Probability and Statistics
Assignment 3

Total Mark:100

Question No.1(10):

A president and a treasurer are to be chosen from a student club consisting of 50 people. How many different choices of officers are possible if
(a) there are no restrictions;
(b) A will serve only if he is president;
(c) B and C will serve together or not at all;
(d) D and E will not serve together?

Solution:

(a) The total number of choices of officers, without any restrictions, is
$^{50}P_2 = 50! / 48!$
$= (50)(49) = 2450$.
(b) Since A will serve only if he is president, we have two situations here: (i) A is selected as the president, which yields 49 possible outcomes for the treasurer's position, or (ii) officers are selected from the remaining 49 people without A, which has the number of choices $^{49}P_2 = (49)(48) = 2352$. Therefore, the total number of choices is $49 + 2352 = 2401$.

(c) The number of selections when B and C serve together is 2. The number of selections when both B and C are not chosen is $^{48}P_2 = 2256$. Therefore, the total number of choices in this situation is $2 + 2256 = 2258$.

(d) The number of selections when D serves as an officer but not E is $(2)(48) = 96$, where 2 is the number of positions D can take and 48 is the number of selections of the other officer from the remaining people in the club except E. The number of selections when E serves as an officer but not D is also $(2)(48) = 96$. The number of selections when both D and E are not chosen is $^{48}P_2 = 2256$. Therefore, the total number of choices is $(2)(96) + 2256 = 2448$. This problem also has another short solution: Since D and E can only serve together in 2 ways, the answer is $2450 - 2 = 2448$.

Question No.2(10):

A California study concluded that following 7 simple health rules can extend a man's life by 11 years on the average and a woman's life by 7 years. These 7 rules are as follows: no smoking, get regular exercise, use alcohol only in moderation, get 7 to 8 hours of sleep, maintain proper weight, eat breakfast, and do not eat between meals. In how many ways can a person
adopt 5 of these rules to follow

(a) if the person presently violates all 7 rules?
(b) if the person never drinks and always eats breakfast?

Solution:

(a) The person presently violates all 7 rules so the person has to adopt 5 out of 7.

The number of combinations of 7 distinct objects taken 5 at a time is

${}^{7}C_{5} = 7! / (5!\ (7-5)!) = 5040 / 240 = 21$

(b) The person never drinks and always eats breakfast so two health rules are always adopted. Hence, there are 3 more rules that the person should adopt. There are 7 – 2 = 5 more health rules that are being tested. So, the person has to adopt 3 out of 5 health rules

The number of combinations of 5 distinct objects taken 3 at a time is

${}^{5}C_{3} = 5! / (3!\ (5-3)!) = 120 / 12 = 10$

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Question No.3:

Married couples have brought 8 seats in a row for a concert in how many ways can they be sitted

(a) No restriction

(b) If all the man sits together to the right of all women

(c) if each couple is to sit together

Solution:

(a) 8! = 40320

(b) 4! = 24

(c) 4! x 2! x 2! x 2! x 2! = 384

Question No.4:

A foreign students club list as its member 2 Canadian, 3 Japanese, 5 Italian and 2 German. If a committee of 4 is selected at random

(a) all nationality is represented

(b) all nationality except Italian is represented

Solution:

(a) Let A be an event that all nationality is represented.

$P(A) = n(A)/n(S)$

$P(A) = ({}^{2}C_{1} \times {}^{2}C_{1} \times {}^{5}C_{1} \times {}^{2}C_{1}) / {}^{12}C_{4}$

$= 0.08$

(b) Let B be an event all nationality except Italian is represented.

$P(B) = n(B)/n(S)$

$= (({}^{2}C_{2} \times {}^{3}C_{1} \times {}^{5}C_{0} \times {}^{2}C_{1}) + ({}^{2}C_{1} \times {}^{3}C_{2} \times {}^{5}C_{0} \times {}^{2}C_{1}) + ({}^{2}C_{1} \times {}^{3}C_{1} \times {}^{5}C_{0} \times {}^{2}C_{2})) / {}^{12}C_{4}$

$= 0.04848$

Question No.5:

If 3 books are picked at random from a shelf containing 5 novels, 3 books of poems, and a dictionary,
what is the probability that
(a) the dictionary is selected?
(b) 2 novels and 1 book of poems are selected?

Solution:

(a) Let A be an event that dictionary is selected.

$P(A) = n(A) / n(S)$

$= (({}^{5}C_{1} \times {}^{3}C_{1} \times {}^{1}C_{1}) + ({}^{5}C_{2} \times {}^{3}C_{0} \times {}^{1}C_{1}) + ({}^{5}C_{0} \times {}^{3}C_{2} \times {}^{1}C_{1})) / ({}^{9}C_{3})$

$= 0.333$

(b) Let B be an event that 2 novels and 1 book of poems are selected.

$P(B) = n(B) / n(S)$

$= ({}^{5}C_{2} \times {}^{3}C_{1} \times {}^{1}C_{0}) / ({}^{9}C_{3})$

$= 0.357$

Question No.6:

The probabilities that a service station will pump gas into 0, 1, 2, 3, 4, or 5 or more cars during a certain 30-minute period are 0.03, 0.18, 0.24, 0.28, 0.10, and 0.17, respectively. Find the probability that in this 30-minute period
(a) more than 2 cars receive gas;
(b) at most 4 cars receive gas;

(c) 4 or more cars receive gas.

Solution:

$P(X=0) = 0.03$

$P(X=1) = 0.18$

$P(X=2) = 0.24$

$P(X=3) = 0.28$

$P(X=4) = 0.1$

$P(X>=5) = 0.17$

(a)

$P(X>2) = P(X=3) + P(X=4) + P(X>=5)$

$= 0.28 + 0.1 + 0.17$

$= 0.55$

(b)

$P(X<=2) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$= 0.03 + 0.18 + 0.24 + 0.28 + 0.1$

$= 0.83$

(c)

$P(X>=4) = P(X=4) + P(X>=5)$

$= 0.1 + 0.17$

$= 0.27$

Question No.7:

An allergist claims that 50% of the patients she tests are allergic to some type of weed. What is the probability that
(a) exactly 3 of her next 4 patients are allergic to
weeds?
(b) none of her next 4 patients is allergic to weeds?

Solution:

(a)

Let WWWW' denotes the event that first 3 patients are allergic to weeds but 4^{th} patients are not allergic to weeds.

Let A denotes the event that exactly 3 of next 4 patients are allergic to weeds.

$P(A) = P(WWWW') + P(WWW'W) + P(WW'WW) + P(W'WWW)$

$= P(W'). (P(W))^3 + P(W'). (P(W))^3 + P(W'). (P(W))^3 + P(W'). (P(W))^3$

$= 4. P(W'). (P(W))^3$

$= 4. (0.5). (0.5)^3$

$= 4. (0.5)^4$

$= 0.25$

(b)

Let B denotes the event that None of next 4 patients is allergic to weeds.

$P(B) = P(W'W'W'W')$

$= (P(W'))^4$

$= (0.5)^4$

$= 0.0625$

Question No.8:

The table below represents the college degrees awarded in a recent academic year by gender.

	Bachelor's	**Master's**	**Doctorate**
Men	573,079	211,381	24,341
Women	775,424	301,264	21,683

Choose a degree at random. Find the probability that it is
a. A bachelor's degree
b. A doctorate or a degree awarded to a woman
c. A doctorate awarded to a woman
d. Not a master's degree

Solution:

n(S) = 573079 + 211381 + 24341 + 775424 + 301264 + 21683 = 1907172

(a)

Let A be the event that the chosen degree is a bachelor's degree.

P(A) = (573079 + 775424)/ 1907172 = 1348503 / 1907172 = 0.707

(b)

Let B be the event that the chosen degree is a doctorate or a degree awarded to a woman.

P(B) = (24341 + 775424 + 301264 + 21683)/ 1907172 = 1122712/ 1907172 = 0.589

(c)

Let C be the event that the chosen degree is a doctorate awarded to a woman.

P(C) = 21683/ 1907172 = 0.011

(d)

Let D be the event that the chosen degree is not a Masters.

P(D) = (211381 + 301264) / 1907172 = 512645/ 1907172 = 0.269

Question No.9:

On a hospital staff, there are 4 dermatologists, 7 surgeons, 5 general practitioners, 3 psychiatrists, and 3 orthopedic specialists. If a doctor is selected at random, find the probability that the doctor is

a. A psychiatrist, surgeon, or dermatologist
b. A general practitioner or surgeon
c. An orthopedic specialist, a surgeon, or a dermatologist
d. A surgeon or dermatologist

Solution:

n(S) = 4 + 7 + 5 + 3 + 3 = 22

(a)

Let A be the event that a doctor is a psychiatrist, surgeon, or dermatologist

P(A) = (3 + 7 + 4)/ 22 = 14/22 = 0.636

(b)

Let B be the event that a doctor is a general practitioner or surgeon

P(B) = (5+7)/22 = 12/22 = 0.545

(c)

Let C be the event that a doctor is an orthopedic specialist, a surgeon, or a Dermatologist

P(C) = (3 + 7 + 4) / 22 = 14/22 = 0.636

(d)

Let D be the event that a doctor is a surgeon or dermatologist

P(D) = (4 + 7)/22 = 11/22 = 0.5

Question No.10(10):

If one card is drawn from an ordinary deck of cards, find the probability of getting the following.
a. A king or a queen or a jack
b. A club or a heart or a spade
c. A king or a queen or a diamond
d. An ace or a diamond or a heart
e. A 9 or a 10 or a spade or a club

Solution:

(a)

Let us denote the event of selecting a card from the deck and getting a king or a queen or a jack by E.

The three events of selecting a king, queen and jack are mutually exclusive hence using the addictive rule 1 of probability and formula of empirical probability, we can write:

P(E)= P(king) + P(queen) + P(jack)

= (4/52) + (4/52) + (4/52)

= 3/13

(b)

Let us denote the event of selecting a card from the deck and getting a club or a heart or a spade by E.

The three events of selecting a club, heart and spade are mutually exclusive hence using the addictive rule 1 of probability and formula of empirical probability, we can write:

P(E)= P(club) + P(heart) + P(spade)

= (13/52) + (13/52) + (13/52)

= 3 / 4

(c)

Let us denote the event of selecting a card from the deck and getting a king or a queen or a diamond by E.

The three events of selecting a king, queen and diamond are not mutually exclusive we can write:

P(E)= P(king) + P(queen) + P(diamond) – P (king and queen) – P (king and diamond) – P (queen and diamond) + P (king and queen and diamond)

P (king and queen and diamond) = 0

P (king and queen) = 0

P(E)= (4/52) + (4/52) + (13/52) -(1/52) -(1/52)

=19/52

(d)

Let us denote the event of selecting a card from the deck and getting an ace or a diamond or a heart by E.

The three events of selecting an ace, a diamond and a heart are not mutually exclusive we can write:

P(E)= P(ace) + P(diamond) + P(heart) – P (ace and diamond) – P (ace and heart) – P (heart and diamond) + P (ace and diamond and heart)

P (ace and diamond and heart) = 0

P (heart and diamond) = 0

P(E)= (4/52) + (13/52) + (13/52) -(1/52) -(1/52)

=7/13

(e)

Let us denote the event of selecting a 9 or a 10 or a spade or a club by E.

A: Selecting a card and getting a 9

B: Selecting a card and getting a 10

C: Selecting a card and getting a spade

D: Selecting a card and getting a club

P(E)= P (A or B or C or D)

= P(A) + P(B) + P(C) + P(D) – P (A and C) -P (A and D) – P (B and C) -P (B and D)

= ((4/52) +(4/52) +(13/52) +(13/52)) -((1/52) +(1/52) +(1/52) +(1/52))

= (34/52) -(4/52)

= 30/52 = 15/26

Question No.11(5):

From past experience, a stockbroker believes that under present economic conditions a customer will invest in tax-free bonds with a probability of 0.6, will invest in mutual funds with a probability of 0.3, and will invest in both tax-free bonds and mutual funds with a probability of 0.15. At this time, find the probability that a customer will invest
(a) in either tax-free bonds or mutual funds;
(b) in neither tax-free bonds nor mutual funds.

Solution:

$P(T) = 0.6$

$P(M) = 0.3$

$P(T \cap M) = 0.15$

(a) $P(T \cup M) = P(T) + P(M) - P(T \cap M)$

$= 0.6 + 0.3 – 0.15$

$= 0.75$

(b) $P(T \cap M) = 1 - P(T \cup M)$

$= 1 - 0.75$

$= 0.25$

Question No.12(5):

The following table shows the distribution of body mass index (BMI) and age groups among male adults in a certain country.

	Normal or low BMI	**Overweight**	**Obese**	**Total**
Age<30	0.09	0.06	0.05	0.20
Age>=30	0.20	0.32	0.28	0.80
Total	0.29	0.38	0.33	1.00

a. What is the probability that a person selected at random from the group will be obese?
b. A person, selected at random from this group, is found to be obese. What is the probability that this person is younger than age 30?

Solution:

(a) Let A be the event that group of people will be obese

P(A) = n(A)/n(S)

= (0.05+0.28)/1.00

= 0.33/1.00 = 0.33

(b) Let B be the event that person is younger than age 30.

P(B|A) = p (A∩B)/P(A)

= 0.05 / 0.32

= 0.156

Question No.13:

The probability that a regularly scheduled flight departs on time is $P(D) = 0.83$; the probability that it arrives on time is $P(A) = 0.82$; and the probability that it departs and arrives on time is $P(D \cap A) = 0.78$. Find the probability that a plane

(a) Arrives on time, given that it departed on time, and
(b) Departed on time, given that it has arrived on time.

Solution:

Let A be the event that regularly scheduled flight arrives on time.

Let D be the event that regularly scheduled flight departs on time.

P(A) = 0.82

P(D) = 0.83

P (D ∩A) = 0.78

(a) P(A|D) = P (D∩A) / P(D)

= 0.78 / 0.83

= 0.94

(b) P(A|D) = P (D∩A) / P(A)

= 0.78 / 0.82

= 0.95

<u>Question No.14(10):</u>

The medal distribution from the 2008 Summer Olympic Games for the top 23 countries is shown below.

	Gold	**Silver**	**Bronze**
United States	36	38	36
Russia	23	21	28
China	51	21	28
Great Britain	19	13	15
Others	173	209	246

Choose 1 medal winner at random.
a. Find the probability that the winner won the gold medal, given that the winner was from the United States.
b. Find the probability that the winner was from the United States, given that she or he won a gold medal.

Solution:

n(S) = 36 +38 + 36 + 23 + 21 + 28 + 51 + 21 + 28 + 19 + 13 + 15 + 173 + 209 + 246 = 957

(a)

Probability that the winner was from United States is:

P (United States) = (36 + 38 + 36) / 957

= 110/957

= 0.115

Probability that the winner was from United States and won gold medal is:

P (United States gold medalist) = 36/957

= 0.0376

P (Gold medalist | United States) = P (United States gold medalist) / P (United States)

= 0.0376 / 0.115

= 0.327

(b)

Probability that the winner was from United States is:

P (Gold medalist) = (36 +23 + 51 + 19 + 173) / 957

= 302/957

= 0.3156

Probability that the winner was from United States and won gold medal is:

P (United States gold medalist) = 36/957

= 0.0376

P (United States | Gold medalist) = P (United States gold medalist) / P (Gold medalist)

= 0.0376 / 0.3156

= 0.1191

Question No.15(5):

The concept of conditional probability has countless uses in both industrial and biomedical applications. Consider an industrial process in the textile industry in which strips of a particular type of cloth are being produced. These strips can be defective in two ways, length and nature of texture. For the case of the latter, the process of identification is very complicated. It is known from historical information on the process that 10% of strips fail the length test, 5% fail the texture test, and only 0.8% fail both tests. If a strip is selected randomly from the process and a quick measurement identifies it as failing the length test, what is the probability that it is texture defective?

Solution:

Let A be the event that strips fail the length test.
Let B be the event that strips fail the texture test.

$P(A) = 10\% = 0.1$

$P(B) = 5\% = 0.05$

$P(A \cap B) = 0.8\% = 0.008$

$P(B|A) = P(A \cap B) / P(A)$

$= 0.008 / 0.1$

$= 0.08$