1-(b) What is the size of matrix A? Write A. (10pt)

$$A = \begin{bmatrix} 1 & x_1 & x_1^2 & x_1^3 & \cdots & x_1^d \\ 1 & x_2 & x_2^2 & x_2^3 & \cdots & x_2^d \\ 1 & x_3 & x_3^2 & x_3^3 & \cdots & x_3^d \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 1 & x_n & x_n^2 & x_n^3 & \cdots & x_n^d \end{bmatrix}$$

size of $A = (d+1) \times n$

1-(c) Let $d+1 = n$, then, A becomes a square matrix. Compute the determinant of A. (40pt in total, Derivation: 30pt, Answer: 10pt, Hint: Vandermonde Matrix.)

$$A = \begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \cdots & x_2^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_1^{n-1} \end{bmatrix} \quad (\because d+1 = n)$$

$$= \begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-1} \\ 0 & x_2 - x_1 & x_2^2 - x_1^2 & \cdots & x_2^{n-1} - x_1^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & x_n - x_1 & x_n^2 - x_1^2 & \cdots & x_n^{n-1} - x_1^{n-1} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & x_2 - x_1 & x_2(x_2 - x_1) & \cdots & x_2^{n-2}(x_2 - x_1) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & x_n - x_1 & x_n(x_n - x_1) & \cdots & x_n^{n-2}(x_n - x_1) \end{bmatrix}$$

$$= \left(\prod_{i=2}^{n} (x_i - x_1) \right) \begin{bmatrix} 1 & x_2 & \cdots & x_2^{n-2} \\ 1 & x_3 & \cdots & x_3^{n-2} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & \cdots & x_n^{n-2} \end{bmatrix}$$

$$\therefore A = \left(\prod_{i=2}^{n} (x_i - x_1) \right) \prod_{2 \le i < j \le n} (x_j - x_i)$$

$$= \prod_{1 \le i < j \le n} (x_j - x_i)$$

1-(d) What is the condition that makes the determinant of A non-zero? (10pt)

determinant of A is non-zero if $x_j \neq x_i$

1-(e) Assume that the determinant of A is non-zero, then, what is the solution of linear equation, $A\mathbf{w} = \mathbf{y}$, with respect to $\mathbf{w}$? (10pt)

$$Aw = y$$

$$A^T A w = A^T y$$

$$\therefore w = (A^T A)^{-1} A^T y$$