The factor structure of short maturity options

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Abstract

Equity index options with less than one month to maturity have surged in popularity over the past two decades, particularly for the very shortest maturities. S&P 500 (SPX) options with fewer than 10 days to maturity represented 17% of all SPX option trading volume in 1996 as compared to around 70% today. Despite this popularity, research on such short maturities remains limited. In this paper, I examine deleveraged daily returns to short maturity contracts written on three major U.S. indices: S&P 500, Nasdaq 100, and Russell 2000. I estimate Instrumental Principal Components Analysis factor models and find evidence for a low-dimensional factor structure that explains over 95% of the variation in the cross-section of option returns. I apply two complementary approaches to interpret the latent factors and conclude they primarily provide compensation for exposure to the forward-looking higher-order moments of these indices, namely risk-neutral variance and skewness. Based on this interpretation, I propose a tradable factor model which outperforms previously proposed models from the literature and industry practice. Using this factor model, I quantify the contributions to the expected return for options with various levels of moneyness, maturity, and type (call/put) from exposure to the underlying index, variance, and skewness. I document substantial contributions to the expected return from exposure to these higher-order moments, with significant variation across moneyness, maturity and type.

Keywords: Short maturity options, Option returns, Higher-order moments

JEL Codes: G12, G13

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1 Introduction

The bulk of total trading volume in the U.S. equity index options market consists of options with 30 days or fewer to maturity. Option maturities of 10 days or fewer by themselves constitute over two-thirds of this total trading volume, a stark change from just two decades prior, when it made up less than one-fifth. This change was largely driven by the rollout of non-standard weekday expirations, a nascent but burgeoning segment of the options market. Despite this emergent popularity, options of such short maturities remain understudied by the literature. This article fills this gap by examining the returns on these contracts and relating them to a fundamental object in derivatives pricing: the risk-neutral distribution. The seminal work of Breeden and Litzenberger (1978) explicitly connects option prices to the forward-looking risk-neutral distribution and vise-versa. Less obvious is the connection between the risk-neutral distribution and option returns. In this article, I study the relationship between the returns to these short maturity equity index options and changes in the higher-order moments of their implied risk-neutral distributions.

Since their introduction on U.S. public exchanges in 1983, equity index options have been the subject of much discussion and interest from academics and practitioners alike. In particular, the market for S&P 500 Index options (and formerly S&P 100/OEX options) has served as the proving ground for many innovations in the options market. Whether it's the rollout of a new method for calculating final settlement prices or the listing of long-dated maturities (LEAPS), such innovations take hold in the equity index options space before branching out to other option markets.[1]

In more recent history, perhaps the single most notable and impactful innovation is that of the short maturity, non-standard expiration contracts, known colloquially as "weeklies". These are call and put option contracts listed between one to five weeks prior to maturity, expiring on a fixed weekday (i.e. Wednesday weeklies are options expiring each Wednesday) at market close. Weeklies augment the traditional standard monthly expiration contracts for equity indices; these traditional expiries settle before market open on the third Friday of each month or the closest trading day preceding it if the Friday lands on a market holiday. In 2005, the Chicago Board of Exchange (CBOE) listed the first set of regular non-standard expirations on U.S. option exchanges: Friday weeklies written on the S&P 500 Index (SPX).[2]

[1] On June 19th, 1987, major US futures exchanges and equity option exchanges began computing S&P 500 settlement prices for associated options and futures contracts using the Special Opening Quotation (SOQ) methodology, a settlement price based off the official opening prices of index constituents. In 1990, U.S. options exchanges listed the first LEAPS (Long-term Equity AnticiPation Securities) on select equity indices. LEAPS are option contracts with very long-dated maturities (2 - 10 years).

[2] I consider quarterly expirations, the first listed expirations for many options written on equity indices and equity index futures as another standard expiration.

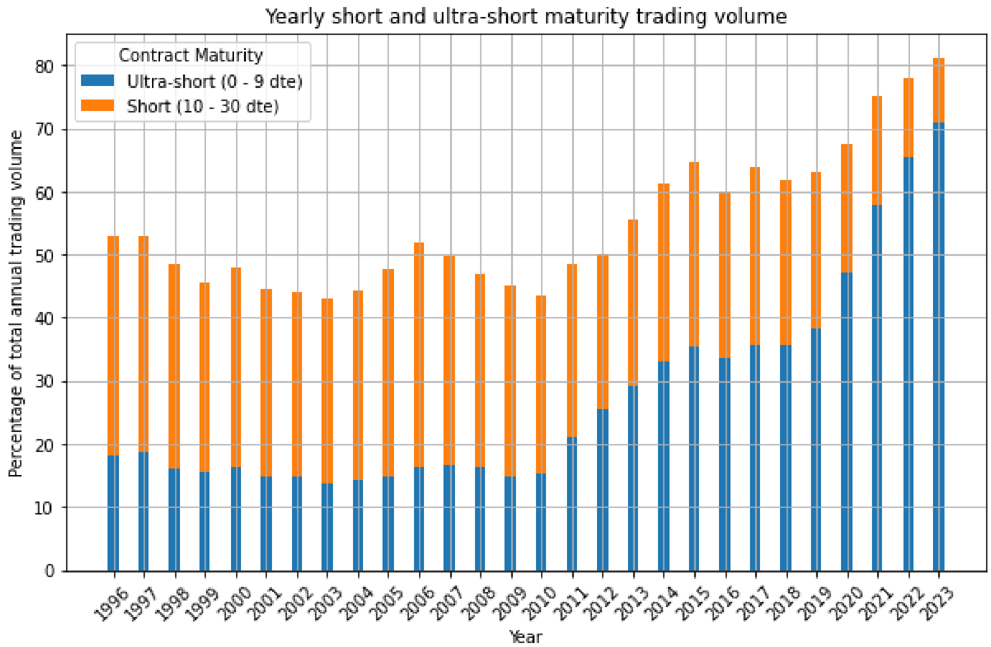

Figure 1. Total trading volume of S&P 500 equity index options by year for short maturities (orange) and ultra-short maturities (blue).

These weekly Friday expirations proved popular with traders prompting CBOE to list Friday weeklies for other major U.S. equity indices, select ETFs, and individual equities. In the decade following their introduction, the Friday weeklies proved to be a smash hit, capturing non-trivial segments of trading volume wherever they are introduced.[3] In response, CBOE began to list weeklies expiring on other weekdays. In 2016, CBOE introduced Wednesday SPX weeklies with Monday weeklies following only a year later. By the end of 2022, CBOE filled out the trading week having introduced Tuesday and Thursday SPX expirations earlier that year. By the end of 2024, there is a set of listed S&P 500, Nasdaq 100 and Russell 2000 options expiring every single trading day. This innovation to the options market has been highly consequential, shifting trading towards the very shortest maturities. The primary focus of this paper is to understand the risks priced into these maturities which have since taken over some of the world's most liquid options markets.

The introduction of these weekly contracts has dramatically transformed the makeup of trading volume in the options market. The adoption and emergent popularity of these contracts is striking - not just for the SPX options market where they were first introduced - but in whichever market they are listed. To aid our discussion on these contracts, I refer to options expiring in 10 and 30 days as short maturity options (S maturities for

[3] The Boom in Zero-Day Options Is Coming for Tesla and Nvidia. *Wall Street Journal - September 9, 2024*

short) and options expiring in fewer than 10 days I refer to as ultra-short maturities (US maturities). The listing of various new weekly expirations coincides with dramatic changes in the composition of trading volume. Figure 1 plots the annual trading volume of short and ultra-short maturities from 1996 to 2023. In 1996, only around 17% of all SPX option trading volume was in US maturities; by 2023, this figure ballooned to over 70%. Other options markets which received weekly expirations see a similar trend of trading shifting towards US maturity contracts.

Despite their overwhelming popularity, such contracts are, as of yet, not well-understood. This is partly due to the fact that most research examining the options market typically only study more conventional maturities - contracts with between 1 month to 2 years to expiration. In this paper, I explicitly focus on equity index options with less than 1 month to maturity and examine the risks priced into this heavily traded segment of the options market. To this end, I examine the daily returns of equity index options written on the most followed US equity indices using complimentary approaches from the factor model literature. I estimate two sets of factor models on the excess deleveraged return for options written on the S&P 500 (SPX), Nasdaq 100 (NDX), and Russell 2000 (RUT). I first estimate the Instrumental Principal Components Analysis (IPCA) latent factor model of Kelly, Pruitt, and Su (2019) for each index. The estimation procedure for IPCA delivers a set of extracted latent factors - potentially allowing for an interpretation of these factors using known risk exposures. The latent factors primarily appear to encode compensation for bearing variance, skewness and jump tail risk. This interpretation informs the specification of my second set of factor models which use tradable and easily interpretable factors. These factor models have the upshot that they are easier to economically interpret as compared to models which employ latent factors.

My results suggest that a five factor model, consisting of the underlying index's return, two volatility factors (straddles constructed from options with 9 days and 30 days to maturity) and two skewness factors (skewness assets constructed from options with 9 days and 30 days to maturity), is the most successful in explaining the variation in the return of these shorter maturity contracts compared to other models which employ observable and tradable factors. In particular, factor models that include factors sensitive to variance and skewness over a 9-day horizon offer substantial improvement in explaining returns in US maturities over models which only use factors relating to 1-month moments such as the VIX and CBOE's SKEW Index or the returns to a straddle and skewness asset constructed using 1-month maturity options. This is in contrast to approaches studying conventional maturities (1 month to 2 years) which typically just use factors exposed to forward-looking 1-month moments as factors.

Using a reduced form model, I attribute fluctuations in the five tradable factors to the return of underlying index and innovations to its risk-neutral forward-looking variance and skewness. I find substantial heterogeneity across maturity, moneyness and option type (call/put) in how innovations to variance and skewness affect option returns. I quantify the contribution of these innovations to expected option returns and find relatively large contributions attributable to innovations in these forward-looking higher-order moments. In particular, index options which are further out-of-the-money usually attribute a greater fraction of their expected return to changes in higher-order moments relative to a similar option further in-the-money. In general, the magnitude of this fraction is generally increasing in moneyness for calls and decreasing for puts. This result resembles a recent finding by Fournier, Jacobs, and Orłowski (2024) who find similar variation in the contribution of variance risk in their study of SPX option returns.

This paper contributes to four strands of the asset pricing literature. The first is the relatively scant but burgeoning literature on options with shorter maturities. Common approaches include parametric and semi-parametric approaches to option pricing (Bandi & Renò 2024; Todorov 2019), examining the return profile of various option trading strategies (Almeida, Freire, & Hizmeri 2024; Johannes, Kaeck, Seeger, & Shah 2024; Vilkov 2024), and considering implications for market participants and trading activity in options and the underlying market (Brogaard, Han, & Won 2023; Dim, Eraker, & Vilkov 2024). This paper's contribution in this vein is most similar to the first two approaches. Prior work such as Almeida et al. (2024) generally find substantial variance risk premia implied by option returns; however, the amount this exposure affects returns is usually not explicitly quantified. Using a reduced form model, I provide estimates for the average contributions of both variance and skewness to the expected return on different categories of options.

The second stand is the vast literature relating option prices and returns to the risk-neutral measure. Breeden and Litzenberger (1978) establishes the explicit connection between option prices and the risk-neutral distribution: prices pin down a risk neutral distribution and vice-versa. The connection the risk neutral distribution shares with option returns is more subtle but nonetheless salient. In particular, the literature consensus considers the risk-neutral forward-looking variance as an important risk exposure and determinant for option returns. This stems from the fact that the price of any vanilla option, all else equal, is increasing in expected variance of the underlying asset's price path. This observation has motivates studies such as those by Jones (2006) and Constantinides, Jackwerth, and Savov (2013) which employ changes in the VIX as a risk factor in their factor models of option returns. These models find large risk premia associated with variance and economically significant contributions to option returns. In some aspects, the models I develop in this paper are an extension to these earlier models which consider only the first two moments of returns. Like prior models, I fundamentally model option returns as compensation for bearing risks associated with the underlying asset and its variance. However, unlike most prior work, I also give serious consideration to skewness risk as a priced factor. While skewness risk has been identified as a driver of asset returns as early as Arditti (1967) and Kraus and Litzenberger (1976), its role as a determinant of option returns is under-explored relative to the role of variance. Kozhan, Neuberger, and Schneider (2013) and Bali, Chabi-Yo, and Murray (2022) are two examples from the literature which examine the pricing of skewness risk in the cross-section of equity options. For certain kinds of options, for example, out-of-the-money call options, I find the contribution of skewness risk to the expected return is large.

The third strand of literature is the broad work that seek to understand option returns using a factor model approach. Recent advancements in factor model methodologies more broadly have mitigated some of the issues present in earlier work. For the markets I focus on here, factor models are potentially more flexible and less susceptible than their no-arbitrage model counterparts to potentially large errors due to model mis-specification (Giglio, Kelly, & Xiu 2022; Israelov & Kelly 2017). The factors employed by these models can either be latent or observable; factor models featuring both types of factors have been successfully employed in explaining option returns. Empirically, models featuring latent factors tend to have lower error but are more difficult to interpret. Goyal and Saretto (2022), Büchner and Kelly (2022), Horenstein, Vasquez, and Xiao (2022), among many others utilize a low-dimensional set of latent factors to explain the cross-section of option returns. Coval and Shumway (2001), Jones (2006), and Fournier et al. (2024) are among many examples which employ observable factors such as the underlying and a forward-looking variance factor. In this paper, I estimate factor models of both types: a model of latent factors extracted from option characteristics and an interpretable factor model for option returns consisting of exposure to the underlying, and the returns to assets exposed to risk-neutral variance and skewness. These two sets of factor models are complementary; the interpretation of the latent factors informs the choice of factors in the observable factor model.

The final strand of literature is large body of work applying dimensionality reduction methods to problems in empirical asset pricing (Chamberlain and Rothschild (1982); Kozak, Nagel, and Santosh (2020); Lettau and Pelger (2020), among many others). It is well-known that option returns depend greatly on characteristics such as moneyness and maturity which may vary greatly over the contract's life, rendering the idea of static betas implausible. However, the betas for contracts which are of similar characteristics are also likely to be related. In light of this observation, many factor model-based approaches employ some form of dimensionality reduction on option characteristics to incorporate dynamic betas in a parsimonious way. This paper uses the Instrumental Principal Components Analysis (IPCA) model of Kelly et al. (2019) as its main empirical workhorse which allows for dynamic option betas to be easily implemented by specifying the betas as linear functions of these potentially time-varying contract characteristics. Both Goyal and Saretto (2022) and Büchner and Kelly (2022) investigate delta-hedged returns through the lens of an IPCA model that features both latent factors and dynamic betas that are functions of contract characteristics. Other approaches include specifying the option betas as non-linear functions of state variables (Jones 2006), using a shrinkage estimator to control for time variation in the factor exposures (Shafaati, Chance, & Brooks 2021), and implementing other PCA-based methods of computing conditional betas from contract characteristics (Z. Chen, Roussanov, Wang, & Zou 2024).

The aim of this article is to examine the factor structure embedded in the returns of equity index options with less than a month to maturity. To this end, I focus on the one-day deleveraged holding returns of these options. Although these contracts have ballooned in popularity, they remain relatively under-studied. In this paper, I estimate the risks embedded into these options and decompose the expected return into components corresponding to movements in the underlying index and fluctuations in its higher order moments. In the sections that follow, I first give an overview of the motivating theory in section 2. Since I want to estimate factor models on option returns, I construct a time series of leverage-adjusted portfolios from these short-maturity contracts in section 3. As noted by Constantinides et al. (2013), delevering option returns can simplify the econometrics considerably, allowing for linear factor models to be estimated on a set of returns more akin to the conventional stock and bond returns on which factor models have been traditionally estimated. With these deleveraged portfolios in hand, I proceed with empirical work starting in section 4 where I estimate a series of IPCA models and assess the models using standard metrics from the factor model literature. In section 5, I take two distinct approaches to interpreting the latent factors extracted in section 4. In the first approach, I use known risk factors from prior factor model-based approaches and regress them against the extracted latent factors. In the second, I examine the factor loadings across various buckets of options grouped by maturity, moneyness and type. Using these interpretations, I propose my own 5 factor model consisting of daily returns to the underlying index, two variance assets, and two skewness assets. In section 6, I propose and estimate a reduced-form model of variance and skewness asset returns and relate them to innovations in the risk-neutral moments of the underlying. Combining this reduced-form model with the 5 factor model from the prior section, I estimate the fraction of the expected return attributable to fluctuations in forward-looking variance and skewness. Lastly, the final section provides concluding remarks and some future directions for this research.

2 Theory

2.1 Factor models of option returns

I study option returns using a factor model approach, one of the most common approaches in empirical asset pricing today. Although frequently used to examine other asset classes, a factor structure for option returns can be easily motivated in several ways. One source of motivation is general as it stems from the universal nature of the framework which is independent of the asset class. Instead, it relies only on the assumption of no-arbitrage and the existence of a stochastic discount factor, m_{t+1}, satisfying for any return $R_{i,t+1}$:

$$\mathbb{E}_t(m_{t+1}R_{i,t+1}) = 1 \tag{1}$$

If we assume that m_{t+1} in (1) is a linear function of systematic risk factors $\mathbf{F}_{t+1}$, then the cross section of returns satisfies a conditional factor model:

$$R_{i,t+1} - R^f_{t+1} = \alpha_{i,t} + \boldsymbol{\beta}'_{i,t}\mathbf{F}_{t+1} + \varepsilon_{i,t+1} \tag{2}$$

where $\mathrm{E}\left[\varepsilon_{i,t+1}\mathbf{F}_{t+1}\right] = \mathbf{0}$, $\alpha_{i,t} = 0$, $\varepsilon_{i,t+1}$ is mean zero for all i and each t, and R^f_{t+1} is the one-day gross risk-free rate. A more asset class-specific motivation emerges from the fact that option prices, and therefore returns, are exposed to attributes of the underlying asset's stochastic process. Indeed, many modern hedging frameworks rely on this intuition, conceptualizing the change in an option price as a function of changes in the price and volatility of the underlying asset (Bates 2005). For instance, Carr and Wu (2020) formalize this insight into a continuous time framework linking short-term fluctuations in the option price directly to variation in the underlying's price and the option's implied volatility. Denoting the change in an option's price by ΔO_{t+1}, a conventional decomposition emerges from a Taylor expansion along the underlying's price and volatility to obtain

$$\Delta O_{t+1} \approx \frac{\partial O_t}{\partial S_t}\Delta S_{t+1} + \frac{\partial^2 O_t}{\partial S_t^2}\left(\Delta S_{t+1}\right)^2 + \frac{\partial O_t}{\partial \mathrm{IV}_t}\Delta \mathrm{IV}_{t+1} \tag{3}$$

where ΔS_{t+1} and $\Delta\mathrm{IV}_{t+1}$ are the changes in the underlying's price and the option's implied volatility respectively. The partial derivatives of O_t in (3) will be familiar to readers as option "Greeks" which provide practitioners with a practical avenue to hedge option exposures in a straightforward way. Normalizing both sides of (3) and performing light algebraic manipulation provides an expression for option returns:

$$\frac{\Delta O_{t+1}}{O_t} \approx \left[\frac{\partial O_t}{\partial S_t}\frac{S_t}{O_t}\right]\frac{\Delta S_{t+1}}{S_t} + \left[\frac{\partial^2 O_t}{\partial S_t^2}\frac{S_{t+1}^2}{O_t}\right]\left(\frac{\Delta S_{t+1}}{S_t}\right)^2 + \left[\frac{\partial O_t}{\partial \mathrm{IV}_t}\frac{\mathrm{IV}_t}{O_t}\right]\frac{\Delta \mathrm{IV}_{t+1}}{\mathrm{IV}_t} \tag{4}$$

The linear approximation in (4) motivates a factor structure for option returns with factor loadings being functions of option characteristics. Substituting $R_{S,t+1} = \Delta S_{t+1}/S_t$ and $R_{\mathrm{IV},t+1} = \Delta \mathrm{IV}_{t+1}/\mathrm{IV}_t$ makes the factor structure clear:

$$\frac{\Delta O_{t+1}}{O_t} \approx \left[\frac{\partial O_t}{\partial S_t}\frac{S_t}{O_t}\right] R_{S,t+1} + \left[\frac{\partial^2 O_t}{\partial S_t^2}\frac{S_t^2}{O_t}\right] R_{S,t+1}^2 + \left[\frac{\partial O_t}{\partial \mathrm{IV}_t}\frac{\mathrm{IV}_t}{O_t}\right] R_{\mathrm{IV},t+1} \tag{5}$$

Note approximation in (5) is valid regardless of the underlying asset of the option. It is also valid regardless of the option's time to maturity and moneyness. This suggests that a shared factor structure across options of differing in type (call or put), moneyness, and time to maturity. It's not just practitioners that utilize, either explicitly or implicitly, a factor structure to conceptualize the risks baked into option returns; many formal option pricing models do so as well. Consider the stochastic process that underlies the classic model of Black and Scholes (1973):

$$\frac{\mathrm{d}S_t}{S_t} = \mu \mathrm{d}t + \sigma \mathrm{d}B_t \tag{6}$$

where S_t, μ and σ are the price, drift and volatility of the underlying and B_t follows a Brownian motion. This implies a one factor model where the risk being priced stems from the Brownian motion B_t. Other popular option pricing models imply their own factor models. In the case of the Heston (1993) model, we have a similar drift-diffusion process for S_t, but with an additional process which makes volatility stochastic. Let B_t^1 and B_t^2 be two Brownian motions and V_t be the level of volatility. In the Heston model, S_t has the dynamics:

$$\begin{aligned} dS_t &= \mu S_t dt + \sqrt{V_t} S_t dB_t^1 \\ dV_t &= k\left(\theta - V_t\right) dt + \sigma \sqrt{V_t} dB_t^2 \\ dB_t^1 dB_t^2 &= \rho dt \end{aligned} \tag{7}$$

Here we have two risk factors - one associated with the diffusive component of S_t and the other with the volatility process V_t. In general, fully specified option pricing models imply a set of risk factors for option returns that are related to the higher-order moments of the underlying.

These theoretical and practical insights have provided the literature with ample motivation to examine option returns using a factor model approach where the factors correspond to higher order moments of the underlying asset. This approach leads researchers to model returns as generated by factor models of the form

$$\text{OptionReturn}_{i,t+1} = \alpha_{i,t+1} + \beta_{1,i,t} R_{t+1}^{\text{underlying}} + \beta_{2,i,t} F_{t+1}^{2} + \beta_{3,i,t} F_{t+1}^{3} + \dots \tag{8}$$

where F_{t+1}^{k} denotes a factor corresponding to the k-th moment of the underlying. Typically, the moments employed look over a one-month horizon. Classic examples include Coval and Shumway (2001) and Jones (2006) who use zero-delta straddle returns and changes in the VIX Index (one-month forward-looking volatility of the S&P 500) respectively as priced variance factors. Other papers, such as Zhu (2015) and Liang and Du (2024) consider the forward-looking skewness of the index, measured by the CBOE SKEW Index (a linear transformation of one-month forward-looking skewness of the S&P 500), as a priced skewness factor. As we will explore in later sections, for options with very short maturities, considering factors sensitive to moments over shorter horizons (specifically 9-day forward-looking variance and skewness) helps explain variation in returns and their inclusion as priced factors produces models with lower pricing errors, in particular for US maturity options.

2.2 Higher order moments and option prices

To precisely investigate the relationship between risk-neutral moments and option returns, I extract moments from option prices. Specifically, I obtain the risk-neutral moments over the 9-day and 30-day horizon by pricing an asset with payoff equal to either the realized variance or realized skewness over that horizon. To obtain risk-neutral variance, I replicate the fair price of a variance claim with maturity m, $V_t^{(m)}$, as in Bondarenko (2014). Such an asset has payoff equal to the realized variance from time t to maturity m days later at time $t+m$. R_{t+1}^{f} is the one-day accrued interest of a 1-month Treasury bill. The price of this variance claim is simply the discounted risk-neutral expectation of realized variance. I price this claim via replication as in Neuberger (1994) and Jiang and Tian (2005):

$$V_t^{(m)} = \frac{1}{1+mR_{t+1}^{f}} \mathbb{E}_t^{\mathbb{Q}} \left(\text{RV}_{t\to t+m}\right) = \frac{2}{1+R_{t+1}^{f}} \left(\int_0^{F_t^{(m)}} \frac{P_t(K,m)}{K^2} dK + \int_{F_t^{(m)}}^{\infty} \frac{C_t(K,m)}{K^2} dK \right) \tag{9}$$

where $\mathbb{Q}$ is the risk-neutral measure, $F_t^{(m)}$ is the $t+m$ forward price of the underlying at time t, and $\text{RV}_{t\to t+m}$ denotes the realized variance over the next m days: time t to $t+m$. $P_t(K,m)$ and $C_t(K,m)$ are the put and call prices for options expiring at $t+m$ corresponding to strike K. Similarly, we can construct an asset with payoff equal to the realized skewness from time t to $t+m$, $\text{RS}_{t\to t+m}$, using the methodology of Neuberger (2012). Specially, the price of this asset $W_t^{(m)}$ is given by:

$$
\begin{aligned}
W_t^{(m)} &= \frac{1}{1+mR_{t+1}^f}\mathbb{E}_t^{\mathbb{Q}}\left(\mathrm{RS}_{t\to t+m}\right) \\
&= \frac{1}{V_t^{(m)}}\left[6\int_{F_t^{(m)}}^{\infty}\frac{K-F_t^{(m)}}{F_t^{(m)}K^2}C_t(K,m)dK - \int_0^{F_t^{(m)}}\frac{F_t^{(m)}-K}{F_t^{(m)}K^2}P_t(K,m)dK\right]
\end{aligned} \tag{10}
$$

For my purposes, I construct the moment claims $V_t^{(m)}$ and $W_t^{(m)}$ for $m = 9d$ (9 days) and $m = 30d$ (30 days). I define the one-day change in these prices by $\Delta V_{t+1}^{(m)}$ and $\Delta W_{t+1}^{(m)}$ respectively. Intuitively, these quantities represent changes in the market's expectation of variance and skewness in an underlying asset over a fixed forward-looking horizon of m days.

Since the variance claim is purely exposed to realized variance over some horizon, we can obtain the price of variance forwards as considered by Dew-Becker, Giglio, Le, and Rodriguez (2017). The variance forward has payoff equal to the realized variance that occurs over the next 30 days excluding the first 9 days. I derive the price of this asset $V_t^{(30d-9d)}$ using the variance claim prices of the 30-day and 9-day variance asset in equation (11):

$$
V_t^{(30d-9d)} = \frac{1}{1+30R_t^f}\left[\mathbb{E}_t^{\mathbb{Q}}\left(\mathrm{RV}_{t\to t+30}\right) - \mathbb{E}_t^{\mathbb{Q}}\left(\mathrm{RV}_{t\to t+9}\right)\right] = V_t^{(30d)} - V_t^{(9d)} \tag{11}
$$

Unlike the variance forward, we cannot construct an analogous skewness claim by simply taking the difference between the two skewness claims. The reason is due to a general fact of probability theory: measures of skewness are not additive in the same way measure of variance are. For example, the realized variance of a continuous-time process from time t_0 and t_1 is typically measured by its quadratic variation $QV_{t_0\to t_1}$. This variance additive in the sense that for $t_2 > t_1$, the quadratic variation from t_0 to t_2 is obtained by the sum:

$$
QV_{t_0\to t_2} = QV_{t_0\to t_1} + QV_{t_1\to t_2}
$$

For realized skewness, we do not observe a similar type of linearity. Instead, to obtain a skewness forward, I normalize the price of an asset with payoff equal to the realized third cumulant by risk-neutral expectation of realized variance between time $t+9$ and $t+30$:

$\mathbb{E}^{\mathbb{Q}}(\mathrm{RV}_{t+9\to t+30})$. The price of the skewness forward is given by:

$$W_t^{(30d-9d)} = \frac{1}{V_t^{(30d-9d)}}\left(\left[6\int_{F_t^{(30d)}}^{\infty}\frac{K-F_t^{(30d)}}{F_t^{(30d)}K^2}C_t(K,30)dK - \int_0^{F_t^{(30d)}}\frac{F_t^{(30d)}-K}{F_t^{(30d)}K^2}P_t(K,30)dK\right] - \left[6\int_{F_t^{(9d)}}^{\infty}\frac{K-F_t^{(9d)}}{F_t^{(9d)}K^2}C_t(K,9)dK - \int_0^{F_t^{(9d)}}\frac{F_t^{(9d)}-K}{F_t^{(9d)}K^2}P_t(K,9)dK\right]\right) \tag{12}$$

The payoff of this asset is the realized skewness of the underlying between time $t+9$ and $t+30$.

The two types of moment claims have a natural interpretation with respect to the volatility surface. The variance claims $V_t^{(m)}$ reflect the overall level of the variance at each maturity m. The skewness claims $W_t^{(m)}$ reflect the slope of implied volatility smile at each maturity. Each of the skewness assets are constructed from a portfolio that is long out-of-the-money call options and short out-of-the-money put options. If the volatility smile were flat, as is the case in the classic Black and Scholes (1973) model, then the value of the two integrals in (10) would be equal and the skewness asset would have zero price. Empirically, equity indices possess an option-implied skewness which is generally negative due to higher implied volatility of out-of-the-money put options relative to call options which are similarly out-of-the-money.

To fully utilize the data, I use linear interpolation to obtain the values of $V_t^{(30d)}$, $V_t^{(9d)}$, $W_t^{(30d)}$, and $W_t^{(9d)}$ using the prices of variance claims $V_t^{(m)}$ and skewness claims $W_t^{(m)}$ whenever there are sufficiently close maturities m_1 and m_2 which bracket the target maturity from above and below.[4]

Figure 2 show the prices of these variance and skewness claims constructed from SPX options. On the left, I plot $W_t^{(30d)}$ and $W_t^{(9d)}$. Both skewness time-series tend to spike downwards during crises when the risk-neutral skew of the index is dominated by the greater likelihood and risk premia associated with a left tail event such as in 2018's "Volmageddon" episode or the 2020 pandemic stock market crash. The skewness of the SPX at both horizons is negative. This is the norm for equity indices which, under the risk-neutral distribution, generally feature a larger left tail compared to its right tail. The right side of the figure plots $V_t^{(30d)}$ and $V_t^{(9d)}$. Large values for these time-series reflect elevated forward-looking uncertainty for the corresponding horizon. As documented by Kozhan et al. (2013), the risk-neutral variance (skewness) for SPX is, on average, larger (more negative) than the variance

[4] I consider two maturities m_1 and m_2 sufficiently close to m if $0.75m \leq m_1 \leq m$ and $m \leq m_2 \leq 1.25m$. CBOE uses a similar construction for the VIX on days when SPX options with exactly 30 days to maturity are not available.

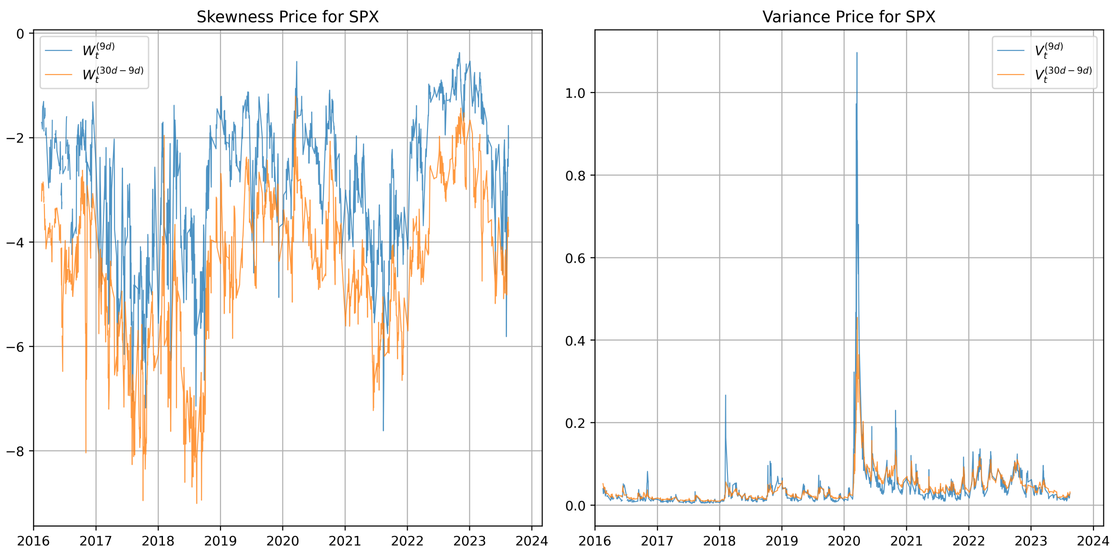

Figure 2. 9-day (blue) and 30-day minus 9-day (orange) forward-looking risk neutral moments of the S&P 500 Index extracted from option prices; plotted variances are annualized.

(skewness) that is realized.

For the rest of this article, I use annualized measures of variance using CBOE's methodology. I compute the un-annualized variance by first computing the integrals in (9) for the desired maturity or by interpolating it using nearby maturities. Then I multiply that risk-neutral variance by 252 divided by number of business days to maturity. For the variance forward $V_t^{(30d-9d)}$, I multiply this by 252/21.

3 The data and option panel construction

3.1 Data description

I obtain data on option contract prices, option greeks, and implied volatilities from OptionMetrics for the most widely followed equity indices in the United States: S&P 500 (SPX), Nasdaq-100 (NDX), and Russell 2000 (RUT). As of 2024, each of these equity indices have an option expiration every single trading day. The contracts are cash-settled European-style equity index call and put options. For all three indices, OptionMetrics's coverage begins in January 1996 and ends in August 2023. In 1996, each equity index had only a single option expiration each month. This monthly expiration date is commonly referred to as the standard expiration and occurs on the third Friday of each month with the options being cash-settled before market open. If the third Friday happens to be a market holiday, the expiration date falls on the closest preceding trading day. As my focus is on short maturity options, I only consider options with 30 days or fewer to expiration for inclusion in the final option panel.

Other data include risk-free rates and option market derived indices. To compute risk-free rates earned over a day, I obtain 1-month T-bill prices from Bloomberg and divide the one-day accrued interest by the T-bill's 3PM mid-price. From CBOE, I obtain time series of various option market derived indices: the VIX Index, a one-month forward-looking measure of SPX volatility, the SKEW Index, a one-month forward-looking measure of SPX skewness, and the VVIX Index, a one-month forward-looking measure of VIX volatility.

On the options data, I impose the standard filters following Constantinides et al. (2013) and Goyal and Saretto (2022). These filters serve to eliminate contracts with prices which are either obviously erroneous, violate no-arbitrage conditions, or are so deep or out of the money that their extrinsic value is close to zero. Specifically, I eliminate contracts which (1) have zero bid price, or (2) have mid prices that cannot be inverted to compute a Black-Scholes implied volatility (BSIV), or (3) have a BSIV lower than 2% or greater than 200%, or (4) have a recorded bid price strictly greater than the ask price, or (5) have a Black-Scholes delta with absolute value below 0.01 or above 0.99, or (6) imply put-call parity violations.[5]

I apply one final set of filters to the data: the removal of extremely deep in-the-money (ITM) and extremely deep out-of-the-money (OTM) options. This removes options that are very thinly traded which have essentially zero value (options deeply OTM) or have little value beyond their intrinsic value (options deeply ITM). I compute the cutoff using the risk-neutral volatility of the underlying implied by options with shared expiration. Specifically, I compute $\text{VIX}_{ind,t}(\tau)$, the risk-neutral annualized volatility of the underlying from time t to $t+\tau$ for index ind, using out-of-the-money options expiring in τ business days. Using the valuation approach of Neuberger (1994), I compute $\text{VIX}_{ind,t}(\tau)$ from the price of a log contract which is easily priced via the replication result of Carr and Madan (1998).

$$\text{VIX}_{ind,t}(\tau) = \sqrt{\frac{2}{\tau/252}\mathbb{E}_t^{\mathbb{Q}}\left(-\log R_{t\to t+\tau}^{ind}\right)} = \sqrt{\frac{2}{\tau/252}\sum_j \frac{p^{ind}(K_j)}{K_j^2}} \tag{13}$$

The sum in (13) is taken over all OTM options expiring at time $t+\tau$ written on index ind, with $p^{ind}(K_j)$ denoting the price of an OTM option of strike K_j.[6] $R_{t\to t+\tau}^{ind}$ denotes the return of index ind over time t to $t+\tau$ and $\mathbb{E}_t^{\mathbb{Q}}$ is the expectation taken over the conditional risk-neutral distribution.To convert this to a risk-neutral volatility over the life of the option

[5] Specifically, I remove option quotes which imply a profitable put-call parity arbitrage using the ask price of the options for long positions and bid price for short positions.

[6] For the sum in (13), I consider a call option to be OTM if their strike is greater than or equal to the forward price. For a put option, I consider it to be OTM if the strike is strictly below the forward price.

Underlying	Monday	Tuesday	Wednesday	Thursday	Friday
SPX	2017	2022	2016	2022	2005
NDX	2020	2022	2019	2023	2010
RUT	2021	2024	2021	2024	2010
VIX	-	-	2016	-	-
TLT	2024	-	2023	-	2010
GLD	2024	-	2023	-	2010
WTI	2023	2024	2023	2024	2016

Table 1. Intial listing year of weekday, non-standard expiry contracts ("weeklies") across underlying assets. A standard option expiry for the underlying assets above is typically the third Friday of each month (excluding the VIX which has its standard expiry on the third Wednesday of each month).

$[t, t+\tau]$, I multiply $\text{VIX}_{ind,t}(\tau)$ by $\sqrt{\tau/252}$. With this quantity in hand for all available expirations and times t, I keep only option quotes with strikes less than 4 times the risk-neutral volatility over the remaining life of the option from the spot price. The rationale for this cutoff is empirically motivated by the fact that nearly all trading volume for equity index options is concentrated inside this range. For SPX options, 89% of all trading volume from 1996 to 2023 falls into this range. I observe very similar proportions for NDX and RUT options.

Over the course of the prior two decades, many non-standard expiration dates were listed, starting with the SPX Friday weeklies in 2005.[7]. Since the introduction of the Friday weeklies, the equity index options market saw precipitous growth in the listing of other weekday contracts. Other equity indices in 2010 saw the listing of their own Friday weeklies. New weekday expirations followed in 2016, when CBOE listed the first Wednesday weeklies for SPX and VIX. By 2024, three widely referenced U.S. equity indices, SPX, NDX, and RUT would have a dedicated options expiration every weekday. Table 1 summarizes the rollout timeline for these US equity indices.

Following successful launches of weeklies written on various equity-linked indices, exchanges began listing weeklies for popular ETFs and futures contracts covering a variety of asset classes. The last three rows of 1 show the rollout timeline of these weeklies for three heavily traded securities outside of equity markets: TLT (iShares 20+ Year Treasury Bond ETF), GLD (SPDR Gold Shares), and WTI (West Texas Intermediate - crude oil futures). Today, the scope of tradable weekly expirations is staggering with around 600 or so individual equities and ETFs obtaining weekly maturities since 2010.[8] At time of writing, there

[7] https://www.cboe.com/insights/posts/the-evolution-of-same-day-options-trading/

[8] The Options Clearing Corporation, a U.S. clearing house specializing in equity derivatives clearing and

are several proposals under review at Securities and Exchange Commission to add Monday and Wednesday expirations to other commodity-linked ETFs such as SLV (silver) and UNG (natural gas).[9] In addition, there are active discussions around listing non-Friday weeklies for individual equities, highlighting the ever-expanding proliferation of such expirations.[10]

3.2 Construction of option panel

It is well-known options provide levered exposure to their underlying assets. For this reason, raw option returns are typically orders of magnitude greater in scale relative to the underlying asset's return (Frazzini & Pedersen 2022). To effectively estimate factor models on such assets, prior work in empirical asset pricing often adjusts the option returns in some way as opposed to simply working with raw option returns. Examining these adjusted returns instead of the raw returns, alleviates well-known issues related to the estimation of expected option returns and risk premia stemming from measurement errors in option quotes (Broadie, Chernov, & Johannes 2009; Duarte, Jones, & Wang 2024). There are two general approaches to producing an adjusted series of option returns considered by researchers: delta-hedged returns and deleveraged returns.

An option's delta-hedged return is the return to a strategy that holds the option while dynamically hedging the option using a known hedge ratio, generally the Black-Scholes delta. The dynamic hedging portion of this strategy aims to remove exposure to the underlying with the hedging frequency usually chosen to be daily. An option's deleveraged, or leverage-adjusted, return is the return to a portfolio consisting of the option and the risk-free asset (one month Treasury bill). The option's portfolio weight is inversely proportional to a measure of its embedded leverage which I compute as in Frazzini and Pedersen (2022). Intuitively, options that provide greater leverage receive a smaller weight when forming the portfolios on which the deleveraged return is computed. Cao and Han (2013), Karakaya (2014), Goyal and Saretto (2022), Büchner and Kelly (2022) among others consider delta-hedged option returns when estimating factor models on the cross-section of option returns. Constantinides et al. (2013), Gruenthaler, Lorenz, and Meyerhof (2022), Frazzini and Pedersen (2022) among others use the second approach, deleveraging option returns by their embedded leverage. I opt for this second approach, as in the context of my dataset, which consists of purely of very short maturity option contracts, the choice of hedging frequency is unclear (Bertsimas,

settlement services, maintains a list of stock and ETF tickers with actively listed weeklies: Link.

[9]Several exchanges including Nasdaq and BOX, have petitioned the SEC for a rule change in its Short Term Option Series Program. This change will allow for the listing of additional Monday and Wednesday expirations for certain commodity-linked ETFs.

[10]Zero-Day Options Boom Will Only Grow Even As Some Investors Fear Disaster. *Bloomberg - May 6, 2024*

Kogan, & Lo 2000; Sepp 2012).

I create a panel of deleveraged one-day option returns for a range of target moneyness values and business days to maturity using options with fewer than 22 business days to expiration for each equity index and option type. In the spirit of Constantinides et al. (2013), I compute these one-day returns in my panel as the one-day deleveraged return on contracts with standardized characteristics via kernel-based weighting scheme. At time t, each panel observation is the one-day deleveraged return for a option with the following standardized characteristics: maturity (1 to 21 business days to maturity) and target moneyness value ranging from 0.9 to 1.1 (spaced by 0.01), option type (call or put), and underlying index.[11] There are $21 \times 21 \times 2 = 882$ possible combinations of characteristics per underlying index (21 possible business days to expiration multiplied by the 21 target moneyness values for both puts and calls). The methodology of constructing the option panel proceeds in two steps. In the first step, I deleverage each option's one-day return from the raw one-day option returns I compute from the OptionMetrics data. I can compute each option's deleveraged return from these raw option returns and the daily holding returns (accrued interest) from the 1-month Treasury bill. Denote option i's gross raw one-day return at time $t+1$ by $R^{\text{opt}}_{i,t+1}$ and the one-day gross risk-free rate by R^f_{t+1}. The deleveraged return $R_{i,t+1}$ is given by

$$R^{\text{delev}}_{i,t+1} = \left(\eta^{-1}_{i,t} R^{\text{opt}}_{i,t+1} + (1 - \eta^{-1}_{i,t}) R^f_{t+1}\right) - 1 \tag{14}$$

where $\eta_{i,t}$ is the Frazzini and Pedersen (2022) embedded leverage measure.[12] In the second step, I construct a panel of deleveraged one-day returns for put and call options with fixed maturity τ (1-21 business days to maturity) and exact moneyness K (values between 0.9 and 1.1 spaced out by 0.01) by interpolating on deleveraged option returns nearby in standardized moneyness. Starting with calls, the deleveraged call option return from this panel with moneyness K, maturity τ, and index ind by $R(K, \tau, \text{Call}, ind)$ is computed from the deleveraged call returns in the first step at time t with maturity τ and strikes close to the target moneyness K:

$$R_{t+1}(K, \tau, \text{Call}, ind) = \sum_i w_{i,t}(K) R^{\text{delev}}_{i,t+1} \tag{15}$$

where I use $K_{i,t}$ to denote the moneyness of the ith option at time t from the OptionMetrics data. The weights $w_{i,t}(K)$ in (15) are computed as a function of distance from K. I define

[11] Moneyness, occasionally referred to as simple moneyness, is the strike price divided by level of the index.

[12] Note the portfolio weights in (14) are always between 0 and 1. This is because Frazzini and Pedersen (2022) compute the embedded leverage $\eta_{i,t}$ as the absolute value of an option's price elasticity with respect to the underlying asset which is always greater than 1.

the weights in (16) where φ denotes the standard Gaussian density.

$$w_{i,t+1}(K) = \varphi\left(\frac{K_{i,t} - K}{VIX_{ind,t}(\tau)\sqrt{\tau/252}}\right) \qquad ind \in \{SPX, NDX, RUT\} \tag{16}$$

As before, $VIX_{ind,t}(\tau)\sqrt{\tau/252}$ is risk-neutral standard deviation of the corresponding underlying index's return over the option's life. Thus, the quantity passed into the Gaussian density is the distance of an option's moneyness from K in standard deviation units. I allow only options nearby in the sum in equation (15), by first computing the weights as in (16) and then removing all weights lower than 0.01 and normalizing the remaining weights to sum to one before computing the panel return using equation (15). I follow an analogous computation for the daily deleveraged return on put options in my option panel.

This procedure interpolates a panel of deleveraged one-day option returns on contracts with standardized characteristics using a Gaussian weighting scheme. To ensure the interpolated returns in the panel are as close to the deleveraged returns from the data as possible, I compute the standardized option return in (15) only when there are at least two sufficiently close options with the desired maturity that bracket the target moneyness K.[13] Following this procedure produces a cross-sectional panel of deleveraged one-day call and put returns on contracts for a range of moneyness values $(0.9, 0.91, \ldots, 1.09, 1.1)$, times to maturity (1-21 BDTE), option types (call/put), and equity indices (SPX, NDX, RUT). For the sake of expositional compactness going forward, I use $R_{i,t+1}$ to a denote the gross one-day delevered option return i from the option panel. Each subscript i is a stand-in for the 4-tuple of target standardized characteristics: moneyness, BDTE, type and index. This standardized panel of deleveraged option returns serves as the main dataset of interest for my empirical analysis in subsequent sections.

One of the primary analyses of this paper is how option returns can vary by the moneyness, maturity, and type of the contract. To this end, I introduce some useful conventions to more easily discuss options with similar characteristics. I sort options of the same underlying index into 20 buckets. For each option in my option panel, contract i at time t is in one of these buckets. Define the set $\mathcal{B}(a_1, a_2, a_3, a_4)$ as the subset of the option panel corresponding to the bucket with maturity a_1, moneyness a_2, type a_3, and index a_4. Buckets are defined along 4 contract characteristics:

1. Maturity ($a_1 \in \{US, S\}$) - Option contracts are either of ultra-short maturity (0 - 9 dte) or short maturity (10 - 30 dte).

[13] I consider an option with moneyness K' to be sufficiently close to K if the distance between K' and K is less than $VIX_{ind,t}(\tau)\sqrt{\tau}/8$. This ensures we are using sufficiently local information relative to our target moneyness K.

2. Moneyness ($a_2 \in \{\text{DOTM}, \text{OTM}, \text{ATM}, \text{ITM}, \text{DITM}\}$) - Contracts are divided into five groups based on the absolute value of their Black-Scholes delta $|\Delta_{i,t}|$: DOTM ($0 \leq |\Delta_{i,t}| < 0.2$), OTM ($0.2 \leq |\Delta_{i,t}| < 0.4$), ATM ($0.4 \leq |\Delta_{i,t}| < 0.6$), ITM ($0.6 \leq |\Delta_{i,t}| < 0.8$), and DITM ($0.8 \leq |\Delta_{i,t}| \leq 1$).

3. Option type ($a_3 \in \{\text{Call}, \text{Put}\}$) - Contract's type as either a call or put option.

4. Equity index ($a_4 \in \{\text{SPX}, \text{NDX}, \text{RUT}\}$) - Option's underlying equity index.

There are 20 buckets for each equity index making for a total of 60 buckets. Every standardized option return in our panel, identified by the pair $(i, t+1)$, belongs to one of these 60 buckets. Before turning to the empirical analyses, I first briefly discuss the constructed panel and give an overview of its summary statistics.

In table 2, I present summary statistics of the standardized panel broken down my the underlying index. I interpolate their contract characteristics, such as its option's Black-Scholes IV (BSIV) or its Black-Scholes delta, by computing the weighted average over those characteristics using the same weighting scheme as in (15). The primary characteristics of the interpolated contracts are as expected. The average business days to maturity (BDTE), for both puts and calls, is around 2 weeks. This is consistent with when weeklies are listed; these contracts generally list on exchanges between 2-5 weeks prior to expiration. The average standardized moneyness for both put and call options in the sample are negative, reflecting the fact that the majority of call (put) options in the OptionMetrics data are in-the-money (out-of-the-money). The average Black-Scholes implied volatilities across indices are typical values for equity indices with at-the-money options ranging from 15% to 30% for large portions of the sample. Turning to the embedded leverage column, we get a sense of the substantial leverage built into options. For instance, SPX call options have an average embedded leverage of 45, implying a 45% return on typical call option if the index rises 1%. For put options, the degree of leverage is similar relative to calls across indices. The summary statistics for the greeks are typical for option data on equity index options. OptionMetrics reports vegas in terms of the change in option price in cents to a 1 percent increase in BSIV. To remove this dependence on the level of the underlying, I rescale this reported vega to be a price elasticity by dividing the reported vega by price of the option multiplied 100. Adjusting the vega in this way yields the percent change in option price for a 1% change in BSIV. The number of observations (Nobs) for each index, indicates a fairly balanced sample between puts and calls. Notably, the three equity indices I consider here are one of only a handful of assets to have an options expiration every day as of 2024.

Turning to returns, table 3 shows the daily deleveraged return in basis points (bps) for put options (moneyness ≤ 1) and call options (moneyness ≥ 1) written on SPX. As

	BDTE	Std. Moneyness	BSIV	Emb. Lev.	Delta	Gamma	Vega	Nobs
Call options:								
SPX	12.146	-0.595	0.207	45.371	0.563	0.003	0.123	497,763
	(5.693)	(1.796)	(0.114)	(43.120)	(0.354)	(0.003)	(0.178)	
NDX	12.246	-0.292	0.263	33.453	0.533	0.001	0.084	408,407
	(5.534)	(1.361)	(0.114)	(27.973)	(0.331)	(0.001)	(0.112)	
RUT	11.724	-0.288	0.251	35.638	0.530	0.005	0.089	289,003
	(5.543)	(1.442)	(0.109)	(27.843)	(0.341)	(0.004)	(0.115)	
Put options:								
SPX	11.835	-0.235	0.210	41.428	-0.471	0.003	0.101	563,161
	(5.750)	(2.048)	(0.107)	(31.960)	(0.370)	(0.003)	(0.103)	
NDX	12.073	-0.197	0.261	30.927	-0.476	0.001	0.071	425,875
	(5.602)	(1.485)	(0.112)	(21.675)	(0.339)	(0.001)	(0.074)	
RUT	11.510	-0.157	0.255	32.096	-0.483	0.005	0.073	309,098
	(5.587)	(1.579)	(0.109)	(21.786)	(0.349)	(0.004)	(0.074)	

Table 2. Means and standard deviations (in parentheses) for the various characteristics of the equity index options on the S&P 500 (SPX), Nasdaq-100 (NDX) and Russell 2000 (RUT) from OptionMetrics. BDTE refers to business days to expiration, BSIV is the standard Black-Scholes implied volatility, Emb. Lev. is the embedded leverage is computed as in Frazzini and Pedersen (2022). Std. Moneyness is the natural logarithm of simple moneyness divided by risk-neutral volatility over the option's life. Delta, gamma, and vega are the Black-Scholes greeks and Nobs denotes the number of observations retained from the OptionMetrics sample after data filters are applied.

expected, all SPX put options in the sample have negative average return. The same is true of put options written on NDX and RUT. This is broadly consistent with prior theoretical and empirical work (Coval and Shumway (2001), Broadie, Chernov, and Johannes (2007), Johannes et al. (2024), among many others). Note table 3 omits moneyness/time to maturity combinations with fewer than a year's worth (250 observations) of returns data. This occurs mostly for extreme levels of moneyness such as 0.9 or 1.1 at very short times to maturity such as 2 BDTE. Such options are extremely deep out-of-the-money options and are usually removed by one of the data filters on the OptionMetrics sample. Across maturities, put options deepest out-of-the-money are the most starkly negative. Although the average SPX return in the sample is 3.6 bps, the average deleveraged return on a put option with 0.9 moneyness and 5 BDTE (one week) is around -30 bps. As I show in section 6, this is due to combination of a negative index beta and their large embedded exposure to variance which command highly negative risk premia. At-the-money options have magnitudes similar to the average SPX daily return, a by-product of their deleveraging procedure. Lastly, call options that are deeper out-of-the-money have negative expected returns while call options

	Deleveraged put returns (bps)						*Deleveraged call returns (bps)*					
Moneyness	0.90	0.92	0.95	0.98	0.99	1.00	1.00	1.01	1.02	1.05	1.08	1.10
Business days to expiration												
2			-23.31	-7.72	-6.38	-5.65	6.89	9.90	5.23			
5	-30.26	-22.54	-14.17	-9.14	-7.78	-5.62	2.14	1.87	2.00	-1.60		
10	-14.10	-11.00	-8.16	-4.92	-3.82	-2.79	1.11	1.29	1.41	-0.26	2.46	
15	-5.36	-4.56	-2.94	-1.20	-0.68	-0.23	1.60	1.89	2.35	1.70	1.29	6.69
21	-11.71	-11.24	-10.07	-8.25	-7.60	-6.91	2.00	1.45	0.78	-0.97	-0.84	-1.06

Table 3. Average daily returns (in basis points) for deleveraged put (with moneyness $\leq$ 1) and call (moneyness $\geq$ 1) options written on SPX. Deleveraged option exposures have the target moneyness and maturity as described in this section. Only moneyness-maturity returns with more than 250 observations in the sample included in table. SPX avg. return: 3.59 bps.

at-the-money have positive expected return. This is consistent with Bakshi, Madan, and Panayotov (2010) finding for longer maturity index options: on average, index call options have positive returns at-the-money but this return is decreasing in strike price, eventually becoming negative once the strikes are sufficiently deep out-of-the-money.

Finally, I examine the betas of our delevered returns with respect to the underlying index. A theoretical point noted by Constantinides et al. (2013) is that if the Black and Scholes (1973) model is exact, deleveraged returns (measured over short horizons) as above should produce a time series of deleveraged call (put) returns which have beta 1 (-1) with respect to the underlying asset. Interestingly, this allows for a indirect "unit beta"-based test of the Black and Scholes (1973) model. Figure 3 plots the betas from running time-series regressions on the delevered SPX option returns: each dot represents the index beta from the time-series regression with one regression for each option type, maturity, and moneyness combination from the panel. The betas are closest in absolute value to 1 for options that are at or in-the-money and become smaller as we move further out-of-the-money. The betas computed from the options for the other two indices exhibit similar patterns. Note for the empirical analysis it is of little concern that the betas are not exactly one as I always control for index exposure; rather, the point of the procedure is to produce a panel of option returns with magnitudes that are more equity-like and amenable to a factor model approach.

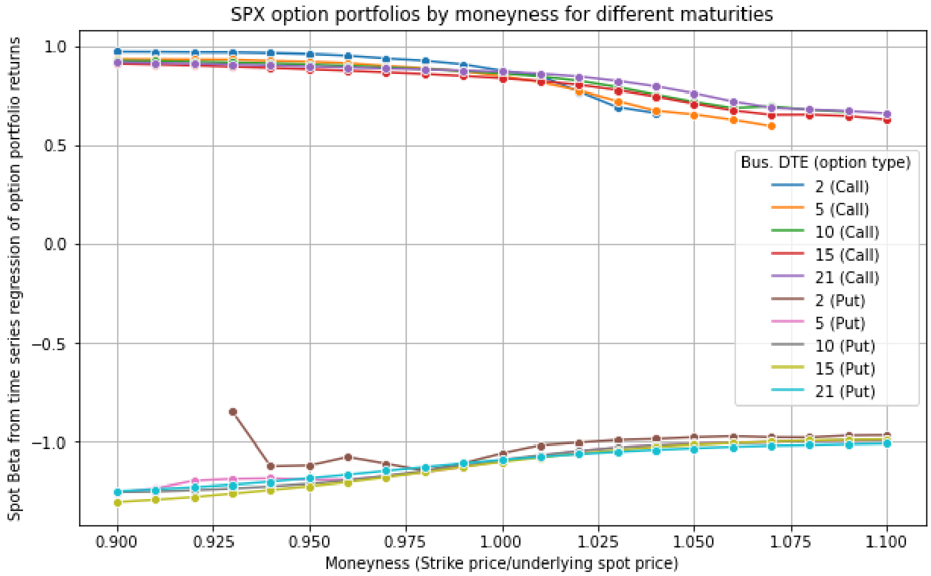

Figure 3. Betas estimated from time series regressions of SPX excess daily returns on the daily delevered option returns. Each point plots an estimated beta. The x-axis represents the moneyness of the contract with color jointly representing the different maturity and type combinations.

4 IPCA: Model estimation and inference

In this section, I outline, estimate, and analyze IPCA (instrumental principal components analysis) factor models on my option panel data. IPCA is a latent factor model that implements a principal components analysis (PCA)-based method for extracting latent factors and factor loadings from asset characteristics developed by Kelly et al. (2019). PCA-based approaches have enjoyed large popularity in recent work (Giglio et al. 2022), although using PCA to extract risk factors specifically was used as early as Chamberlain and Rothschild (1982). IPCA is one of several contemporary innovations on this earlier approach.[14]

4.1 IPCA: Model setup

The IPCA model has several key features which are useful in estimating the factor structure of option returns. First, IPCA, and PCA-based methods more broadly, provide a parsimonious way to reduce the dimension of the potentially large range of characteristics that relate to option returns. Second, the model incorporates time-varying betas which are estimated as a function of these characteristics. Time-varying betas prove to be particularly plausible and relevant in this context. Fundamental characteristics relevant for option returns, such

[14]Other approaches which come to mind include risk premia PCA of Lettau and Pelger (2020) or the tensor PCA of Babii, Ghysels, and Pan (2024).

as an option's moneyness or time to maturity can move much over an option's life, making static betas intuitively implausible. In addition, Büchner and Kelly (2022) note substantial advantages in and out-of-sample for factor models of option returns which allow for betas to be time-varying. Lastly, IPCA can robustly handle imbalanced panels like the ones I examine here.[15]

I estimate separate IPCA models for each of the three equity indices. The IPCA model assumes excess returns on N_t assets over time period $t \in \{1, \dots, T\}$ are generated by the K-factor model:

$$\begin{aligned} R_{i,t+1} - R^f_{t+1} &= \alpha_{i,t} + \beta'_{i,t} F_{t+1} + \beta^{mkt}_{i,t} R^{mkt}_{ind,t+1} + E_{i,t+1} \\ &= \left(Z'_{i,t}\Gamma_\alpha\right) + \left(Z'_{i,t}\Gamma_\beta\right) F_{t+1} + \left(Z'_{i,t}\Gamma_\delta\right) R^{mkt}_{ind,t+1} + E_{i,t+1} \end{aligned} \tag{17}$$

I always include one pre-specified factor, $R^{mkt}_{ind,t+1}$, the one-day return of the underlying equity index ind. The vector F_{t+1} consists of the $K-1$ latent factors to be estimated by IPCA. $\alpha_{i,t}$ $\beta'_{i,t}$, and $\beta^{mkt}_{i,t}$ are asset specific time-varying intercepts and factor loadings. $Z_{i,t} \in \mathbb{R}^{L+1}$ is a vector of L observable characteristics and a constant. These characteristics $Z_{i,t}$ are linearly mapped to $\alpha_{i,t}$, $\beta_{i,t}$ and $\beta^{mkt}_{i,t}$ by the matrices Γ_α, Γ_β, and Γ_δ respectively. Intuitively, each beta is "linear-in-characteristics"; that is, it is a linear function of the L characteristics plus an intercept. For example, the equity index beta is given by:

$$\beta^{mkt}_{i,t} = \sum_{j=1}^{L+1} \Gamma_\delta[j] \times Z_{i,t}[j]$$

where $x[j]$ denotes the j-th entry of a vector x.

Turning to estimation, IPCA estimates a pair of objects, the latent factors F_{t+1} and Γ matrices. If equation (17) is estimated with the restriction that $\Gamma_\alpha = \mathbf{0}_{L+1}$, one can estimate a factor model without an intercept. Since I always enforce the inclusion of the underlying return as a pre-specified factor, a one-factor IPCA model contains no latent factors. The one-factor IPCA model is similar in spirit to linear models like conditional CAPM. The conditional beta on the underlying, $\beta^{mkt}_{i,t}$ is given by

$$\beta^{mkt}_{i,t} = Z'_{i,t}\Gamma_\beta$$

which in turn describes a conditional one-factor model in the index return:

$$R_{i,t+1} = (Z'_{i,t}\Gamma_\beta) R^{mkt}_{ind,t+1} + \varepsilon_{i,t+1}$$

[15] Over time several factors, primarily the range of listed expirations increasing, spacing between strike prices narrowing, and range of listed strike prices increasing have contributed to the imbalance in the panel.

As we are working with options, the set of relevant observable characteristics to include in the vector $Z_{i,t}$ is fairly apparent. Unlike common equity or corporate bonds, options and similar derivatives are unique in the sense that they possess unambiguously relevant characteristics for their returns. For instance, where an option lies on the volatility surface (pinned down by moneyness and time to maturity) does much to characterize the risk profile of that particular contract (Karakaya 2014). Following the literature, I consider the following baseline set $B_{i,t}$ of observable characteristics:

$$B_{i,t} = \begin{pmatrix} \text{BSIV}_{i,t} \\ \text{EmbLev}_{i,t} \\ \text{Delta}_{i,t} \\ \text{Gamma}_{i,t} \\ \text{Vega}_{i,t} \\ \text{ttm}_{i,t} \\ \text{Std. Moneyness}_{i,t} \end{pmatrix}$$

$\text{BSIV}_{i,t}$ refers to the Black-Scholes implied volatility of a particular option i at time t. Time to maturity ($\text{ttm}_{i,t}$) and standardized moneyness ($\text{Std. Moneyness}_{i,t}$) conveys the option's location on the volatility surface. $\text{Delta}_{i,t}$ $\text{Gamma}_{i,t}$ and $\text{Vega}_{i,t}$ are the risk sensitivities (so-called "greeks") computed from the Merton (1973) model. It is plausible that calls and puts with certain characteristics might load onto our risk factors differently. Following Büchner and Kelly (2022), I accommodate this possibility by including in $Z_{i,t}$ those same 6 characteristics in $B_{i,t}$ interacted with a dummy isPut_i which is 1 if option i is a put option and 0 otherwise. Our full set of characteristics $Z_{i,t}$ is a vector consisting of a constant and 14 characteristics:

$$Z_{i,t} = \begin{pmatrix} 1 \\ B_{i,t} \\ B_{i,t} \times \text{isPut}_i \end{pmatrix} \tag{18}$$

4.2 IPCA: Model estimation and results

Kelly et al. (2019) derive first order conditions from minimizing the sum of squared model errors:

$$\min_{\Gamma, \mathbf{F}_{t+1}} \sum_{t=1}^{T-1} \left(\mathbf{R}^e_{t+1} - Z_t \Gamma \mathbf{F}_{t+1}\right)' \left(\mathbf{R}^e_{t+1} - Z_t \Gamma \mathbf{F}_{t+1}\right) \tag{19}$$

where $\Gamma = [\Gamma_\beta : \Gamma_\delta : \Gamma_\alpha]$ and $\mathbf{F}_{t+1} = [F_{t+1}, R^{mkt}_{ind,t+1}, 1]$. $\hat{\Gamma}$ and $\hat{F}_{t+1}$ are the estimated counterparts. $\mathbf{R}^e_{t+1}$ is the vector of excess returns at time $t+1$. The values of F_{t+1} and Γ that minimize the objective in (19) satisfy the first-order conditions:

$$\begin{aligned} \hat{F}_{t+1} &= \left(\Gamma'_\beta Z'_t Z_t \Gamma_\beta\right)^{-1} \Gamma'_\beta Z'_t \left(\mathbf{R}^e_{t+1} - Z_t\Gamma_\delta - Z_t\Gamma_\alpha\right) \quad \forall t \\ \text{vec}\left(\hat{\Gamma}'\right) &= \left(\sum_{t=1}^{T} \left[Z_t \otimes \mathbf{F}'_{t+1}\right]' \left[Z_t \otimes \mathbf{F}'_{t+1}\right]\right)^{-1} \left(\sum_{t=1}^{T} \left[Z_t \otimes \mathbf{F}'_{t+1}\right]' \mathbf{R}^e_{t+1}\right). \end{aligned} \tag{20}$$

The latent factors F_{t+1} and Γ matrices are estimated via alternating least squares.[16] One point of note is that the first order conditions (20) above identify a solution to (19) only up to multiplication by a rotation matrix. For instance, if Γ_β is part of the matrix Γ which minimizes the objective in (20), so will $P\Gamma_\beta$ for any rotation matrix P. To pin down a unique solution, I follow Kelly et al. (2019) and impose the normalization that $\Gamma_\beta\Gamma'_\beta$ is the identity matrix. This identification assumption is not a restriction on the model, but simply a means to pin down its unique parameters. As is common in the factor model literature, I assess my estimated models on total R^2s and the magnitudes of alphas relative to the magnitudes of excess returns. I compute the total R-squared of the IPCA model as in Büchner and Kelly (2022) and Goyal and Saretto (2022):

$$\text{Total } R^2 = 1 - \frac{\sum_{i,t}\left(R_{i,t+1} - Z'_{i,t}\hat{\Gamma}_\beta \hat{F}_{t+1}\right)^2}{\sum_{i,t} R^2_{i,t+1}} \tag{21}$$

An important aspect in the study of factor models are the alphas, the component of expected returns not explained by the risk loadings. Indeed, many formal statistical tests of factor models focus on testing whether the alphas are jointly zero (Gibbons, Ross, & Shanken 1989; Pesaran & Yamagata 2024), with alphas significantly different from zero indicating either mis-pricing (in the case the factor model is correctly specified) or the returns are not fully spanned by the factors. Formally, I conduct inference on the null hypothesis:

$$\mathbb{H}_0 : \alpha_{i,t} = 0 \qquad \forall i, t$$

Or equivalently in the context of IPCA:

$$\mathbb{H}_0 : \Gamma_\alpha = \mathbf{0}_{L+1}$$

[16] Specific implementation details can be found in the publicly available code furnished by Matthias Buechner and Leland Bybee: Github repository.

where $L+1$ is the number of characteristics in $Z_{i,t}$ and $\mathbf{0}_{L+1}$ is the zero vector in $\mathbb{R}^{L+1}$. As I am conducting inference on a conditional factor model, the classic Gibbons et al. (1989) test does not apply. Instead, following Kelly et al. (2019), I apply the wild bootstrap of Liu (1988) and Wu (1986) with 5000 replications. To describe briefly, I first compute Γ_α and compute the Wald statistic $\Gamma_\alpha'\Gamma_\alpha$. Then for each bootstrap iteration, I compute the same Wald statistic after estimating the model using the bootstrapped sample generated under the null. The p-value for rejecting $\mathbb{H}_0$ is then the fraction of bootstrapped Wald statistics greater than the Wald statistic of the unrestricted estimated model. Details of the bootstrapping procedure are left to appendix A.

Hypothesis testing for other null hypotheses proceeds similarly via wild bootstrap. For instance, to test if all betas for the j-th latent factor are jointly zero, one can test the following null hypothesis:

$$\mathbb{H}_0 : \beta_{i,t}^j = 0 \qquad \forall i, t$$

where $\beta_{i,t}^j$ is the loading j-th latent factor. This is equivalent to testing the null hypothesis below:

$$\mathbb{H}_0 : \Gamma_\beta^j = \mathbf{0}_{L+1}$$

where Γ_β^j is on the j-th column of Γ. The relevant test statistic is again a familiar Wald statistic ${\Gamma_\beta^j}'\Gamma_\beta^j$ with p-values computed via bootstrap.

I estimate the IPCA model in (17) both with and without an intercept separately for each equity index (SPX, NDX and RUT) using 0 to 3 latent factors. The model without an intercept is the model in (17) where Γ_α is restricted to be a vector of zeroes. Goodness of fit measures, R^2 and mean absolute pricing error (MAPE), are reported in table 4. R^2 is computed as in (21) and the MAPE of a model is the average taken over the absolute value of the model's pricing errors as in (22). N_{ind} in equation (22) refers to the number of observations pertaining to the equity index from the option panel used to estimate the model.

$$MAPE = \frac{1}{N_{ind}} \sum_{i,t} \left| R_{i,t+1} - Z_{i,t}'\hat{\Gamma}_\alpha - \hat{\Gamma}_\beta \hat{F}_{t+1} - Z_{i,t}'\hat{\Gamma}_\delta R_{t+1}^{mkt} \right| \tag{22}$$

Table 4 reports the R^2 and MAPE for IPCA models with 1 - 4 factors in panel A and B. Note as the equity index return is always included as a factor, models with only a single factor do not feature any latent factors. The columns in table 4 headed by a number indicate the total number of factors for each IPCA model. Beginning with the one-factor models, two salient facts stand out. The first is that a fairly large proportion of the variation is explained just by the equity index return; for the S&P 500 options, around 83% of the variation is explained by the one-factor model with similar results indicated by one-factor

No. Factors:	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	$\mathbb{E}\lvert R_{i,t+1}\rvert$	N_{ind}
	R^2				$10000\times$ MAPE					
Panel A - IPCA without intercept ($\Gamma_\alpha = \mathbf{0}_{L+1}$)										
Equity Index										
SPX	0.829	0.956	0.975	0.980	26.66	14.35	11.54	9.90	83.81	961,948
NDX	0.854	0.955	0.974	0.982	33.04	17.81	13.34	10.40	116.13	822,069
RUT	0.839	0.943	0.964	0.975	32.01	18.34	13.82	11.61	104.76	599,252
	R^2				$10000\times$ MAPE					
Panel B - IPCA with intercept ($\Gamma_\alpha \neq \mathbf{0}_{L+1}$)										
Equity Index										
SPX	0.830	0.956	0.975	0.979	26.34	14.38	11.54	10.18	83.81	961,948
NDX	0.854	0.956	0.974	0.983	32.80	17.81	13.33	10.39	116.13	822,069
RUT	0.840	0.923	0.965	0.976	31.28	22.31	13.90	11.59	104.76	599,252
	P-value for $\mathbb{H}_0$				$10000 \times \mathbb{E}\lvert\alpha_{i,t}\rvert$ when $\Gamma_\alpha \neq \mathbf{0}_{L+1}$					
Panel C - Hypothesis testing ($\mathbb{H}_0 : \Gamma_\alpha = \mathbf{0}_{L+1}$)										
Equity Index										
SPX	0.016	0.000	0.012	0.006	2.25	2.94	3.27	2.75	84.38	961,948
NDX	0.008	0.004	0.020	0.014	2.54	5.38	2.80	4.19	116.13	822,069
RUT	0.006	0.004	0.012	0.024	4.16	5.35	4.04	2.25	104.76	599,252

Table 4. IPCA estimation results for options written on the three major US equity indices: SPX (S&P 500), NDX (Nasdaq-100) and RUT (Russell 2000). R-squareds (computed as in (21)) and MAPE (expressed in basis points) for IPCA models without an intercept (Panel A) and with an intercept (Panel B). Columns headed with a number represent results for a IPCA model with that number of total factors. $\mathbb{E}|R_{i,t+1}|$ is the mean absolute value of returns for the option panel corresponding to the equity index. N_{ind} is the number of observations pertaining to the equity index in the option panel used to estimate the model. Panel C reports the p-values for the test the alphas are jointly zero for each IPCA model and the average absolute value over the alphas from the unrestricted model ($\Gamma_\alpha \neq 0$) in basis points. P-values are computed using the wild bootstrap procedure of Liu (1988) as in Goyal and Saretto (2022) and Büchner and Kelly (2022) using 5000 bootstrap samples. The p-value is the fraction of bootstrapped samples b under the null for which $\hat{\Gamma}^{b\prime}_\alpha \hat{\Gamma}^b_\alpha$ exceeds $\hat{\Gamma}'_\alpha \hat{\Gamma}_\alpha$ where $\hat{\Gamma}^b_\alpha$ and $\hat{\Gamma}_\alpha$ are the estimated values of $\hat{\Gamma}^b_\alpha$ and Γ_α using the bootstrapped sample and option panel respectively. See the appendix A for details on the implementation.

models estimated on other equity index options. The second fact to note is that MAPE for the one-factor IPCA models is relatively high compared to the typical magnitudes of the deleveraged option returns. The average magnitude of the option return corresponding to each equity index is shown in the column headed by $\mathbb{E}|R_{i,t+1}|$. For instance, the average magnitude for SPX options is around 84 bps. The MAPEs for the one-factor SPX IPCA models, estimated with and without an intercept, are both around 27 bps, approximately one-third of the typical magnitude of the returns in the sample. One-factor models estimated on other indices show similarly large errors relative to the size of the returns in the data.

Models utilizing latent factors (columns (2) - (4) in table 4), demonstrate significant improvements in pricing performance and R-squareds. Looking at the 2-factor models, we see that just including one latent factor combined with the market index pushes the R^2s to above 90% for all market indices and about halves the pricing error relative to the corresponding one-factor models. As more latent factors are included, pricing performance in particular improves. The 4-factor models (3 latent factors and market index factor), all have pricing errors of around 10 bps for each equity index. These pricing errors are about 8 to 13 percent of the average magnitude of their corresponding option returns. Overall, the models demonstrate good in-sample performance with minimal differences between models estimated with and without an intercept. These results suggest that an IPCA model featuring latent factors and the equity index return produce models that explain the variation in returns well and do so with fairly low pricing error, on the order of one-tenth of the average magnitude of realized returns. I take the four-factor IPCA models in column (4), hereafter the 3L+MKT models (3 latent factors and the market index return), as the benchmark IPCA models for each equity index.

4.3 IPCA: model alphas

In this subsection, I examine the alphas of my benchmark IPCA model for SPX, the SPX 3L+MKT model. I first employ a dynamic factor model version of the Gibbons-Ross-Shanken test. Formally, I test the null hypothesis

$$\mathbb{H}_0 : \alpha_{i,t} = 0 \qquad \forall i, t \tag{23}$$

using the econometric test described in section 4.2. Panel C of table 4 displays the P-values for the null hypothesis in (23) for IPCA models between 1 and 4 total factors. Panel C also presents the average magnitudes of the alphas in each of the unrestricted models ($\Gamma_\alpha \neq 0$). At a 1% percent level of significance, we must reject the null hypothesis in (23) for all the factor models considered and thus cannot exclude the possibility that some subset of the alphas are non-zero. However, upon deeper examination of the benchmark 3L+MKT models, I find that these alphas are not economically significant, or at the very least, are in some sense small. After transaction costs are taken into account, the arbitrage opportunities suggested by the alphas disappear.

Before proceeding further, it is instructive to review how the alphas from a factor model can be traded. Given a factor model, one can capture the alpha via a long-short approach. In our context, if $\alpha_{i,t} > 0$, one would adopt the following long-short strategy:

$$\underbrace{R_{i,t+1} - R^f_{t+1}}_{\text{Long}} = \alpha_{i,t} + \underbrace{\beta'_{i,t} F_{t+1} + \beta^{mkt}_{i,t} R^{mkt}_{t+1}}_{\text{Short}} \tag{24}$$

If $\alpha_{i,t} < 0$, the long and short positions in equation (24) are reversed. For the benchmark 3L+MKT models, the average absolute value of the alphas range between 2 - 5 bps for our three equity indices. Although this is on the order of the average daily excess equity index return, after transaction costs are paid, the average return of an alpha capturing strategy like (24) is negative. The reason for this is that option markets feature sizable total transaction costs. The literature estimates this transaction cost to be around 50% - 100% of the bid-ask spread (Bali, Beckmeyer, Moerke, & Weigert 2023). Since implementing the long-short strategy in (24) requires we trade an option and the latent factors twice (round trip), a trader aiming to exploit this alpha incurs very significant transaction costs. For the sake of a back-of-the-envelope calculation, I adopt the following generous assumptions:

- Underlying index is costless to trade (trading the market index factor incurs zero transaction costs).
- I assume 75% of bid-ask is paid each way.
- Lastly, I assume latent factors cost 1 bps to trade round trip.[17]

I compute the average alpha captured by the long-short strategy in (24) net of the transaction costs computed using these assumptions, using OptionMetrics's end-of-day bid-ask quotes to estimate the transaction costs to trading the delevered option. The average alpha net of transaction costs for each of the 60 buckets across all equity indices are negative. Figure 4 shows the average net alpha captured by the long-short strategy on delevered SPX option returns for each of the 20 buckets. For all SPX buckets, transaction costs on average subsume the alphas of the SPX 3L+MKT model. Although transaction costs especially hamper the alpha capture strategy in (24) across all options, they are especially impactful for contracts which are either deep-in-the-money or deep-out-of-the-money as bid-ask spreads

[17]For comparison, trading the S&P 500 via SPY ETF costs around 25 cents per 10,000 dollars of the ETF, or 0.25 bp per trade. As we will see in a later section, the factors mainly reflect exposure to higher order moments of the underlying index. Trading such exposures has similarly high transaction costs as those faced in the equity index options market, making this assumption particularly generous.

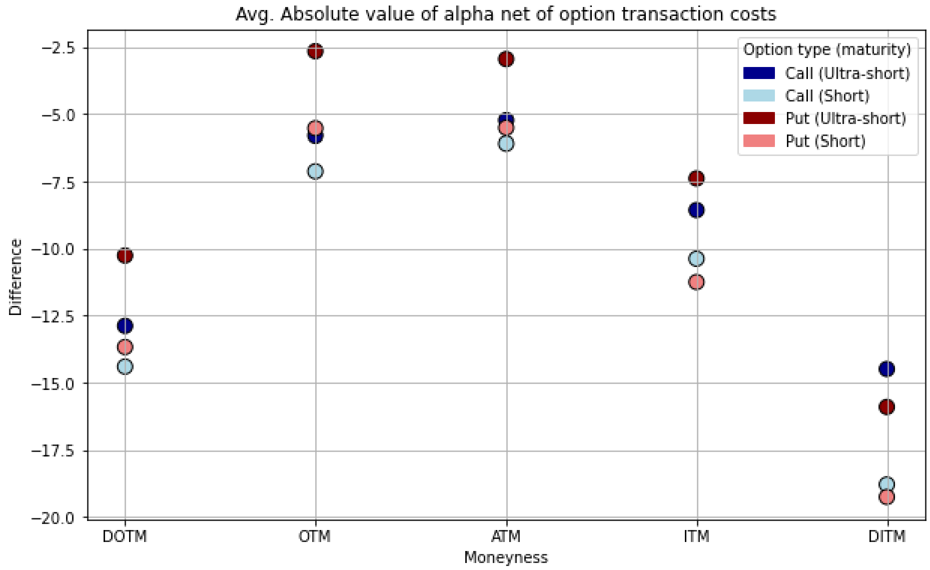

Figure 4. Average absolute alphas of 3L+MKT net of transaction costs in basis points for the 20 SPX equity index option buckets.

are widest for these buckets. Even for options at-the-money, where contracts are typically the most liquid and bid-ask spreads are tightest, the transaction costs still prove too high for a market participant to exploit in any economically practical sense. These patterns are present in the other two equity indices I examine. The general conclusion I draw from this analysis of the 3L+MKT model alphas is that, while some alphas are likely non-zero, they are in an economic sense small, certainly too small to realistically trade using conventional long-short arbitrage.

Overall, the benchmark 3L+MKT IPCA models do well at explaining returns in the sample. Computing the pricing errors out-of-sample suggest my results not due to over-fitting. Table 5 compares the pricing errors in and out-of-sample for the SPX IPCA models. The out-of-sample errors are all within a single basis point of their in-sample counterparts. Furthermore, each 3L+MKT model has small alphas in an economic sense, that is, the alphas are small enough in magnitude that attempting to realize them is thwarted by the transaction costs required to isolate them. The large fraction of variation explained by the 3L+MKT models, their lack of economically realizable alphas, and overall pricing performance suggest a relatively low-dimensional factor structure of option returns which is captured by the index return and three latent factors. Although not directly comparable, this finding is consistent those by Karakaya (2014), Christoffersen, Fournier, and Jacobs (2018), and Büchner and Kelly (2022). These studies examine factor models estimated on delta-hedged option returns and find similar evidence of a low-dimensional factor structure.

Despite the relative effectiveness of IPCA models and latent factor models more broadly,

Number of Total Factors :	1	2	3	4
Mean abs. pricing error ($\Gamma_\alpha \neq 0$, in-sample)	26.34	14.38	11.54	10.18
Mean abs. pricing error ($\Gamma_\alpha \neq 0$, out-of-sample)	27.11	13.46	10.42	9.62
Total R^2 ($\Gamma_\alpha \neq 0$, in-sample)	0.830	0.956	0.975	0.979
Total R^2 ($\Gamma_\alpha \neq 0$, out-of-sample)	0.824	0.956	0.972	0.977

Table 5. In-sample and out-of-sample mean absolute pricing error (in basis points) and R-squareds for the SPX IPCA models with an intercept. Out-of-sample errors and R-squareds are computed using a rolling window approach. Appendix A provides implementation details.

they share a common trait detrimental to attempts at economic interpretation; the factors that are extracted from estimating the model are, in some sense, purely statistical ones. The extracted factors emerge out of correlated fluctuations in the cross-section of delevered option returns and may not correspond to conventional factors considered by the finance literature previously. More succinctly, the estimated latent factors are agnostic to their economic sources - a potential setback when attempting to identify the risks that are priced. To recover some economic interpretability from the estimated IPCA models, I pursue two complementary approaches in the following section. One approach involves analyzing the relationship between the recovered latent factors and factors uncovered by the literature relevant to option returns. The other approach examines the potentially informative variation in the unconditional latent factor loadings across option returns in different buckets.

5 Interpretation of latent factors

In this section, I provide some economic interpretation of the extracted latent factors from the 3L+MKT models. I first consider two complementary approaches to interpreting the 3 latent factors from the SPX 3L+MKT model, then I propose my own factor model consisting entirely of observable and economically interpretable factors. I primarily focus on the 3L+MKT model estimated from SPX options for the analysis in this section as it is the index with the most associated observations in my sample. As a robustness check, I provide some similar results for the NDX and RUT indices in appendix B.

5.1 Latent factor regressions

Our baseline 3L+MKT model for SPX options features three latent factors. One approach to uncovering the risks baked into the recovered latent factors is to regress them on known option market risk factors from prior work studying individual equity and equity index

Factor	Related Moment	Construction	Reference
- Level (S/US)	Variance (30/9-day horizon)	One-day return of an ATM straddle constructed from options with 30/9 DTE.	Coval and Shumway (2001)
+ Skew (S/US)	Skew (30/9-day horizon)	One-day return to vega-neutral portfolio long OTM calls and short OTM puts with 30/9 DTE. Includes small position in underlying to render portfolio delta-neutral.	Bali and Murray (2013)
- Left tail jump index (LT)	Kurtosis of jump distribution	Ratio of deep OTM 9-day SPX put price to forward price of the underlying.	Bollerslev and Todorov (2011)
+ Right tail jump index (RT)	Kurtosis of jump distribution	Ratio of deep OTM 9-day SPX call price to forward price of underlying	"
- ΔVIX	SPX volatility (30-day horizon)	Portfolio of OTM 30-day SPX options	CBOE (2019)
- ΔVVIX	VIX volatility (30-day horizon)	Portfolio of OTM 30-day VIX options	"

Table 6. A summary of the risk factors formed from SPX options in $\mathbf{X}_{t+1}$ used to estimate regressions in (25) for SPX 3L+MKT factors. + (-) indicates the risk factor earns positive (negative) risk premium in sample.

options. For n^{th} latent factor F^n_{t+1} from the SPX 3L+MKT model, I estimate:

$$F^n_{t+1} = a + \mathbf{b}'\mathbf{X}_{t+1} + \varepsilon_{t+1} \tag{25}$$

where $\mathbf{X}_{t+1}$ is a vector containing known risk factors from the literature. In particular, the risk factors I include in $\mathbf{X}_{t+1}$ were employed in prior studies which either directly modeled option returns using factor models (Bali & Murray 2013; Coval & Shumway 2001) or are implied by formal option pricing models such as the Heston (1993) model. Roughly the risk factors can be labeled as factors corresponding to the underlying's variance, skewness, tail risk, or volatility-of-volatility.

Table 6 summarizes the risk factors included in $\mathbf{X}_{t+1}$. All risk factors are constructed from SPX options with the exception of the change in the $VVIX$, which is constructed from 1-month maturity VIX options. I consider two types of variance exposed factors: a straddle-based factor constructed using the methodology in Coval and Shumway (2001) and the daily change in the VIX. I construct two time series of the straddle factor: the daily return of an at-the-money zero-delta straddle constructed from options with 30 days to maturity (Level^{S}) and 9 days to maturity (Level^{US}). These factors are meant to capture exposure to the level of forward-looking variance at horizons relevant to S and US maturity options respectively.

Specifically, let $C^{\text{ATM, S}}_t$ and $P^{\text{ATM, S}}_t$ denote the time t price of an ATM call and put option with 30 days to maturity.[18] Their prices the next trading day are $C^{\text{ATM, S}}_{t+1}$ and $P^{\text{ATM, S}}_{t+1}$. Then the Level^{S} factor at time $t+1$ is constructed as the one-day return to a portfolio holding

[18] The ATM call and put options selected have the same strike price. The strike price is chosen such that it is the closest traded strike price to the corresponding forward price of the underlying at time t.

$\omega_t^{\text{ATM, S}}$ units of $C_t^{\text{ATM, S}}$ and one unit of $P_t^{\text{ATM, S}}$:

$$\text{Level}_{t+1}^{\text{S}} = \frac{\omega_t^{\text{ATM, S}}(C_{t+1}^{\text{ATM, S}} - C_{t+1}^{\text{ATM, S}}) + (P_{t+1}^{\text{ATM, S}} - P_{t+1}^{\text{ATM, S}})}{\omega_t^{\text{ATM, S}} C_t^{\text{ATM, S}} + P_t^{\text{ATM, S}}} \tag{26}$$

As in Coval and Shumway (2001), $\omega_t^{\text{ATM, S}}$ is chosen such that the Black-Scholes delta of a straddle portfolio is zero at time t. Doing this makes the straddle neutral with respect to the small moves in underlying. The Level^{US} factor is constructed using options with 9 days to maturity at time t. $C_t^{\text{ATM, US}}$ and $C_t^{\text{ATM, US}}$ are the time t price of an ATM call and put option with 9 days to maturity. Analogously, define $\text{Level}_{t+1}^{\text{US}}$:

$$\text{Level}_{t+1}^{\text{US}} = \frac{\omega_t^{\text{ATM, US}} C_{t+1}^{\text{ATM, US}} + P_{t+1}^{\text{ATM, US}}}{\omega_t^{\text{ATM, US}} C_t^{\text{ATM, US}} + P_t^{\text{ATM, US}}} \tag{27}$$

where $\omega_t^{\text{ATM, US}}$ is again chosen to make the Black-Scholes delta of the straddle zero at time t. As expected, I observe negative average returns for the level factors constructed from options with both 9 and 30 days to maturity. This is consistent with general findings that straddles heavily load heavily on variance which is known to possess highly negative risk premium.

Another popular volatility factor for option returns is changes in the VIX or its square as in the factor models of Jones (2006) and Fournier et al. (2024). As my focus is on daily returns, I use the daily change in the VIX:

$$\Delta\text{VIX}_{t+1} = \text{VIX}_{t+1} - \text{VIX}_t \tag{28}$$

The next set of factors I consider are related to the skewness of the underlying's risk neutral distribution. Specifically, I construct the one-day return in the skewness asset from Bali and Murray (2013). The skewness asset is a portfolio consisting of a long OTM call position, short OTM put position and a small position in the underlying. The positions in the options are chosen to make the portfolio vega-neutral at construction in order to remove exposure to changes in implied volatility. A small position in the underlying is taken to render the portfolio delta-neutral. Doing so, we get an asset with returns determined by changes in the skewness of the risk-neutral distribution of the underlying or, in terms of implied volatilities, the slope of the volatility smile. Like before, I compute the returns to the skewness assets constructed using options with 9 and 30 days to maturity to get risk factors exposed to skewness at horizons relevant to US and S maturity options. Let $C_t^{\text{OTM, S}}$ and $P_t^{\text{OTM, S}}$ be the time t price of an OTM call and put option with 30 days to maturity.[19]

[19] I choose the strike of the OTM call (put) to be the traded strike with Black-Scholes delta closest to 0.1

I use $C_t^{\text{OTM, US}}$ and $P_t^{\text{OTM, US}}$ analogously to denote the price of an OTM call and put option with 9 days to maturity. Formally, I compute $\text{Skew}_{t+1}^{\text{S}}$ as follows:

$$\text{Skew}_{t+1}^{\text{S}} = \frac{\omega_t^{\text{OTM, S}}\left[C_{t+1}^{\text{OTM, S}} - C_t^{\text{OTM, S}}\right] - \left[P_{t+1}^{\text{OTM, S}} - P_t^{\text{OTM, S}}\right] + x_t^{\text{S}}\left[S_{t+1} - S_t\right]}{\left|\omega_t^{\text{OTM, S}} C_t^{\text{OTM, S}} - P_t^{\text{OTM, S}} + x_t^{\text{S}} S_t\right|} \tag{29}$$

For the skewness asset, I compute the return by dividing by the absolute value of the combined cost of constructing the skewness asset rather than simply the combined cost. This follows Bali and Murray (2013) who point out that the price of constructing the skewness asset need not be positive. In my own sample, I do observe some skewness assets with negative prices albeit very infrequently. I denote the Black-Scholes vega of the call and put option above by $\mathcal{V}_t^{\text{C}}$ and $\mathcal{V}_t^{\text{P}}$. To render the skewness asset at time t vega-neutral, $\omega_t^{\text{OTM, S}}$ is chosen to satisfy:

$$\mathcal{V}_t^{\text{C}} \omega_t^{\text{OTM, S}} - \mathcal{V}_t^{\text{P}} = 0 \tag{30}$$

To render the skewness asset at time t delta-neutral, I choose x_t^S such that the net delta of the time t portfolio is zero. The $\text{Skew}_{t+1}^{\text{US}}$ skewness factor is constructed analogously using options with 9 days to maturity. The skewness assets from which our skewness factors are computed are long skewness assets. As discussed in Bali and Murray (2013), the constructed skewness asset rises (falls) in value if the skewness of the risk-neutral distribution increases (decreases). On average, the returns to both skew factors are positive. This is consistent with general empirical findings such as Harvey and Siddique (2000) and Langlois (2020) which find that systematic skewness commands an economically significant and positive risk premium. At the index level, this translates to investors preferring positive skewness or having an aversion to states of the world where the index is more negatively skewed. I interpolate level and skew factors using nearby options whenever there are option maturities which bracket either the target 9 or 30 day maturity used to construct the factors.

The level and skew factors constructed so far map onto the forward-looking second and third moment of the daily SPX return respectively. In the context of short maturity options, US maturity options might be more sensitive to moments at the 9-day horizon while S maturity options more to moments which look over the 30 day horizon. The last set of factors do not correspond to the moments of SPX returns per-se, but could nonetheless be relevant for option returns and therefore could be correlated to the latent factors in the SPX 3L+MKT model. I include a set of jump tail measures LT_{t+1} (left tail) and RT_{t+1} (right tail) introduced by Bollerslev and Todorov (2011) in $\mathbf{X}_{t+1}$. LT_{t+1} (RT_{t+1}) embeds the probability

(-0.1).

and intensity of a large negative (positive) jump in S&P 500 Index. Formally, I compute them from the price of a 10% OTM SPX put (call) using options with less than a month to maturity, selecting the maturity τ that is closest to 8 days:

$$\mathrm{LT}_{t+1} = \frac{e^{r_{t+1}} P_{t+1}^{\mathrm{DOTM}}}{(\tau/365) \times FP_{t+1}} \qquad \mathrm{RT}_{t+1} = \frac{e^{r_{t+1}} C_{t+1}^{\mathrm{DOTM}}}{(\tau/365) \times FP_{t+1}} \tag{31}$$

where P_{t+1}^{DOTM} and C_{t+1}^{DOTM} are the prices of the 10% OTM put and call, r_{t+1} is the interest on a risk-free asset over one day, and FP_{t+1} is the forward price of the S&P 500. The intuition behind LT_{t+1} and RT_{t+1} is that options with short maturity which are very deep out-of-the-money would only have non-zero payoff if a large jump were realized (Carr & Wu 2003). Lastly, I also include the one-day change in the VVIX index in $\mathbf{X}_{t+1}$. VVIX measures the volatility of the VIX from VIX options with 1 month to maturity. The VVIX is a proxy for the volatility-of-volatility, a priced risk factor that emerges from many option pricing models which embed stochastic volatility.

Table 7 presents the estimated coefficients of the regression in (25). I regress the three latent factors $F_{t+1}^1, F_{t+1}^2, F_{t+1}^3$ and the SPX daily return from the SPX 3L+MKT model on the vector of known risk factors $\mathbf{X}_{t+1}$. Examining the estimates from the regression on the first latent factor, I find a significant negative coefficient on the left tail jump factor, indicating that our first latent factor embeds compensation for bearing left tail jump risk. The first latent factor also incorporates compensation for bearing other risks such as very near-term volatility $\mathrm{Level}_{t+1}^{\mathrm{US}}$ and very near-term skewness of the index $\mathrm{Skew}_{t+1}^{\mathrm{US}}$. The positive coefficient on $\mathrm{Skew}_{t+1}^{\mathrm{S}}$ and negative coefficient $\mathrm{Skew}_{t+1}^{\mathrm{US}}$ could indicate the first latent factor also incorporates compensation for bearing risks associated with the term structure of skewness. Overall, the first latent factor seems to embed compensation for exposure to left tail jumps, volatility and skewness around the ultra-short horizon.

Turning to the second latent factor, there are positive coefficients on the US factors ($\mathrm{Level}_{t+1}^{\mathrm{US}}$ and $\mathrm{Skew}_{t+1}^{\mathrm{US}}$), but negative coefficients on the S factors ($\mathrm{Level}_{t+1}^{\mathrm{S}}$ and $\mathrm{Skew}_{t+1}^{\mathrm{S}}$). This factor appears to be encoding risks associated with the term structure of the level of volatility and skewness. Alternatively, an intuitive interpretation is to view it as a maturity factor like those identified by Karakaya (2014) and Büchner and Kelly (2022). US maturity options are more sensitive to the very near term component of the term structure and S maturities more sensitive to the 30-day horizon of the volatility and skewness. Indeed, as shown in a later analysis, on average US maturity options have larger loadings on F_{t+1}^2 compared to their S maturity counterparts. Lastly, I turn to the most easily interpretable factor, F_{t+1}^3, which likely embed compensation for bearing variance risk as evinced by the negative coefficients on both level factors and $\Delta\mathrm{VIX}_{t+1}$.

Dependent variable (SPX 3L+MKT Factors):	(F^1_{t+1})	(F^2_{t+1})	(F^3_{t+1})	(MKT)
Risk factor:				
LT_{t+1}	-2.483***	-0.479**	0.032	0.091***
	(0.715)	(0.149)	(0.080)	(0.014)
RT_{t+1}	2.289*	0.477**	-0.150	-0.064***
	(0.959)	(0.164)	(0.108)	(0.014)
$\mathrm{Level}^{\mathrm{S}}_{t+1}$	0.334	-0.111***	-0.124***	0.014***
	(0.192)	(0.030)	(0.023)	(0.003)
$\mathrm{Level}^{\mathrm{US}}_{t+1}$	-0.168**	0.038***	-0.034***	-0.001
	(0.060)	(0.010)	(0.009)	(0.001)
$\mathrm{Skew}^{\mathrm{S}}_{t+1} \times 100$	-0.254***	-0.053***	-0.007	0.012***
	(0.070)	(0.015)	(0.007)	(0.001)
$\mathrm{Skew}^{\mathrm{US}}_{t+1} \times 100$	0.281***	0.038**	-0.039***	0.015***
	(0.070)	(0.014)	(0.008)	(0.002)
$\Delta\mathrm{VIX}_{t+1}$	-0.031	-0.005*	-0.008***	-0.001***
	(0.018)	(0.003)	(0.001)	(0.000)
$\Delta\mathrm{VVIX}_{t+1}$	0.006	0.001	0.001***	-0.000
	(0.003)	(0.001)	(0.000)	(0.000)
Constant	-0.026**	-0.007***	-0.003**	0.003***
	(0.009)	(0.002)	(0.001)	(0.000)
Factor Mean (basis points)	17.67	16.28	11.99	4.13
Factor Sharpe Ratio	0.018	0.044	0.050	0.034
Number of Observations	2691	2691	2691	2691
R^2	0.297	0.225	0.752	0.958

Table 7. OLS estimates of the coefficients in equation (25) for the daily realizations of the 3 latent factors and the MKT factor (daily SPX return) from the SPX 3L+MKT. The Sharpe ratio reported is the un-annualized daily Sharpe ratio. Standard errors are computing using the heteroskedasticity robust methodology of White (1980). One, two and three stars denote significance at the 5%, 1% amd 0.1% level respectively.

To summarize, the regression results reported in table 7 shed light on the types of risks priced into the latent factors, and by extension, the cross-section of SPX option returns. The first latent factor primarily captures compensation for left tail jump risk and very near-term volatility and skewness; this is consistent with some of the crash risk factors considered by Constantinides et al. (2013) and Bates (2008). The second latent factor could be viewed as a maturity factor; generally, it possesses positive exposure to factors constructed from US maturity options and negative exposure to those constructed from S maturity options. The third latent factor is most straightforwardly interpretable as a variance risk factor, capturing compensation for exposure to the overall level of the volatility surface across maturities.

5.2 Unconditional factor betas

In this subsection, I examine the unconditional factor loadings of the SPX 3L+MKT model. As pointed out by Goyal and Saretto (2022) and Büchner and Kelly (2022), the variation in latent factor loadings can shed light on what risks these factors pick up. For instance, if a latent factor loads very highly on US maturity DOTM and OTM puts relative to other option maturities, one might posit that this latent factor embeds risks related to skewness or negative jumps in the underlying. To this end, I compute the average factor loadings as the unconditional betas over the buckets introduced in section 3.2 for the SPX options returns.

Recall the buckets split each index's option returns along moneyness a_1 (DOTM, OTM, ATM, ITM, DITM), maturity a_2 (US or S) and type a_3 (call/put) for a total of 20 buckets. Each SPX bucket is the set $\mathcal{B}(a_1, a_2, a_3, SPX)$. I estimate the unconditional beta on the n-th latent factor, F^n_{t+1}, for options in $\mathcal{B}(a_1, a_2, a_3, SPX)$ by calculating the sample average over their conditional betas $\beta^n_{i,t}$:

$$\mathbb{E}\left[\beta^n_{i,t} | (i,t) \in \mathcal{B}(a_1, a_2, a_3, SPX)\right] \approx \frac{1}{|\mathcal{B}(a_1, a_2, a_3, SPX)|} \sum_{(i,t) \in \mathcal{B}(a_1,a_2,a_3,SPX)} \beta^n_{i,t} \tag{32}$$

Figure 5 plots these unconditional loadings for each bucket with the darker shaded bars corresponding to US maturities (blue for calls, red for puts) and lighter shades to S maturities. The first latent factor (top panel) shows highly negative loadings for the DOTM put options for both S and US maturities. In particular, the most negative loading is on the DOTM US put bucket where jump risks and skewness are most saliently priced (Andersen, Fusari, & Todorov 2017; Bollerslev & Todorov 2011). This is consistent with the evidence from the regressions in the previous subsection and bolsters the interpretation that the first latent factor broadly encodes compensation for bearing skewness and variance risk at the 9-day horizon, both of which are driven largely by jumps. The lower magnitude of the unconditional beta for DOTM puts at S maturities can be explained by the diminishing effect of jumps on DOTM put option prices as the time to maturity becomes longer as shown by Carr and Wu (2003).

The second latent factor (middle panel) displays clear patterns by maturity. For two option types with the same moneyness category, the US maturities are always larger (less negative in the case of the put options), than their S maturity counterparts. Again, this is consistent with the prior subsection's interpretation of the second latent factor as a maturity-linked factor. The notion of a maturity factor appears in quite a few studies of option returns such as Karakaya (2014) and Büchner and Kelly (2022). Such a maturity factor is often constructed from the returns to a long-short portfolio on longer maturity options (long) and

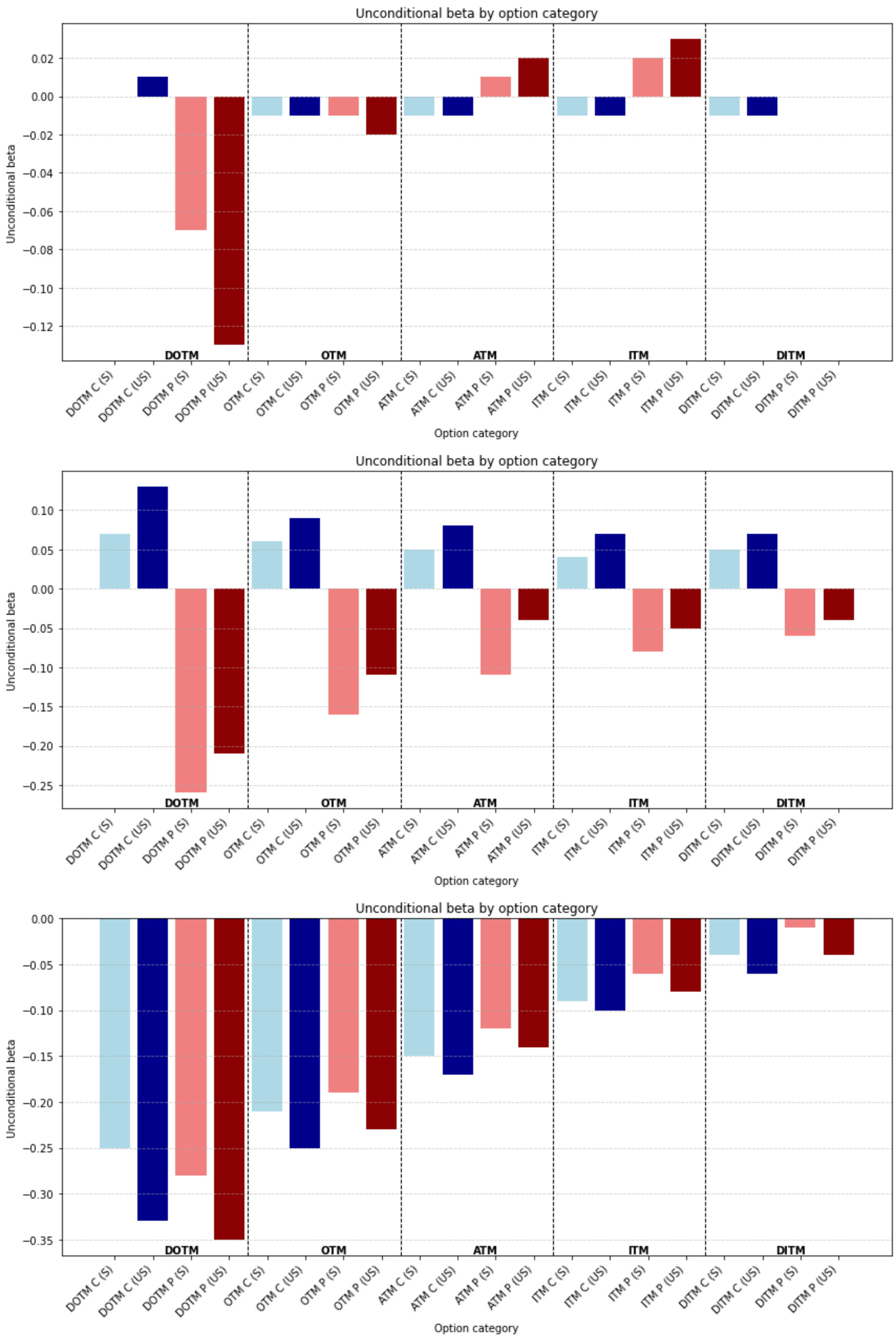

Figure 5. Unconditional betas of the SPX 3L+MKT model for each of the the 20 SPX option buckets for the latent factors: F^1_{t+1} (top), F^2_{t+1} (middle), F^3_{t+1} (bottom). The y-axis is the unconditional beta computed as in (32) and x-axis shows the corresponding bucket. US maturity calls (puts) are plotted as dark blue (dark red) bars and S maturity calls (puts) are plotted as light blue (light red) bars.

shorter maturity options (short). Although a maturity-linked factor appears in quite a few factor models of index options returns, there is disagreement on its economic basis.

The third latent factor (bottom panel) presents a very striking pattern across moneyness. The most negative loadings are on the DOTM buckets and steadily become less negative as we move from DOTM buckets to DITM buckets where the unconditional betas are close to zero. If the third latent factor primarily embeds compensation for bearing variance risk, we should expect two things from the unconditional betas. First, we should observe negative loadings across the board, as all options are mechanically long variance. Second, we should see that DOTM options have the most negative loadings and DITM to have the least negative loadings. Both are displayed by the unconditional betas for F^3_{t+1} in the bottom panel of figure 5.

At first, the reason why this moneyness pattern provides evidence of a variance risk factor is not immediately clear, especially since in many option pricing models the option's vega is highest at-the-money. However, this is not the case for options which are deleveraged. By deleveraging the option by its embedded leverage, the smallest vega exposures in the option panel occur for options deepest in-the-money and is increasing in how far out-the-money the option's strike price is. Table 7 shows that the third latent factor is negatively related to the level factors and the daily change in the VIX. Overall, the evidence from the unconditional betas support the interpretation that the third latent factor primarily provides compensation for exposure to SPX variance.

To summarize, the patterns in the unconditional betas for the SPX 3L+MKT model corroborate the interpretations put forth in section 5.1. The first latent factor encodes variance and skewness risk over the 9-day horizon, a horizon where jumps affect higher-order moments more saliently. The second latent factor appears to be a maturity linked factor which negatively loads onto put returns but positively loads onto call returns. Lastly, the unconditional betas and regression evidence suggest the third latent factor is likely exposure to the overall level of variance across maturities.

5.3 5MOM: A moment-based interpretable factor model

Latent factor models such as IPCA have found widespread adoption in the literature, in no small part, due to their pricing performance and ability to explain the variation in returns across a wide variety of asset classes (Giglio et al. 2022). However, a common drawback of these models is that they do not immediately shed light on any underlying economic risks. To interpret these latent factors and understand the risk exposures they embed, analyses of their factor loadings or covariances to known risk factors like those conducted earlier are necessary. Even if the latent factors can be interpreted, there are myriad situations facing researchers and market participants alike which calls for a factor model using easily interpretable and tradable factors. Here, I build off the interpretations of the latent factor models offered thus far and propose my own factor model. I evaluate the pricing performance of this model relative to other models proposed in the literature and industry practice.

I propose a 5-factor model of SPX option returns consisting entirely of easily tradable and interpretable factors.[20] The 5-factor model uses both level factors (Level$^{\mathrm{US}}$ and Level$^{\mathrm{S}}$), both skewness factors (Skew$^{\mathrm{US}}$ and Skew$^{\mathrm{S}}$), and underlying index return $R^{mkt}_{SPX,t+1}$. $\mathbf{F}^{SPX}_{t+1}$ stacks these factors into a five element vector. As this model employs factors that are sensitive to 5 moments - expected market return, risk-neutral volatility at the 9-day and 30-day horizon, and risk-neutral skewness at the 9-day and 30-day horizon - I hereafter refer to this model as 5MOM (5 moments). Equation (33) specifies the structure of the model. The factor model adopts the linear-in-characteristics dynamic betas of the IPCA model to implement time-varying betas. I use the same vector of characteristics $Z_{i,t}$ used to estimate the IPCA models defined in (18).

$$\begin{aligned} R_{i,t+1} - R^f_{t+1} &= \boldsymbol{\beta}'_{i,t}\mathbf{F}^{SPX}_{t+1} + E_{i,t+1} \\ &= \left(Z'_{i,t}\Gamma_\beta\right)\mathbf{F}^{SPX}_{t+1} + E_{i,t+1} \end{aligned} \tag{33}$$

As in earlier analyses, I focus my analysis on SPX options here. The choice of factors is motivated by the interpretations of the latent factors offered in this section. The two volatility factors, Level$^{\mathrm{US}}$ and Level$^{\mathrm{S}}$, correspond to the risks priced in the third latent factor. As shown in table 7, a large fraction of the variation in the third latent factor is explained by the returns on the two level factors. The level factors have negative coefficients indicating the third latent factor awards compensation for bearing index variance risk. The skewness factors, Skew$^{\mathrm{US}}$ and Skew$^{\mathrm{S}}$, are a component of the risks priced into the first and second latent factors. The first latent factor negatively co-moves with both the left tail jump factor and Skew$^{\mathrm{US}}$. In addition, the large negative loadings on DOTM put options (figure 5) suggest there is compensation for bearing very near-term skewness which could be largely driven by negative jumps. The second latent factor is a maturity-linked factor, co-moving negatively with Level$^{\mathrm{S}}$ and Skew$^{\mathrm{S}}$ and positively with Level$^{\mathrm{US}}$ and Skew$^{\mathrm{US}}$.

I estimate the 5MOM model on the SPX option panel using the same IPCA methodology used to estimate the latent factor models. The primary difference in estimation here is that I do not estimate any latent factors, only the dynamic betas. The 5MOM model has an R^2 of

[20]By easily tradable, I refer to the fact that the factors can be constructed using only at most 2 options and some trading in the underlying. Compare to this to the VIX or CBOE SKEW Index both of which require a portfolio of dozens of options to replicate.

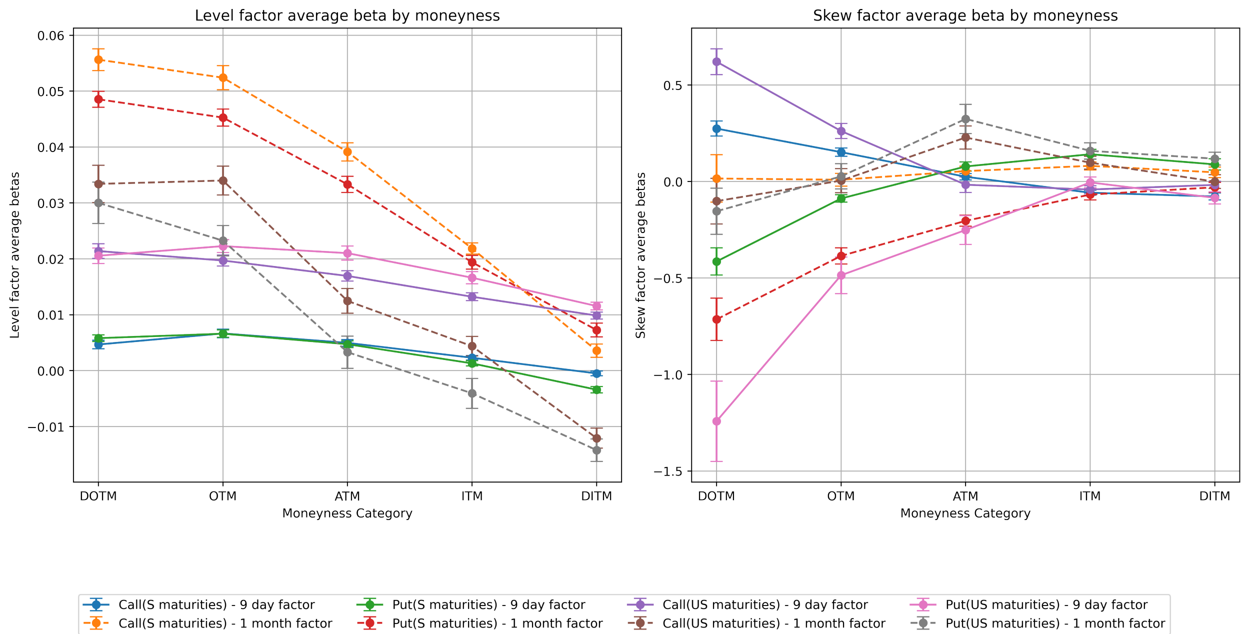

Figure 6. Unconditional betas of the 20 SPX buckets for the 5MOM model. Left plot presents unconditional betas for Level^{US} (solid lines) and Level^{S} (dashed lines). Right plot presents unconditional betas for Skew^{US} (solid lines) and Skew^{S} (dashed lines). X-axis shows bucket moneyness while color encodes bucket type and maturity (see legend). Standard errors computed using wild bootstrap (see appendix A for implementation details). Appendix B presents plots of the unconditional betas for the NDX 5MOM and RUT 5MOM models.

0.952 and an MAPE of 13.4 bps. The MAPE of the SPX 5MOM model is on par with that of the 2 or 3 factor IPCA models in table 4 and trails the benchmark 3L+MKT model by only 3 bps. Estimating a 3-factor model (SPX 3MOM) consisting of the SPX return and just the 1-month horizon moment factors, Level^{S} and Skew^{S} offers an instructive comparison. The SPX 3MOM model uses the same linear-in-characteristics conditional betas structure as (33) and has a MAPE of 14.18 bps. The overall improvement in pricing performance from including the additional factors (Level^{US} and Skew^{US}) is small on average, but the improvement in pricing US maturity options is noticeable. The MAPEs for S maturity options for 5MOM and 3MOM are 11.57 bps and 11.69 bps respectively. US maturities see more significant improvement, highlighting the relevance of these 9 day horizon moments: the MAPE for 3MOM model is 22.78 bps and is 19.04 bps for the 5MOM model, a 16% improvement in pricing performance.

To get a sense of the factor loadings, I compute the unconditional betas of my observable factors $\mathbf{F}_{t+1}^{SPX}$. Like in equation (32), I estimate the unconditional betas as cross-sectional averages of the conditional betas in the 5MOM model for each of the 20 SPX option buckets. Figure 6 plots the unconditional betas from the 5MOM model with two standard deviation error bands around the point estimate for each SPX bucket. I compute bootstrapped standard errors via wild bootstrap (see appendix A). The plot on the left is the average beta for the level factors corresponding to the factor constructed using options with 9 days (solid lines) and 30 days (dashed lines) to maturity. On the right, I plot the average betas on the two skew factors for the 20 buckets. For betas on the level (variance) factors, we notice that nearly all the betas are positive. This is sensible since our panel consists entirely of returns on net long option positions and all vanilla options are inherently long variance; however, this positive exposure is decreasing as we move from DOTM to DITM buckets. For the skew factors, the loadings are all negative for put buckets, with the exception of the DITM put buckets. The loadings are generally positive for call buckets, although there is greater heterogeneity in sign. Like for the level factors, we notice a clear trend along moneyness. As buckets move deeper into the money, the magnitudes of the skew exposures get smaller. In particular, the DOTM call buckets have the largest exposure to index skewness, underscoring the sensitivity to the right tail of the risk-neutral distribution these call options possess. In appendix B, I present the same plot but for 5MOM models estimated on the NDX and RUT index options. The conclusions I draw from the SPX unconditional betas are robust to estimating the 5MOM model using options and factors corresponding to other two equity indices.

I compare the performance of my 5MOM model vis-à-vis other factor models from the literature on the basis of R-squareds and MAPEs. As shown in equation (5), many option pricing models used in industry imply a dynamic factor structure, providing us with an additional basis of comparison beyond factor models from the literature. Models implied from standard industry practice, which I refer to as practitioner models are derived from the Taylor expansion along the underlying price and option's implied volatility:

$$\Delta O_{i,t+1} \approx \underbrace{\frac{\partial O_{i,t+1}}{\partial S_t}}_{\text{Delta}} \Delta S_{t+1} + \underbrace{\frac{\partial^2 O_{i,t+1}}{\partial S_t^2}}_{\text{Gamma}} (\Delta S_{t+1})^2 + \underbrace{\frac{\partial O_{i,t+1}}{\partial \text{IV}_{i,t}}}_{\text{Vega}} \Delta \text{IV}_{i,t+1} \tag{34}$$

where $\Delta O_{i,t+1}$ is the change in the price of option i. This can be rearranged to obtain a factor structure for option returns:

$$\frac{\Delta O_{i,t+1}}{O_{i,t}} \approx \underbrace{\left[\frac{\partial O_{i,t}}{\partial S_t}\frac{S_t}{O_{i,t}}\right]}_{\text{Delta}} \frac{\Delta S_{t+1}}{S_t} + \underbrace{\left[\frac{\partial^2 O_{i,t}}{\partial S_t^2}\frac{S_t^2}{O_{i,t}}\right]}_{\text{Gamma}} \left(\frac{\Delta S_{t+1}}{S_t}\right)^2 + \underbrace{\left[\frac{\partial O_{i,t}}{\partial \text{IV}_{i,t}}\frac{\text{IV}_{i,t}}{O_{i,t}}\right]}_{\text{Vega}} \frac{\Delta \text{IV}_{t+1}}{\text{IV}_{i,t}} \tag{35}$$

To evaluate factor models from the literature, I take the proposed factors and estimate a factor model using the same linear-in-characteristics structure as the 5MOM and IPCA models. Endowing this structure on the betas of these models makes the comparison fair, as many of the factor models mentioned here were originally estimated using static betas rather than time-varying ones. For the proposed factor models I construct the factors $\{F_{t+1}^n\}_{n=1}^K$ and estimate the model:

$$R_{i,t+1} - R_{t+1}^f = \sum_{n=1}^{K} \left(\Gamma'_\beta Z_{i,t}\right) F_{t+1}^n + v_{i,t+1} = \sum_{n=1}^{K} \beta_{i,t}^n F_{t+1}^n + v_{i,t+1} \tag{36}$$

where $Z_{i,t}$ is the same set of characteristics used to estimate the IPCA models defined in (18).

The factor model literature on index options is broad and has considered a wide range of factors with factors relating to forward-looking variance emerging as the most common. A long linage of work, from Coval and Shumway (2001) onwards, consider a variance factor of some kind when estimating factor models of option returns. Ang, Hodrick, Xing, and Zhang (2006) and Büchner and Kelly (2022) make use of the Coval and Shumway (2001) zero-beta straddle as a long variance factor. Other work such as Jones (2006), and more recently by Fournier et al. (2024), use changes in the VIX to construct a variance factor. I consider factor models which incorporate variance in both of these ways here. Other factor models I estimate include those from the literature incorporate the CBOE's SKEW Index and the betting-against-beta (BAB) factor of Frazzini and Pedersen (2022). I also consider factor models employing the volatility-of-volatility as a risk factor, proxied here by the VVIX index.

Table 8 presents R^2 and MAPEs of various factor models adapted from the literature and industry practice. Panel A shows the results for factor models which incorporate known factors from the academic literature. I include the four-factor model (FF4) consisting of the three Fama and French (1993) factors and the momentum factor of Carhart (1997). I also consider factor models which combine the FF4 factors with the Coval and Shumway (2001) straddle and Frazzini and Pedersen (2022) BAB factor. Other models I estimate include the classic CAPM market factor with various combinations of one-day changes in the forward-looking 1-month indices computed by CBOE (VIX, SKEW, VVIX). From the MAPEs reported, we find that models which incorporate a variance factor have lower MAPEs relative to factor models which do not. Out of the models considered from the literature in panel A, the factor model combining the FF4 factors with the straddle and BAB factor (the benchmark FF6 model considered by Büchner and Kelly (2022)) produces the lowest hedging error both for the SPX sample as a whole as well as for the US and S maturity subsamples.

Panel B presents the results for the practitioners' models implied by (35). The delta

Factor model	R^2	MAPE	MAPE (S)	MAPE (US)
Panel A: Prior literature				
FF4 (Carhart 1997)	0.857	26.752	24.411	36.284
FF4 + Straddle	0.944	16.731	14.673	25.109
FF4 + Straddle + BAB	0.944	16.617	14.556	25.005
CAPM	0.850	25.741	23.462	35.018
CAPM + VIX	0.901	21.542	19.197	31.087
CAPM + VIX + SKEW	0.902	21.564	19.217	31.120
CAPM + VIX + VVIX	0.903	21.670	19.293	31.348
CAPM + VIX + SKEW + VVIX	0.903	21.653	19.276	31.329
Panel B: Practitioners' model				
Delta hedging	0.811	27.790	25.497	35.675
Delta and Gamma hedging	0.869	26.092	23.488	35.039
Delta, Gamma and Vega hedging	0.853	25.744	19.881	45.846
Panel C: IPCA (1-4 factors)				
1	0.829	26.661	24.461	35.141
2	0.956	14.355	13.020	19.501
3	0.975	11.546	10.440	15.808
3L+MKT	0.979	9.901	8.493	15.332
Panel D: 3MOM/5MOM				
3MOM (MKT+Level$^{\mathrm{S}}$ + Skew$^{\mathrm{S}}$)	0.956	13.855	11.662	22.784
5MOM	0.966	12.930	11.427	19.046

Table 8. R^2 and mean absolute pricing error (MAPE) for the full SPX options panel. MAPE (S) and MAPE (US) denote the MAPE for the S and US maturity subsample respectively. All pricing error averages reported in basis points. The daily absolute return for the SPX panel is 84 basis points.

hedging, delta and gamma hedging, and delta, gamma and vega hedging models in panel B refer to the factor model in (35) using the first 1, 2 and 3 terms as factors respectively. Overall, the models in panel B are generally outperformed by the other models considered in panel A, C and D. For reference, Panel C reports results for the IPCA model using 0 - 3 latent factors combined with the SPX return as a pre-specified factor. The latent factors in the IPCA model are in principle tradable, although as the first order conditions in (20) imply, they are portfolios of consisting of a large number of options. Transaction costs make such a portfolio infeasible to trade in practice owing to the high transaction costs from transacting in the options market.

Finally, panel D reports the hedging performance of the SPX 3MOM model (daily index return, Level$^{\mathrm{S}}$, and Skew$^{\mathrm{S}}$) and the SPX 5MOM model introduced in the prior subsection. Out of the non-latent factor models, they demonstrate the best performance across both the S maturity and US maturity subsamples. The 5MOM model only trails the benchmark 3L+MKT model by around 3 basis points for both subsamples while explaining a comparable amount of the variation in the SPX option panel. Notably the performance for US maturity option returns are improved by around 3 bps ($\approx$ 16% reduction in pricing error) relative to the 3MOM model while the performance in hedging the S maturity options is essentially unchanged, suggesting that factors sensitive to moments over the 9-day horizon contain relevant and important pricing information for US maturity options.

An important caveat to interpreting the 5MOM model is the fact that the factors do not purely load onto a single higher-order moment at a particular horizon. For instance, the Level^{S} factor (Coval and Shumway (2001) straddle factor) does not load purely on forward-looking variance at the 30-day horizon. Instead, as I show in the following section, Level^{S} is also exposed to changes in forward-looking skewness and some residual market exposure due to the fact that the straddle position used to compute Level^{S} is not dynamically hedged intra-day to maintain zero exposure to the underlying. Another source of residual exposure to the underlying comes from the fact that the option pricing model used to compute the delta of the position might be mis-specified. For instance, Bates (2005) notes that, at some maturities the Black-Scholes deltas are biased. These details complicate attributing option returns to innovations to forward-looking moments in a precise and direct way. In the following section, I make headway on this issue using a reduced form model where I can measure the contribution of these moments to the tradable factors in the 5MOM models. This enables a decomposition of the returns in the option panel to innovations in the underlying index and the model-free measures of forward-looking higher-order moments constructed in section 2 in a more direct way.

6 Decomposition of option returns

In this section, I decompose the expected option returns in the panel into exposures to the underlying as well as its second and third moments over the 9-day and 30-day horizon. To do this, I estimate risk prices using a factor model-like approach to quantify the contribution of the daily innovations in these moments to the variance and skewness asset returns constructed in the previous section. Using these estimated contributions, I can combine these results with the estimated 5MOM models to quantify precisely the contribution of different moment exposures for different buckets of option returns in my panel. The upshot of this approach is that I can assess relative importance of these risk factors for determining expected option returns, filling a gap in the literature that typically relates variance and skewness to option prices rather than returns.

6.1 Risk prices of moments

My analyses in the prior section suggest the importance of using assets that reflect risk exposures to variance and skewness over different horizons. In particular, the 5MOM model uses daily returns to the variance (level) and skewness (skew) assets constructed from 9-day and 30-day maturity options as factors. In this subsection, we directly quantify how changes in the moments of the risk-neutral distribution affect the returns to the variance and skewness assets using a reduced-form model. Before proceeding, I define two additional pieces of notation: $\text{Level}^{(m)}_{ind,t+1}$ and $\text{Skew}^{(m)}_{ind,t+1}$. $\text{Level}^{(m)}_{ind,t+1}$ is the one-day holding return to a zero-beta straddle computed as in equation (27) using options written on index *ind* of maturity m. Likewise, $\text{Skew}^{(m)}_{ind,t+1}$ is the one-day holding return to the skewness asset computed as in equation (29) using options on index *ind* with maturity m. Using prior definitions, I can relate them to our level and skew returns used to estimate the SPX 5MOM model from equation (25):

$$\text{Level}^{\text{US}}_{t+1} = \text{Level}^{(9d)}_{SPX,t+1} \qquad \text{Level}^{\text{S}}_{t+1} = \text{Level}^{(30d)}_{SPX,t+1}$$

and

$$\text{Skew}^{\text{US}}_{t+1} = \text{Skew}^{(9d)}_{SPX,t+1} \qquad \text{Skew}^{\text{S}}_{t+1} = \text{Skew}^{(30d)}_{SPX,t+1}.$$

I construct these variance and skewness asset returns using options with maturity m equal to 1-30 days (1d - 30d) and 2-12 months (2M-12M) for options on the SPX, NDX, and RUT.[21] I retain only time series with at least 250 return observations. Applying this filter, leaves me with 186 time series of variance and skewness asset returns.

The literature often uses assets of similar construction to proxy for factors exposed to higher order moments; in particular, straddles are used as a stand-in for long variance assets (B. Chen, Gan, & Vasquez 2023; Dew-Becker, Giglio, & Kelly 2021). Of course, a natural question is in what way these assets exposed to such moments. To explore this, I use the prices of the variance and skewness claims $V^{(m)}_{ind,t}$ and $W^{(m)}_{ind,t}$, replicated from a portfolio of options written on index *ind* using equations (9) and (10) respectively. As discussed in section 2, the prices of these variance (skewness) claims are the annualized realized variance (skewness) over the next m days for equity index *ind*. Their prices are the risk-neutral expectations of realized variance and skewness and are informative of the real-time forward-looking expectations of the options market. As these assets give pure exposure to a specific moment and horizon, I can use their prices to obtain the fair price of variance and skewness

[21] For maturities m greater than or equal to 2 months (60 days), the level and skew returns are calculated with options within 2 weeks of the exact target maturity. The target maturities greater than 30 days to expiration are multiples of 30 from 60 (2 months) to 360 (12 months).

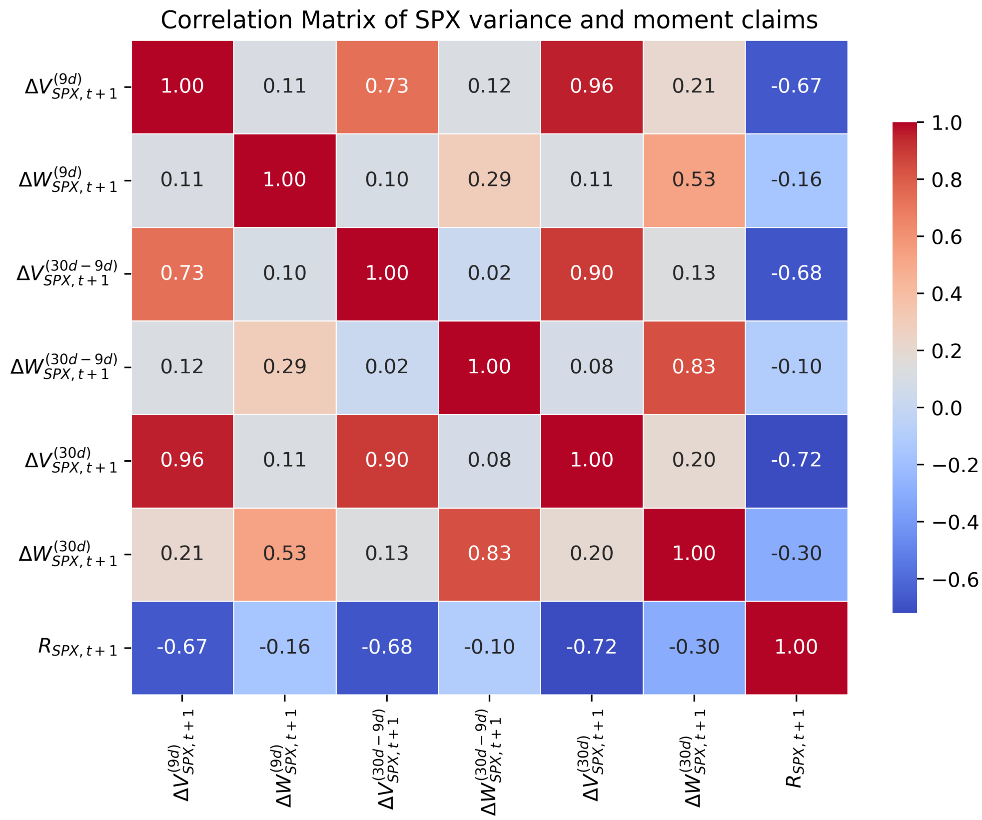

Figure 7. Correlation matrix for changes in the prices of variance and skewness claims written on the S&P 500 Index.

forwards; specifically, I use equations (11) and (12) to compute $V_{ind,t}^{(30d-9d)}$ and $V_{ind,t}^{(30d-9d)}$, the prices of assets with payoff equal to the annualized realized variance and skewness respectively that occurs over the next 30 days and excluding the first 9 days for index *ind*. With these prices in hand, I examine the moment-related exposures of our level and skew factors. Denote the vector of daily changes in the prices of the variance and skewness claims written on index $ind \in \{\text{SPX, NDX, RUT}\}$ by $\Delta\mathbf{M}_{ind,t+1}$:

$$\Delta\mathbf{M}_{ind,t+1} = \left[\Delta V_{ind,t+1}^{(9d)}, \Delta V_{ind,t+1}^{(30d-9d)}, \Delta W_{ind,t+1}^{(9d)}, \Delta W_{ind,t+1}^{(30d-9d)}\right]'$$

Figure 7 plots the correlation matrix for the components of $\Delta\mathbf{M}_{SPX,t+1}$, the market return, and 30-day horizon risk-neutral variance and skewness for SPX. The correlations for the SPX index return with innovations in the variance risk-neutral moments, $\Delta V_{SPX,t+1}^{(9d)}$, $\Delta V_{SPX,t+1}^{(30d)}$ and $\Delta V_{SPX,t+1}^{(30d-9d)}$, are all very strongly negative ranging from -0.72 to -0.67. This is consistent with the well-documented "leverage effect", the tendency for negative asset returns to increase that asset's volatility. The correlation between $\Delta V_{SPX,t+1}^{(9d)}$ and $\Delta V_{SPX,t+1}^{(30d)}$

is high at 0.96 indicating that innovations to the shorter horizons of the variance term structure are likely driven by common shocks. For the skewness innovations, the correlations to the index return are still negative but weak with the strongest correlation being with 30-day horizon skewness $\Delta W^{(30d)}_{SPX,t+1}$ at -0.3. The correlation between $\Delta W^{(9d)}_{SPX,t+1}$ and $\Delta W^{(30d)}_{SPX,t+1}$ is 0.53. This suggests a term structure of skewness which is driven by overlapping, although not identical sources, as appears to be the case for variance. Lastly, I find that innovations between variance and skewness are positively correlated although mild. The correlation between $\Delta V^{(9d)}_{SPX,t+1}$ and $\Delta W^{(9d)}_{SPX,t+1}$ is very weak at 0.11 and between $\Delta V^{(30d)}_{SPX,t+1}$ and $\Delta W^{(30d)}_{SPX,t+1}$ is only slightly stronger at 0.2.

To understand how innovations to forward-looking moments relate to $\text{Level}^{(m)}_{ind,t+1}$ and $\text{Skew}^{(m)}_{ind,t+1}$, I build a reduced form model relating these returns to innovations in their underlying index's corresponding risk-neutral moments $\Delta\mathbf{M}_{ind,t+1}$. Let $\mathbf{I} = \{SPX, NDX, RUT\}$ and denote one-day return of index ind by $R^{mkt}_{ind,t+1}$. I use $\overline{\Delta\mathbf{M}_{ind,t+1}}$ to denote $\Delta\mathbf{M}_{ind,t+1}$ subtracted by its unconditional mean. I estimate the following model of returns for $\text{Level}^{(m)}_{ind,t+1}$ and $\text{Skew}^{(m)}_{ind,t+1}$ using the available range of maturities m and indices ind:

$$\text{Level}^{(m)}_{ind,t+1} - R^f_{t+1} = \boldsymbol{\beta}^{(m)\prime}_{ind,level}\overline{\Delta\mathbf{M}_{ind,t+1}} + \beta^{(m)}_{ind,Level,mkt}R^{mkt}_{ind,t+1} + \eta^{(m)}_{ind,Level,t+1} \quad \forall ind \in \mathbf{I} \tag{37}$$

$$\text{Skew}^{(m)}_{ind,t+1} - R^f_{t+1} = \boldsymbol{\beta}^{(m)\prime}_{ind,Skew}\overline{\Delta\mathbf{M}_{ind,t+1}} + \beta^{(m)}_{ind,Skew,mkt}R^{mkt}_{ind,t+1} + \eta^{(m)}_{ind,Skew,t+1} \quad \forall ind \in \mathbf{I} \tag{38}$$

$$\begin{aligned}\mathbb{E}\left(j^{(m)}_{ind,t+1}\right) = {} & \beta^{(m)}_{ind,j,V^{(9d)}}\lambda_{V^{(9d)}} + \beta^{(m)}_{ind,j,V^{(30d-9d)}}\lambda_{V^{(30d-9d)}} + \beta^{(m)}_{ind,j,W^{(9d)}}\lambda_{W^{(30d-9d)}} + \\ & \beta^{(m)}_{ind,j,W^{(30d-9d)}}\lambda_{W^{(30d-9d)}} + \beta^{(m)}_{ind,j,mkt}\lambda_{mkt} + v^{(m)}_{ind,j} \quad \forall ind \in \mathbf{I}, \forall j \in \{\text{Level}, \text{Skew}\}\end{aligned} \tag{39}$$

where $\boldsymbol{\beta}^{(m)}_{ind,j}$ is the vector

$$\left[\beta^{(m)}_{ind,j,V^{(9d)}}, \beta^{(m)}_{ind,j,V^{(30d-9d)}}, \beta^{(m)}_{ind,j,W^{(9d)}}, \beta^{(m)}_{ind,j,W^{(30d-9d)}}\right]' \qquad j \in \{\text{Level}, \text{Skew}\}.$$

The structure of the model is similar to, although not quite, a standard linear factor model with static betas. The key difference is that the factors are actually index dependent. In equations (37) and (38), I regress the level and skew returns on index-specific risk factors; the regressors are the moment innovations $\Delta\mathbf{M}_{ind,t+1}$ and the daily index return $R^{mkt}_{ind,t+1}$ pertaining to the same underlying index ind as $\text{Level}^{(m)}_{ind,t+1}$ and $\text{Skew}^{(m)}_{ind,t+1}$. This alters the interpretation of the lambda terms in the final equation. In a standard factor model, each lambda in equation (39) is price of risk or risk premium (if the factor is a return) associated with exposure to a specific asset return or factor. In my model, the lambdas do retain some "price of risk" interpretation; however, it is a price of risk in a more local sense. The lambdas in this context encode compensation for bearing risks associated with exposure to the moments of a corresponding equity index. The signs of the coefficients of $\Delta\mathbf{M}_{ind,t+1}$ reveal the sign of the risk premium associated with variance and skewness risk. The coefficient λ_{index} has the familiar equity risk premium interpretation. As the underlying equity indices are all fairly well-diversified equity indices, the λ_{index} I recover here can be seen an estimate of risk premium associated with the broad equity market.

To close the model, I impose the economically motivated restrictions on the coefficients of $\Delta V^{(30d-9d)}_{ind,t+1}$

$$\beta^{(m)}_{ind,level,V^{(30d-9d)}} = 0, \qquad \beta^{(m)}_{ind,skew,V^{(30d-9d)}} = 0 \qquad\qquad \forall m \leq 9d \tag{40}$$

and on the coefficients of $\Delta W^{(30d-9d)}_{ind,t+1}$

$$\beta^{(m)}_{ind,level,W^{(30d-9d)}} = 0, \qquad \beta^{(m)}_{ind,skew,W^{(30d-9d)}} = 0 \qquad\qquad \forall m \leq 9d. \tag{41}$$

The economic reasoning for these restrictions is straightforward; for assets constructed from options that expire in 9 days or fewer, the risk-neutral dynamics of the underlying after the options expire should not effect their prices. The full reduced form model is specified by equations (37) - (41).

I estimate this model in two stages; I first estimate equations (37) and (38) using ordinary least squares to obtain the betas in both sets of equations. Implementing the economic restrictions imposed by (40) and (41) can be done simply by omitting the restricted variables from the regression. Then I estimate the lambdas by regressing the expected return on the betas estimated from the first stage. Equation (39) describes how exposure to each risk factor in $\Delta\mathbf{M}_{ind}$ and the underlying index contributes to the expected return. I show the summary statistics for the betas obtained by estimating equation (37) and (38) in table 9. Panel A presents summary statistics for the betas estimated for regressions on one-day straddle returns $\text{Level}^{(m)}_{ind,t+1}$. Overall, the level time series are heavily exposed to changes in forward-looking variance $\Delta V^{(9d)}_{ind,t+1}$ and $\Delta V^{(30d-9d)}_{ind,t+1}$ with larger changes being associated with high straddle returns. The average betas on $\Delta V^{(9d)}_{ind,t+1}$ and $\Delta V^{(30d-9d)}_{ind,t+1}$ for the straddles are 1.19 and 1.05 respectively. Panel B shows the summary statistics for the betas estimated from the returns on the skewness asset. Unlike the straddle returns, the variance exposure is close to zero on average for the skewness asset one-day returns $\text{Skew}^{(m)}_{ind,t+1}$. This is due to the fact the skewness asset is vega-neutral at construction. For the skewness assets, the betas on $\Delta W^{(9d)}_{ind,t+1}$ and $\Delta W^{(30d-9d)}_{ind,t+1}$ nearly all positive.

I show the estimated model of expected returns in equation (42) multiplied by 100 on

	$\Delta V^{(9d)}_{ind,t+1}$	$\Delta W^{(9d)}_{ind,t+1}$	$\Delta V^{(30d-9d)}_{ind,t+1}$	$\Delta W^{(30d-9d)}_{ind,t+1}$	$R^{mkt}_{ind,t+1}$
Panel A: Estimated betas from regressions on $\text{Level}_{ind,t+1}$					
Mean	1.193	0.022	1.052	0.005	0.570
Standard Deviation	1.533	0.030	0.802	0.008	1.169
25th Percentile	0.340	0.005	0.000	0.000	0.163
50th Percentile	0.873	0.021	0.961	0.003	0.478
75th Percentile	1.346	0.028	1.782	0.006	0.827
Panel B: Estimated betas from regressions on $\text{Skew}_{ind,t+1}$					
Mean	-0.039	0.003	-0.140	0.001	1.848
Standard Deviation	0.079	0.002	0.113	0.002	0.067
25th Percentile	-0.100	0.002	-0.245	-0.001	1.806
50th Percentile	-0.068	0.003	-0.132	0.000	1.830
75th Percentile	0.031	0.005	0.000	0.000	1.884

Table 9. Summary statistics for the estimated betas from the regressions for $\text{Level}^{(m)}_{ind,t+1}$ and $\text{Skew}^{(m)}_{ind,t+1}$ in equations (37) and (38) respectively.

both sides to express the expected return in percentages. $\widehat{\mathbb{E}}\left(j^{(m)}_{ind,t+1}\right)$ denotes the fitted expected return of the estimated model for $j \in \{\text{Level}, \text{Skew}\}$. The t-statistics for the test if the coefficient is zero are given in square brackets for each coefficient. All test statistics and inference apply the wild bootstrap procedure proposed by Liu (1988) and Mammen (1993) and take into account that the betas from the first stage regressions are estimated. The R-squared of the estimated expected return equation (42) is 0.47.

$$\begin{aligned}\widehat{\mathbb{E}}\left(j^{(m)}_{ind,t+1} \times 100\right) = \underbrace{-0.29}_{[-6.59]}\widehat{\beta}^{(m)}_{ind,j,V^{(9d)}} + \underbrace{6.39}_{[2.80]}\widehat{\beta}^{(m)}_{ind,j,W^{(9d)}} - \underbrace{0.07}_{[-4.15]}\widehat{\beta}^{(m)}_{ind,j,V^{(30d-9d)}} \\ + \underbrace{3.15}_{[1.27]}\widehat{\beta}^{(m)}_{ind,j,W^{(30d-9d)}} + \underbrace{0.07}_{[6.07]}\widehat{\beta}^{(m)}_{ind,j,mkt}\end{aligned} \tag{42}$$

Examining the risk prices above, we see that variance risk over the following 9 days and 9 to 30 days ahead are strongly priced. Both the coefficients on $\widehat{\beta}^{(m)}_{ind,j,V9}$ and $\widehat{\beta}^{(m)}_{ind,j,V30-V9}$ are statistically significant at the 0.1% level. The price of risk for variance exposures have the correct negative sign reflecting the presence of a negative variance risk premium. The estimated prices of risk associated with skewness exposure also have the correct positive sign although only the coefficient on $\widehat{\beta}^{(m)}_{ind,j,W9}$ is statistically significant. Overall, the signs of my estimated prices of risk are in line with a representative investor with any utility function displaying decreasing marginal utility of wealth and non-increasing absolute risk aversion as discussed in Kraus and Litzenberger (1976). An investor with such a utility function should display a preference for positive skewness and an aversion to variance at the index level which is consistent with the estimated prices of risk in (42). Intuitively, a preference for positive skewness has a behavioral basis as well. Experimental studies conducted by Ebert and Wiesen (2011) and Ebert (2015) show a preference for positively skewed lotteries over negatively skewed ones even when controlling for other moments of the lotteries' payoff distributions. Lastly, the coefficient on $\widehat{\beta}^{(m)}_{i,j,mkt}$ of 0.07 implies a monthly average market index return of 1.47%. Although ostensibly high, the sample is mostly populated by data from the 2014-2023 period when equity indices (particularly the Nasdaq-100) experienced markedly higher returns relative to history.

There is a plethora of ways the literature measures risk premia associated with higher-order moments. A popular approach in the literature synthesizes the variance and skewness payoffs from options. Jiang and Tian (2005), Carr and Lee (2009), Bollerslev, Gibson, and Zhou (2011), Neuberger (2012), Kozhan et al. (2013), Choi, Mueller, and Vedolin (2017), Heston and Todorov (2023) among many others take this approach across a wide variety of asset classes. A related approach looks at the returns of traded derivatives purely exposed to higher-order moments; examples from the literature are Dew-Becker et al. (2017) and Konstantinidi and Skiadopoulos (2016) who consider the hold-to-maturity returns of S&P 500 variance swaps. My model offers a novel and complementary angle in examining these higher-order related risk premia. I use coefficients of equation (42) and the estimated betas to obtain the variance and skewness risk premium embedded in a typical straddle or skewness asset. By a typical straddle or skewness asset, I refer to a straddle (skewness asset) with the betas equal to the average betas reported in panel A (panel B) of table 9. The average straddle has a beta with respect to $\Delta V^{(9d)}_{ind,t+1}$ equal to 1.193. Since $\lambda_{V^{(9d)}} = -0.0029$, exposure to $\Delta V^{(9d)}_{ind,t+1}$ for a typical straddle loses on average around 0.35% per day or 7.3% per month. By the same computation, I calculate how much a typical straddle loses to innovations in $V^{(30d-9d)}$. A typical straddle with beta equal to 1.052 with respect to $\Delta V^{(30d-9d)}_{ind,t+1}$ loses 1.6% per month. Generally, the typical straddle has betas close to 1 with respect to both $\Delta V^{(9d)}_{ind,t+1}$ and $\Delta V^{(30d-9d)}_{ind,t+1}$ and on average sees a negative return due to these exposures to innovations in variance. These exposures to variance constitute the bulk of the average negative returns of straddles and, although not directly comparable, the returns are roughly similar to the monthly holding returns of the variance swaps studied by Konstantinidi and Skiadopoulos (2016). Turning to a typical skewness asset, I find the exposure to innovations in skewness, $\Delta W^{(9d)}_{ind,t+1}$ and $\Delta W^{(30d-9d)}_{ind,t+1}$, contribute 1.9 and 0.2 basis points to the daily expected return. On a monthly (annual) basis, this is 0.40% (4.83%) and 0.07% (0.79%) respectively. In particular, innovations in 9-day skewness is the economically and statistically significant component of the skewness asset return. Innovations to skewness between 9 and 30 days

	$\Delta V^{(9d)}_{SPX,t+1}$	$\Delta W^{(9d)}_{SPX,t+1}$	$\Delta V^{(30d-9d)}_{SPX,t+1}$	$\Delta W^{(30d-9d)}_{SPX,t+1}$	$R_{SPX,t+1}$	Return	Fitted Return
$\text{Level}^{\text{US}}_{t+1}$	-36.6	25.8	0	0	-8.8	-24.8	-19.6
$\text{Skew}^{\text{US}}_{t+1}$	4.3	1.1	0	0	13.2	12.8	18.6
$\text{Level}^{\text{S}}_{t+1}$	-12	12.2	-15.1	0.2	-0.8	-35.8	-15.5
$\text{Skew}^{\text{S}}_{t+1}$	2.9	0.8	1.4	-0.2	12.8	11.3	17.8

Table 10. Contributions (in basis points) to the expected return of the option-based factors in the 5MOM model for SPX computed from the estimated reduced form model. Model implied expected return (fitted return) and average return (return) in-sample are also in basis points.

ahead contribute little and cannot be statistically distinguished from zero.

Variance and skewness assets owe a substantial portion of their expected returns to innovations in variance and skewness respectively. I quantify the variance exposure in a typical straddle and skewness asset using the betas and lambdas recovered from estimating the reduced form model in equations (37) to (41). Table 10 shows the contribution of each of the components in $\Delta \mathbf{M}_{SPX,t+1}$ to the option-based factors of the SPX 5MOM model in basis points. I find the variance risk premium embedded in straddles to be large; $\Delta V^{(9d)}_{i,t+1}$ contributes -7.8% on a monthly basis to $\text{Level}^{\text{US}}_{t+1}$. The returns to the 30-day straddle $\text{Level}^{\text{S}}_{t+1}$ also owes its highly negative return to variance exposure stemming from both $\Delta V^{(9d)}_{i,t+1}$ and $\Delta V^{(30d-9d)}_{i,t+1}$. Interestingly, the model estimates high skewness exposure for both $\text{Level}^{\text{US}}_{t+1}$ and $\text{Level}^{\text{S}}_{t+1}$ perhaps suggesting that straddle returns embed exposure to higher-order moments beyond variance. Turning to skewness asset returns, I find that $\text{Skew}^{\text{US}}_{t+1}$ and $\text{Skew}^{\text{S}}_{t+1}$, while both having positive exposure to innovations in skewness are even more greatly affected by the return of the underlying index. The skewness exposure on $\text{Skew}^{\text{US}}_{t+1}$ contributes around 1.1 basis points each day or 2.8% per year. This estimate is close to the risk premium associated with skewness estimated by Harvey and Siddique (2000), Langlois (2020), among others. Overall, the risk premia associated with innovations to higher-order moments implied by my reduced form model are quantitatively similar to those estimated by the literature for variance and skewness.

6.2 Decomposing option returns

The estimated model in the prior subsection allows us to decompose the factors of the 5MOM models into exposures to the underlying index, its variance, and its skewness. I use $\mathbf{F}^{ind}_{t+1}$ to denote the 4 option-based factors in the 5MOM model for index *ind*:

$$\mathbf{F}^{ind}_{t+1} = \left[\text{Level}^{(9d)}_{ind,t+1}, \text{Level}^{(30d)}_{ind,t+1}, \text{Skew}^{(9d)}_{ind,t+1}, \text{Skew}^{(30d)}_{ind,t+1}\right]' \qquad ind \in \mathbf{I}$$

For any of these option-based factors, I can estimate the contribution to their expected return stemming from the moment innovations in $\Delta\mathbf{M}_{ind,t+1}$ and the return to index ind using equation (42). For instance, the contribution of innovations in $V^{(9d)}_{SPX,t+1}$ to $\text{Level}^{(30d)}_{SPX,t+1}$ is the quantity:

$$\widehat{\beta}^{(30d)}_{SPX,level,V^{(9d)}}\widehat{\lambda}_{V^{(9d)}}$$

This quantity is the component of the unconditional expected return of $\text{Level}^{(9d)}_{SPX,t+1}$ attributed to changes in the 9-day risk-neutral expected variance $V^{(9d)}_{SPX,t+1}$. Define $\mathbf{G}^{ind}_{\ell}$ by

$$\mathbf{G}^{ind}_{\ell} = \left[\widehat{\beta}^{(9d)}_{ind,Level,\ell}\widehat{\lambda}_{\ell}, \widehat{\beta}^{(30d)}_{ind,Level,\ell}\widehat{\lambda}_{\ell}, \widehat{\beta}^{(9d)}_{ind,Skew,\ell}\widehat{\lambda}_{\ell}, \widehat{\beta}^{(30d)}_{ind,Skew,\ell}\widehat{\lambda}_{\ell}\right]' \qquad \ell \in \mathbf{L}$$

where $\mathbf{L} = \{V^{(9d)}, W^{(9d)}, V^{(30d-9d)}, W^{(30d-9d)}, mkt\}$. $\mathbf{G}^{ind}_{\ell}$ consists of the contributions to the unconditional expected returns of the factors in $\mathbf{F}^{ind}_{t+1}$ stemming from exposure to $\ell \in \mathbf{L}$. Using these quantities, I can decompose the daily unconditional expected return of the 5MOM factors for any index ind into innovations to risk-neutral moments and the daily index return. It is a decomposition in the sense we can take the sum across $\ell \in \mathbf{L}$ to recover the expected returns estimated in equation (39):

$$\hat{\mathbb{E}}\left(\mathbf{F}^{ind}_{t+1}\right) = \sum_{\ell\in\mathbf{L}} \mathbf{G}^{ind}_{\ell} \tag{43}$$

I take this decomposition of the F^{ind}_{t+1} factors and use this to decompose the returns of the option panel. I start with the estimated 5MOM model for index $ind \in \mathbf{I}$ and take expectations to obtain $\hat{y}_{i,t+1}$, the unconditional expected return of option i at time $t+1$.

$$\hat{y}_{i,t+1} = \hat{\boldsymbol{\beta}}'_{i,t}\mathbb{E}\left(\mathbf{F}^{ind}_{t+1}\right) + \hat{\beta}^{mkt}_{i,t}\mathbb{E}\left(R^{mkt}_{ind,t+1}\right) \tag{44}$$

The subscript i is a stand-in for the 4-tuple of characteristics (moneyness category, maturity group, option type, and underlying index) described in section 3 that uniquely identifies each option return in the panel at time $t+1$. I substitute the sample averages for the unconditional expectations to evaluate equation (44). Rearranging terms yields the following expression for $\hat{y}_{i,t+1}$:

$$\hat{y}_{i,t+1} = \hat{\boldsymbol{\beta}}'_{i,t}\mathbf{G}^{ind}_{V^{(9d)}} + \hat{\boldsymbol{\beta}}'_{i,t}\mathbf{G}^{ind}_{W^{(9d)}} + \hat{\boldsymbol{\beta}}'_{i,t}\mathbf{G}^{ind}_{V^{(30d)}} + \hat{\boldsymbol{\beta}}'_{i,t}\mathbf{G}^{ind}_{W^{(30d)}} + \hat{\boldsymbol{\beta}}'_{i,t}\mathbf{G}^{ind}_{mkt} + \hat{\beta}^{mkt}_{i,t}\mathbb{E}(R^{mkt}_{ind,t+1}) \tag{45}$$

To obtain the contribution of a moment $\ell \in \mathbf{L}$, I retain only the terms above relating to ℓ. For instance, the contribution of $\Delta V^{(9d)}_{ind,t+1}$ to the expected return is given by $\hat{\boldsymbol{\beta}}'_{i,t}\mathbf{G}^{ind}_{V^{(9d)}}$. Since the behavior and risk loadings of option contracts are highly heterogeneous across moneyness, maturity, and option type, it is useful to consider the contribution of different risk sources ℓ across options of different characteristics. I calculate the contributions of $\ell \in \mathbf{L}$ to expected returns for each of the 60 buckets in my option panel. Specifically, for $\ell \in \mathbf{L}$ and $ind \in \mathbf{I}$, define $Q_\ell(a_1, a_2, a_3, ind)$,

$$Q_\ell(a_1, a_2, a_3, ind) = \frac{\hat{\mathbb{E}}\left[\hat{\boldsymbol{\beta}}'_{i,t}\mathbf{G}_\ell^{ind} + \mathbf{1}_{\ell=mkt}\hat{\beta}_{i,t}^{mkt}\lambda_{mkt}|(i,t) \in \mathcal{B}(a_1, a_2, a_3, ind)\right]}{\left|\hat{\mathbb{E}}\left[\hat{y}_{i,t+1}|(i,t) \in \mathcal{B}(a_1, a_2, a_3, ind)\right]\right|} \tag{46}$$

where $\hat{\mathbb{E}}\left[\cdot|(i,t) \in \mathcal{B}(a_1, a_2, a_3, ind)\right]$ is the sample average taken over option returns in bucket $\mathcal{B}(a_1, a_2, a_3, ind)$. $\mathbf{1}_{\ell=mkt}$ is an indictor function equal to 1 if $\ell = mkt$ and 0 otherwise. Equation (46) computes $Q(a_1, a_2, a_3, ind)$: the component of the return attributable to ℓ. Computing this as the expected contribution normalized by the absolute value of the expected return gives a sense of the relative importance of each component.

Table 11 reports $100 \times Q_\ell(a_1, a_2, a_3, ind)$ in the columns headed by $\ell \in \mathbf{L}$ for the 20 SPX buckets. As an example, $Q_{V^{(30d-9d)}}(US, OTM, Call, SPX)$ in table 11 is -11.9%. This indicates exposure to the forward-looking variance of SPX between the next 30 days and excluding the next 9 days has a sizable negative effect on the expected return, around 12% of the magnitude of the bucket's expected return. The figures reported in the square brackets are bootstrapped t-statistics for the test if the estimated quantity is statistically different from zero. The bootstrapping procedure takes into account the conditional betas of the IPCA models and coefficients in equations (37), (38), and (39) are estimated. I use 5,000 bootstrap iterations to compute all standard errors and t-statistics. The last three columns of table 11 presents the fitted expected return ($\widehat{\text{Return}}$), average residuals of the 5MOM model (Residual), and the in-sample average return (Return) in basis points computed over each bucket. The rounded parentheses in the column headed by $\widehat{\text{Return}}$ show the standard error of the bootstrapped samples. Appendix B contains tables 12 and 13 which compute the same quantities as table 11 but for equity index options written on the Nasdaq 100 (NDX) and Russell 2000 (RUT) respectively. For buckets of all indices (tables 11 - 13), I find significant heterogeneity in how innovations to different moments affect expected returns across maturity, moneyness and option type.

Taken together, the results for the SPX, NDX and RUT indices suggest a few key findings on the link between option returns and innovations in forward-looking moments. First, the contributions from changes to risk-neutral variances, $\Delta V_{ind,t+1}^{(9d)}$ and $\Delta V_{ind,t+1}^{(30d-9d)}$, to the expected return are large and statistically significant for many buckets with substantial variation across buckets, even for those of the same index and type. Option pricing theory suggests that contributions from variance should be negative across all buckets as all options

Maturity	Type	Risk source (ℓ): Moneyness	$\Delta V^{(9d)}_{SPX,t+1}$	$\Delta W^{(9d)}_{SPX,t+1}$	$\Delta V^{(30d-9d)}_{SPX,t+1}$	$\Delta W^{(30d-9d)}_{SPX,t+1}$	$R^{mkt}_{SPX,t+1}$	$\widehat{\text{Return}}$	Residual	Return
S	C	DOTM	7.11	19.76	-14.73	0.19	87.68^{***}	5.56	-2.24	3.32
			[0.54]	[1.59]	[-1.64]	[0.06]	[4.43]	(0.09)		
		OTM	-2.85	14.99^{*}	-11.94^{**}	0.16	99.64^{***}	6.52	1.72	8.24
			[-0.5]	[2.5]	[-2.75]	[0.1]	[16.24]	(0.05)		
		ATM	-5.04	8.74^{**}	-6.64^{**}	0.01	102.94^{***}	7.74	0.09	7.83
			[-1.55]	[2.68]	[-2.79]	[0.01]	[39.84]	(0.04)		
		ITM	-6.7^{*}	6.07^{**}	-3.9^{*}	-0.14	104.67^{***}	5.48	-0.14	5.34
			[-2.19]	[2.7]	[-2.27]	[-0.17]	[66.23]	(0.03)		
		DITM	-5.82^{*}	-0.26	0.38	-0.17	105.87^{***}	3.86	-0.04	3.82
			[-2.56]	[-0.82]	[0.56]	[-0.59]	[52.22]	(0.03)		
	P	DOTM	-27.99^{*}	-1.76	-10.66	0.74	-60.33^{*}	-16.54	7.52	-9.01
			[-2.24]	[-0.98]	[-1.58]	[0.35]	[-2.03]	(0.1)		
		OTM	-21.66^{**}	2.93	-11.84^{*}	0.67	-70.1^{***}	-10.47	4.04	-6.43
			[-2.9]	[1.52]	[-2.22]	[0.39]	[-4.81]	(0.06)		
		ATM	-9.54^{**}	5.11^{*}	-9.29^{**}	0.46	-86.74^{***}	-8.59	0.53	-8.06
			[-2.67]	[2.43]	[-3.03]	[0.46]	[-22.74]	(0.07)		
		ITM	2.46	6.52^{*}	-7.06^{**}	0.27	-102.18^{***}	-5.54	0.04	-5.5
			[0.72]	[2.55]	[-3.02]	[0.34]	[-28.84]	(0.06)		
		DITM	7.7^{**}	1.56	-3.51^{**}	0.15	-105.9^{***}	-4.36	-1.4	-5.76
			[2.69]	[1.91]	[-3.01]	[0.39]	[-29.5]	(0.04)		
US	C	DOTM	13.75	17.13	-7.22	0.26	76.08^{**}	8.92	-6.88	2.04
			[1.05]	[1.52]	[-1.59]	[0.17]	[3.26]	(0.14)		
		OTM	0.25	13.62^{*}	-5.71^{**}	0.08	91.76^{***}	8.82	0.75	9.57
			[0.05]	[2.44]	[-2.57]	[0.09]	[12.1]	(0.1)		
		ATM	-1.8	7.3^{*}	1.35	-0.31	93.47^{***}	10.47	-1.67	8.79
			[-0.56]	[2.27]	[1.05]	[-0.54]	[16.93]	(0.09)		
		ITM	-5.16^{*}	5.11^{*}	0.88	-0.17	99.33^{***}	8.47	-1.18	7.29
			[-2.31]	[2.52]	[1.4]	[-0.62]	[36.38]	(0.06)		
		DITM	-8.18^{*}	2.49	5.1^{**}	-0.08	100.67^{***}	3.58	-0.8	2.78
			[-2.04]	[1.53]	[2.64]	[-0.11]	[33.74]	(0.04)		
	P	DOTM	-35.06^{*}	-2.71	-3.43	0.16	-58.96^{*}	-19.77	5.5	-14.27
			[-2.45]	[-1.13]	[-1.67]	[0.24]	[-2.05]	(0.35)		
		OTM	-26.56^{***}	3.15	-2.68^{*}	0.01	-73.92^{***}	-11.77	0.83	-10.94
			[-3.99]	[1.91]	[-2.3]	[0.03]	[-7.17]	(0.16)		
		ATM	-13.35^{**}	8.06^{**}	5.66^{**}	-0.67	-99.69^{***}	-7.32	-2.43	-9.75
			[-2.69]	[3.22]	[2.95]	[-0.95]	[-26.76]	(0.09)		
		ITM	-1.9	7.29^{**}	4.14^{**}	-0.36	-109.17^{***}	-6.96	-0.4	-7.35
			[-0.59]	[2.58]	[2.7]	[-0.75]	[-17.42]	(0.11)		
		DITM	-4.62	2.21^{*}	6.26^{***}	-0.35	-103.5^{***}	-6.13	-2.48	-8.61
			[-1.62]	[2.1]	[3.58]	[-0.63]	[-43.8]	(0.05)		

Table 11. Return decomposition for delevered SPX option returns across buckets characterized by $\Theta = (a_1, a_2, a_3, SPX)$ expressed as a percentage. Left-most three columns identify the bucket and columns headed by $\ell \in \{\Delta V^{(9d)}_{SPX,t+1}, \Delta W^{(9d)}_{SPX,t+1}, \Delta V^{(30d-9d)}_{SPX,t+1}, \Delta W^{(30d-9d)}_{SPX,t+1}, R^{mkt}_{SPX,t+1}\}$ report $100 \times Q_\ell(\Theta)$. T-statistics for the null hypothesis that $Q_\ell(\Theta)$ is zero are presented in square brackets. Right-most three columns present estimates of the expected return ($\widehat{\text{Return}}$), MAPE (Residual), and average return in the bucket (Return) in basis points. Rounded parentheses show the standard errors for the column headed by ($\widehat{\text{Return}}$). All standard errors and t-statistics are calculated using the wild bootstrap approach proposed by Liu (1988) and Mammen (1993) using 5,000 iterations. Statistical significance at the 5%, 1%, and 0.1% denoted by 1, 2 and 3 stars respectively.

are mechanically long variance which has negative risk premium (Bollerslev, Tauchen, & Zhou 2009; Carr & Wu 2009). This is generally consistent with the columns for $\Delta V^{(9d)}_{ind,t+1}$ and $\Delta V^{(30d-9d)}_{ind,t+1}$ in tables 11, 12, and 13; most of the estimated relative contributions have the correct negative sign and are statistically significant. The observed heterogeneity in relative contributions is stark and appears primarily linked to the moneyness of the bucket and the option type. Options which are DOTM and OTM generally incur a more negative contribution to expected returns. For SPX and NDX call and put options in particular, the contributions from $\Delta V^{(9d)}_{ind,t+1}$ and $\Delta V^{(30d-9d)}_{ind,t+1}$ generally become less negative as one moves along moneyness categories from the DOTM bucket to the DITM bucket. With the exclusion of the RUT buckets, the difference in contribution between a DOTM bucket and its corresponding DITM bucket are statistically significant at the 5% level.[22] The combined contribution from innovations in variance is the sum of the contributions in the $\Delta V^{(9d)}_{ind,t+1}$ and $\Delta V^{(30d-9d)}_{ind,t+1}$ columns. Doing so, I find that DOTM put options are highly exposed to variance; exposure to variance constitutes anywhere from 18% to upwards of 43% of the negative average returns on these put buckets. Just the contributions from $\Delta V^{(9d)}_{ind,t+1}$, a portion of the variance term structure highly exposed to jumps, ranges from around 9% of expected negative return to as much as 40%. Interestingly, this stands in contrast to DOTM call buckets which, with the exception the RUT buckets, which see significantly less negative contributions from $\Delta V^{(9d)}_{ind,t+1}$ than their corresponding DOTM put buckets.

Second, like variance, exposure to skewness also possesses similarly sizable variation by moneyness across option returns; however, unlike variance, the sign of this exposure appears to differ by option type. Contribution from skewness exposure, specifically exposure to $\Delta W^{(9d)}_{ind,t+1}$, across indices is increasing in moneyness for call options while decreasing in moneyness for put options. For DOTM and OTM buckets, the change in skewness over the 9-day horizon contributes positively to expected call returns and negatively to expected put returns, with the magnitudes of the contributions from $\Delta W^{(9d)}_{ind,t+1}$ for calls in particular, being quantitatively large and significant. For instance, exposure to $\Delta W^{(9d)}_{ind,t+1}$ contributes around 20% of the expected return to DOTM S maturity call options written on SPX while the corresponding DOTM put bucket sees a contribution of -2%. Contributions from $\Delta W^{(30d-9d)}_{ind,t+1}$ are generally statistically insignificant with the exception of a handful of RUT buckets in table 13. Like before, I obtain the total skewness exposure by taking the sum of the $\Delta W^{(9d)}_{ind,t+1}$ and $\Delta W^{(30d-9d)}_{ind,t+1}$. Doing this, I find the role of skewness is highly salient, particularly for NDX options. For DOTM and OTM call options on NDX, the contribution of skewness is second in magnitude only to the contribution of the underlying index.

[22] I bootstrap the p-values for a standard difference in means Welch t-test where I compare the average contribution from the DOTM bucket to that of the corresponding DITM bucket.

Lastly, the relative importance of each risk factor is highly dependent on moneyness and option type across indices; however, the contribution of the 9-day moments, $\Delta V^{(9d)}_{ind,t+1}$ and $\Delta W^{(9d)}_{ind,t+1}$, is fairly similar for buckets that only differ in maturity. Comparing the contributions of the 9-day moments for buckets which only differ in option type, I find substantial asymmetry in the magnitudes of $\Delta V^{(9d)}_{ind,t+1}$ and $\Delta W^{(9d)}_{ind,t+1}$ columns. Typically, exposure to $\Delta V^{(9d)}_{ind,t+1}$ is more negative for a put bucket than an otherwise similar call bucket. This asymmetry could be explained by the fact that $\Delta V^{(9d)}_{ind,t+1}$ is highly sensitive to jumps in the index which disproportionately affect DOTM put options as most jumps in the indices are negative ones (Bollerslev, Todorov, & Xu 2015) and would be consistent with the evidence presented in section 5. Exposure to $\Delta W^{(9d)}_{ind,t+1}$ is generally larger in magnitude for call buckets as compared to otherwise similar put buckets. This exposure is largest for DOTM calls for which the contribution of $\Delta W^{(9d)}_{ind,t+1}$ exceeds the magnitude of the contribution from either $\Delta V^{(9d)}_{ind,t+1}$ or $\Delta V^{(30d-9d)}_{ind,t+1}$, running counter to the trend of a variance risk exposure being the second largest contributor to returns after the underlying index.

7 Conclusion

In this article, I study the determinants of equity index option returns written on the S&P 500, Nasdaq 100, and Russell 2000 maturing in 30 days or less and relate this to fundamental information from the implied risk-neutral distribution: its forward-looking moments. Options with 30 days or less are a heavily traded section of the maturity profile which, in recent years, has come to constitute the bulk of all traded contracts in the options markets of all three equity indices. At time of writing, a large majority of trading volume is concentrated in options expiring in just 10 days or fewer. Despite this overwhelming popularity, the literature, up until recently, has focused on the more conventional band of maturities which lie between 1 month out to around 2 years. I contribute to the nascent yet burgeoning literature that aims to understand this rapidly expanding market by conducting a through analysis of the daily deleveraged returns to these contracts.

I first estimate a series of latent IPCA factor models with time-varying betas and find that a factor model consisting of a low-dimensional set of four factors (3 latent factors and the underlying index return) can accurately describe the cross-section of the excess deleveraged returns to these short maturity option returns. Although I fail to reject the null hypothesis that the alphas of these latent factor models are zero, I find that the alphas are small in an economic sense as, on average, the arbitrage opportunities suggested by the alphas disappears once transaction costs are taken into account. Using complementary approaches, I interpret the latent factors that emerge from the four factor IPCA model. My findings suggest that these latent factors award compensation for bearing risks associated with exposure to higher-order moments of the underlying.

Motivated by the interpretation of these latent factors, I propose and estimate my own factor models for these returns, one for each equity index, which are both easily interpretable and tradable. The five factors in these models are the daily return of the underlying index and two pairs of factors exposed to the risk-neutral forward-looking variance and skewness of that index at two distinct horizons. The first pair consists of a variance and skewness factor: the daily returns to the Coval and Shumway (2001) zero-delta straddle and the Bali and Murray (2013) skewness asset respectively constructed using options with 30 days to maturity. The second pair consists of the returns to the same variance and skewness assets which only differ in the sense they are constructed using options with 9 days to maturity. The first pair of factors are exposed to fluctuations in variance and skewness at the typical 1-month horizon examined by the literature, while the second pair provides factors with exposure to innovations in variance and skewness that look over the shorter 9-day horizon. I show that incorporating exposures to these shorter horizon moments helps more accurately model the returns to options with fewer than 10 days to maturity. I find a 5 factor model of consisting of the daily return to these variance and skewness factors with the daily return of underlying index explains the variation in the cross-section of option returns well and delivers lower pricing errors relative to existing factor models proposed by the literature and implied by industry practice.

Lastly, I propose and estimate a reduced form model which enables a decomposition of expected option returns. This decomposition explicitly quantifies the portion of the expected return from to fluctuations to the underlying index *ind* and two pairs of its forward-looking risk-neutral moments. These moments consist of variance and skewness over the next 9 days, $V^{(9d)}_{ind,t+1}$ and $W^{(9d)}_{ind,t+1}$, and variance and skewness over the next 30 days excluding the first 9 days, $V^{(30d-9d)}_{ind,t+1}$ and $W^{(30d-9d)}_{ind,t+1}$ for each equity index *ind*. The resulting decomposition reveals interesting differences across options with different moneyness, maturities, and type. Exposure to both $V^{(9d)}_{ind,t+1}$ and $V^{(30d-9d)}_{ind,t+1}$ generally contributes negatively to expected option returns as options are systematically long index variance, a factor carrying a negative price of risk. The contributions to expected returns from these innovations to variance is highest for DOTM options and is decreasing as one examines options that are closer to or deeper in-the-money. After the return of the underlying, variance exposure is usually the second largest driver of expected returns. For skewness exposure, I find noticeable asymmetry between option types. The contribution from skewness exposure, specifically exposure to $W^{(9d)}_{ind,t+1}$, is statistically significant and large for DOTM and OTM call options, but are generally negative and statistically insignificant for DOTM and OTM put options.

Overall, these findings present several avenues for further research as well as potential applications for market participants. A natural extension is to consider a decomposition of expected returns for options written on other asset classes such as bonds or commodities. The price of risk with regards to variance or skewness in other markets might be substantially different, in which case the composition of expected returns in terms of higher-order moments will differ. Market participants can apply this work in a risk management context. By explicitly quantifying the variance and skewness risk exposure present in option returns, one can more precisely manage the risks in an options portfolio which can be greatly exposed to changes in these higher-order moments.

References

Almeida, C., Freire, G., & Hizmeri, R. (2024). 0dte asset pricing. *Available at SSRN*.

Andersen, T. G., Fusari, N., & Todorov, V. (2017). Short-term market risks implied by weekly options. *The Journal of Finance*, *72*(3), 1335–1386.

Ang, A., Hodrick, R. J., Xing, Y., & Zhang, X. (2006). The cross-section of volatility and expected returns. *The journal of finance*, *61*(1), 259–299.

Arditti, F. D. (1967). Risk and the required return on equity. *The Journal of Finance*, *22*(1), 19–36.

Babii, A., Ghysels, E., & Pan, J. (2024). Tensor pca for factor models.

Bakshi, G., Madan, D., & Panayotov, G. (2010). Returns of claims on the upside and the viability of u-shaped pricing kernels. *Journal of Financial Economics*, *97*(1), 130–154.

Bali, T. G., Beckmeyer, H., Moerke, M., & Weigert, F. (2023). Option return predictability with machine learning and big data. *The Review of Financial Studies*, *36*(9), 3548–3602.

Bali, T. G., Chabi-Yo, F., & Murray, S. (2022). A factor model for stock returns based on option prices. *Available at SSRN 3487947*.

Bali, T. G., & Murray, S. (2013). Does risk-neutral skewness predict the cross-section of equity option portfolio returns? *Journal of Financial and Quantitative Analysis*, *48*(4), 1145–1171.

Bandi, F. M., & Renò, R. (2024). Local edgeworth expansions. *Available at SSRN*.

Bates, D. S. (2005). Hedging the smirk. *Finance Research Letters*, *2*(4), 195–200.

Bates, D. S. (2008). The market for crash risk. *Journal of Economic Dynamics and Control*, *32*(7), 2291–2321.

Bertsimas, D., Kogan, L., & Lo, A. W. (2000). When is time continuous? *Journal of Financial Economics*, *55*(2), 173–204.

Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of political economy*, *81*(3), 637–654.

Bollerslev, T., Gibson, M., & Zhou, H. (2011). Dynamic estimation of volatility risk premia and investor risk aversion from option-implied and realized volatilities. *Journal of econometrics*, *160*(1), 235–245.

Bollerslev, T., Tauchen, G., & Zhou, H. (2009). Expected stock returns and variance risk premia. *The Review of Financial Studies*, *22*(11), 4463–4492.

Bollerslev, T., & Todorov, V. (2011). Tails, fears, and risk premia. *The Journal of Finance*, *66*(6), 2165–2211.

Bollerslev, T., Todorov, V., & Xu, L. (2015). Tail risk premia and return predictability. *Journal of Financial Economics*, *118*(1), 113–134.

Bondarenko, O. (2014). Variance trading and market price of variance risk. *Journal of Econometrics*, *180*(1), 81–97.

Breeden, D. T., & Litzenberger, R. H. (1978). Prices of state-contingent claims implicit in option prices. *Journal of business*, 621–651.

Broadie, M., Chernov, M., & Johannes, M. (2007). Model specification and risk premia: Evidence from futures options. *The Journal of Finance*, *62*(3), 1453–1490.

Broadie, M., Chernov, M., & Johannes, M. (2009). Understanding index option returns. *The Review of Financial Studies*, *22*(11), 4493–4529.

Brogaard, J., Han, J., & Won, P. Y. (2023). How does zero-day-to-expiry options trading affect the volatility of underlying assets? *Available at SSRN 4426358*.

Büchner, M., & Kelly, B. (2022). A factor model for option returns. *Journal of Financial Economics*, *143*(3), 1140–1161.

Cao, J., & Han, B. (2013). Cross section of option returns and idiosyncratic stock volatility. *Journal of Financial Economics*, *108*(1), 231–249.

Carhart, M. M. (1997). On persistence in mutual fund performance. *The Journal of finance*, *52*(1), 57–82.

Carr, P., & Lee, R. (2009). Volatility derivatives. *Annu. Rev. Financ. Econ.*, *1*(1), 319–339.

Carr, P., & Madan, D. (1998). Towards a theory of volatility trading. *Volatility: New estimation techniques for pricing derivatives*, *29*, 417–427.

Carr, P., & Wu, L. (2003). What type of process underlies options? a simple robust test. *The Journal of Finance*, *58*(6), 2581–2610.

Carr, P., & Wu, L. (2009). Variance risk premiums. *The Review of Financial Studies*, *22*(3), 1311–1341.

Carr, P., & Wu, L. (2020). Option profit and loss attribution and pricing: A new framework. *The Journal of Finance*, *75*(4), 2271–2316.

CBOE. (2019). Vix white paper.

Chamberlain, G., & Rothschild, M. (1982). *Arbitrage, factor structure, and mean-variance analysis on large asset markets.* National Bureau of Economic Research Cambridge, Mass., USA.

Chen, B., Gan, Q., & Vasquez, A. (2023). Anticipating jumps: Decomposition of straddle price. *Journal of Banking & Finance*, *149*, 106755.

Chen, Z., Roussanov, N. L., Wang, X., & Zou, D. (2024). Common risk factors in the returns on stocks, bonds (and options), redux. *Bonds (and Options), Redux (January 21, 2024)*.

Choi, H., Mueller, P., & Vedolin, A. (2017). Bond variance risk premiums. *Review of Finance*, *21*(3), 987–1022.

Christoffersen, P., Fournier, M., & Jacobs, K. (2018). The factor structure in equity options. *The Review of Financial Studies*, *31*(2), 595–637.

Constantinides, G. M., Jackwerth, J. C., & Savov, A. (2013). The puzzle of index option returns. *Review of Asset Pricing Studies*, *3*(2), 229–257.

Coval, J. D., & Shumway, T. (2001). Expected option returns. *The journal of Finance*, *56*(3), 983–1009.

Dew-Becker, I., Giglio, S., & Kelly, B. (2021). Hedging macroeconomic and financial uncertainty and volatility. *Journal of Financial Economics*, *142*(1), 23–45.

Dew-Becker, I., Giglio, S., Le, A., & Rodriguez, M. (2017). The price of variance risk. *Journal of Financial Economics*, *123*(2), 225–250.

Dim, C., Eraker, B., & Vilkov, G. (2024). 0dtes: Trading, gamma risk and volatility propagation. *Gamma Risk and Volatility Propagation (January 11, 2024)*.

Duarte, J., Jones, C. S., & Wang, J. L. (2024). Very noisy option prices and inference regarding the volatility risk premium. *The Journal of Finance*.

Ebert, S. (2015). On skewed risks in economic models and experiments. *Journal of Economic Behavior & Organization*, *112*, 85–97.

Ebert, S., & Wiesen, D. (2011). Testing for prudence and skewness seeking. *Management Science*, *57*(7), 1334–1349.

Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of financial economics*, *33*(1), 3–56.

Fournier, M., Jacobs, K., & Orłowski, P. (2024). Modeling conditional factor risk premia implied by index option returns. *The Journal of Finance*, *79*(3), 2289–2338.

Frazzini, A., & Pedersen, L. H. (2022). Embedded leverage. *The Review of Asset Pricing Studies*, *12*(1), 1–52.

Gibbons, M. R., Ross, S. A., & Shanken, J. (1989). A test of the efficiency of a given portfolio. *Econometrica: Journal of the Econometric Society*, 1121–1152.

Giglio, S., Kelly, B., & Xiu, D. (2022). Factor models, machine learning, and asset pricing. *Annual Review of Financial Economics*, *14*(1), 337–368.

Goyal, A., & Saretto, A. (2022). Are equity option returns abnormal? ipca says no. *IPCA Says No (December 12, 2022)*.

Gruenthaler, T., Lorenz, F., & Meyerhof, P. (2022). Option-based intermediary leverage. *Journal of Banking & Finance*, *145*, 106670.

Harvey, C. R., & Siddique, A. (2000). Conditional skewness in asset pricing tests. *The Journal of finance*, *55*(3), 1263–1295.

Heston, S. L. (1993). A closed-form solution for options with stochastic volatility with applications to bond and currency options. *The review of financial studies*, *6*(2), 327–343.

Heston, S. L., & Todorov, K. (2023). Exploring the variance risk premium across assets. *Unpublished working paper*.

Horenstein, A. R., Vasquez, A., & Xiao, X. (2022). Common factors in equity option returns. *Available at SSRN 3290363*.

Israelov, R., & Kelly, B. T. (2017). Forecasting the distribution of option returns. *Available at SSRN 3033242*.

Jiang, G. J., & Tian, Y. S. (2005). The model-free implied volatility and its information content. *The Review of Financial Studies*, *18*(4), 1305–1342.

Johannes, M. S., Kaeck, A., Seeger, N., & Shah, N. (2024). Expected 1dte option returns. *Available at SSRN 4729757*.

Jones, C. S. (2006). A nonlinear factor analysis of s&p 500 index option returns. *The Journal of Finance*, *61*(5), 2325–2363.

Karakaya, M. M. (2014). *Characteristics and expected returns in individual equity options*. The University of Chicago.

Kelly, B. T., Pruitt, S., & Su, Y. (2019). Characteristics are covariances: A unified model of risk and return. *Journal of Financial Economics*, *134*(3), 501–524.

Konstantinidi, E., & Skiadopoulos, G. (2016). How does the market variance risk premium vary over time? evidence from s&p 500 variance swap investment returns. *Journal of Banking & Finance*, *62*, 62–75.

Kozak, S., Nagel, S., & Santosh, S. (2020). Shrinking the cross-section. *Journal of Financial Economics*, *135*(2), 271–292.

Kozhan, R., Neuberger, A., & Schneider, P. (2013). The skew risk premium in the equity index market. *The Review of Financial Studies*, *26*(9), 2174–2203.

Kraus, A., & Litzenberger, R. H. (1976). Skewness preference and the valuation of risk assets. *The Journal of finance*, *31*(4), 1085–1100.

Langlois, H. (2020). Measuring skewness premia. *Journal of Financial Economics*, *135*(2), 399–424.

Lettau, M., & Pelger, M. (2020). Factors that fit the time series and cross-section of stock returns. *The Review of Financial Studies*, *33*(5), 2274–2325.

Liang, F., & Du, L. (2024). Option pricing with dynamic conditional skewness. *Journal of Futures Markets*.

Liu, R. Y. (1988). Bootstrap procedures under some non-iid models. *The annals of statistics*, *16*(4), 1696–1708.

Mammen, E. (1993). Bootstrap and wild bootstrap for high dimensional linear models. *The annals of statistics*, *21*(1), 255–285.

Merton, R. C. (1973). Theory of rational option pricing. *The Bell Journal of economics and management science*, 141–183.

Neuberger, A. (1994). The log contract. *Journal of portfolio management*, *20*, 74–74.

Neuberger, A. (2012). Realized skewness. *The Review of Financial Studies*, *25*(11), 3423–3455.

Pesaran, M. H., & Yamagata, T. (2024). Testing for alpha in linear factor pricing models with a large number of securities. *Journal of Financial Econometrics*, *22*(2), 407–460.

Sepp, A. (2012). An approximate distribution of delta-hedging errors in a jump-diffusion model with discrete trading and transaction costs. *Quantitative Finance*, *12*(7), 1119–1141.

Shafaati, M., Chance, D. M., & Brooks, R. (2021). The cross-section of individual equity option returns. *Available at SSRN 3927410*.

Todorov, V. (2019). Nonparametric spot volatility from options. *The Annals of Applied Probability*, *29*(6), 3590–3636.

Vilkov, G. (2024). 0-dte trading rules. *SSRN working paper*.

White, H. (1980). A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity. *Econometrica: journal of the Econometric Society*, 817–838.

Wu, C.-F. J. (1986). Jackknife, bootstrap and other resampling methods in regression analysis. *the Annals of Statistics*, *14*(4), 1261–1295.

Zhu, C. (2015). Stochastic skewness and index option returns. *Available at SSRN 2563944*.

Appendix A Statistical inference and evaluation for IPCA models

A.1 Hypothesis testing using wild bootstrap

Statistical inference for the IPCA model implements a bootstrapping procedure. For exposition, I first illustrate the procedure as applied to testing the null hypothesis in (23). The null hypothesis in (23) is that $\alpha_{i,t}$ jointly are zero in the model. Note this is equivalent to testing the following null hypothesis:

$$\mathbb{H}_0 : \Gamma_\alpha = \mathbf{0}_{L+1} \tag{47}$$

where $\mathbf{0}_{L+1}$ is the zero vector. I first estimate the unrestricted model

$$y_{i,t+1} = R_{i,t+1} - R^f_{t+1} = \left(Z'_{i,t}\Gamma_\alpha\right) + \left(Z'_{i,t}\Gamma_\beta\right) F_{t+1} + \left(Z'_{i,t}\Gamma_\delta\right) R^{mkt}_{ind,t+1} + E_{i,t+1}$$

and obtain the estimated values $\hat{F}_{t+1}$, $\hat{\Gamma}_\alpha$, $\hat{\Gamma}_\beta$, and $\hat{\Gamma}_\delta$. I compute the Wald statistic W from these estimates:

$$W = \hat{\Gamma}'_\alpha \hat{\Gamma}_\alpha$$

To implement wild bootstrapping as proposed by Liu (1988), I generate bootstrapped samples imposing the null hypothesis (in this case the null sets Γ_α to the zero vector). Let $q_{i,t+1} \sim t(5)$ be independent draws from the student t distribution with 5 degrees of freedom for each observation $(i, t+1)$. Then the observation corresponding to $(i, t+1)$ in the B-th bootstrapped sample $y^B_{i,t+1}$ is given by:

$$y^B_{i,t+1} = \left(Z'_{i,t}\hat{\Gamma}_\beta\right) \hat{F}_{t+1} + \left(Z'_{i,t}\hat{\Gamma}_\delta\right) R^{mkt}_{ind,t+1} + q_{i,t+1}\hat{E}_{i,t+1}$$

Then I estimate the IPCA model using the bootstrapped $y^B_{i,t+1}$ under the null hypothesis. Specifically, I estimate the model

$$y^B_{i,t+1} = \left(Z'_{i,t}\Gamma^B_\beta\right) F^B_{t+1} + \left(Z'_{i,t}\Gamma^B_\delta\right) R^{mkt}_{ind,t+1} + E^B_{i,t+1}$$

to obtain the fitted parameters $\hat{F}^B_{t+1}$, $\hat{\Gamma}^B_\alpha$, $\hat{\Gamma}^B_\beta$, and $\hat{\Gamma}^B_\delta$ corresponding to the B-th bootstrapped sample. For each bootstrapped sample, I compute the Wald-like statistic W^B:

$$W^B = \hat{\Gamma}^{B\prime}_\alpha \hat{\Gamma}^B_\alpha$$

The p-value of the hypothesis test in (47) is equal to the fraction of bootstrapped samples for which W^B exceeds W. In practice, the number of bootstrapped samples in the literature is chosen to be large. Following similar implementations for IPCA models, such as those by Kelly et al. (2019) and Goyal and Saretto (2022), I use 5,000 bootstrapped samples when drawing samples for statistical inference.

Testing similar hypotheses, such as the significance of a factor is analogous. For example, to test the significance of the index return $R^{mkt}_{ind,t+1}$ as a priced factor, we first calculate a similar Wald statistic:

$$W = \hat{\Gamma}'_{\delta}\hat{\Gamma}_{\delta}$$

Then, for each bootstrapped sample, we assume the null and estimate the model:

$$y^B_{i,t+1} = \left(Z'_{i,t}\Gamma_{\alpha}\right) + \left(Z'_{i,t}\Gamma_{\beta}\right) F_{t+1} + E^B_{i,t+1}$$

where the bootstrapped sample $y^B_{i,t+1}$ is generated as

$$y^B_{i,t+1} = \left(Z'_{i,t}\Gamma^B_{\alpha}\right) + \left(Z'_{i,t}\hat{\Gamma}^B_{\beta}\right) \hat{F}^B_{t+1} + q\hat{E}_{i,t+1}$$

where q is a draw from the student t-distributed random variable as before. Then the p-value for our statistical test is again the fraction of bootstrapped samples where W^B_{δ} is larger than W.

$$W^B_{\delta} = \hat{\Gamma}^{B\prime}_{\delta}\hat{\Gamma}^B_{\delta}$$

A.2 Out-of-sample evaluation and estimation

To produce out-of-sample (OOS) predictions of returns, I implement an expanding window approach. Suppose the data in the sample runs from time 0 to T. Then I select a month M_0 in the data which cuts the data into two halves, that is, there is approximately an equal number of observations in the sample prior to and after M_0. To compute the out-of-sample average error of the IPCA model, we can proceed iteratively. First, we estimate the model in equation (48) on a subsample of the full option panel consisting of all observations in the option panel $y_{i,t+1}$ for all times t before month M_0:

$$y_{i,t+1} = \left(Z'_{i,t}\Gamma^{M_0}_{\alpha}\right) + \left(Z'_{i,t}\Gamma^{M_0}_{\beta}\right) F_{t+1} + \left(Z'_{i,t}\Gamma^{M_0}_{\delta}\right) R^{mkt}_{t+1} + E_{i,t+1} \tag{48}$$

Now using the estimated parameters, $\hat{F}^{M_0}_{t+1}$, $\hat{\Gamma}^{M_0}_{\alpha}$, $\hat{\Gamma}^{M_0}_{\beta}$, and $\hat{\Gamma}^{M_0}_{\delta}$, we can compute the out-of-sample errors using $E^{OOS}_{i,t+1}$ for times t in month M_0 by:

$$E^{OOS}_{i,t+1} = \left(Z'_{i,t}\hat{\Gamma}^{M_0}_{\alpha}\right) + \left(Z'_{i,t}\hat{\Gamma}^{M_0}_{\beta}\right) F_{t+1} + \left(Z'_{i,t}\hat{\Gamma}^{M_0}_{\delta}\right) R^{mkt}_{t+1} \tag{49}$$

To compute the OOS errors for the observations in month $M_0 + 1$, one can estimate the IPCA model using all observations prior to month $M_0 + 1$. The OOS errors are then given by:

$$E^{OOS}_{i,t+1} = \left(Z'_{i,t}\hat{\Gamma}^{M_0+1}_{\alpha}\right) + \left(Z'_{i,t}\hat{\Gamma}^{M_0+1}_{\beta}\right) F_{t+1} + \left(Z'_{i,t}\hat{\Gamma}^{M_0+1}_{\delta}\right) R^{mkt}_{t+1} \tag{50}$$

where $\hat{F}^{M_0+1}_{t+1}$, $\hat{\Gamma}^{M_0+1}_{\alpha}$, $\hat{\Gamma}^{M_0+1}_{\beta}$, and $\hat{\Gamma}^{M_0+1}_{\delta}$ are the estimated latent factors and parameters using only observations prior to month $M_0 + 1$.

We continue this process until we cover all observations in the second half of the sample. At the end, we will should have a panel of out-of-sample errors $E^{OOS}_{i,t+1}$ for all $t \geq M_0$. Using the out-of-sample errors, we can compute various summary statistics out-of-sample such as the out-of-sample R-squareds and pricing errors shown in table 5. Out-of-sample pricing errors are computed as sample averages of the out-of-sample errors $E^{OOS}_{i,t+1}$. Out-of-sample R-squareds are computed as:

$$1 - \frac{\sum_{i,t}\left(E^{OOS}_{i,t+1}\right)^2}{\mathbb{E}\left[y_{i,t+1} - \mathbb{E}(y_{i,t+1})\right]^2}$$

Appendix B Figures and tables for other equity indices

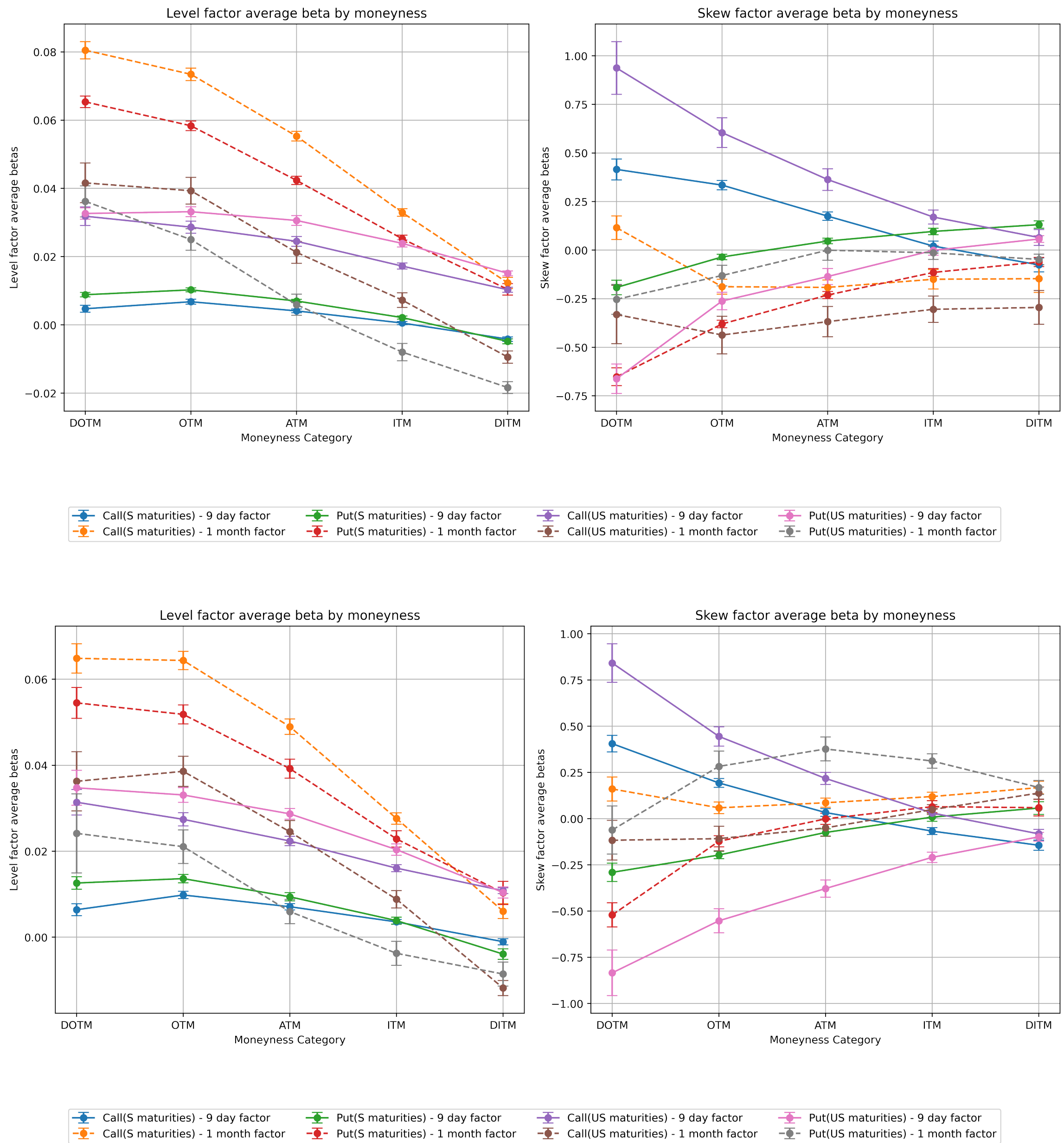

Figure 8. 5MOM moment portfolio unconditional betas for NDX (top) and RUT (bottom).

Maturity	Type	Risk source (ℓ): Moneyness	$\Delta V^{(9d)}_{NDX,t+1}$	$\Delta W^{(9d)}_{NDX,t+1}$	$\Delta V^{(30d-9d)}_{NDX,t+1}$	$\Delta W^{(30d-9d)}_{NDX,t+1}$	$R^{mkt}_{NDX,t+1}$	$\widehat{\text{Return}}$	Residual	Return
S	C	DOTM	-7.93	35.32^{**}	-6.07^{*}	0.8	77.88^{***}	7.74	-1.37	6.36
			[-1.42]	[2.67]	[-2.5]	[0.71]	[4.7]	(0.12)		
		OTM	-21.07^{***}	23.36^{***}	-8.16^{***}	1.38	104.49^{***}	7.7	4.13	11.83
			[-4.99]	[3.56]	[-4.09]	[1.36]	[25.09]	(0.07)		
		ATM	-19.39^{***}	12.37^{***}	-6.4^{***}	1.12	112.29^{***}	7.92	2.21	10.13
			[-5.87]	[3.34]	[-4.05]	[1.38]	[62.74]	(0.06)		
		ITM	-18.52^{***}	3.22^{*}	-5.2^{***}	0.94	119.56^{***}	6.26	1.32	7.58
			[-6.22]	[1.97]	[-3.82]	[1.33]	[32.4]	(0.05)		
		DITM	-21.46^{***}	-14.4^{***}	-5.62^{***}	1.15	140.33^{***}	3.25	1.69	4.94
			[-5.46]	[-3.85]	[-3.73]	[1.49]	[13.15]	(0.06)		
	P	DOTM	-34.08^{**}	-7.32	-6.92	1.38	-53.07^{*}	-12.86	3.12	-9.74
			[-3.18]	[-1.41]	[-1.76]	[0.67]	[-2.3]	(0.08)		
		OTM	-27.68^{***}	0.57	-5.9^{*}	1.12	-68.11^{***}	-11.12	3.2	-7.93
			[-3.67]	[0.2]	[-2.02]	[0.73]	[-5.75]	(0.04)		
		ATM	-17.98^{***}	3.51	-4.28^{*}	0.79	-82.05^{***}	-10.42	2.89	-7.52
			[-3.86]	[1.34]	[-2.2]	[0.78]	[-15.72]	(0.04)		
		ITM	-8.54^{***}	5.19^{*}	-2.88^{*}	0.52	-94.29^{***}	-8.69	2.21	-6.48
			[-3.39]	[1.98]	[-2.43]	[0.84]	[-77.18]	(0.04)		
		DITM	4.68	6.98^{*}	-2.24^{**}	0.42	-109.84^{***}	-5.02	-0.49	-5.51
			[1.68]	[2.16]	[-2.58]	[0.93]	[-28.17]	(0.05)		
US	C	DOTM	-8.46	38.2^{**}	-5.48^{**}	1.06	74.68^{***}	9.32	-3.12	6.2
			[-1.15]	[3.08]	[-3.19]	[1.2]	[5.38]	(0.29)		
		OTM	-18.78^{***}	21.98^{***}	-5.76^{***}	1.17	101.4^{***}	9.83	5.87	15.69
			[-4.06]	[3.79]	[-4.01]	[1.54]	[24.26]	(0.17)		
		ATM	-15.91^{***}	10.97^{**}	-3.64^{***}	0.78	107.81^{***}	10.87	2.88	13.75
			[-4.55]	[3.25]	[-3.4]	[1.38]	[40.42]	(0.15)		
		ITM	-15.04^{***}	3.91^{*}	-2.73^{**}	0.63	113.23^{***}	9.38	1.4	10.78
			[-4.86]	[2.28]	[-2.86]	[1.28]	[32.47]	(0.11)		
		DITM	-23.47^{***}	-7.78^{*}	-3.41	1.02	133.63^{***}	3.97	0.72	4.69
			[-3.81]	[-2.42]	[-1.7]	[1.06]	[12.71]	(0.07)		
	P	DOTM	-39.53^{***}	-12.0^{*}	-3.15^{*}	0.6	-45.92^{*}	-13.33	-3.32	-16.65
			[-4.62]	[-2.02]	[-2.44]	[0.89]	[-2.55]	(0.19)		
		OTM	-28.31^{***}	-1.37	-2.1^{*}	0.39	-68.6^{***}	-12.37	0.96	-11.4
			[-5.09]	[-0.52]	[-2.51]	[0.89]	[-8.42]	(0.12)		
		ATM	-17.71^{***}	1.74	-0.33	0.05	-83.74^{***}	-12.29	2.15	-10.14
			[-4.88]	[0.73]	[-1.44]	[0.56]	[-24.66]	(0.12)		
		ITM	-9.86^{***}	2.83	0.39	-0.05	-93.31^{***}	-11.41	2.11	-9.3
			[-4.7]	[1.34]	[1.78]	[-0.54]	[-48.3]	(0.09)		
		DITM	-4.34^{**}	1.91	1.23^{*}	-0.14	-98.66^{***}	-7.53	-1.45	-8.98
			[-2.63]	[1.07]	[2.28]	[-0.57]	[-71.42]	(0.05)		

Table 12. Return decomposition for delevered NDX option returns across buckets characterized by $\Theta = (a_1, a_2, a_3, NDX)$ expressed as a percentage. Left-most three columns identify the bucket and columns headed by $\ell \in \{\Delta V^{(9d)}_{i,t+1}, \Delta W^{(9d)}_{i,t+1}, \Delta V^{(30d-9d)}_{i,t+1}, \Delta W^{(30d-9d)}_{i,t+1}, R^{mkt}_{NDX,t+1}\}$ report $100 \times Q_\ell(\Theta)$. T-statistics for the null hypothesis that $Q_\ell(\Theta)$ is zero are presented in square brackets. Right-most three columns present estimates of the expected return ($\widehat{\text{Return}}$), MAPE (Residual), and average return in the bucket (Return) in basis points. Rounded parentheses show the standard errors for the column headed by ($\widehat{\text{Return}}$). All standard errors and t-statistics are calculated using the wild bootstrap approach proposed by Liu (1988) and Mammen (1993) using 5,000 iterations. Statistical significance at the 5%, 1%, and 0.1% denoted by 1, 2 and 3 stars respectively.

Maturity	Type	Risk source (ℓ): Moneyness	$\Delta V^{(9d)}_{RUT,t+1}$	$\Delta W^{(9d)}_{RUT,t+1}$	$\Delta V^{(30d-9d)}_{RUT,t+1}$	$\Delta W^{(30d-9d)}_{RUT,t+1}$	$R^{mkt}_{RUT,t+1}$	$\widehat{\text{Return}}$	Residual	Return
S	C	DOTM	-26.54	36.53	-6.83	8.62	88.22^{**}	8.69	-7.22	1.48
			[-1.55]	[1.47]	[-1.18]	[0.61]	[2.93]	(0.12)		
		OTM	-33.46^{*}	27.79^{*}	-10.77^{*}	7.76	108.68^{***}	7.15	0.59	7.74
			[-2.33]	[1.99]	[-1.99]	[0.8]	[8.14]	(0.06)		
		ATM	-25.77^{**}	15.01^{*}	-6.9^{*}	6.74	110.92^{***}	7.42	-0.05	7.36
			[-2.83]	[2.24]	[-2.12]	[0.97]	[18.2]	(0.05)		
		ITM	-20.01^{**}	7.34^{*}	-2.67	6.84	108.51^{***}	6.06	-0.76	5.29
			[-3.06]	[2.1]	[-1.36]	[1.04]	[28.76]	(0.05)		
		DITM	-12.97^{*}	-2.89	6.2^{*}	10.07	99.58^{***}	3.52	-0.88	2.64
			[-1.99]	[-0.89]	[2.54]	[0.99]	[21.12]	(0.06)		
	P	DOTM	-8.77	-13.06	-10.86^{*}	-3.86	-63.46^{**}	-15.48	4.72	-10.76
			[-1.01]	[-1.54]	[-2.09]	[-0.56]	[-3.28]	(0.11)		
		OTM	-21.36^{**}	-3.69	-9.45^{**}	1.31	-66.83^{***}	-9.81	0.46	-9.35
			[-2.97]	[-1.27]	[-3.05]	[0.48]	[-9.67]	(0.07)		
		ATM	-20.87^{***}	3.98^{*}	-6.68^{***}	3.34	-79.78^{***}	-8.03	-2.38	-10.41
			[-4.26]	[2.02]	[-3.6]	[1.25]	[-41.96]	(0.08)		
		ITM	-18.42^{***}	10.89^{**}	-3.68^{*}	5.24	-94.04^{***}	-5.31	-0.15	-5.47
			[-3.55]	[2.67]	[-2.13]	[1.17]	[-23.13]	(0.07)		
		DITM	-4.32	18.84^{**}	-1.49	7.77	-120.79^{***}	-2.32	-1.93	-4.25
			[-0.81]	[3.04]	[-0.81]	[1.34]	[-11.79]	(0.09)		
US	C	DOTM	-22.95	42.23	-6.3	0.29	86.74^{**}	11.15	-13.92	-2.77
			[-1.3]	[1.33]	[-1.48]	[0.08]	[2.65]	(0.36)		
		OTM	-26.95^{*}	30.75	-7.76^{*}	0.69	103.27^{***}	9.26	-3.05	6.21
			[-2.2]	[1.85]	[-2.18]	[0.23]	[6.87]	(0.16)		
		ATM	-23.07^{**}	20.0^{*}	-4.95^{*}	0.88	107.14^{***}	8.56	-2.18	6.38
			[-2.91]	[2.2]	[-2.55]	[0.49]	[13.35]	(0.11)		
		ITM	-18.71^{**}	11.21	-0.44	2.04	105.9^{***}	7.43	-2.88	4.54
			[-3.02]	[1.96]	[-0.63]	[0.88]	[19.45]	(0.07)		
		DITM	-22.82	7.74	13.21	5.67	96.2^{***}	3.09	-2.45	0.64
			[-1.33]	[0.85]	[1.8]	[0.5]	[8.36]	(0.06)		
	P	DOTM	-18.19^{*}	-16.11	-2.59	0.29	-63.4^{***}	-17.05	-1.02	-18.06
			[-2.33]	[-1.75]	[-1.82]	[0.2]	[-3.93]	(0.29)		
		OTM	-37.35^{***}	-5.19	2.37^{*}	7.21	-67.04^{***}	-9.34	-6.22	-15.55
			[-5.74]	[-1.52]	[2.51]	[1.7]	[-14.03]	(0.17)		
		ATM	-51.02^{***}	5.58	10.4^{***}	13.0^{*}	-77.98^{***}	-5.73	-6.24	-11.97
			[-7.88]	[1.81]	[6.72]	[2.35]	[-21.54]	(0.15)		
		ITM	-43.45^{***}	12.36^{**}	13.73^{***}	12.51^{*}	-95.15^{***}	-4.45	-3.44	-7.9
			[-6.48]	[3.28]	[6.36]	[2.02]	[-19.54]	(0.13)		
		DITM	-56.33^{***}	19.08^{***}	28.46^{***}	17.35^{*}	-108.56^{***}	-1.47	-2.78	-4.25
			[-6.09]	[3.77]	[7.36]	[2.19]	[-14.14]	(0.09)		

Table 13. Return decomposition for delevered RUT option returns across buckets characterized by $\Theta = (a_1, a_2, a_3, RUT)$ expressed as a percentage. Left-most three columns identify the bucket and columns headed by $\ell \in \{\Delta V^{(9d)}_{i,t+1}, \Delta W^{(9d)}_{i,t+1}, \Delta V^{(30d-9d)}_{i,t+1}, \Delta W^{(30d-9d)}_{i,t+1}, R^{mkt}_{RUT,t+1}\}$ report $100 \times Q_\ell(\Theta)$. T-statistics for the null hypothesis that $Q_\ell(\Theta)$ is zero are presented in square brackets. Right-most three columns present estimates of the expected return ($\widehat{\text{Return}}$), MAPE (Residual), and average return in the bucket (Return) in basis points. Rounded parentheses show the standard errors for the column headed by ($\widehat{\text{Return}}$). All standard errors and t-statistics are calculated using the wild bootstrap approach proposed by Liu (1988) and Mammen (1993) using 5,000 iterations. Statistical significance at the 5%, 1%, and 0.1% denoted by 1, 2 and 3 stars respectively.