



Dependence Analysis

(CS 3006)

Department of Computer Science,
National University of Computer & Emerging Sciences,
Islamabad Campus



The Big Picture

1. What are our goals?

- **Simple Goal:** Make execution time as small as possible

2. Which leads to:

- Achieve **execution** of **many** (all, in the best case) **instructions** in **parallel**
- But, you have to find **INDEPENDENT** instructions



Data Dependence

- Data must be **produced** and **consumed** in the *correct order*

- Simple **example of data dependence:**

S_1 $PI = 3.14$

S_2 $R = 5.0$

S_3 $AREA = PI * R ** 2$

- Statement **S_3** **cannot** be **moved** before either **S_1** or **S_2** without **compromising** correct **results**



Motivation

- **DOALL loops**: loops whose iterations can execute in parallel

```
for i = 11, 20  
    a[i] = a[i] + 3
```



Examples

```
for i = 11, 20  
    a[i] = a[i] + 3
```

Parallel

```
for i = 11, 20  
    a[i] = a[i-1] + 3
```

NOT Parallel?

```
for i = 11, 20  
    a[i] = a[i-10] + 3
```

Parallel ?



Dependence Analysis

- A **dependence** is a **relationship** between **2 computations** that **places constraints** on their **execution order**
- **Dependence analysis identifies** these **constraints**
- **Constraints** are used to **determine** whether a **particular transformation** can be **applied** without changing the computation's **semantics**
- **2 types of dependences**: **control** and **data** dependences
- ***Both of them must be considered*** when parallelizing programs.



Control Dependence

- There is a **control dependence** between **S1** and **S2**, when **S1** determines whether **S2** will be executed or not

- Example:**

```
S1    IF (T .NE. 0.0)
S2          A = A / T
S3    CONTINUE
```

- Executing **S2** before **S1** could cause a **divide by zero** exception (in this example).
- S2 is conditional** upon the **execution** of the **branch** in **S1**.



Data Dependence

- Two **statements** have a **data dependence** if they **cannot be executed simultaneously** due to **conflicting uses of the same data**.
- **Ensure that data is produced and consumed in the right order:**
 1. do not interchange **loads** and **stores** to the same location
 2. **two stores** take place in the **correct order**
- **Formally:**
 - There is a **data dependence from statement S1 to statement S2 (S2 depends on S1)** if:
 - Both statements **access the same memory location** and at least **one of them stores** onto it, and
 - There is a **feasible run-time execution** path from **S1 to S2**



Load/Store Classification

- Dependences classified in terms of load-store order:

- True dependences

- S_2 depends on S_1 is denoted by $S_1 \delta S_2$

$$\begin{array}{l} S_1 \quad X = \dots \\ S_2 \quad \dots = X \end{array}$$

This is a crucial
dependence!

- Antidependence

- S_2 depends on S_1 is denoted by $S_1 \delta^{-1} S_2$

$$\begin{array}{l} S_1 \quad \dots = X \\ S_2 \quad X = \dots \end{array}$$

- Output dependence

- S_2 depends on S_1 is denoted by $S_1 \delta^0 S_2$

$$\begin{array}{l} S_1 \quad X = \dots \\ S_2 \quad X = \dots \end{array}$$



Data Dependence of Scalar Variables

- True/Flow dependence

$a =$
 $= a$

- Output dependence

$a =$
 $a =$

- Anti-dependence

$= a$
 $a =$

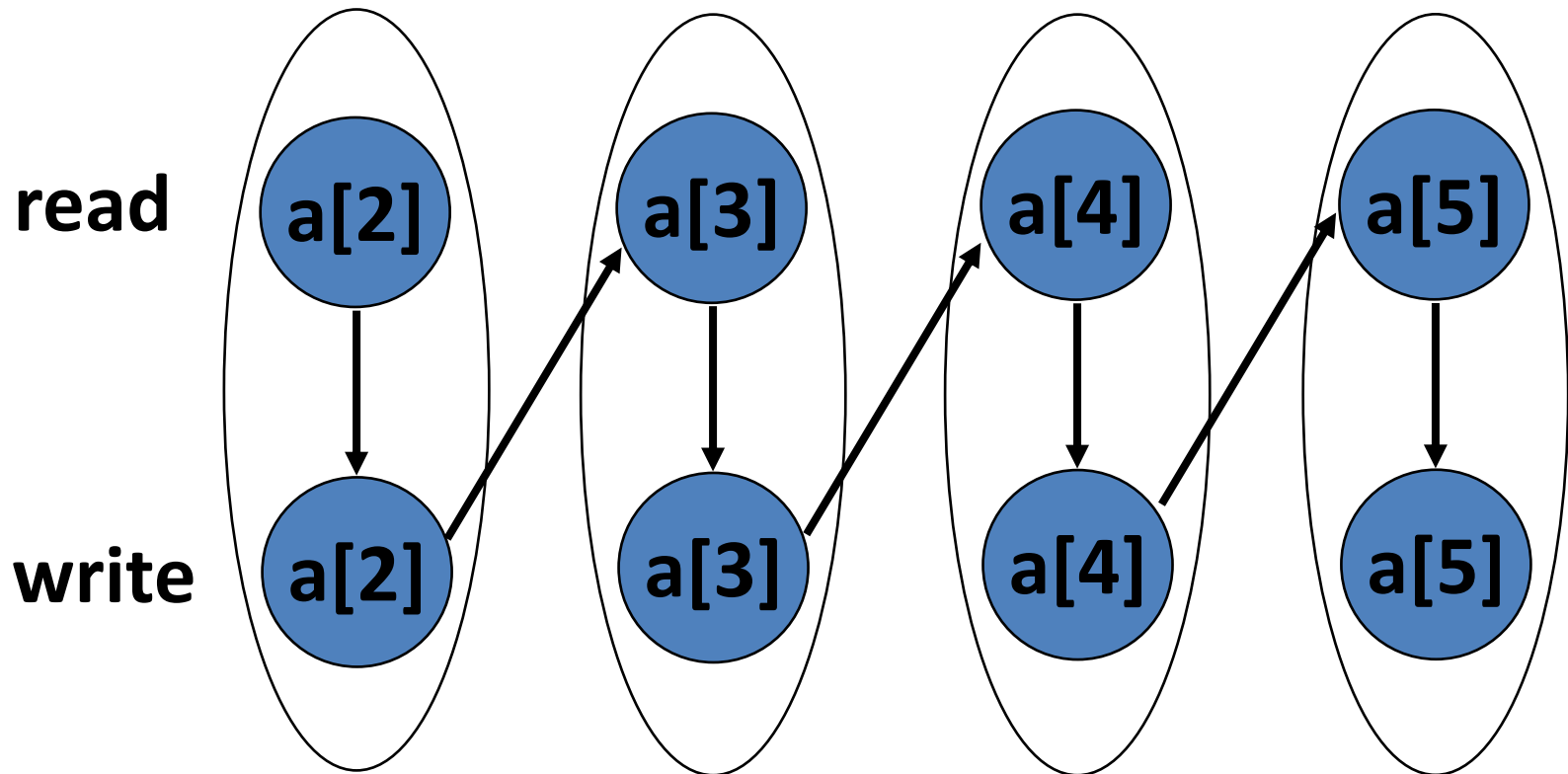
- Input dependence

$= a$
 $= a$

- _ Only data **flow dependences** are **true dependences**.
- _ **Anti** and **output** can be removed by renaming

Array Accesses in a Loop

```
for i = 2, 5  
    a[i] = a[i] + 3
```

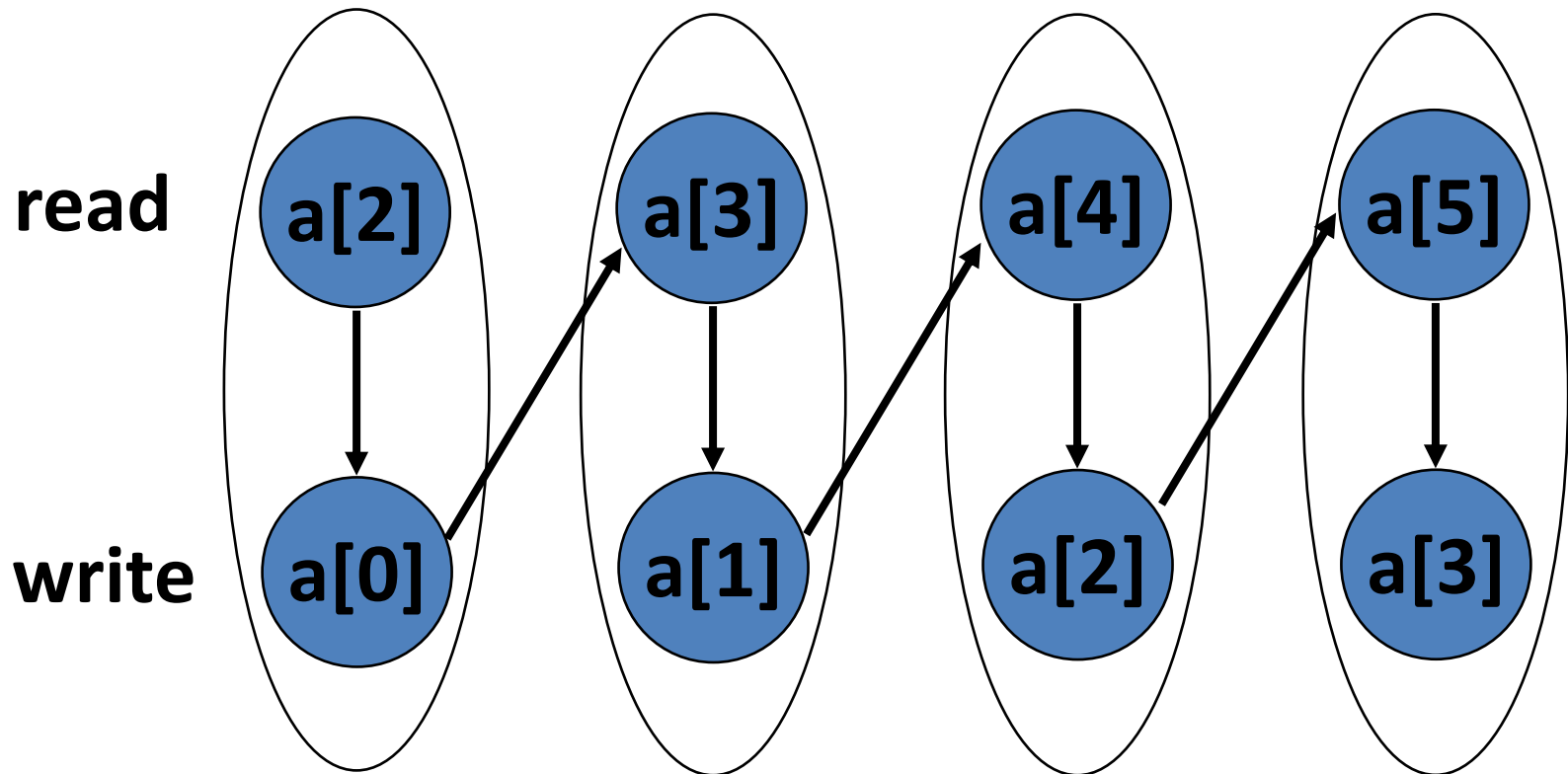




Array Anti-dependence

for $i = 2, 5$

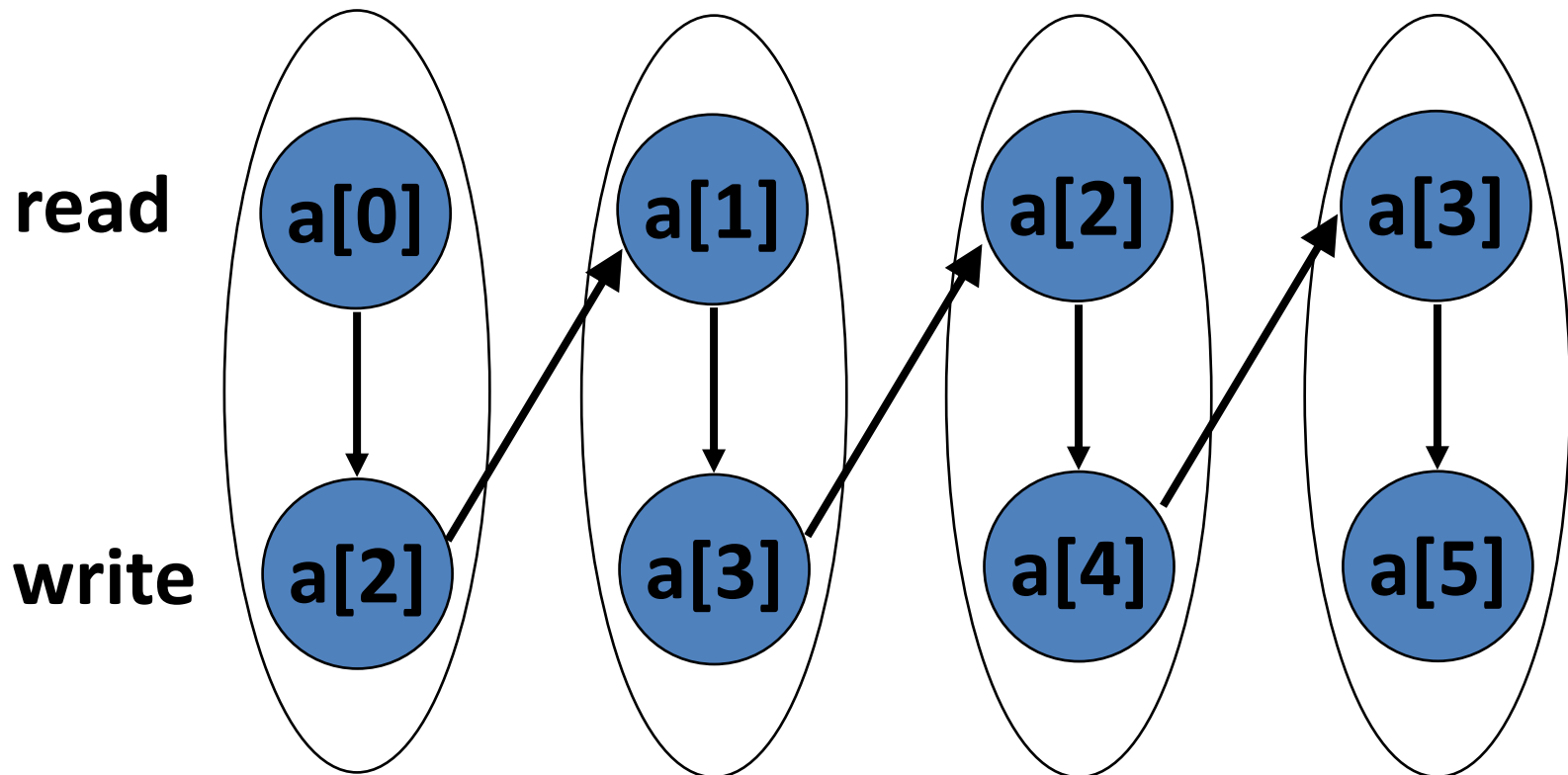
$a[i-2] = a[i] + 3$





Array True-dependence

```
for i= 2, 5  
  a[i] = a[i-2] + 3
```





A Parallel DOALL Loop

- A **loop is fully parallel** if no dependencies flow across iterations:

```
DO I = 2, N  
  A(I) = A(I) + 1  
ENDDO
```

```
DO I = 2, N  
  A(I) = A(I-1) + 1  
ENDDO
```

- Parallel loops** are found through **dependence analysis** and **dependence tests**
- Usually **done** at the **source-code level**, **focus is on arrays**



Recognizing DOALL Loops

- Find data **dependences** in loop
- **Definition:** a **dependence** is **loop-carried** if it **crosses an iteration boundary**
- If there are no loop-carried dependences only then loop is parallelizable



Example: Loop Parallelization

```
Do i=1,n  
  A(i)=5*B(i)+A(i)  
Enddo
```

```
Do i=1,n  
  A(i-1)=5*B(i)+A(i)  
Enddo
```

```
Do i=1,n  
  tmp=5*B(i)  
  A(i)=tmp  
Enddo
```

Which of the following loops are parallelizable?



Dependence in Loops

1: $A(2) = A(1) + B(1)$

2: $A(3) = A(2) + B(2)$

3: $A(4) = A(3) + B(3)$

S1 & S1

1: $A(3) = A(1) + B(1)$

2: $A(4) = A(2) + B(2)$

3: $A(5) = A(3) + B(3)$

4: $A(6) = A(4) + B(4)$

- Let us look at two different loops:

```
DO I = 1, N
S1   A(I+1) = A(I) + B(I)
ENDDO
```

```
DO I = 1, N
S1   A(I+2) = A(I) + B(I)
ENDDO
```

- In both cases, statement S_1 depends on itself
- However, there is a significant difference.
- We need a formalism to describe and distinguish such dependences



Iteration Numbers

- The iteration number of a **loop** is **equal** to the **value** of the **loop index**

- Definition:**

- For an arbitrary loop in which the **loop index** ***I*** runs from **L** to **U** in **steps of S**, the **iteration number** ***i*** of a specific iteration is equal to the **index value** ***I*** on that **iteration**:

Example:

```
DO I = 0, 10, 2  
S1    <some statement>  
ENDDO
```