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Section: BS-DS(N)
Subject: Probability & Statistics
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Assignment 5

## Q1.

The overseas shipment has 5 automobiles out of which 2 have slightly paint blemishes.

So, the automobiles are $\{N_1, N_2, N_3, B_1, B_2\}$

So, here the Sample Space

$S = \{N_1N_2N_3, N_1N_2B_1, N_1N_2B_2, N_1N_3B_1, N_1N_3B_2, N_1B_1B_2, N_2N_3B_1, N_2N_3B_2, N_2B_1B_2, N_3B_1B_2\}$

Let $X$ = No. of automobiles purchased with paint blemishes

$X = 0$, With, $[N_1, N_2, N_3]$

$= 1$, With, $[N_1N_2B_1], [N_1N_2B_2], [N_1N_3B_1], [N_1N_3B_2], [N_2N_3B_1], [N_2N_3B_2]$

$= 2$, With, $[N_1B_1B_2], [N_2B_1B_2], [N_3B_1B_2]$

## Q2.

From the given information, The cumulative distribution of T is,

$$F(t) = \begin{cases} 0, & t < 1 \\ 1/4, & 1 \le t < 3 \\ 1/2, & 3 \le t < 5 \\ 3/4, & 5 \le t < 7 \\ 1, & t \ge 7 \end{cases}$$

a. $P[T=5]$

From the knowing information,

$P[X=x] = F[x] - F[x-1]$

$P[T=5] = P[T \le 5] - P[T \le 4]$

$= F[5] - F[4] = \frac{3}{4} - \frac{1}{2}$

$\boxed{= \frac{1}{4}}$

b. $P[T>3]$

$P[T>3] = 1 - P[T \le 3]$

$= 1 - F[3] = 1 - \frac{1}{2}$

$\boxed{= \frac{1}{2}}$

c. $P[1.4 < T < 6]$

From the knowing information

$P[a < X < b] = F[b] - F[a]$

$P[1.4 < T < 6] = F[6] - F[1.4]$

$= \frac{3}{4} - \frac{1}{4} = \frac{2}{4}$

$\boxed{= \frac{1}{2}}$

Q3.

| $W$ | $P[W=w]$ |
|---|---|
| -3 | 0.1 |
| -2 | 0.25 |
| -1 | 0.3 |
| 0 | 0.15 |
| 1 | d |

a. Value of d

$\sum P[W = w] = 1$

$0.1 + 0.25 + 0.3 + 0.15 + d = 1$

$0.8 + d = 1$

$\boxed{d = 0.2}$

b. $P[-3 \leq W < 0]$

$= P[W = -3] + P[W = -2] + P[W = -1]$

$= 0.1 + 0.25 + 0.3$

$\boxed{= 0.65}$

c. $P[W > -1]$

$= P[W = 0] + P[W = 1]$

$= 0.15 + 0.2$

$\boxed{= 0.35}$

d. $P[-1 < W < 1]$

$= P[W = 0]$

$\boxed{= 0.15}$

Q4.

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 \leq x < 2 \\ 0, & \text{elsewhere} \end{cases}$$

a. Convert the number of hours into units by dividing the number of hours with total number of units.

$$P\left[X \leq \frac{120}{100}\right] = P[X \leq 1.2]$$

$$= \int_0^{1.2} f(x)\,dx = \int_0^1 f(x)\,dx + \int_1^{1.2} f(x)\,dx$$

$$= \int_0^1 x\,dx + \int_1^{1.2} (2 - x)\,dx$$

$$= \frac{x^2}{2}\Big|_0^1 + \left[2\int_1^{1.2} dx - \int_1^{1.2} x\,dx\right]$$

$$=\left[\frac{1}{2}-0\right]+\left[(2\times 0.2)-\left[\frac{x^2}{2}\right]_1^{1.2}\right]$$

$$=0.5+\left[0.4-\left[\frac{1.44}{2}-\frac{1}{2}\right]\right]=0.5+[0.4-0.22]$$

$$=0.5+0.18$$

$$\boxed{=0.68}$$

b. Convert the number of hours into units by dividing the number of hours with total number of units.

$$P\left[\frac{50}{100}\le x\le\frac{100}{100}\right]=P[0.5\le X\le 1]$$

$$=\int_{0.5}^{1} f[x]\,dx=\int_{0.5}^{1} x\,dx$$

$$=\left.\frac{x^2}{2}\right|_{0.5}^{1}$$

$$=\left[\frac{1^2}{2}-\frac{[0.5]^2}{2}\right]=\frac{1}{2}-\frac{0.25}{2}$$

$$=0.5-0.125$$

$$\boxed{=0.375}$$

Q5. $f(y)=\begin{cases} Ky^4[1-y]^3, & 0\le Y\le 1 \\ 0, & \text{else where}\end{cases}$

a. Given probability density function is valid function then the value of the constant

$f[y]\ge 0$ Since, $Y^4[1-Y]^3\ge 0\ \forall\ 0\le Y\le 1$

$$\int_{-\infty}^{\infty} f[x]=1$$

$$K\int_0^1 Y^4[1-Y]^3\,dy=1$$

$$\Rightarrow K\int_0^1[-Y^7+3Y^6-3Y^5+Y^4]\,dy=1$$

$$\Rightarrow K\left[-\int_0^1 Y^7 dy+\int_0^1 3Y^6 dy-\int_0^1 3Y^5 dy+\int_0^1 Y^4 dy\right]=1$$

$\Rightarrow K\left[-\frac{1}{8}+\frac{3}{7}-\frac{1}{2}+\frac{1}{5}\right]=1$

$\Rightarrow \frac{1}{280}K=1$

$\Rightarrow \boxed{K=280}$

b. The aim is to the Probability that at most 50% of the firms make a profit in the first Year:

Here, 50% = 0.5. This implies that the value of Y is from $0 < Y < 0.5$. Therefore,

$$P[Y<0.5]=\int_0^{0.5} f[y]\,dy$$

$$=280\int_0^{0.5} y^4(1-y)^3\,dy$$

$$=280\int_0^{0.5}[y^4-3y^5+3y^6-y^7]\,dy$$

$$=280\left[\int_0^{0.5} y^4\,dy-3\int_0^{0.5} y^5\,dy+3\int_0^{0.5} y^6\,dy-\int_0^{0.5} y^7\,dy\right]$$

$$=280[0.00625-0.00781+0.00335-0.0049]$$

$$=280\times 0.0013$$

$$\boxed{=0.364}$$

c. The aim is to the Probability that at least 80% of the firms make a profit in the first Year

Here, 80% = 0.8

$$P[Y\geq 80\%]=P[Y\geq 0.8]$$

$$=\int_{0.8}^{1} f[y]\,dy$$

$$=280\int_{0.8}^{1} y^4[1-y]^3\,dy$$

$$=280\int_{0.8}^{1}[y^4-3y^5+3y^6-y^7]\,dy$$

$$=280\left[\int_{0.8}^{1} y^4\,dy-3\int_{0.8}^{1} y^5\,dy+3\int_{0}^{1} y^6\,dy\mp\int_{0.8}^{1} y^7\,dy\right.$$

$= 280[0.134464 - 0.36894 + 0.3387 - 0.10403]$

$= 280 \times 0.00019$

$\boxed{= 0.0563}$

Q6. $f(x) = \begin{cases} K(3-x^2), & -1 \le x \le 1 \\ 0, & \text{elsewhere} \end{cases}$

a. $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\Rightarrow K\int_{-1}^{1} (3-x^2)\,dx = 1$

$\Rightarrow K\left[3\int_{-1}^{1} dx - \int_{-1}^{1} x^2 dx\right] = 1$

$\Rightarrow K\left[3 \times x\Big|_{-1}^{1} - \frac{x^3}{3}\Big|_{-1}^{1}\right] = 1 \quad \Rightarrow K\left[6 - \left[\frac{1}{3} + \frac{1}{3}\right]\right] = 1$

$\Rightarrow K\left[6 - \frac{2}{3}\right] = 1 \Rightarrow \frac{16K}{3} = 1$

$\Rightarrow \boxed{K = \frac{3}{16}}$

b. $P[X < 0.5] = \int_{-1}^{0.5} f(x)\,dx$

$= \frac{3}{16}\left[3\int_{-1}^{1/2} dx - \int_{-1}^{1/2} x^2 dx\right]$

$= \frac{3}{16}\left[3\left[\frac{1}{2} + 1\right] - \frac{x^3}{3}\Big|_{-1}^{1/2}\right]$

$= \frac{3}{16}\left[4.5 - \left[\frac{1}{24} + \frac{1}{3}\right]\right] = \frac{3}{16}\left[\frac{9}{2} - \frac{1}{24} - \frac{1}{3}\right]$

$= \frac{3}{16} \times \frac{99}{21}$

$\boxed{= \frac{99}{128}}$

c. $P[|x| > 0.8] = 1 - P[|x| \le 0.8]$

$= 1 - P[-0.8 \le x \le 0.8]$

$$= 1 - \int_{-0.8}^{0.8} f(x)\,dx = 1 - \int_{-0.8}^{0.8} \frac{3}{16}\,(3-x^2)\,dx$$

$$= 1 - \frac{3}{16}\left[3x - \frac{x^3}{3}\right]_{-0.8}^{0.8}$$

$$= 1 - \frac{3}{16}\,[2.2293 + 2.2293]$$

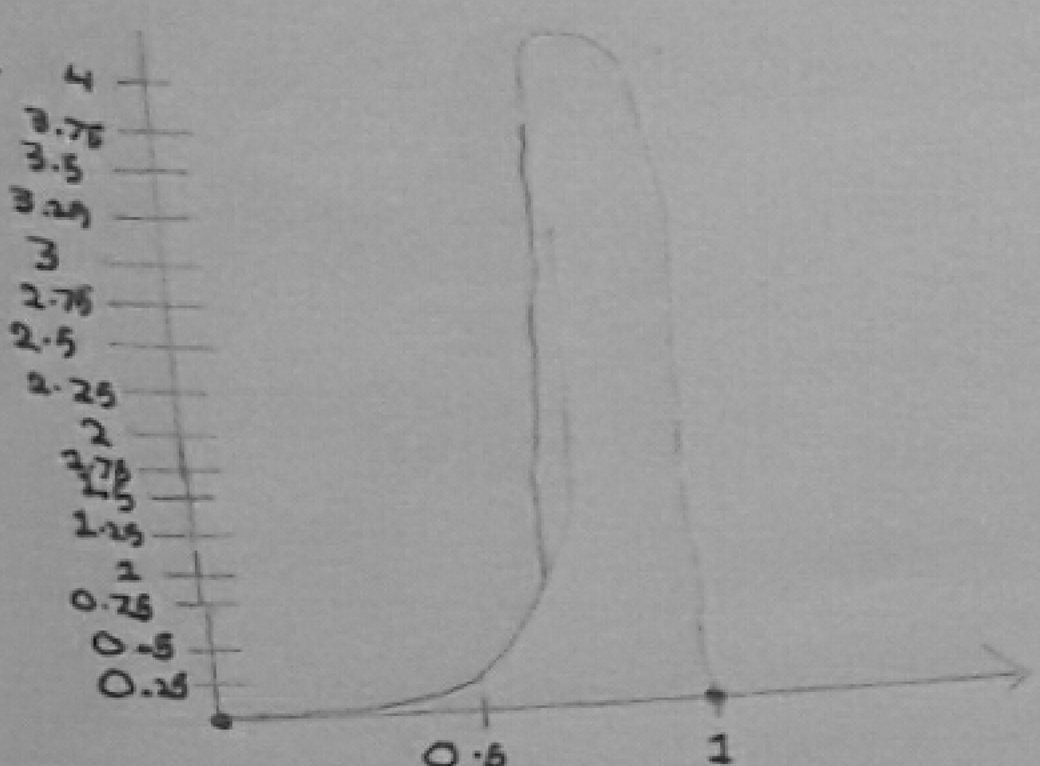
$$= 1 - 0.836$$

$$\boxed{= 0.164}$$

Q7.

a. $$f(x) = \begin{cases} 90x^8(1-x), & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

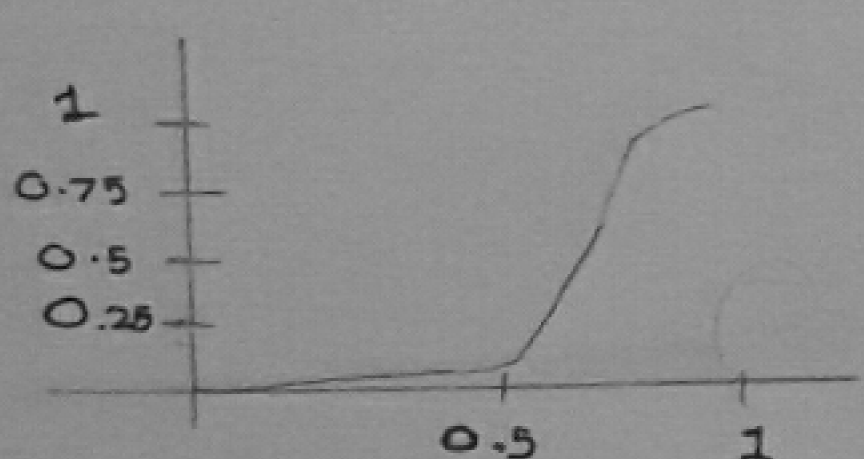
i.




Cdf of $X$ is

$$F(x) = P(X \le x) = \int_0^x f(x)\,dx = \int_0^x 90\,(x^8 - x^9)\,dx = 90\left[\frac{x^9}{9} - \frac{x^{10}}{10}\right]_0^x$$

$$= 10x^9 - 9x^{10}$$



ii. $P[X \leq 0.5)$

$= 10(0.5)^9 - 9(0.5)^{10}$

$\boxed{= 0.0107}$

iii. $P[0.25 < x \leq 0.5)$

$P(0.25 < x \leq 0.5) = F(0.5) - F[0.25)$

We already found the first term. The second term

$F(0.25] = P[X \leq 0.25) = 10(0.25)^9 - 9(0.25)^{10} = 0.00003$

So,

$P[0.25 < x \leq 0.5] = F(0.5) - F(0.25) = 0.01074 - 0.0003$

$\boxed{= 0.01071}$

Since $f(x)$ is continuous, $P[0.25 \leq x \leq 0.5) = P[0.25 < x \leq 0.5)$

$\boxed{= 0.01071}$

6.

$$F[x] = \begin{cases} 0 & x < 0 \\ x^4 & 0 \leq x \leq 1 \\ 1 & x \geq 1 \end{cases}$$

i. $P(0.3 < X < 0.6]$

$P(0.3 < x < 0.6] = F[0.6] - F(0.3)$

$= [(0.6)^4 - (0.3)^4]$

$= 0.1296 - 0.0081$

$\boxed{= 0.1215}$

$$\therefore f(x) = \frac{d}{dx}[F(x)] = \begin{cases} 0 \text{ ; otherwi} \\ 4x^3 \text{ ; } 0 \leq x \leq \\ 0 \text{ ; otherwi} \end{cases}$$

ii. $P[X > a] = 0.4$

$\int_a^\infty f[x]\,dx = 0.4$

$\int_0^1 4x^3\,dx = 0.4$

$(1)^4 - (a)^4 = 0.4$

$1 - a^4 = 0.4 \Rightarrow 1 - 0.4 = a^4$

$a^4 = 0.6 \Rightarrow a = (0.6)^{1/4} \Rightarrow \boxed{a = 0.8801}$