Probability Models for Customer Lifetime Value Analysis

Computer Lab Exercise Solution Notes

June 2024

This document should be read in conjunction with the Excel workbook SSE_2024_lab_solution.xlsx.

PART 1

Question 1

- *Plot the (empirical) survivor function, S(t). Plot the (empirical) retention rate function, r(t).* See the worksheet **Basic Plots**.
- *Fit the BG model to the Blue Apron data using just the data from the first four renewal opportunities / five months. Create plots that compare predicted and actual survival and retention rates over all 12/13 months.*

  See the worksheet **BG (4 ren)**. Not bad!

  Suggested modification to the plots: Draw in a vertical line that makes the divide between the model calibration and validation periods clear (as in the workshop slides).
- *Fit the BG model to the Blue Apron data using all the renewal data. Create plots that compare predicted and actual survival and retention rates over all 12/13 months. What are your conclusions regarding the robustness of the BG model?*

  See the worksheet **BG (all ren)**.

  Looking at the two sets of tracking plots, we see that there's no major improvement. (You might want to create plots that show the "4 renewals" and "all renewals" lines on the same set of axes.)

  The parameter estimates have changed, but not drastically. (The fact that the various plots are similar reflects this.) We can compute the mean and variance of the associated beta distributions (for theta) and see that they are similar — see the worksheet **Mean and variance**.

  More generally, we could compare the plots of the two beta distributions. We do this in the worksheet **Comparing beta distributions**.

  Computing the "all renewals" LL using the "4 renewals" parameters sees the LL increase (in absolute terms) from −1.986344219 to −1.986873032, a 0.0266% change.

  Conclusion: The fact that there's little change suggests that the BG model is robust to changes in the length of the model calibration period, which in turn suggests it is a good characterization of the data-generating process.

Question 2

- *Fit the two-segment model using just the data from the first four renewal opportunities. Create plots that compare predicted and actual survival and retention rates over all 12/13 months.*

  See the worksheet **2 seg G (4 ren)**. The fit is ok, but the forecast isn't that good. Notice how the retention rate plot levels off.
- *Fit the two-segment model using all the renewal data. Create plots that compare predicted and actual survival and retention rates over all 12/13 months. What do you conclude about the robustness of the two-segment model compared to the BG model?*

See the worksheet **2 seg G (all ren)**. The results change drastically as we reduce the length of the model calibration period. Contrast this to the BG model, for which we saw minimal changes in model performance.

Question 3

- *The data reported in the case are monthly, but the underlying subscription process is weekly. Fit a weekly model to these monthly data and assess the performance of the model. How do you interpret the change in the values of γ and δ?*

  See the worksheet **Weekly (4 ren)**. Notice that we first compute weekly churn probabilities (column D) from which we compute the week-level survivor function (column E). We extract the month-level numbers from the survivor function (column K; yes, we should have used some formula making use of the VLOOKUP function) and then compute the monthly model-based P("die") numbers (column J).

  The fit is very similar to that observed in the worksheet **BG (4 ren)**, but not exactly the same.

  The beta distribution now captures the distribution of P(tails) for coins tossed weekly, as opposed to monthly. We therefore expect its mean to be smaller. But will this occur through a change in γ, δ, or both? In light of the analysis presented in Fader et al. (2019)[1], we would expect γ to stay (approximately) the same and δ to increase by a factor of (close to) 4, which is what we see.

Question 4

- *Fit the BG model to the HelloFresh data, first using data from the first four renewal opportunities, then using all the data. Having now analysed two datasets, what do you conclude about the robustness of the BG model?*

  See the worksheet **HF BG (4 ren)** and **HF BG (all ren)**. Once again, the BG model is robust to changes in the length of the model calibration period, which suggests it is a good characterization of the data-generating process.

  Note that δ is now less than 1. This means the beta distributions are U-shaped (**HF Comparing beta distributions**).

- *Repeat using the two-segment model. What do you conclude about the robustness of the two-segment model compared to the BG model?*

  See the worksheets **HF 2 seg G (4 ren)** and **HF 2 seg G (all ren)**. The results mirror what we saw with the Blue Apron data.

  Conclusion: The BG model is a great little model for characterising the duration of a customer's relationship with the firm in a contractual setting.

PART 2

Question 1

- *Using the BG model from Part 1 and the financial data given on p. 8 of the case (and the logic as covered in the lecture), what is the expected value of an as-yet-to-be-acquired Blue Apron customer? [Check: Why is the monthly WACC not 20/12 = 1.67%?]*

  See the worksheet **E(CLV) – all ren**. (See **E(CLV) – 4 ren** for the solution using the parameters from **BG (4 ren)**.)

[1] Fader, Peter S., Bruce G. S. Hardie, Daniel McCarthy, and Ramnath Vaidyanathan (2019), "Exploring the Equivalence of Two Common Mixture Models for Duration Data," *The American Statistician*, **73** (August), 288–295.

We have computed E(CLV) over a 100-year (1200-month) time horizon (as an approximation of an infinite time horizon). What happens if we use a shorter time period? Looking at cells H7:I14, we see that a twenty-year horizon gives us a very similar answer.

Regarding the discount rate: Assuming compounding, an annual rate of 20% is equivalent to a monthly rate of $(1 + 0.2)^{1/12} - 1 = 0.015309$. Note that we have rounded this to d = 0.0153 in our calculations.

Question 2

- *According to the case (Exhibit 9), the expected value of an as-yet-to-be-acquired Blue Apron customer is $133.60. How do you reconcile this with the number you have computed above?*

  With reference to the worksheet **E(CLV) -- 2 seg G (all ren)**, we repeat the E(CLV) calculations undertaken above using the two-segment survivor function in place of that associated with the BG model. We get E(CLV) = $160.42. Why such a drop compared to the number computed using the BG model? It's driven by what happens to the survivor function over time. In the worksheet **Comparing S(t)**, we compare the survivor functions over a 10-year period. The 2 seg G assumes customers die off more quickly, which results in a lower E(CLV).

  But that doesn't account for all of the difference. Reading the case closely, we see that the case writers have chosen not to include the first payment associated with the start of the relationship with the customer; $160.42 − $26.82 = $133.60.

Question 3

- *Compute the distribution of CLV (where CLV is computed to the nearest cent). What is the mean and variance of this distribution?*

  See the worksheets **Distribution of CLV -- all ren (i)** and **Distribution of CLV -- all ren (ii)**.

- *Assuming a CAC of $100, what is the probability that Blue Apron loses money on a newly acquired customer?*

  With reference to the worksheets **Distribution of CLV -- all ren (ii)** and **Calculations**, a new customer has a lifetime value less than $100 if they remain a customer for only 1, 2, or 3 months. Therefore P(loss | CAC = 100) = 0.351 + 0.153 + 0.088 = 0.592 (with rounding).

- *Suppose Blue Apron acquires 10,000 new customers (with a CAC of $100). What is the expected value of this cohort of customers? What is the associated 95% interval?*

  See the worksheet **Calculations**.

- *Repeat for 1,000 new customers. How do you explain the difference in the relative ranges of the 95% intervals?*

  See the worksheet **Calculations**.

  Relative to the magnitude of the mean, the confidence intervals are wider in this case compared to the 10,000 new customers case.

  The coefficient of variation (CV) is defined as the ratio of the standard deviation to the mean. It shows the extent of variability in relation to the mean of the population. We see that the CV for the 1,000 new customers case is $\sqrt{10} = 3.16$ times bigger than that of the 10,000 new customers case. Equivalently, the range divided by the mean for the 1,000 new customers case is $\sqrt{10} = 3.16$ times bigger than that of the 10,000 new customers case.