

LECTURE : 27:

(1)

2.5 : Solutions By Substitution: Continuation

② Bernoulli's Equation:-

The differential Equation

$$\frac{dy}{dx} + P(x)y = f(x)y^n \quad \dots (1)$$

where n is any real no. is called Bernoulli's Equation.
For $n=0$ and $n=1$ Eqn (1) is Linear Equation.

Substitution: For $n \neq 0$ and $n \neq 1$, i.e. if first order D.E
is Bernoulli Equation, then we use the Substitution
 $u = y^{1-n}$ which reduces Bernoulli Equation into a
Linear Equation.

$$\left. \begin{array}{l} u = y^{1-n} \\ \Rightarrow u = \frac{1}{y^{n-1}} \\ \text{or } y = \frac{1}{u^{1/n-1}} \\ \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \end{array} \right\}$$

That is how we Put the
Substitution into Bernoulli Equation
to get- Linear Equation.

Example :- Solve $x \frac{dy}{dx} + y = x^2 y^2$ — (2)

Solu:- Equ(2) is Bernoulli Equ with $n=2$.

$$\text{Let } u = y^{1-n}$$

$$\text{For } n=2, u = y^{1-2}$$

$$u = y^{-1}$$

$$\Rightarrow y = u^{-1}$$

$$\frac{dy}{du} = -\frac{1}{u^2} = -u^{-2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = -u^{-2} \frac{du}{dx}$$

Substitute values in Given D.E (2).

$$\frac{dy}{dx} + \frac{1}{x} y = x y^2$$

$$-u^{-2} \frac{du}{dx} + \frac{u^{-1}}{x} = x u^{-2}$$

$$-\frac{1}{u^2} \frac{du}{dx} + \frac{1}{u x} = \frac{x}{u^2}$$

(\times by $-u^2$)

$$\boxed{\frac{du}{dx} - \frac{u}{x} = -x}$$

→ Equ is now Linear in u
— (3)

$$I.F = e^{-\int y_x dx} = e^{-\ln x} = e^{\ln x^{-1}} = x^{-1} \quad (2)$$

Multiply I.F by Equ (3),

$$x^{-1} \frac{dy}{dx} - \frac{u}{x^2} = -x^{-1}x$$

$$\frac{d}{dx} [x^{-1}u] = -1$$

integrating w.r.t x

$$\int \frac{d}{dx} [x^{-1}u] dx = - \int dx$$

$$x^{-1}u = -x + C$$

$$\frac{u}{x} = -x + C$$

$$u = -x^2 + cx$$

Resubstitute value of u

$$\frac{1}{y} = -x^2 + cx$$

$$y = \frac{1}{-x^2 + cx}$$

Explicit Solution of Bernoulli Equation.

Practice Problems :-

$$1) \frac{dy}{dx} - y = e^x y^2 \quad (\text{Hint: Bernoulli Eqn with } n=2)$$

use $u = y^{1-n} = y^{1-2} = y^{-1}$

$$2) y^{\frac{1}{2}} \frac{dy}{dx} + y^{\frac{3}{2}} = 1, \quad y(0) = 4.$$

$$(\text{Hint: } \frac{dy}{dx} + y^{\frac{3}{2}-\frac{1}{2}} = y^{-\frac{1}{2}}$$

$$\frac{dy}{dx} + y = y^{-\frac{1}{2}}$$

Bernoulli Eqn with $n = -\frac{1}{2}$.

$$\text{use Substitution: } u = y^{1-n} = y^{1+\frac{1}{2}} = y^{\frac{3}{2}}$$

$$y = u^{\frac{2}{3}} \quad)$$

Related Exercise Questions :-

Ex 2.5

Q: 15 - 22