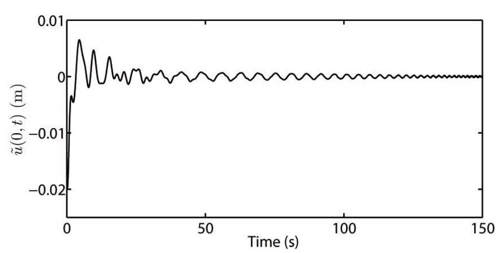

Fig. 7. Estimation error $\tilde{u}(0,t)=u(0,t)-\hat{u}(0,t)$ at the cage.

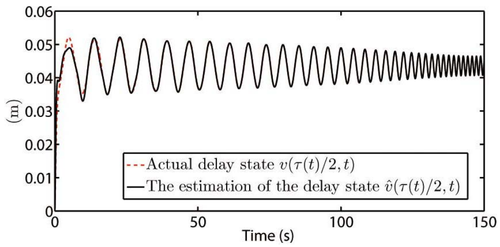

Fig. 8. Sensor delay state $v(\tau(t)/2, t)$, i.e., the future state of the output signal $y(t + \tau(t)/2)$ (red dashed line) and the estimation $\hat{v}(\tau(t)/2, t)$ (black line).

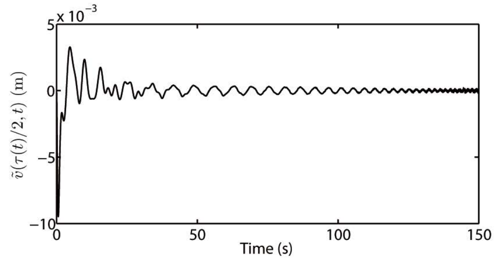

Fig. 9. Estimation error $\tilde{v}(\tau(t)/2, t) = v(\tau(t)/2, t) - \hat{v}(\tau(t)/2, t) = u(0, t - \tau(t)/2) - \hat{v}(\tau(t)/2, t) = y(t + \tau(t)/2) - \hat{v}(\tau(t)/2, t)$ of the sensor delay state $v(\tau(t)/2, t)$, i.e., the future output state $y(t + \tau(t)/2)$.

system output $y(t)$. From Fig. 9, we can see that the estimation error $\tilde{v}(\tau(t)/2, t)$ is convergent to zero fast.

According to the above simulation results, we know that the proposed state observer can compensate the time-varying sensor delay to estimate the distributed states of the wave PDE on a time-varying domain effectively and have the robustness to the model parameter errors and additive measurement noise.

VI. Conclusion and Future Work

In this brief, we propose a full-order state observer which can compensate a time-varying sensor delay of arbitrary delay length to estimate the vibration states and the sensor delay states in a mining cable elevator under a time-varying sensor delay. The dynamics of the mining cable elevator, including a varying-length cable and a cage with a time-varying sensor delay, is represented as a wave PDE-ODE-transport PDE coupled system, where the PDEs are on time-varying domains. An infinite-dimensional observer is designed to estimate all states of such a hybrid system only using boundary values at the input and output of the overall system without measurements at the connection points of the PDEs and the ODE. The exponential stability of the observer error system is proved via Lyapunov analysis. The simulation results verify that the proposed observer converges to the vibration states and sensor delay states fast.

In this brief, the observer is designed based on a model with completely known model parameters. One open problem is considering some unknown model parameters in the delay-compensation observer design for a PDE system. In the future work, the adaptive technology may be incorporated to build an adaptive delay-compensation observer for a mining cable elevator with some unknown physical parameters. In addition, based on the observer design in this brief, an output-feedback controller of a wave PDE, which can compensate the time-varying sensor delay and have robustness to actuation delays, will be built through taking the proximal reflection [3], [4] into account.

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