Course Name:Linear Algebra

Course Code: MT 104

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Topic: Introduction and System of Linear Equations

Recommended Books

- **Textbook** David C.Lay, Linear Algebra and its Applications 3rd Edition
- **Reference Book** Elementary Linear Algebra With Applications (Howard Anton)

Marks Distribution (100 Marks)

- **Quiz: 15 Marks**
- **Assignments: 5 Marks**
- **Midterm: 30 Marks**
- **Final: 50 Marks**

Course Overview

- **Linear Equations**
  - System of Linear Equations
  - Row Reduction & Echelon Form (Guass Elimination and Guass Jordan Method)
  - Vector Equation and Matrix Equation
  - Linear Independence & Linear Transformations
- **Matrix Algebra**
  - Matrix Operations & Determinants
  - Inverse of Matrices & Invertible Matrices
  - Partition Matrices & Matrix Factorization (LU Decompositions)
  - Cramers Rule
- **Vector Spaces**
  - Vector Space & Subspaces
  - Null Spaces, Column Space and Row Spaces
  - Bases
- **Eigenvalues & Eigenvectors**
- **Orthogonality, Least Square and Quadratic Forms**

Some Applications of Linear Algebra

- Graph Theory
- Cryptography
- Network Models
- Computer Graphics

Cryptography

- Encryption and decryption require the use of some secret information, usually referred to as a key.
- **Example** Let the message be

  "PREPARE TO NEGOTIATE"
- We assign a number for each letter of the alphabet.

Thus the message becomes:

P	R	E	P	A	R	E	*	T	O	*	N	E	G	O	T	I	A	T	E
16	18	5	16	1	18	5	27	20	15	27	14	5	7	15	20	9	1	20	5

- Since we are using a 3 by 3 matrix, we break the enumerated message above into a sequence of 3 by 1 vectors:

$$\begin{bmatrix}16\\18\\5\end{bmatrix} \begin{bmatrix}16\\1\\18\end{bmatrix} \begin{bmatrix}5\\27\\20\end{bmatrix} \begin{bmatrix}15\\27\\14\end{bmatrix} \begin{bmatrix}5\\7\\15\end{bmatrix} \begin{bmatrix}20\\9\\1\end{bmatrix} \begin{bmatrix}20\\5\\27\end{bmatrix}$$

By multiplying encoding matrix to this matrix we will encrypt the msg

$$\begin{bmatrix} -3 & -3 & -4 \\ 0 & 1 & 1 \\ 4 & 3 & 4 \end{bmatrix} \begin{bmatrix} 16 & 16 & 5 & 15 & 5 & 20 & 20 \\ 18 & 1 & 27 & 27 & 7 & 9 & 5 \\ 5 & 18 & 20 & 14 & 15 & 1 & 27 \end{bmatrix}$$

$$\begin{bmatrix} -122 & -123 & -176 & -182 & -96 & -91 & -183 \\ 23 & 19 & 47 & 41 & 22 & 10 & 32 \\ 138 & 139 & 181 & 197 & 101 & 111 & 203 \end{bmatrix}$$

Now to decrypt the msg we have to multiply this matrix to Inverse of encoding matrix

- The inverse of this encoding matrix, the decoding matrix, is:

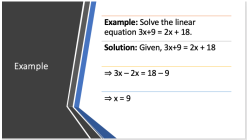
$$\begin{bmatrix} 1 & 0 & 1 \\ 4 & 4 & 3 \\ -4 & -3 & -3 \end{bmatrix}$$

Recall Some Basic Concept

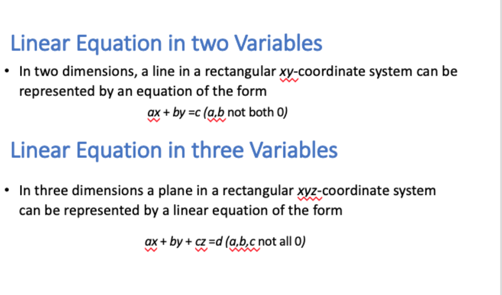
- What is a mathematical expression?
- What is an equation?
- What is a Linear Equation(in 1D, 2D, 3D)?
- Think about trignometric, logrithmic and exponential finctions??

Linear Equations and Their Solutions

- Linear Equation in One Variable



- Linear Equation in Two Variables



Linear Equations and Their Solutions

- Linear Equation in **n** Variables

An equation in n variables is said to be **linear** if it is equivalent to an equation of the form

$$\boxed{a_1x_1 + a_2x_2 + \cdots + a_nx_n = b}$$

where $x_1, x_2, \ldots, x_n$ are n distinct variables, $a_1, a_2, \ldots, a_n, b$ are constants, and at least one of the a's is not zero.

System of Linear Equations

A finite set of linear equations is called a ***system of linear equations*** or, more briefly, a ***linear system***. The variables are called ***unknowns***.

Example:

The following system has unknowns ***x*** and ***y***

$$\begin{aligned} 5x + y &= 3 \\ 2x - y &= 4 \end{aligned}$$

System of m Linear Equations in n unknowns

- A linear system of m equations in n unknowns $x_1, x_2, \dots, x_n$ is a set of equations of the form

$$\left.\begin{array}{r} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{array}\right\} \dots\dots\dots (1)$$

$a_{11}, \dots, a_{mn}$ are called the coefficients of the system.

- The set of numbers $x_1, x_2, \dots, x_m$ that satisfies all equations of system (1) is **Solution** of system.
- If all $b_j = 0; j = 1, .., m$, then the system (1) is called **Homogeneous System**.
- If all $b_j \neq 0; j = 1, .., m$, then the system (1) is called **Nonhomogeneous System**.
- Homogeneous System always has at least one solution $x_1 = x_2 = \dots = x_m = 0$ called **Trivial Solution**
- Nonhomogeneous System may have solutions or may not have solution
- If a system has at least one solution it is called **Consistent System** otherwise **Inconsistent System**

Types of Solution

- Every system of linear equation has either

i. No solution, or

ii. exactly one solution, or

iii. infinitely many solutions

- There are three ways to solve system of linear equations in two variables
- Substitution method
- Elimination method
- Graphing

Substitution Method:

$$\begin{cases} 5x-2y=-3 \\ y=3x \end{cases}$$

$$5x-2y=-3$$
$$5x-2(3x)=-3$$
$$5x-6x=-3$$
$$-x=-3$$
$$x=3$$

$$y=3x$$
$$y=3(3)$$
$$y=9$$

Solution $(3,9)$

Elimination Method:

$$\begin{cases} 3x+2y=9 \\ 2x+6y=6 \end{cases}$$

$$-2\,(3x+2y=9)$$
$$3\,(2x+6y=6)$$

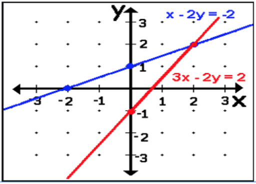
$$-6x-4y=-18$$
$$+\ 6x-18y=18$$

$$-22y=0$$
$$y=0$$

if $y = 0$, then $3x + 2(0) = 9$ and $x = 3$

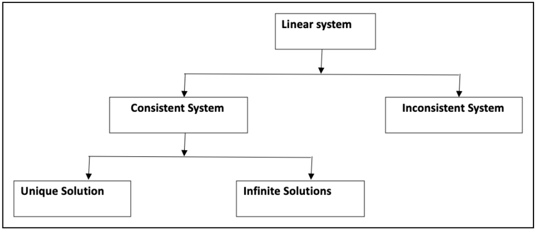
$(3,0)$

Graphing:



Consistent System & Inconsistent System

A system of equations is consistent if there is a solution, and it is inconsistent if there is no solution. However, a consistent system of equations does not mean a unique solution, that is, a consistent system of equations may have a unique solution or infinite solutions (Figure 1).



Example: A Linear System with Unique Solution: $x - y = 1$ $2x + y = 6$ Simultaneously solving both the equations give the unique solution $x=7/3$, $y= 4/3$. Geometrically, this means that the lines represented by the equations in the system intersect at the single point $(7/3,4/3)$.	**Example: A Linear System with No Solution:** $x + y = 4$ $3x + 3y = 6$ We can eliminate x from the second equation by adding −3 times the first equation to the second equation. This yields $0 = -6$ Thus the system has no solution. Geometrically, this means that the lines corresponding to the equations in the original system are parallel and distinct.

Example: A Linear System with Infinie many Solutions:

$$4x - 2y = 1$$
$$16x - 8y = 4$$

We can eliminate x from the second equation by adding −4 times the first equation to the second. This yields

$$0=0$$

There is no restriction on the values of x and y. Geometrically, this means the lines corresponding to the two equations in the original system coincide.

Now Think About

- What would be solution if you have one equation in two variables? Or two equations in three variables? Or more generally less number of equations in more variables?
- What could be situation if you have three equations in two variables? Or four equations in three variables? Or more generally more number of equations in less variables?