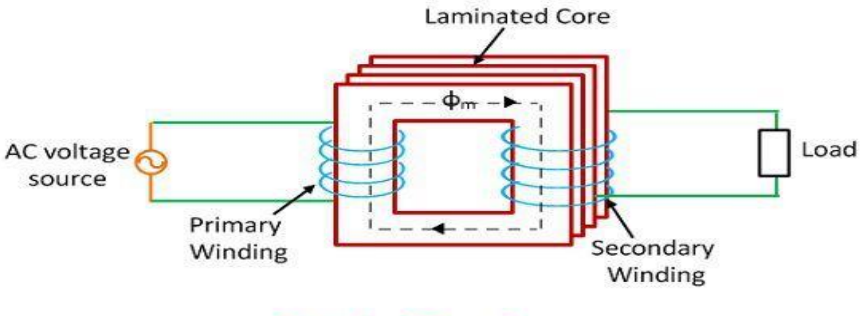
Electrical Transformer

Definition: The transformer is the static device which works on the principle of electromagnetic induction. It is used for transferring the electrical power from one circuit to another without any variation in their frequency. In electromagnetic induction, the transfer of energy from one circuit to another takes places by the help of the mutual induction. i.e the flux induced in the primary winding is linked with the secondary winding.

Construction of an Electrical Transformer

The primary winding, secondary winding and the magnetic core are the three important of the transformer. These coils are insulated from each other. The main flux is induced in the primary winding of the transformer. This flux passes through the low reluctance path of the magnetic core and linked with the secondary winding of



Electrical Transformer

the transformer.

Transformer Working

Consider the T_1 and T_2 are the numbers of the turn on the primary and the secondary winding of the transformer shown in the figure above. The voltage is applied to the primary winding of the transformer because of which the current is induced in it. The current causes the magnetic flux which is represented by the dotted line in the above figure.

The flux induces in the primary winding because of self-induction. This flux is linked with the secondary winding because of mutual induction. Thus, the emf is induced in the secondary winding of the transformer. The power is transferred from the primary winding to the secondary winding. The frequency of the transferred energy also remains same.

EMF Equation of an Electrical Transformer

The emf induced in each winding of the transformer can be calculated from its emf equation.

$$\emptyset = \emptyset_m \operatorname{Sin} \omega t \dots\dots\dots equ(1)$$

The linking of the flux is represented by the faraday law of electrmagentic induction. It is expressed as,

$$e = -\frac{d}{dt}(\emptyset t) = -T\frac{d}{dt}\emptyset = -T\frac{d}{dt}(\emptyset_m Sin\omega t)$$

$$= -T\omega\emptyset_m Cos\omega t = -T\omega\frac{d}{dt}\emptyset_m\left(\omega t - \frac{\pi}{2}\right)\dots\dots\dots equ(2)$$

$$e = E_m Sin\left(\omega t - \frac{\pi}{2}\right) \ldots \ldots \ldots . . equ(3)$$

The above equation may be written as,

where $E_m = 4.44\omega\Phi_m$ = maximum value of e. For a sine wave, the r.m.s value of e.m.f is given

$$E_{rms} = E = \frac{E_m}{\sqrt{2}}$$

$$E = \frac{T\omega\emptyset_m}{\sqrt{2}} = \frac{T(2\pi f)\emptyset_m}{\sqrt{2}}$$

$$E = 4.44\emptyset_m fT \ldots \ldots \ldots \ldots equ(4)$$

by

The emf induced in their primary and secondary winding is expressed as,

$$E_1 = 4.44\emptyset_m fT_1 \ldots \ldots \ldots \ldots equ(5)$$

$$E_2 = 4.44\emptyset_m fT_2 \ldots \ldots \ldots \ldots equ(6)$$

The secondary RMS voltage is

Where φ_m is the maximum value of flux in Weber (Wb), f is the frequency in hertz (Hz) and E_1 and E_2 in volts.

If, B_m = maximum flux density in the magnetic circuit in Tesla (T)

$$B_m = \frac{\emptyset_m}{A}$$

A = area of cross-section of the core in square meter (m^2)

The winding which has the higher number of voltage has high voltage while the primary winding has low voltage.

Voltage Ratio and Turns Ratio

The ratio of E/T is called volts per turn. The primary and secondary volts per turns is given by the formula

$$\frac{E_1}{T_1} = 4.44\emptyset_m f \ldots \ldots \ldots . equ(1) \quad \frac{E_2}{T_2} = 4.44\emptyset_m f \ldots \ldots \ldots \ldots equ(2)$$

The equation (1) and (2) shows that the voltage per turn in both the winding is same, i.e.

$$\frac{E_1}{T_1} = \frac{E_2}{T_2} \dots\dots\dots\dots equ(3)$$

$$\frac{E_1}{E_2} = \frac{T_1}{T_2} \dots\dots\dots\dots equ(4)$$

The ratio T_1/T_2 is called the turn ratio. The turn ratio is expressed as

$$\frac{E_1}{E_2} = \frac{T_1}{T_2} = a \dots\dots\dots\dots equ(5)$$

The ratio of primary to secondary turn which equals to primary to secondary induced voltage indicates how much the primary voltage lowered or raised. The turn ratio or induced voltage ratio is called the transformation ratio, and it is denoted by the symbol a. Thus,

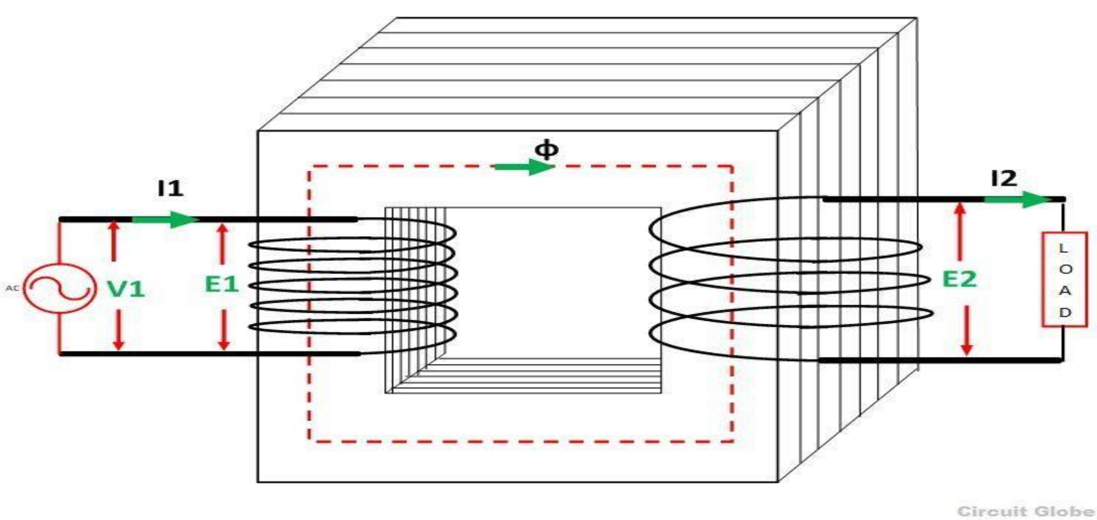
$$\frac{V_1}{V_2} = \frac{T_1}{T_2} = a \dots\dots\dots\dots. equ(6)$$

The any desired voltage ratio can be obtained by shifting the number of turns.

Working Principle of a Transformer

The basic principle on which the transformer works is **Faraday's Law of Electromagnetic Induction** or mutual induction between the two coils. The working of the transformer is explained below. The transformer consists of two separate windings placed over the laminated silicon steel core.

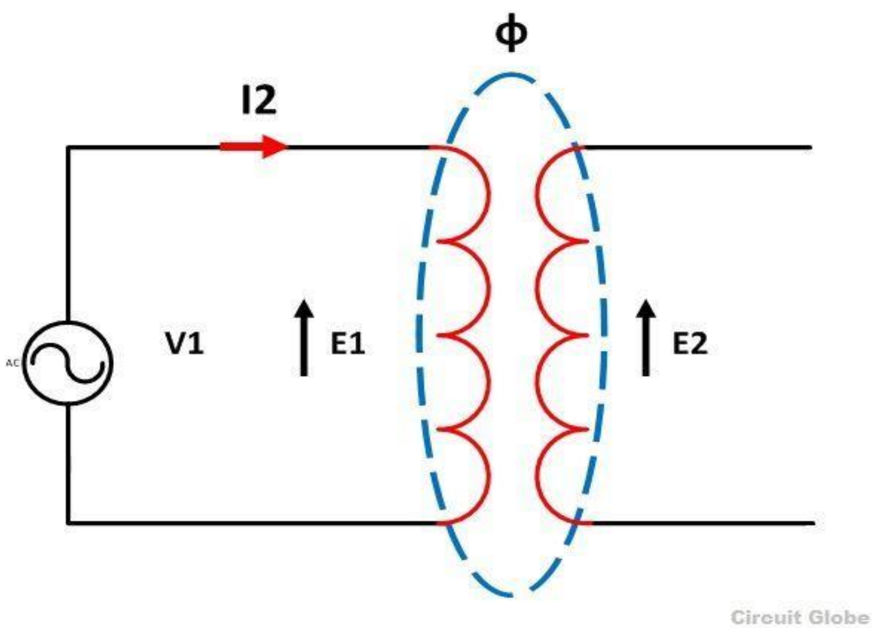
The winding to which AC supply is connected is called primary winding and to which load is connected is called secondary winding as shown in the figure below. It works on the **alternating current only** because an alternating flux is required for mutual induction between the two windings.



Contents:

- Transformer on DC supply
- Turn Ratio
- Transformation Ratio

When the AC supply is given to the primary winding with a voltage of V_1, an alternating flux ϕ sets up in the core of the transformer, which links with the secondary winding and as a result of it, an emf is induced in it called **Mutually Induced emf**. The direction of this induced emf is opposite to the applied voltage V_1, this is because of the Lenz's law shown in the figure below:



Physically, there is no electrical connection between the two windings, but they are magnetically connected. Therefore, the electrical power is transferred from the primary circuit to the secondary circuit through mutual inductance.

The induced emf in the primary and secondary windings depends upon the rate of change of flux linkage that is ($N\,d\phi/dt$).

$d\phi/dt$ is the change of flux and is same for both the primary and secondary windings. The induced emf E_1 in the primary winding is proportional to the number of turns N_1 of the primary windings ($E_1 \propto N_1$). Similarly induced emf in the secondary winding is proportional to the number of turns on the secondary side. ($E_2 \propto N_2$).

Transformer on DC supply

As discussed above, the transformer works on AC supply, and it cannot work not DC supply. If the rated DC voltage is applied across the primary winding, a constant magnitude flux will set up in the core of the transformer and hence there will not be any self-induced emf generation, as for the linkage of flux with the secondary winding there must be an alternating flux required and not a constant flux.

According to Ohm's Law

$$\text{Primary Current} = \frac{\text{DC applied voltage}}{\text{Resistance of primary winding}}$$

The resistance of the primary winding is very low, and the primary current is high. So this current is much higher than the rated full load primary winding current. Hence, as a result, the amount of heat produced will be greater and therefore, eddy current loss (I^2R) loss will be more.

Because of this, the insulations of the primary windings will get burnt, and the transformer will get damaged.

Turn Ratio

It is defined as the ratio of primary to secondary turns.

$$\text{Turn ratio} = \frac{N_1}{N_2}$$

If $N_2 > N_1$ the transformer is called **Step-up transformer**

If $N_2 < N_1$ the transformer is called **Step down transformer**

Transformation Ratio

The transformation ratio is defined as the ratio of the secondary voltage to the primary voltage. It is denoted by K.

$$K = \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

As ($E_2 \propto N_2$ and $E_1 \propto N_1$)

This is all about the working of the transformer.

EMF Equation of a Transformer

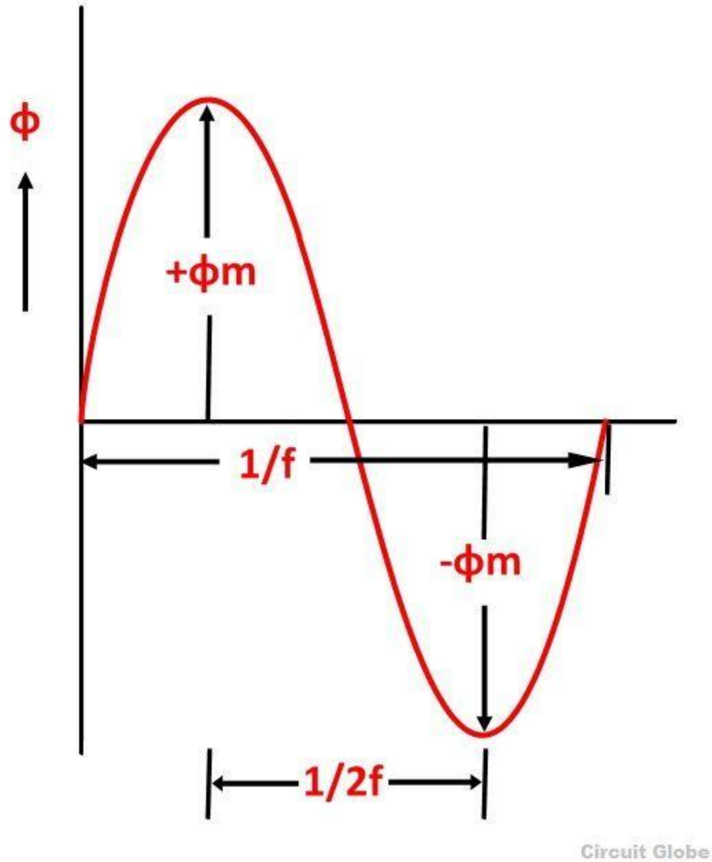
When a sinusoidal voltage is applied to the primary winding of a transformer, alternating flux ϕ_m sets up in the iron core of the transformer. This sinusoidal flux links with both primary and secondary winding. The function of flux is a sine function.

The rate of change of flux with respect to time is derived mathematically.

The derivation of the **EMF Equation** of the transformer is shown below. Let

- ϕ_m be the maximum value of flux in Weber
- f be the supply frequency in Hz
- N_1 is the number of turns in the primary winding
- N_2 is the number of turns in the secondary winding

Φ is the flux per turn in Weber



As shown in the above figure that the flux changes from + ϕ_m to – ϕ_m in half a cycle of 1/2f seconds.

By Faraday's Law

Let E_1 be the emf induced in the primary winding

$$E_1 = -\frac{d\psi}{dt} \ldots \ldots \ldots (1)$$

Where $\Psi = N_1\phi$

$$\text{Therefore, } E_1 = -N_1 \frac{d\varphi}{dt} \ \ldots\ldots\ldots(2)$$

Since ϕ is due to AC supply $\phi = \phi_m$ Sinwt

$$E_1 = -N_1 \frac{d}{dt} (\varphi_m \text{ Sinwt})$$

$$E_1 = -N_1 w\varphi_m \text{ Coswt}$$

$$E_1 = N_1 w\varphi_m \text{ Sin}(wt - \pi/2) \ldots\ldots\ldots.(3)$$

So the induced emf lags flux by 90 degrees.

Maximum valve of emf

$$E_1 max = N_1 w\varphi_m \ldots\ldots\ldots.(4)$$

But $w = 2\pi f$

$$E_1 max = 2\pi f N_1 \varphi_m \ldots\ldots\ldots.(5)$$

Root mean square RMS value is

$$E_1 = \frac{E_{1max}}{\sqrt{2}} \ldots\ldots\ldots..(6)$$

Putting the value of E_1max in equation (6) we get

$$E_1 = \sqrt{2}\pi f N_1 \varphi_m \ldots\ldots..(7)$$

Putting the value of $\pi = 3.14$ in the equation (7) we will get the value of E_1 as

$$E_1 = 4.44 f N_1 \varphi_m \ldots\ldots\ldots(8)$$

Similarly

$$E_2 = \sqrt{2}\pi f N_2 \varphi_m$$

Or

$$E_2 = 4.44 f N_2 \varphi_m \ldots\ldots\ldots(9)$$

Now, equating the equation (8) and (9) we get

$$\frac{E_2}{E_1} = \frac{4.44fN_2\varphi_m}{4.44fN_1\varphi_m}$$

Or

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

The above equation is called the **turn ratio** where K is known as the transformation ratio.

The equation (8) and (9) can also be written as shown below using the relation

($\phi m = B_m \times A_i$) where A_i is the iron area and B_m is the maximum value of flux density.

$$E_1 = 4.44N_1 fB_mA_i \ \text{Volts} \quad \text{and} \quad E_2 = 4.44N_2fB_mA_i \ \text{Volts}$$

$$\frac{\text{R.M.S value}}{\text{Average value}} = \text{Form factor} = 1.11$$

For a sinusoidal wave

Here 1.11 is the form factor.

Ideal Transformer

Definition: The transformer which is free from all types of losses is known as an ideal transformer. It is an imaginary transformer that has no core loss, no ohmic resistance, and no leakage flux. The ideal transformer has the following important characteristic.

1. The resistance of their primary and secondary winding becomes zero.
2. The core of the ideal transformer has infinite permeability. The infinite permeable means less magnetizing current requires for magnetizing their core.
3. The leakage flux of the transformer becomes zero, i.e. the whole of the flux induces in the core of the transformer links with their primary and secondary winding.
4. The ideal transformer has 100 percent efficiency, i.e., the transformer is free from hysteresis and eddy current loss.

The above mention properties are not possible in the practical transformer. **In an ideal transformer, there is no power loss. Therefore, the output power is equal to the input power.**

$$E_2 I_2 \cos\varphi = E_1 I_1 \cos\varphi \quad \text{or} \quad E_2 I_2 = E_1 I_1$$

OR

$$\frac{E_2}{E_1} = \frac{I_1}{I_2}$$

Since $E_1 \infty N_2$ and $E_1 \infty N_1$, also E_1 is similar to V_1 and E_2 is similar to V_2

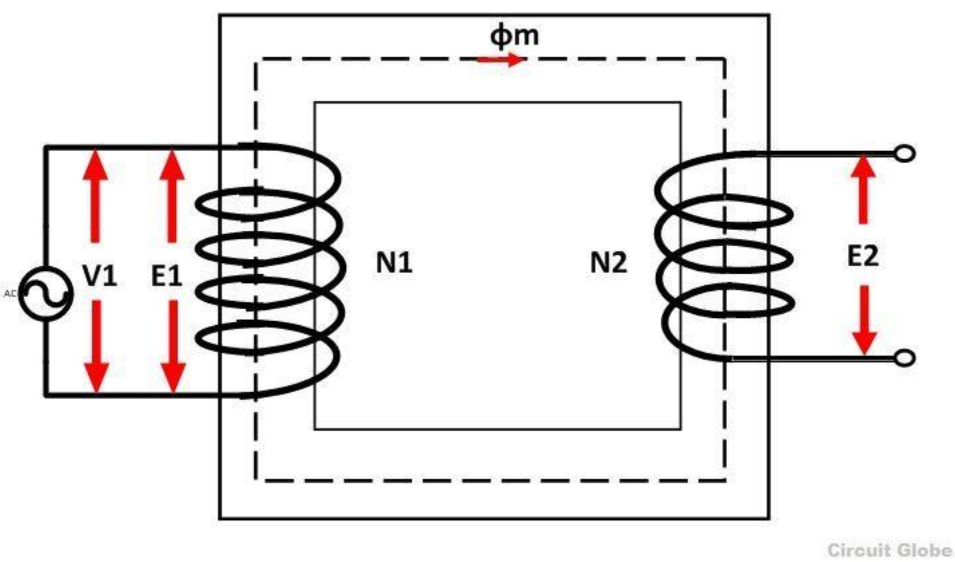
Therefore, the transformation ratio will be given by the equation shown below

$$\frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K$$

The primary and the secondary currents are inversely proportional to their respective turns.

Behavior of Ideal Transformer

Consider the ideal transformer shown in the figure below:



The voltage source V_1 is applied across the primary winding of the transformer. Their secondary winding is kept open. The N_1 and N_2 are the numbers of turns of their primary and secondary winding.

The current I_m is the magnetizing current flows through the primary winding of the transformer. The magnetizing current produces the flux φ_m in the core of the transformer.

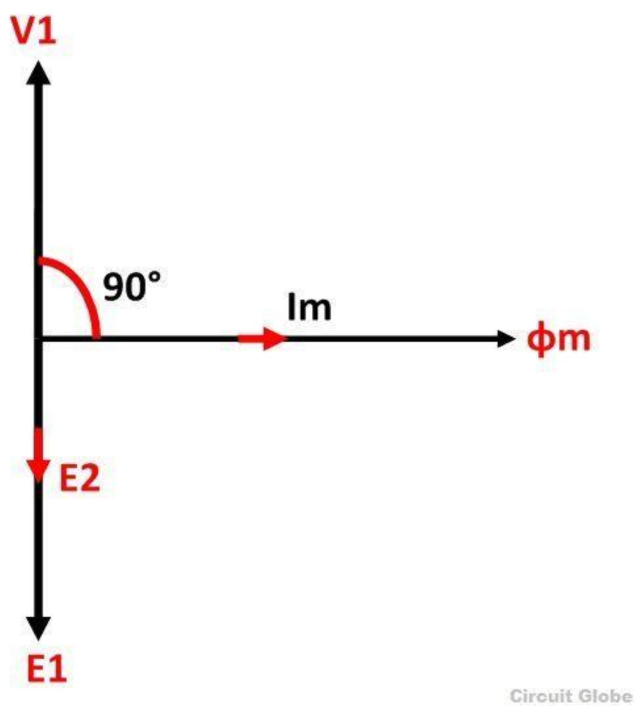
As the permeability of the core is infinite the flux of the core link with both the primary and secondary winding of the transformer.

The flux link with the primary winding induces the emf E_1 because of self-induction. The direction of the induced emf is inversely proportional to the applied voltage V_1. The emf E_2 induces in the secondary winding of the transformer because of mutual induction.

Phasor Diagram of Ideal Transformer

The phasor diagram of the ideal transformer is shown in the figure below. As the coil of the primary transformer is purely inductive the magnetizing current induces in the transformer lag 90º by the input voltage V_1.

The E_1 and E_2 are the emf induced in the primary and secondary winding of the transformer. The direction of the induced emf inversely proportional to the applied voltage.



Phasor Diagram of an Ideal Transformer

Point to Remember

The input energy of the transformer is equal to its output energy. The power loss in the ideal transformer becomes zero.

Types of Losses in a Transformer

There are various types of losses in the transformer such as iron loss, copper loss, hysteresis loss, eddy current loss, stray loss, and dielectric loss. The hysteresis losses occur because of the variation of the magnetization in the core of the transformer and the copper loss occurs because of the transformer winding resistance.

The various types of losses are explained below in detail.

Contents:

- Iron Losses
- Hysteresis Loss
- Eddy Current Loss
- Copper Loss Or Ohmic Loss
- Stray Loss
- Dielectric Loss

Iron Losses

Iron losses are caused by the alternating flux in the core of the transformer as this loss occurs in the core it is also known as **Core loss**. Iron loss is further divided into hysteresis and eddy current loss.

Hysteresis Loss

The core of the transformer is subjected to an alternating magnetizing force, and for each cycle of emf, a hysteresis loop is traced out. Power is dissipated in the form of heat known as hysteresis loss and given by the equation shown below:

$$P_h = K\eta B_{max}^{1.6} f V \quad \text{watts}$$

Where

- $K\eta$ is a proportionality constant which depends upon the volume and quality of the material of the core used in the transformer,
- f is the supply frequency,
- Bmax is the maximum or peak value of the flux density.

The iron or core losses can be minimized by using silicon steel material for the construction of the core of the transformer.

Eddy Current Loss

When the flux links with a closed circuit, an emf is induced in the circuit and the current flows, the value of the current depends upon the amount of emf around the circuit and the resistance of the circuit.

Since the core is made of conducting material, these EMFs circulate currents within the body of the material. These circulating currents are called **Eddy Currents**. They will occur when the conductor experiences a changing magnetic field. As these currents are not responsible for doing any useful work, and it produces a loss (I^2R loss) in the magnetic material known as an **Eddy Current Loss**. The eddy current loss is minimized by making the core with thin laminations.

The equation of the eddy current loss is given as:

$$P_e = K_e B_m^2 t^2 f^2 V \quad \text{watts}$$

Where,

- K_e – coefficient of eddy current. Its value depends upon the nature of magnetic material like volume and resistivity of core material, the thickness of laminations
- B_m – maximum value of flux density in wb/m^2
- T – thickness of lamination in meters
- F – frequency of reversal of the magnetic field in Hz
- V – the volume of magnetic material in m^3

Copper Loss Or Ohmic Loss

These losses occur due to ohmic resistance of the transformer windings. If I_1 and I_2 are the primary and the secondary current. R_1 and R_2 are the resistance of primary and secondary winding then the copper losses occurring in the primary and secondary winding will be $I_1^2R_1$ and $I_2^2R_2$ respectively.

Therefore, the total copper losses will be

$$P_c = I_1^2R_1 + I_2^2R_2$$

These losses varied according to the load and known hence it is also known as variable losses. Copper losses vary as the square of the load current.

Stray Loss

The occurrence of these stray losses is due to the presence of leakage field. The percentage of these losses are very small as compared to the iron and copper losses so they can be neglected.

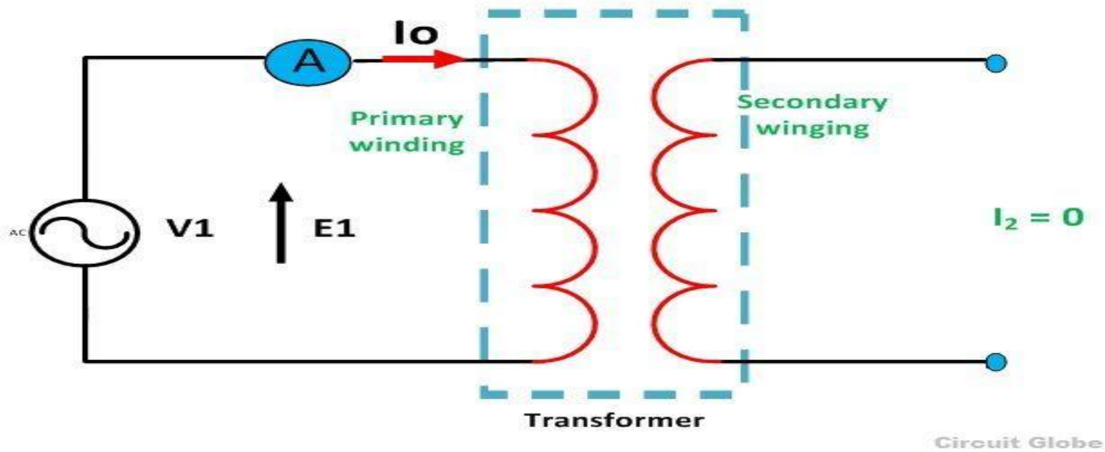
Dielectric Loss

Dielectric loss occurs in the insulating material of the transformer that is in the oil of the transformer, or in the solid insulations. When the oil gets deteriorated or the solid insulation gets damaged, or its quality decreases, and because of this, the efficiency of the transformer gets affected.

Transformer on No Load Condition

When the transformer is operating at no load, the secondary winding is open-circuited, which means there is no load on the secondary side of the transformer and, therefore, current in the secondary will be zero. While primary winding carries a small current I_0 called no-load current which is **2 to 10% of the rated current**.

This current is responsible for supplying the iron losses (hysteresis and eddy current losses) in the core and a very small amount of copper losses in the primary winding. The angle of lag depends upon the losses in the transformer. The power factor is very low and varies from **0.1 to 0.15**.



The no-load current consists of two components:

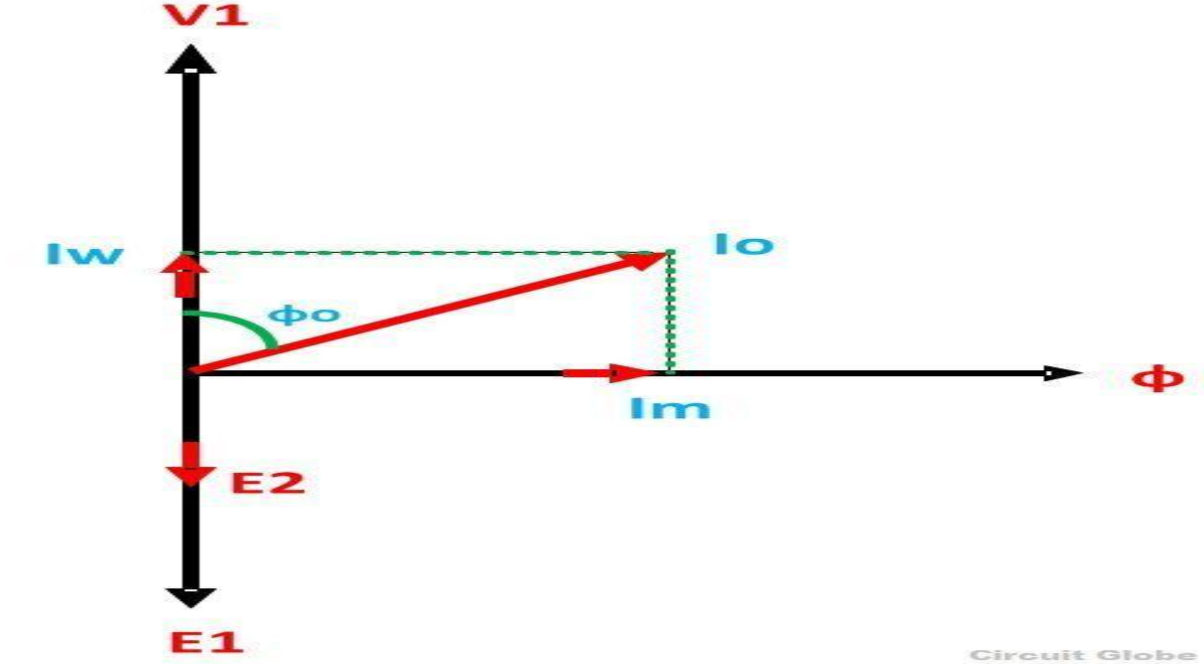
- **Reactive or magnetizing component I_m**
(It is in quadrature with the applied voltage V_1. It produces flux in the core and does not consume any power).

- **Active or power component I_w**, also know as a working component
(It is in phase with the applied voltage V_1. It supplies the iron losses and a small amount of primary copper loss).

The following steps are given below to draw the phasor diagram:

1. The function of the magnetizing component is to produce the magnetizing flux, and thus, it will be in phase with the flux.
2. Induced emf in the primary and the secondary winding lags the flux ϕ by 90 degrees.
3. The primary copper loss is neglected, and secondary current losses are zero as
$I_2 = 0$.
Therefore, the current I_0 lags behind the voltage vector V_1 by an angle ϕ_0 called the no-load power factor angle and is shown in the phasor diagram above.
4. The applied voltage V_1 is drawn equal and opposite to the induced emf E_1 because the difference between the two, at no load, is negligible.
5. Active component I_w is drawn in phase with the applied voltage V_1.

6. The phasor sum of magnetizing current I_m and the working current I_w gives the no-load current I_0.



7. From the phasor diagram drawn above, the following conclusions are made

Working component $I_w = I_0 \mathrm{Cos}\varphi_0$

No load current $I_0 = \sqrt{I_w^2 + I_m^2}$

Magnetizing component $I_m = I_0 \ \mathrm{Sin}\varphi_0$

Power factor $\mathrm{Cos}\ \varphi_0 = \frac{I_w}{I_0}$

No load power input $P_0 = V_1 I_0 \mathrm{Cos}\varphi_0$

This is all about transformer in no-load condition.

Transformer On Load Condition

When the transformer is on the loaded condition, the secondary of the transformer is connected to load. The load can be resistive, inductive or capacitive. The current I_2 flows through the secondary winding of the transformer. The magnitude of the secondary current depends on the terminal voltage V_2 and the load impedance. The phase angle between the secondary current and voltage depends on the nature of the load.

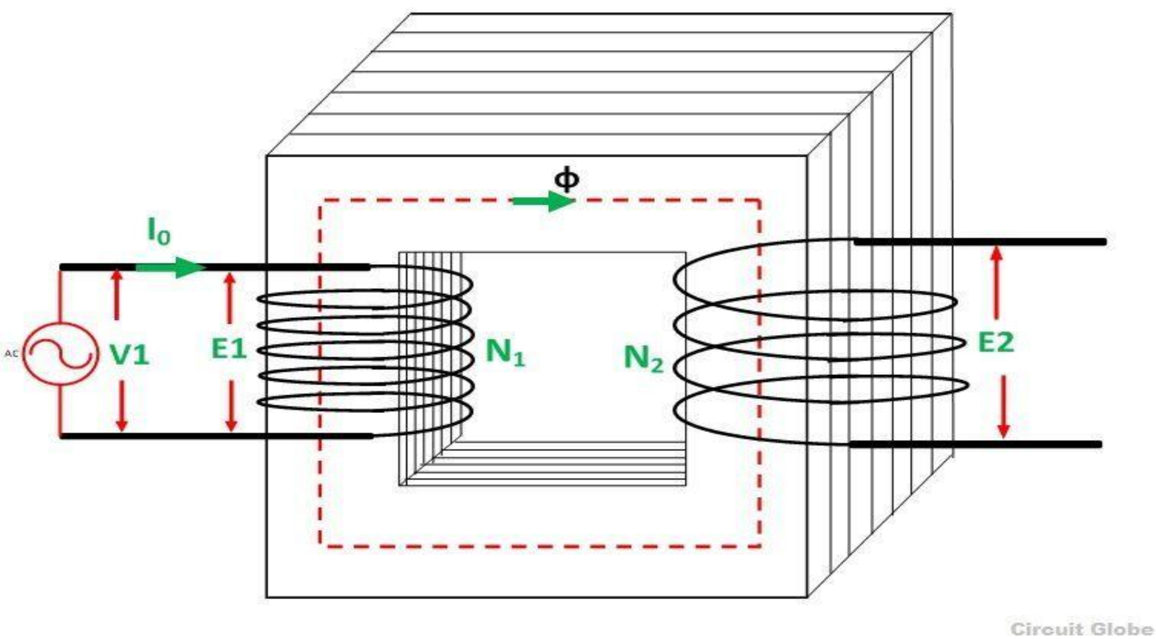
Contents:

- Operation of the Transformer on Load Condition
- Phasor Diagram of Transformer on Inductive Load
- Steps to draw the phasor diagram
- Phasor Diagram of Transformer on Capacitive Load
- Steps to draw the phasor diagram at capacitive load

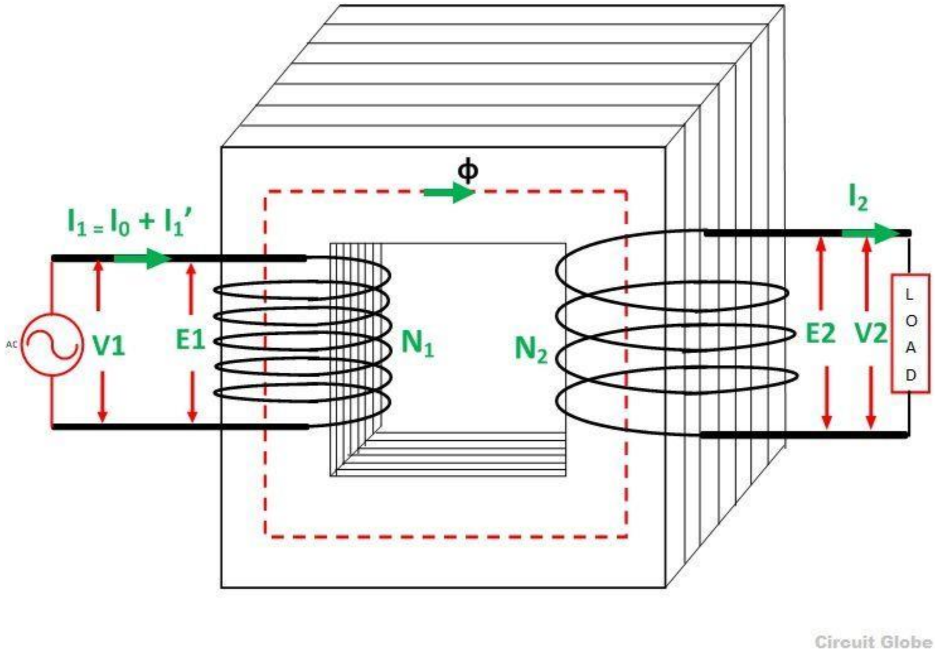
Operation of the Transformer on Load Condition

The Operation of the Transformer on Load Condition is explained below:

- When the secondary of the transformer is kept open, it draws the no-load current from the main supply. The no-load current induces the magnetomotive force N_0I_0 and this force set up the flux Φ in the core of the transformer. The circuit of the transformer at no load condition is shown in the figure below:



  - When the load is connected to the secondary of the transformer, I_2 current flows through their secondary winding. The secondary current induces the magnetomotive force N_2I_2 on the secondary winding of the transformer. This force set up the flux φ_2 in the transformer core. The flux φ_2 opposes the flux φ, according to **Lenz's law**.



- As the flux φ_2 opposes the flux φ, the resultant flux of the transformer decreases and this flux reduces the induced EMF E_1. Thus, the strength of the V_1 is more than E_1 and an additional primary current I'_1 drawn from the main supply.
  The additional current is used for restoring the original value of the flux in the core of the transformer so that $V_1 = E_1$. The primary current I'_1 is in phase opposition with the secondary current I_2. Thus, it is called the **primary counter-balancing current**.
- The additional current I'_1 induces the magnetomotive force $N_1I'_1$. And this force set up the flux φ'_1. The direction of the flux is the same as that of the φ and it cancels the flux φ_2 which induces because of the MMF N_2I_2

Now, $N_1I_1' = N_2I_2$

$$I_1' = \left(\frac{N_2}{N_1}\right) I_2 = KI_2$$

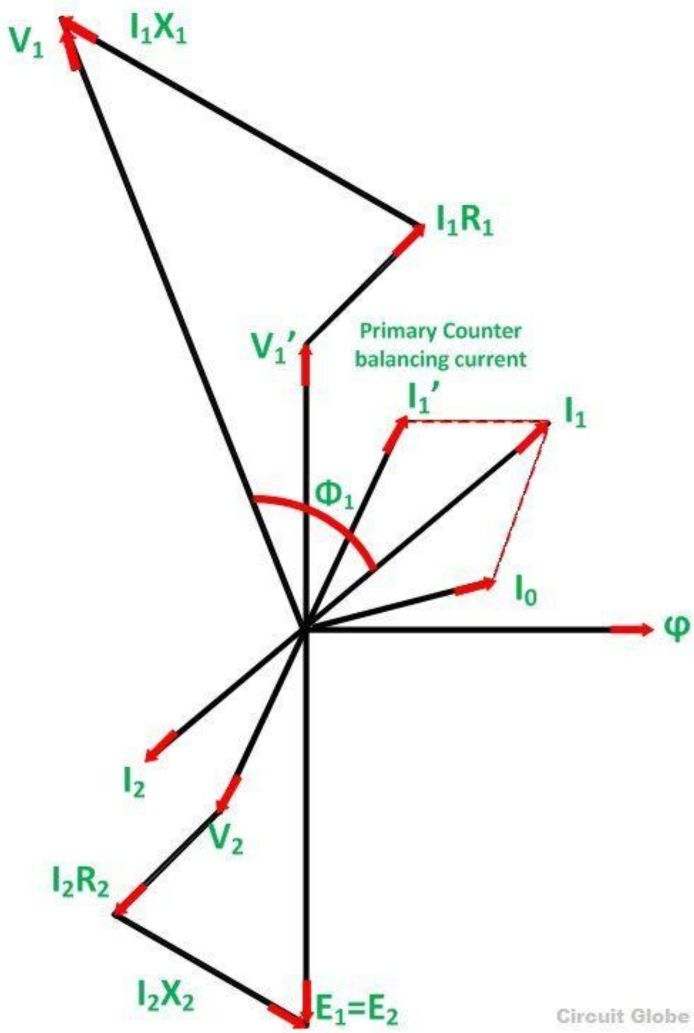
Therefore,

- The phase difference between V_1 and I_1 gives the power factor angle ϕ_1 of the primary side of the transformer.
- The power factor of the secondary side depends upon the type of load connected to the transformer.
- If the load is inductive as shown in the above phasor diagram, the power factor will be lagging, and if the load is capacitive, the power factor will be leading. The total primary current I_1 is the vector sum of the currents I_0 and I_1'. i.e

$$\overline{I_1} = \overline{I_0} + \overline{I_1'}$$

Phasor Diagram of Transformer on Inductive Load

The phasor diagram of the actual transformer when it is loaded inductively is shown below:



Phasor Diagram of the Transformer on Inductive Load

Steps to draw the phasor diagram

- Take flux ϕ, a reference
- Induces emf E_1 and E_2 lags the flux by 90 degrees.
- The component of the applied voltage to the primary equal and opposite to induced emf in the primary winding. E_1 is represented by V_1'.
- Current I_0 lags the voltage V_1' by 90 degrees.
- The power factor of the load is lagging. Therefore current I_2 is drawn lagging E_2 by an angle ϕ_2.
- The resistance and the leakage reactance of the windings result in a voltage drop, and hence secondary terminal voltage V_2 is the phase difference of E_2 and voltage drop.

$V_2 = E_2$ – voltage drops
$I_2 R_2$ is in phase with I_2 and I_2X_2 is in quadrature with I_2.

- The total current flowing in the primary winding is the phasor sum of I_1' and I_0.
- Primary applied voltage V_1 is the phasor sum of V_1' and the voltage drop in the primary winding.
- Current I_1' is drawn equal and opposite to the current I_2

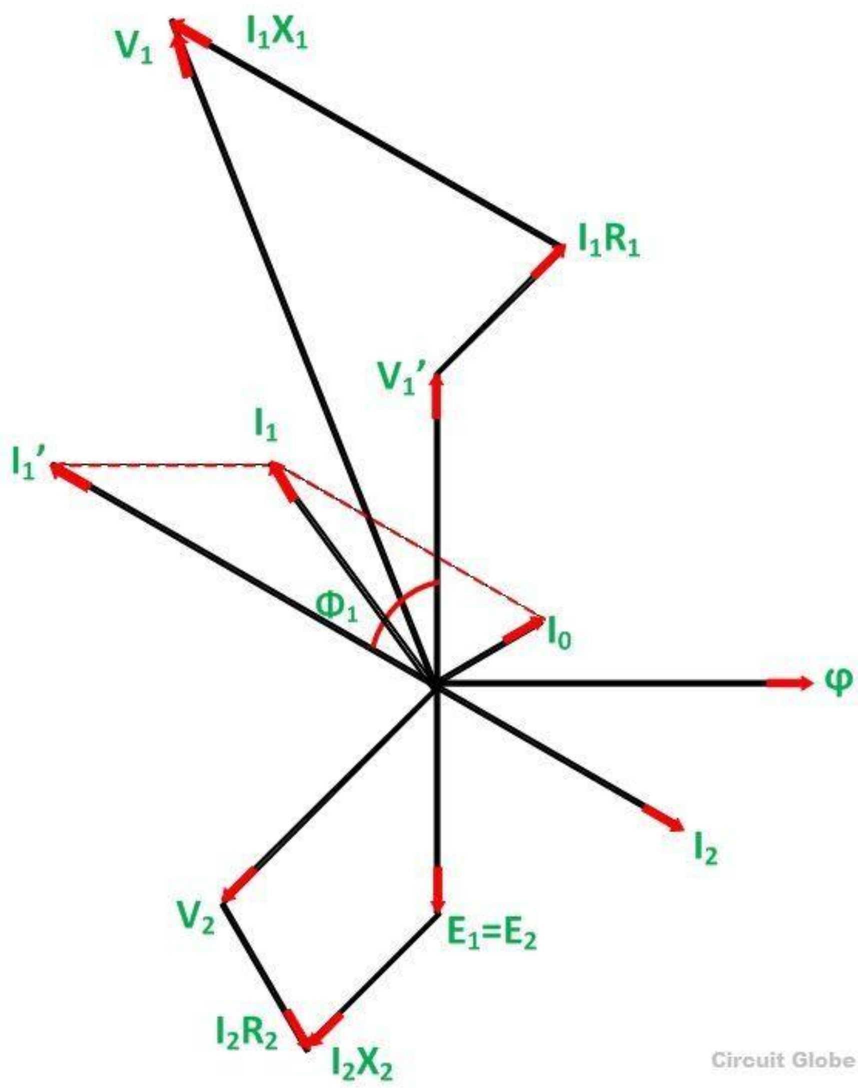
$V_1 = V_1' +$ voltage drop
I_1R_1 is in phase with I_1 and I_1X_I is in quadrature with I_1.

- The phasor difference between V_1 and I_1 gives the power factor angle ϕ_1 of the primary side of the transformer.
- The power factor of the secondary side depends upon the type of load connected to the transformer.
- If the load is inductive as shown in the above phasor diagram, the power factor will be lagging, and if the load is capacitive, the power factor will be leading. Where I_1R_1 is the resistive drop in the primary windings I_2X_2 is the reactive drop in the secondary winding

Similarly

Phasor Diagram of Transformer on Capacitive Load

The Transformer on the Capacitive load (leading power factor load) is shown below in the phasor diagram.



Phasor Diagram of the Transformer on Capacitive Load

Steps to draw the phasor diagram at capacitive load

- Take flux ϕ a reference
- Induces emf E_1 and E_2 lags the flux by 90 degrees.
- The component of the applied voltage to the primary equal and opposite to induced emf in the primary winding. E_1 is represented by V_1'.
- Current I_0 lags the voltage V_1' by 90 degrees.
- The power factor of the load is leading. Therefore current I_2 is drawn leading E_2
- The resistance and the leakage reactance of the windings result in a voltage drop, and hence secondary terminal voltage V_2 is the phasor difference of E_2 and voltage drop.

$V_2 = E_2$ – voltage drops
$I_2 R_2$ is in phase with I_2 and I_2X_2 is in quadrature with I_2.

- Current I_1' is drawn equal and opposite to the current I_2
- The total current I_1 flowing in the primary winding is the phasor sum of I_1' and I_0.
- Primary applied voltage V_1 is the phasor sum of V_1' and the voltage drop in the primary winding.

$V_1 = V_1$' + voltage drop
I_1R_1 is in phase with I_1 and I_1X_I is in quadrature with I_1.

- The phasor difference between V_1 and I_1 gives the power factor angle ϕ_1 of the primary side of the transformer.
- The power factor of the secondary side depends upon the type of load connected to the transformer.

This is all about the phasor diagram on various loads.

Resistance and Reactance of the Transformer

The Resistance of the transformer is defined as the internal resistance of both primary and secondary windings. In an actual transformer, the primary and the secondary windings have some resistance represented by R_1 and R_2 and the reactances by X_1 and X_2. Let K be the transformation ratio.

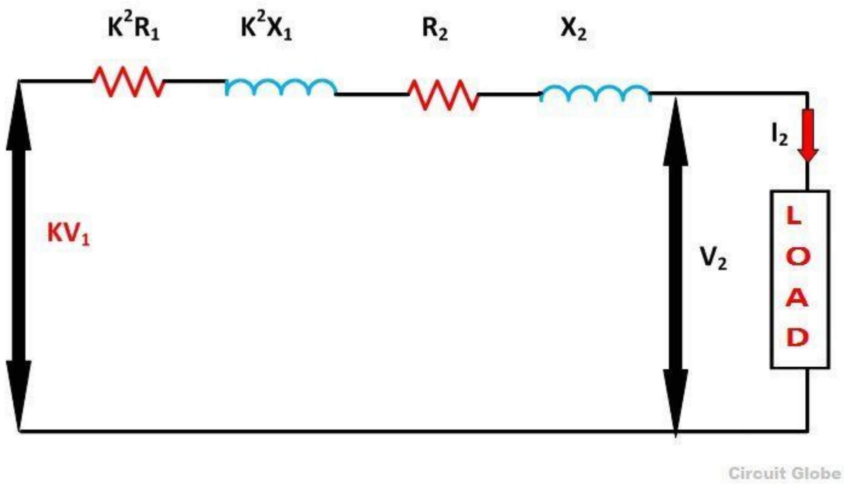
To make the calculations easy the resistances and reactances can be transferred to either side, which means either all the primary terms are referred to the secondary side, or all the secondary terms are referred to the primary side.

The resistive and the reactive drops in the primary and secondary side are represented as follows

- Resistive drop in the secondary side = I_2R_2
- Reactive drop in the secondary side = I_2X_2
- Resistive drop in the primary side = I_1R_1
- Reactive drop in the primary side = I_1X_1

Primary Side Referred to Secondary Side

Since the transformation ratio is K, the primary resistive and reactive drop as referred to secondary side will be K times, i.e., K I_1R_1 and K I_1X_1 respectively. If I_1 is substituted equal to KI_2 then we have primary resistive, and reactive drop referred to secondary side equal to $K^2I_2R_1$ and $K^2I_2X_1$ respectively.



The total resistive drop in a transformer

$$K^2 I_2 R_1 + I_2 R_2 = I_2 (K^2 R_1 + R_2)$$

The total reactive drop in a transformer

$$K^2 I_2 X_1 + I_2 X_2 = I_2 (K^2 X_1 + X_2)$$

The terms

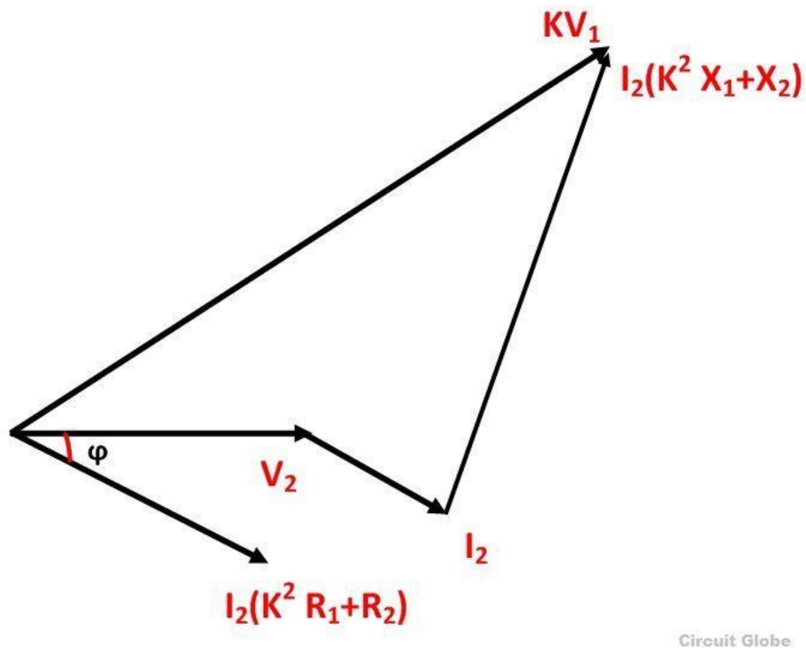
$(K^2 R_1 + R_2)$ And $(K^2 X_1 + X_2)$

represent the equivalent resistance and reactance of the transformer referred to the secondary side.

Where

$$\left(K^2R_1 + R_2\right) = R_{02} \text{ And } (K^2X_1 + X_2) = X_{02}$$

Thus,



<u>From the phasor diagram shown above the equation can be formed as</u>

$$KV_1 = \sqrt{(V_2 + I_2R_{02}Cos\varphi + I_2X_{02}Sin\varphi)}^2 + \sqrt{(I_2X_{02}Cos\varphi - I_2X_{02}Sin\varphi}^2$$

Where V_2 is the secondary terminal voltage and I_2 is secondary current lagging behind the terminal voltage V_2 by an angle ϕ.

Since the term

$$(I_2X_{02}Cos\varphi - I_2X_{02}Sin\varphi)$$

is very small and is neglected as compared to the term

$$(V_2 + I_2R_{02}Cos\varphi - I_2X_{02}Sin\varphi)$$

Now the equation becomes

$$KV_1 = V_2 + I_2R_{02}Cos\varphi + I_2R_{02}Sin\varphi$$

Or

$$V_2 = KV_1 - I_2R_{02}Cos\varphi - I_2X_{02}Sin\varphi \text{..........(1)}$$

Where V_1 is the applied voltage to the primary winding

If the load on the secondary side of the transformer is purely resistive then $\phi = 0$ and the equation (1) becomes

$$V_2 = KV_1 - I_2R_{02}$$

If the load on the secondary side of the transformer is capacitive then ϕ should be taken as negative, and the equation (1) becomes

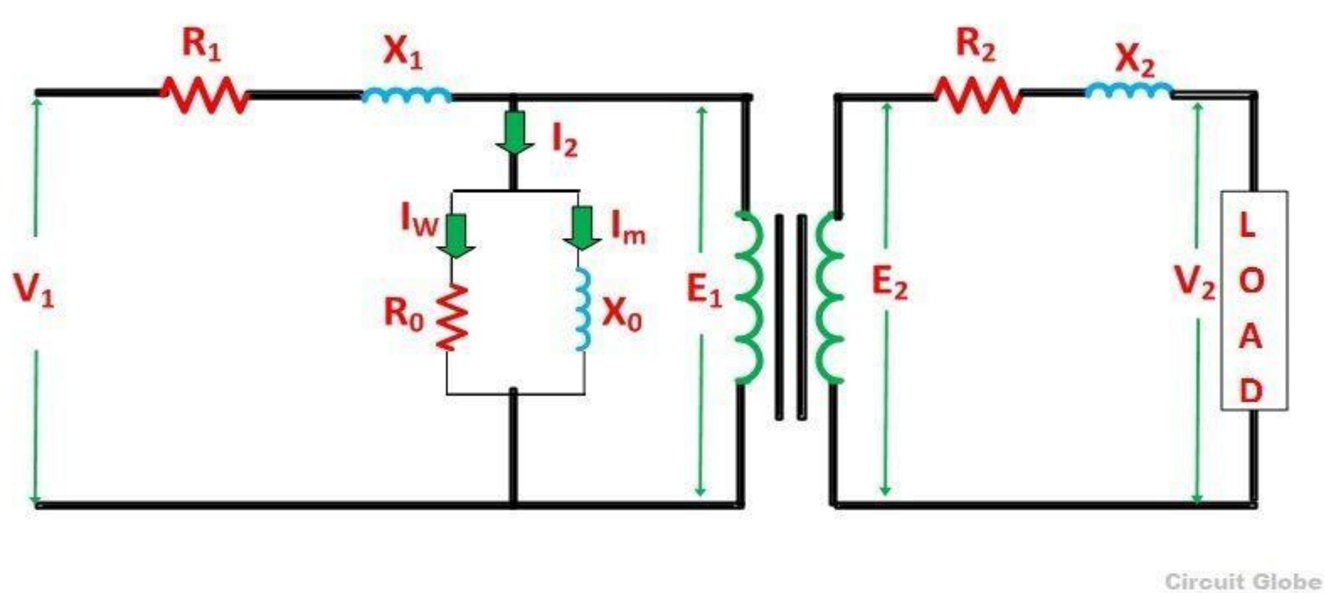
$$V_2 = KV_1 - I_2R_{02}Cos\varphi + I_2X_{02}Sin\varphi$$

Therefore this will be the load voltage.

Equivalent Circuit of a Transformer

The equivalent circuit diagram of any device can be quite helpful in the pre-determination of the behavior of the device under the various condition of operation. It is simply the circuit representation of the equation describing the performance of the device.

The simplified equivalent circuit of a transformer is drawn by representing all the parameters of the transformer either on the secondary side or on the primary side. The equivalent circuit diagram of the transformer is shown below:



EQUIVALENT CIRCUIT DIAGRAM OF A TRANSFORMER

Contents:

- Equivalent Circuit When all the Quantities are Referred to Primary side
- Equivalent Circuit When all the Quantities are Referred to Secondary side

Let the equivalent circuit of a transformer having the transformation ratio **$K = E_2/E_1$**

The induced emf E_1 is equal to the primary applied voltage V_1 less primary voltage drop. This voltage causes current I_0 no-load current in the primary winding of the transformer. The value of no-load current is very small, and thus, it is neglected.

Hence, $I_1 = I_1'$. The no-load current is further divided into two components called **magnetizing current** (I_m) and **working current** (I_w).

These two components of no-load current are due to the current drawn by a non-inductive resistance R_0 and pure reactance X_0 having voltage E_1 or (V_1 – primary voltage drop).

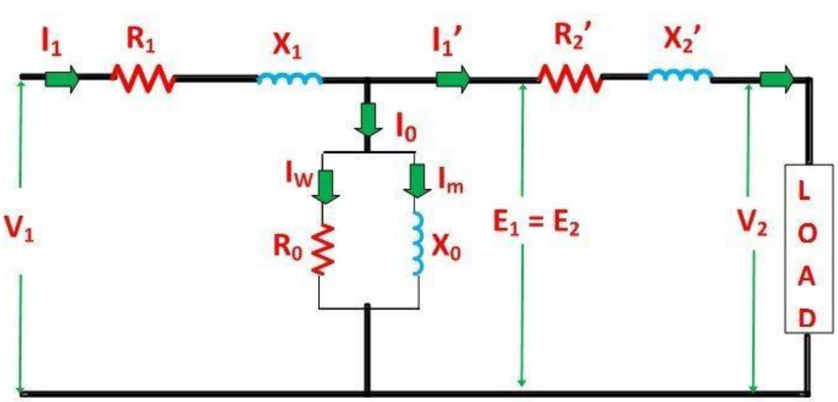
The secondary current I_2 is

$$I_2 = \frac{I_1'}{K} = \frac{I_1 - I_0}{K}$$

The terminal voltage V_2 across the load is equal to the induced emf E_2 in the secondary winding less voltage drop in the secondary winding.

Equivalent Circuit when all the quantities are referred to Primary side

In this case, to draw the equivalent circuit of the transformer all the quantities are to be referred to the primary as shown in the figure below:



Circuit Diagram of Transformer when all the Secondary Quantities are Referred to Primary Side

The following are the values of resistance and reactance given below

Secondary resistance referred to the primary side is given as:

$$R_2' = \frac{R_2}{K^2}$$

The equivalent resistance referred to the primary side is given as:

$$R_{ep} = R_1 + R_2'$$

Secondary reactance referred to the primary side is given as:

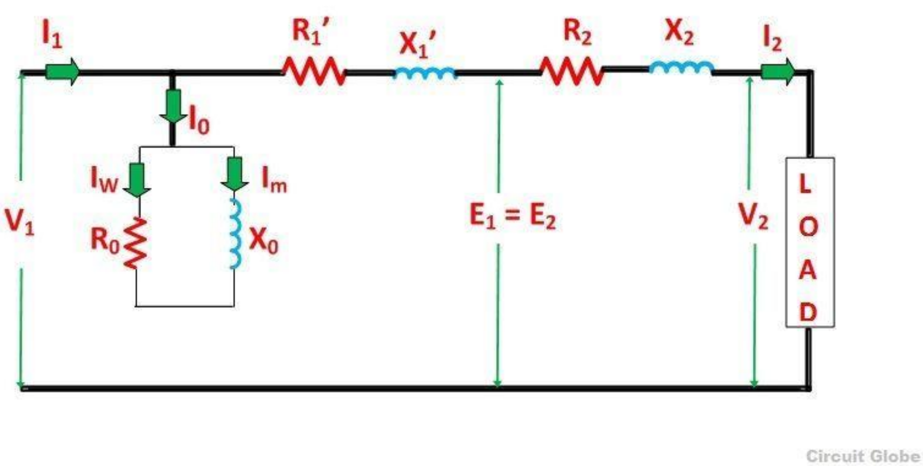
$$X_2' = \frac{X_2}{K^2}$$

The equivalent reactance referred to the primary side is given as:

$$X_{ep} = X_1 + X_2'$$

Equivalent Circuit when all the quantities are referred to Secondary side

The equivalent circuit diagram of the transformer is shown below when all the quantities are referred to the secondary side.



Circuit Diagram of Transformer When All the Primary Quantities are Referred to Secondary Side

The following are the values of resistance and reactance given below

Primary resistance referred to the secondary side is given as

$$R_1' = K^2 R_1$$

The equivalent resistance referred to the secondary side is given as

$$R_{es} = R_2 + R_1'$$

Primary reactance referred to the secondary side is given as

$$X_1' = K^2 X_1$$

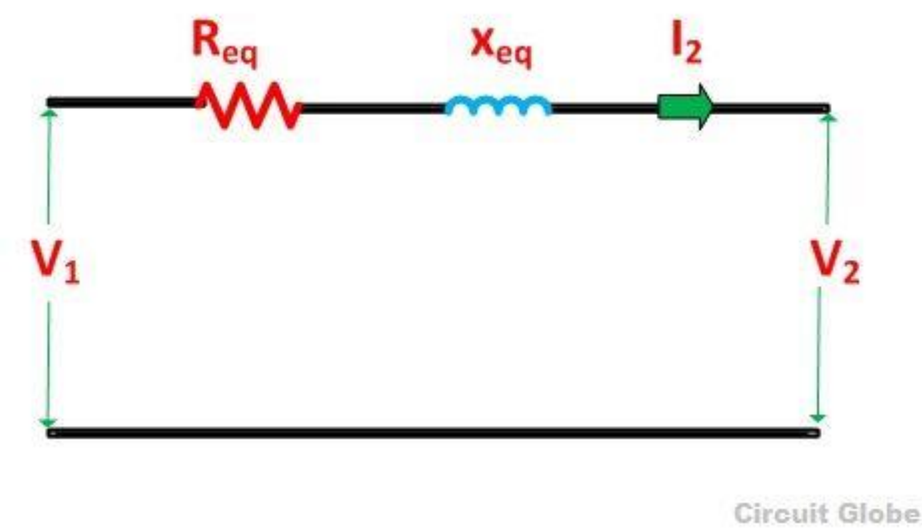
The equivalent reactance referred to the secondary side is given as

$$X_{eq} = X_2 + X_I'$$

No-load current I_0 is hardly **3 to 5%** of full load rated current, the parallel branch consisting of resistance R_0 and reactance X_0 can be omitted without introducing any appreciable error in the behavior of the transformer under the loaded condition.

Further simplification of the equivalent circuit of the transformer can be done by neglecting the parallel branch consisting of R_0 and X_0.

The simplified circuit diagram of the transformer is shown below:



Simplified Equivalent Circuit Diagram of a Transformer

This is all about the equivalent circuit of the Transformer.

Transformer Efficiency

The **Efficiency** of the transformer is defined as the ratio of useful output power to the input power. The input and output power are measured in the same unit. Its unit is either in Watts (W) or KW. **Transformer efficiency** is denoted by **η**.

$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{\text{output power}}{\text{output power} + \text{losses}}$$

$$\eta = \frac{\text{output power}}{\text{output power} + \text{iron losses} + \text{copper losses}}$$

$$\eta = \frac{V_2 I_2 \text{Cos}\varphi_2}{V_2 I_2 \text{Cos}\varphi_2 + P_i + P_c}$$

Where,

- V_2 – Secondary terminal voltage
- I_2 – Full load secondary current
- $\text{Cos}\phi_2$ – power factor of the load
- P_i – Iron losses = hysteresis losses + eddy current losses
- P_c – Full load copper losses = $I_2^2 R_{es}$

Consider, the x is the fraction of the full load. The efficiency of the transformer regarding x is expressed as

$$\eta_x = \frac{x \text{ X output}}{x \text{ X output} + P_i + x^2 P_c} = \frac{x\, V_2 I_2 \text{Cos}\varphi_2}{x\, V_2 I_2 \text{Cos}\varphi_2 + P_i + x^2 I_2^2 R_{es}}$$

The copper losses vary according to the fraction of the load.

Maximum Efficiency Condition of a Transformer

The efficiency of the transformer along with the load and the power factor is expressed by the given relation:

$$\eta = \frac{V_2 I_2 \text{Cos}\varphi_2}{V_2 I_2 \text{Cos}\varphi_2 + P_i + I_2^2 R_{es}} = \frac{V_2 \text{Cos}\varphi_2}{V_2 \text{Cos}\varphi_2 + P_i/I_2 + I_2 R_{es}} \dots\dots\dots (1)$$

The value of the terminal voltage V_2 is approximately constant. Thus, for a given power factor the Transformer efficiency depends upon the load current I_2. In equation (1), the numerator is constant and the transformer efficiency will be maximum if the denominator with respect to the variable I_2 is equated to zero.

$$\frac{d}{dI_2} = \left(V_2 \ Cos\varphi_2 + \frac{P_i}{I_2} + I_2 R_{es} \right) = 0 \qquad or \qquad 0 - \frac{P_i}{I_2^2} + R_{es} = 0$$

Or

$$I_2^2 R_{es} = P_i \ \ldots\ldots\ldots..(2)$$

i.e

Copper losses = Iron losses

Thus, the transformer will give the maximum efficiency when their copper loss is equal to the iron loss.

$$\eta_{max} = \frac{V_2 I_2 Cos\varphi_2}{V_2 I_2 Cos\varphi_2 + 2P_i} \qquad as\ (P_c = P_i)$$

From equation (2) the value of output current I_2 at which the transformer efficiency will be maximum is given as

$$I_2 = \sqrt{\frac{P_i}{R_{es}}}$$

If x is the fraction of full load KVA at which the efficiency of the transformer is maximum then,

Copper losses = x^2P_c(where P_c is the full load copper losses)

Iron losses = P_i

For maximum efficiency

$\mathbf{x^2 \ P_c = P_i}$
Therefore

$$x = \sqrt{\frac{P_i}{P_c}} \ldots\ldots\ldots\ldots..(3)$$

Thus, output KVA corresponding to maximum efficiency

$$\eta_{max} = x \text{ X full load KVA} \ldots \ldots \ldots \ldots (4)$$

Putting the value of x from the above equation (3) in equation (4) we will get,

$$\eta_{max} = \sqrt{\frac{P_i}{P_c}} \text{ X full load KVA}$$

$$\eta_{max} = \text{ Full load KVA X } \sqrt{\frac{\text{iron losses}}{\text{copper losses at full load}}} \ldots \ldots \ldots \ldots . (5)$$

The above equation (5) is the maximum efficiency condition of the transformer.

Voltage Regulation of a Transformer

Definition: The voltage regulation is defined as the change in the magnitude of receiving and sending voltage of the transformer. The voltage regulation determines the ability of the transformer to provide the constant voltage for variable loads.

When the transformer is loaded with continuous supply voltage, the terminal voltage of the transformer varies. The variation of voltage depends on the load and its power factor.

Mathematically, the voltage regulation is represented as:

$$\text{Voltage Regulation} = \frac{E_2 - V_2}{E_2}$$

$$\text{\% Voltage Regulation} = \frac{E_2 - V_2}{E_2} \text{ x } 100$$

where,
E_2 – secondary terminal voltage at no load
V_2 – secondary terminal voltage at full load

The voltage regulation by considering the primary terminal voltage of the transformer is expressed as,

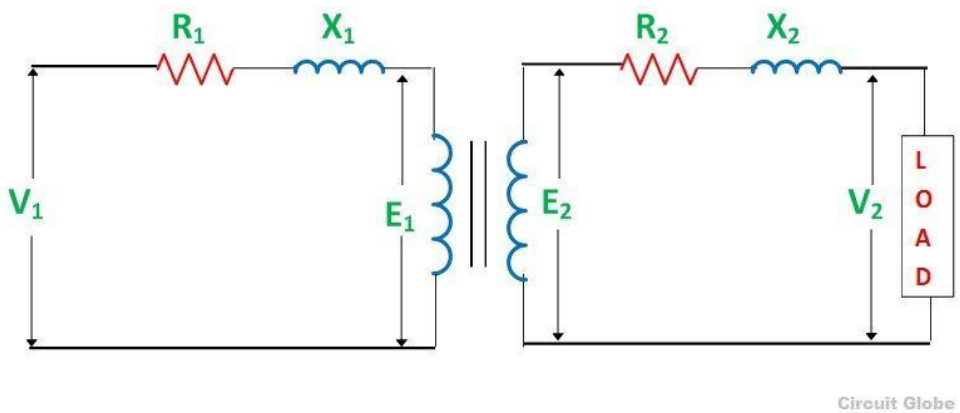
$$\% \text{ Voltage Regulation} = \frac{V_1 - E_1}{V_1} \times 100$$

Let us understand the voltage regulation by taking an example explained below:

If the secondary terminals of the transformer are open-circuited or no load is connected to the secondary terminals, the no-load current flows through it.

If the no current flows through the secondary terminals of the transformer, the voltage drops across their resistive and reactive load become zero. The voltage drop across the primary side of the transformer is negligible.

If the transformer is fully loaded, i.e., the load is connected to their secondary terminal, the voltage drops appear across it. The value of the voltage regulation should always be less for the better performance of the transformer.



From the circuit diagram shown above, the following conclusions are made

- The primary voltage of the transformer is always greater than the induced emf on the primary side. $\mathbf{V_1 > E_1}$
- The secondary terminal voltage at no load is always greater than the voltage at full load condition. $\mathbf{E_2 > V_2}$

By considering the above circuit diagram, the following equations are drawn

$$V_1 = I_1R_1\cos\varphi_1 + I_1X_1\sin\varphi_1 + E_1$$

$$E_2 = I_2R_2\cos\varphi_2 + I_2X_2\sin\varphi_2 + V_2$$

The approximate expression for the no-load secondary voltage for the different types of the load is

$$E_2 = I_2R_{02}Cos\varphi_2 + I_2X_{02}Sin\varphi_2 + V_2$$

OR

$$E_2 - V_2 = I_2R_{02}Cos\varphi_2 + I_2X_{02}Sin\varphi_2$$

OR

$$\frac{E_2 - V_2}{E_2} \text{ x } 100 = \frac{I_2R_{02}}{E_2} \text{ x } 100 \text{ } Cos\varphi_2 + \frac{I_2X_{02}}{E_2} \text{ x } 100 \text{ } Sin\varphi_2$$

1. For inductive load
   Where,

$\frac{I_2R_{02}}{E_2}$ x 100 is a percentage resistance drop

$\frac{I_2X_{02}}{E_2}$ x 100 is a percentage reactance drop

- For Capacitive load

$$E_2 = I_2R_{02}Cos\varphi_2 - I_2X_{02}Sin\varphi_2 + V_2$$

OR

$$E_2 - V_2 = I_2R_{02}Cos\varphi_2 - I_2X_{02}Sin\varphi_2$$

OR

$$\frac{E_2 - V_2}{E_2} \text{ x } 100 = \frac{I_2R_{02}}{E_2} \text{ x } 100 \text{ } Cos\varphi_2 - \frac{I_2X_{02}}{E_2} \text{ x } 100 \text{ } Sin\varphi_2$$

Open Circuit and Short Circuit Test on Transformer

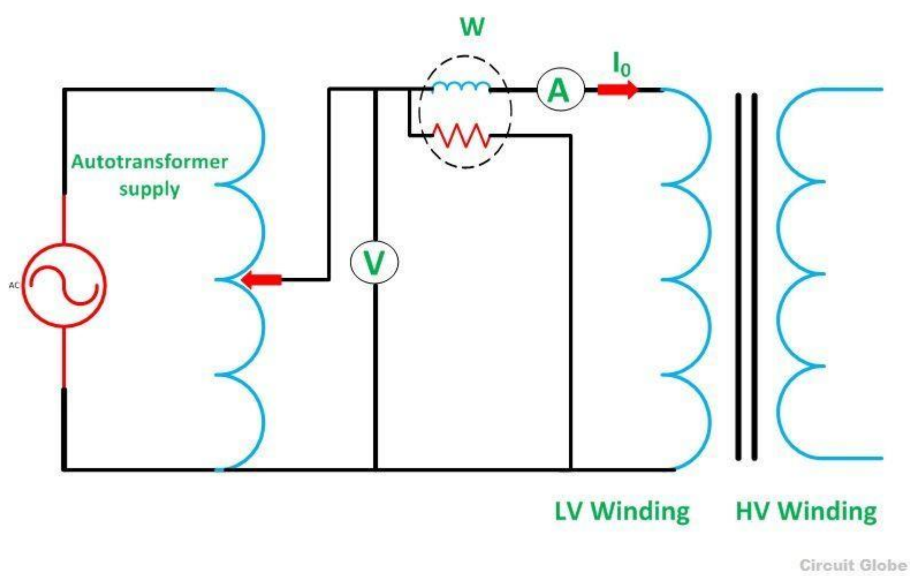
The open circuit and short circuit test are performed for determining the parameter of the transformer like their efficiency, voltage regulation, circuit constant etc. These tests are performed without the actual loading and because of this reason the very less power is required for the test. The open circuit and the short circuit test gives a very **accurate result** as compared to the full load test.

Contents:

- Open Circuit Test
- Calculation of Open Circuit Test
- Short Circuit Test
- Calculation of Short Circuit Test

Open Circuit Test

The purpose of the open-circuit test is to determine the no-load current and losses of the transformer because of which their no-load parameters are determined. This test is performed on the primary winding of the transformer. The wattmeter, ammeter and the voltage are connected to their primary winding. The nominal rated voltage is supplied to their primary winding with the help of the ac source.



Circuit Diagram of Open Circuit Test on Transformer

The secondary winding of the transformer is kept open, and the voltmeter is connected to their terminal. This voltmeter measures the **secondary induced voltage**. As the secondary of the transformer is open, thus no-load current flows through the primary winding.

The value of no-load current is very small as compared to the full rated current. The copper loss occurs only on the primary winding of the transformer because the secondary winding is open. The reading of the wattmeter only represents the core and iron losses. The core loss of the transformer is the same for all types of loads.

Calculation of open-circuit test

Let,

- W_0 – wattmeter reading
- V_1 – voltmeter reading
- I_0 – ammeter reading

Then the iron loss of the transformer $P_i = W_0$ and

$$W_0 = V_1 I_0 \cos\varphi_0 \quad \text{.............}(1)$$

The no-load power factor is

$$\cos\varphi_0 = \frac{W_0}{V_1 I_0}$$

Working component I_w is

$$I_w = \frac{W_0}{V_1} \ldots \ldots \ldots \ldots (2)$$

Putting the value of W_0 from the equation (1) in equation (2) you will get the value of the working component as

$$I_w = I_0 \cos\varphi_0$$

Magnetizing component is

$$I_m = \sqrt{I_0^2 - I_w^2}$$

No-load parameters are given below:

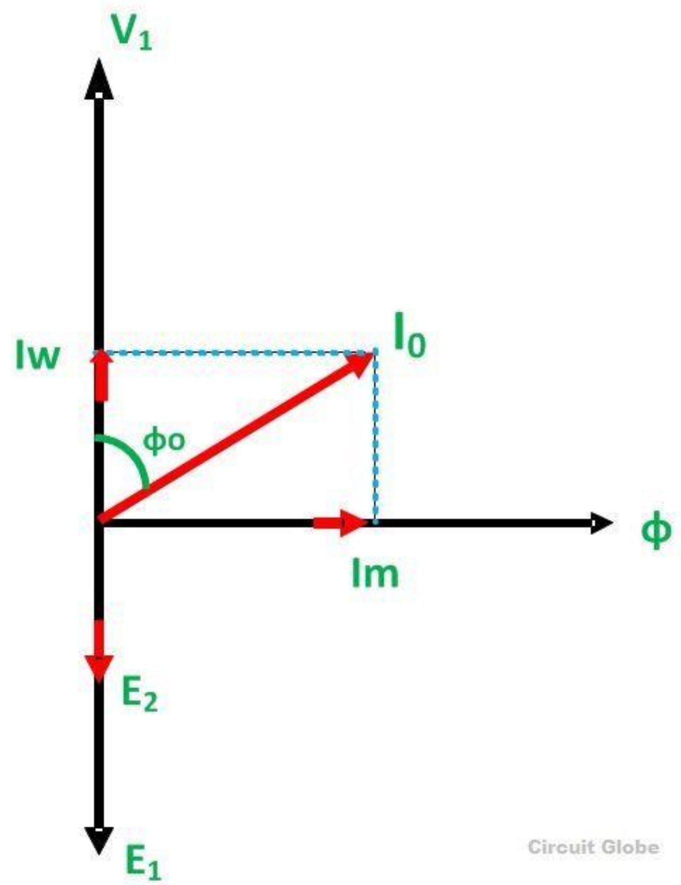
Equivalent exciting resistance is

$$R_0 = \frac{V_1}{I_w}$$

Equivalent exciting reactance is

$$X_0 = \frac{V_1}{I_m}$$

The phasor diagram of the transformer at no load or when an open circuit test is performed is shown below



Phasor Diagram of Open Circuit Test

The iron losses measured by the open circuit test is used for calculating the efficiency of the transformer.

Short Circuit Test

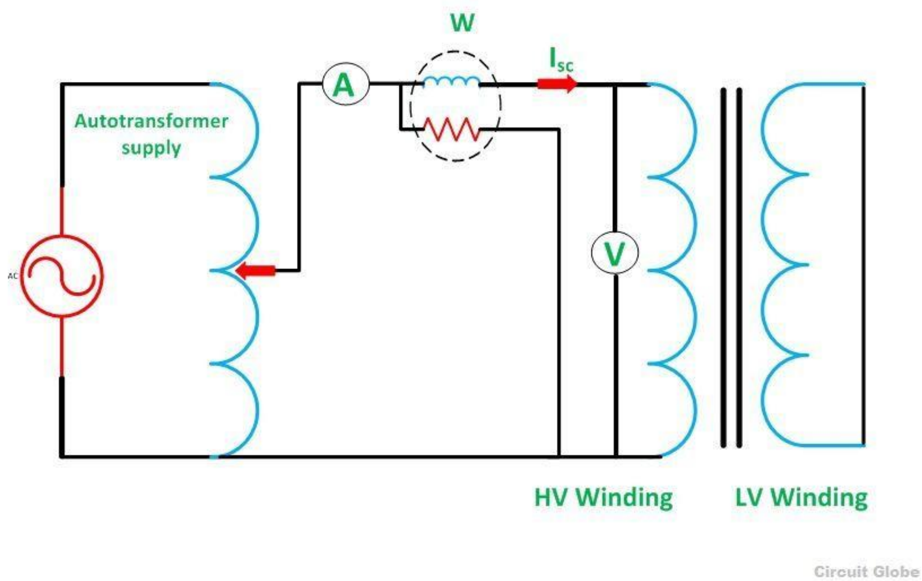
The short circuit test is performed for determining the below mention parameter of the transformer.

- It determines the copper loss occur on the full load. The copper loss is used for finding the efficiency of the transformer.
- The equivalent resistance, impedance, and leakage reactance are known by the short circuit test.

The short circuit test is performed on the secondary or high voltage winding of the transformer. The measuring instrument like wattmeter, voltmeter and ammeter are connected to the high voltage winding of the transformer. Their primary winding is short-circuited by the help of thick strip or ammeter which is connected to its terminal.

The low voltage source is connected across the secondary winding because of which the full load current flows from both the secondary and the primary winding of the transformer. The full load current is measured by the ammeter connected across their secondary winding.

The circuit diagram of the short circuit test is shown below:



Circuit Diagram of Short Circuit Test on Transformer

The low voltage source is applied across the secondary winding, which is approximately **5 to 10%** of the normal rated voltage. The flux is set up in the core of the transformer. The magnitude of the flux is small as compared to the normal flux.

The iron loss of the transformer depends on the flux. It is less occur in the short circuit test because of the low value of flux. The reading of the wattmeter only determines the copper loss occurred, in their windings. The voltmeter measures the voltage applied to their high voltage winding. The secondary current induces in the transformer because of the applied voltage.

Calculation of Short Circuit Test

Let,

- W_c – Wattmeter reading
- V_{2sc} – voltmeter reading
- I_{2sc} – ammeter reading

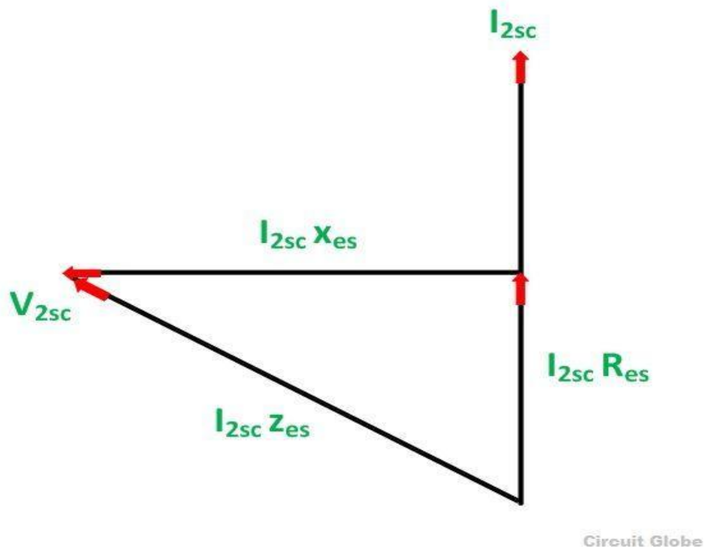
Then the full load copper loss of the transformer is given by

$$P_c = \left(\frac{I_{2fl}}{I_{2sc}}\right)^2 W_c \qquad \text{And} \qquad I_{2sc}^2 R_{es} = W_c$$

Equivalent resistance referred to the secondary side is

$$R_{es} = \frac{W_c}{I_{2sc}^2}$$

The phasor diagram of the short circuit test of the transformer is shown below



Phasor Diagram of Short Circuit Test

From the phasor diagram

$$I_{2sc}Z_{es} = V_{2sc}$$

Equivalent impedance referred to the secondary side is given by

$$Z_{es} = \frac{V_{2sc}}{I_{2sc}}$$

The equivalent reactance referred to the secondary side is given by

$$X_{es} = \sqrt{(Z_{es})^2 - (R_{es})^2}$$

The voltage regulation of the transformer can be determined at any load and power factor after knowing the values of $\mathbf{Z_{es}}$ and $\mathbf{R_{es}}$.

In the short circuit test the wattmeter record, the total losses, including core loss but the value of core loss are very small as compared to copper loss so the core loss can be neglected.