Dynamic Programming

→ Greedy & Dynamic

– both solve optimization problem

In greedy, we follow a method to find optimal e.g. dijiskra, krusked

In dynamic, we find all possible solution then select the right one

↳ we use recursive formula, (not recursion) we use iteration

. It uses principle of optimality

We can solve with sequence of decision.

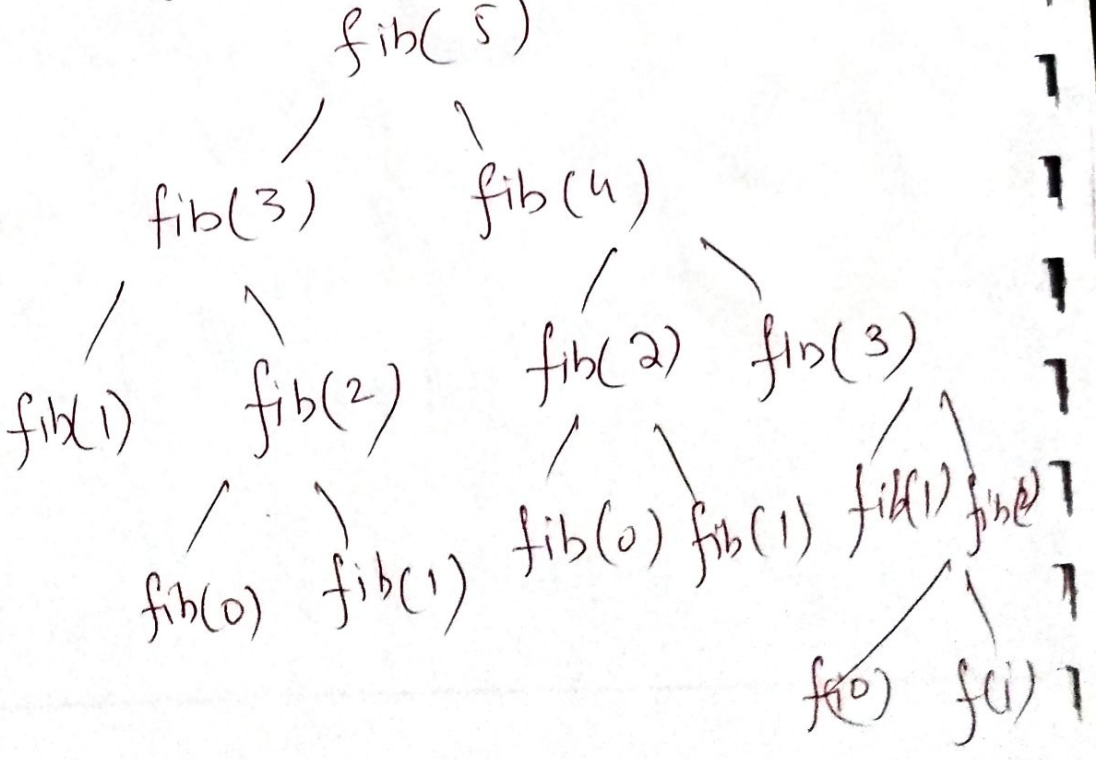
In greedy, it's taken 1 time

e.g $$fib(n) = \begin{cases} 0 & \text{if } n<0 \\ 1 & \text{if } n=1 \\ fib(n-2)+fib(n-1) & \text{if } n>1 \end{cases}$$

```
int fib(int n)
    if (n<=1)
        return n;
    return fib(n-2) + fib(n-1);
```

0 1 1 2 3 5 8 13

assume f(5) ⇒ 9 out 15 calls



fib(n-2) + fib(n-1)

calling 2 time (assume)
n-2 = n-1

$T(n) = 2T(n-1) + 1$

So $O(2^n)$

exponential time (too much)

if we see here, it is been called fib(1) many time.

we can reduce this. (by memorization)

let's take array.

-1	-1	-1	-1	-1	-1
0	1	2	3	4	5

-1	1	-1	-1	-1	-1

f 0 1 -1 -1 -1 -1

0 1 1 -1 -1 -1

0 1 1 2 -1 -1

0 1 1 2 3 -1

0 1 1 2 3 5

Only 6 call n+1 calls

So its O(n)

Polynomal to linear

Some differe is always there

we use loop / iterati.

```
int fib(int n)
    if (n <= 1)
        retu n

f[0] = 0  F(1) = 1

for (int i = 2; i <= n i++)
{   F[i] = F[i-2] + F[i-1]
}
return F[n];
```

0-1 Knapsack

$n = 8 \quad P = \{1, 2, 5, 6\}$

$n = 4 \quad w = \{2, 3, 4, 5\}$

2^n if we see all solutions

Table = V

		obj ↓ / w →	0	1	2	3	4	5	6	7	8
P_i	w_i	0	0	0	0	0	0	0	0	0	0
1	2	1	0	0	1	1	1	1	1	1	1
2	3	2	0	0	1	2	2	3	3	3	3
5	4	3	0	0	1	2	5	5	6	7	7
6	5	4	0	0	1	2	5	6	6	7	8

$V[i, w] = \max \{ V[i-1, w], V[w - w_i] + P_i \}$

$[\, i-1, \quad w - w[i] + P[i] \,]$

$V[4,1] = \max \{ V[3,1], V[3, 1-5] + 9 \}$

$[3,1], [3,-4]$

$V[4,5] = \{3,5\}, [3, 5-5+6]$

$= \max [3,5], [3,6]$

5 (5) ✓

Now we need sequence of decision

$x_1 \; x_2 \; x_3 \; x_4$

what to include what not to include

from table check max profit in last add obj 4

$= (8)$.

$8 - 6 = 2$.

check 2 in 3rd row.

check if it exist in row 2 then don't include it.

$(2) - 2 = 0$.

if not then add obj 2

$x_1 \; x_2 \; x_3 \; x_4$

$0 \; 1 \; 0 \; 1$