MT-205 Probability and Statistics

Assignment No# 04

Total Marks: 100

Question No 01: A flight from Pittsburgh to Charlotte has a 90% on-time record. From Charlotte to Jacksonville, North Carolina, the flight is on time 80% of the time. The return flight from Jacksonville to Charlotte is on time 50% of the time and from Charlotte to Pittsburgh, 90% of the time. Consider a round trip from Pittsburgh to Jacksonville on these flights. Assume the flights are independent. (10 Marks)

a) What is the probability that all 4 flights are on time?
b) What is the probability that at least 1 flight is not on time?
c) What is the probability that at least 1 flight is on time?
d) Which events are complementary?

Solution:

a) What is the probability that all 4 flights are on time?

P (all 4 flights are on time) = (0.90) (0.80) (0.50) (0.90) = 0. 324 or 32.4%

b) What is the probability that at least 1 flight is not on time?

P (at least 1 flight is not on time) = (0.10) (0.80) (0.50) (0.90) + (0.10) (0.20) (0.50) (0.90) + (0.10) (0.80) (0.20) (0.90) + (0.10) (0.20) (0.50) (0.10)

= 0.036 + 0.009 + 0.009 + 0.001

P (at least 1 flight is not on time) = 0.055

c) What is the probability that at least 1 flight is on time?

P (at least 1 flight is on time) = 1 – {(1- 0.9) (1 - 0.80) (1 - 0.50) (1 - 0.90)}

= 1 – 0.0001 = 0.999

d) Which events are complementary?

Events (a) and (b) are complementary

Question No 02: A production process produces an item. On average, 15% of all items produced are defective. Each item is inspected before being shipped, and the inspector misclassifies an item 10% of the time. What proportion of the items will be "classified as good"? What is the probability that an item is defective given that it was classified as good? (5 Marks)

Solution:

P (good) = 0.9 * 0.85+0.1(0.15) = 0.78

P (defect | good) = P (defect ∩ good) / P(good) = (0.1*0.15)/0.78 = 0.019

Question No 03: In an experiment to study the relationship of hypertension and smoking habits, the following data are collected for 180 individuals: (5 Marks)

	Non Smokers	Moderate Smokers	Heavy Smokers
Hypertension (H)	21	36	30
Non hypertension (NH)	48	26	19

If one of these individuals is selected at random, find the probability that the person is

a) experiencing hypertension, given that the person is a heavy smoker
b) a nonsmoker, given that the person is experiencing no hypertension

Solution:

a) experiencing hypertension, given that the person is a heavy smoker

Let Hs denote Heavy Smoker,

P (Hs) = total no. of heavy smokers / total no. of individuals

= 49 / 180

P (H ∩ Hs) = 30 /180

P (H | Hs) = P (H ∩ Hs) / P (Hs) = (30 /180) / (49 / 180)

= 30 / 49 = 0.6122

b) a nonsmoker, given that the person is experiencing no hypertension

Let Ns denote Non Smoker,

P(NH) = total no of Non Hypertension / total no. of individuals

= 93 / 180

P (Ns ∩ NH) = total no of non-smokers experienced Non hypertension / total

= 48 / 180

P (Ns | NH) = P (Ns ∩ NH) / P (NH) = (48 /180) / (93 / 180)

= 48 / 93 = 0.5161

Question No 04: A batch of 500 containers for frozen orange juice contains 5 that are defective. Two are selected, at random, without replacement from the batch. (10 Marks)

a) What is the probability that the second one selected is defective given that the first one was defective?
b) What is the probability that both are defective?
c) What is the probability that both are acceptable?
d) What is the probability that the third one selected is defective given that the first and second ones selected were defective?

e) What is the probability that the third one selected is defective given that the first one selected was defective and the second one selected was okay?
f) What is the probability that all three are defective?

Solution:

Let, A= first container drawn at random is defective, B= second container drawn at random is defective

C= third container drawn at random is defective

a) What is the probability that the second one selected is defective given that the first one was defective?

$P(B|A) = 4/499$

b) What is the probability that both are defective?

$P(A \cap B) = P(A).\ P(B|A) = (5/500)\ (4/499) = 1/12475 = 8.016 \times 10^{-5}$

c) What is the probability that both are acceptable?

$P(A') = 1 - P(A) = 1 - 5/500 = 495/500 = 99 / 100$

$P(B'|A') = 494/499$

$P(A' \cap B') = P(B'|A').\ P(A') = (99/100).\ (494/499) = 0.98$

d) What is the probability that the third one selected is defective given that the first and second ones selected were defective?

No. of detective containers is reduced by two

$P(C|AB) = 3/498$

e) What is the probability that the third one selected is defective given that the first one selected was defective and the second one selected was okay?

$P(C|B'A) = 4/498 = 2/249$

f) What is the probability that all three are defective?

$P(A\ B\ C) = P(C|AB).\ P(AB) = (1/166) \times 8.016 \times 10^{-5} = 4.83 \times 10^{-7}$

Question No 05: A certain federal agency employs three consulting firms (A, B, and C) with probabilities 0.40, 0.35, and 0.25, respectively. From past experience it is known that the probability of cost overruns for the firms are 0.05, 0.03, and 0.15, respectively. Suppose a cost overrun is experienced by the agency. (10 Marks)

a) What is the probability that the consulting firm involved is company C?
b) What is the probability that it is company A?

Solution:

P(A) = 0.40 P(B) = 0.35 P(C) = 0.25

Let, D denotes the event company experience cost overrun

P (D|A) = 0.05 P(D|B) = 0.03 P(D|C) = 0.15

a) What is the probability that the consulting firm involved is company C?

$$P(C|D) = P(D|C). P(C) / (P(D|A). P(A) + P(D|B). P(B) + P(D|C). P(C))$$

$$= 0.15 \text{ X } 0.25 / (0.15 \text{ X } 0.25 + 0.03 \text{ X } 0.35 + 0.05 \text{ X } 0.4)$$

$$=0.0375/0.068 = 0.551$$

b) What is the probability that it is company A?

$$P(A|D) = P(D|A). P(A) / (P(D|A). P(A) + P(D|B). P(B) + P(D|C). P(C))$$

$$= 0.05 \text{ X } 0.4 / (0.15 \text{ X } 0.25 + 0.03 \text{ X } 0.35 + 0.05 \text{ X } 0.4)$$

$$=0.02/0.068 = 0.294$$

Question No 06: A construction company employs two sales engineers. Engineer 1 does the work of estimating cost for 70% of jobs bid by the company. Engineer 2 does the work for 30% of jobs bid by the company. It is known that the error rate for engineer 1 is such that 0.02 is the probability of an error when he does the work, whereas the probability of an error in the work of engineer 2 is 0.04. Suppose a bid arrives and a serious error occurs in estimating cost. Which engineer would you guess did the work? Explain and show all work. (5 Marks)

Solution:

Let, E1 denotes the event that 1^{st} engineer does the work

E2 denotes the event that 2^{nd} engineer does the work and

A denotes the event that error occurs during work

P(E1) = 0.7 P(E2) = 0.3

P(O|E1) = 0.02 P(O|E2) = 0.04

$$P(O) = P(O|E1). P(E1) + P(O|E2). P(E2) = 0.02 \text{ X } 0.7 + 0.3 \text{ X } 0.04 = 0.026$$

$$P(E1|O) = P(O|E1). P(E1) / P(O) = 0.014 / 0.026 = 0.538$$

$$P(E2|O) = P(O|E2). P(E2) / P(O) = 0.012 / 0.026 = 0.462$$

Question No 07: For married couples living in a certain suburb, the probability that the husband will vote on a bond referendum is 0.21, the probability that the wife will vote on the referendum is 0.28, and the probability that both the husband and the wife will vote is 0.15. What is the probability that: (5 Marks)

a) at least one member of a married couple will vote?
b) a wife will vote, given that her husband will vote?
c) a husband will vote, given that his wife will not vote?

Solution:

Let, H denotes the event that husband will vote on a bond referendum
W denotes the event that wife will vote on a bond referendum
B denotes the event that both will vote on a bond referendum

$P(H) = 0.21$ $P(W) = 0.28$ $P(B) = 0.15 = P(H \cap W)$

a) at least one member of a married couple will vote?

$P(H \cup W) = P(H) + P(W) - P(H \cap W) = 0.21 + 0.28 - 0.15$

$P(H \cup W) = 0.34$

b) a wife will vote, given that her husband will vote?

$P(W \mid H) = P(H \cap W) / P(H) = 0.15 / 0.21 = 0.7143$

c) a husband will vote, given that his wife will not vote?

$P(W') = 1 - P(W) = 1 - 0.28 = 0.72$

$P(H \cap W') = P(H) - P(H \cap W) = 0.21 - 0.15 = 0.06$

$P(H|W') = P(H \cap W') / P(W') = 0.06 / 0.72 = 1/12 = 0.0833$

Question No 08: In a bolt factory machine A, B and C manufacture 25%, 35%, and 40% of the total output, respectively of their outputs 5, 4 and 2% respectively are defective bolts. (5 Marks)

a) What is the probability that a bolt is selected all random and found to be defective?
b) What is the probability that the bolt came from machine A?

Solution:

$P(A) = 0.25$ $P(B) = 0.35$ $P(C) = 0.40$

Let, D represent the event that the bolt is defective

$P(D|A) = 0.05$ $P(D|B) = 0.04$ $P(D|C) = 0.02$

a) What is the probability that a bolt is selected all random and found to be defective?

$P(D) = P(D|A).\ P(A) + P(D|B).\ P(B) + P(D|C).\ P(C)$

$= 0.25 \times 0.05 + 0.35 \times 0.04 + 0.40 \times 0.02$

$P(D) = 0.034$

b) What is the probability that the bolt came from machine A?

$P(A|D) = P(A).\ P(D|A) / P(D) = 0.25 \text{ X } 0.05 / 0.034 = 0.362$

Question No 09: A certain system can experience three different types of defects. Let A_i ($i = 1,2,3$) denote the event that the system has a defect of type i. Suppose that $P(A_1)=0.12$, $P(A_2)=0.07$, $P(A_3)=0.05$, $P(A_1 \cup A_2)=0.13$, $P(A_1 \cup A_3)=0.14$, $P(A_2 \cup A_3)=0.10$, $P(A_1 \cap A_2 \cap A_3)=0.01$

a) What is the probability that the system does not have a type 1 defect?
b) What is the probability that the system has both type 1 and type 2 defects?
c) What is the probability that the system has both type 1 and type 2 defects but not a type 3 defect?
d) What is the probability that the system has at most two of these defects?

(10 Marks)

<u>Solution:</u>

a) What is the probability that the system does not have a type 1 defect?

$P(A_1') = 1 – P(A_1) = 1- 0.12 = 0.88$

b) What is the probability that the system has both type 1 and type 2 defects?

$P (A_1 \cap A_2) = P(A_1) + P(A_2) – P (A_1 U A_2) = 0.12 + 0.07 – 0.13$

$= 0.06$

c) What is the probability that the system has both type 1 and type 2 defects but not a type 3 defect?

$P (A_1 \cap A_2 \cap A_3') = P (A_1 \cap A_2) – P (A_1 \cap A_2 \cap A_3) = 0.06 – 0.01$

$= 0.05$

d) What is the probability that the system has at most two of these defects?

Let, T=at most two defects

$P(T) = 1- P (A_1 \cap A_2 \cap A_3) = 1- 0.01 = 0.99$

Question No 10: 3 members of a private country club have been nominated for the office of the president. The probability that Mr. A will be elected is 0.3, the probability that Mr. B will be elected is 0.5 and the probability that Mr. C will be elected is 0.2. Should Mr. A be elected the probability for an increase in membership is 0.8. Should Mr. B or Mr. C be elected, the increase in fees are 0.1 and 0.4. What is the probability that there will be an increase in membership fees?

(5 Marks)

<u>Solution:</u>

$P(A) = 0.3$ $P(B) = 0.5$ $P(C) = 0.2$

Let, D represent the event for an increase in membership

$P(D|A) =0.8$ $P(D|B) = 0.1$ $P(D|C) = 0.4$

$P(D) = P(D|A). P(A) + P(D|B). P(B) + P(D|C). P(C) = 0.8 \times 0.3 + 0.1 \times 0.5 + 0.4 \times 0.2$

$P(D) = 0.37$

Question No 11: A student has three different e-mail accounts. Most of her messages, in fact 70%, come into account #1, whereas 20% come into account #2 and the remaining 10% into account #3. Of the messages coming into account #1, only 1% are spam, compared to 2% and 5% for account #2 and account #3, respectively. What is the student's overall spam rate, i.e., what is the probability a randomly selected e-mail message received by her is spam? (5 Marks)

Solution:

Let, A1= Account#1 A2=Account#2 A3=Account#3

$P(A1) = 0.70$ $P(A2) = 0.20$ $P(A3) = 0.10$

Let, D represents the event that the messages are spam,

$P(D|A1) = 0.01$ $P(D|A2) = 0.02$ $P(D|A3) = 0.05$

$$P(D) = P(D|A1).\ P(A1) + P(D|A2).\ P(A2) + P(D|A3).\ P(A3)$$
$$= 0.01 \times 0.70 + 0.02 \times 0.20 + 0.02 \times 0.10 = 0.016$$

Question No 12: The concept of conditional probability has countless uses in both industrial and biomedical applications. Consider an industrial process in the textile industry in which strips of a particular type of cloth are being produced. These strips can be defective in two ways, length and nature of texture. For the case of the latter, the process of identification is very complicated. It is known from historical information on the process that 10% of strips fail the length test, 5% fail the texture test, and only 0.8% fail both tests. If a strip is selected randomly from the process and a quick measurement identifies it as failing the length test, what is the probability that it is texture defective? (5 Marks)

Solution:

Let, A be the event that strips fail the length test,

B be the event that strips fail the texture test

$P(A) = 0.1$ $P(B) = 0.05$ $P(A \cap B) = 0.008$

$$P(B|A) = P(A \cap B) / P(A) = 0.008/0.1$$
$$= 0.08$$

Question No 13: The following table summarizes the analysis of sample of galvanized steel for coating weight and surface roughness: (10 Marks)

		Coating Weight	
		high	low
Surface	high	12	16
roughness	Low	88	34

a. If the coating weight of a sample is high, what is the probability that the surface roughness is high?

b. If the surface roughness of a sample is high, what is the probability that the coating weight is high?

c. If the surface roughness of a sample is low, what is the probability that the coating weight is low?

Solution:

Let, W be the event that coating weight of a sample is high
R be the event that the surface roughness is high

$P(W) = 12+88/ 12+16 + 88+34 = 100/150 = 2/3$

$P(R) = 12+16 / 150 = 14/ 75 = 0.1866$

$P(W \cap R) = 12/ 120 = 2/ 25 = 0.08$

a. If the coating weight of a sample is high, what is the probability that the surface roughness is high?

$P(R|W) = P(W \cap R). P(W) = (2/25) / (2/3) = 3/25 = 0.12$

b. If the surface roughness of a sample is high, what is the probability that the coating weight is high?

$P(W|R) = P(W \cap R). P(R) = (2/25) / (14/75) = 3/7 = 0.4286$

c. If the surface roughness of a sample is low, what is the probability that the coating weight is low?

$P(W' \cap R') = 34$ $P(R') = 88+ 34 = 122$

$P(W'|R') = P(W' \cap R') / P(R') = 34/ 122 = 17/61 = 0.2787$

Question No 14: At a certain gas station, 40% of the customers use regular gas (A1), 35% use plus gas (A2), and 25% use premium (A3). Of those customers using regular gas, only 30% fill their tanks (event B). Of those customers using plus, 60% fill their tanks, whereas of those using premium, 50% fill their tanks. a. What is the probability that the next customer will request plus gas and fill the tank (A2 U B)? b. What is the probability that the next customer fills the tank? c. If the next customer fills the tank, what is the probability that regular gas is requested? Plus? Premium? (10 Marks)

Solution:

$P(A1) = 0.40$ $P(A2) = 0.35$ $P(A3) = 0.25$

$P(B|A1) =0.3$ $P(B|A2) = 0.6$ $P(B|A3) = 0.5$

a) What is the probability that the next customer will request plus gas and fill the tank (A2 U B)?

$P (A2 \cup B) = P(A2). P(B|A2) = 0.35 \times 0.6 = 0.21$

b) What is the probability that the next customer fills the tank?

$P(B) = P(B|A1) P(A1) + P(B|A2). P(A2) + P(B|A3). P(A3)$

$P(B) = 0.4 \times 0.3 + 0.35 \times 0.6 + 0.25 \times 0.5 = 0.455$

c) If the next customer fills the tank, what is the probability that regular gas is requested? Plus? Premium?

$P(A1|B) = P (A1 \cap B) / P(B) = 0.12 / 0.455 = 0.264$

$P(A2|B) = P (A2 \cap B) / P(B) = 0.21 / 0.455 = 0.462$

$P(A3|B) = P (A3 \cap B) / P(B) = 0.125 / 0.453 = 0.275$