Q8. a. $P(x) = P[\text{1st digit is } x] = \log_{10}\left[\frac{x+1}{x}\right],\ x = 1, 2, \ldots, 9$

$$P(x) = \sum_{x=1}^{9} \log_{10}\left[\frac{x+1}{x}\right]$$

$$P(x) = \log_{10}\left[\frac{2}{1} \times \frac{3}{2} \times \cdots \times \frac{10}{9}\right]$$

$$P(x) = \log_{10} 10$$

$$\boxed{P(x) = 1}$$

b.

| $X$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| $P(x)$ | 0.301 | 0.176 | 0.124 | 0.096 | 0.079 | 0.066 | 0.0580 | 0.0512 | 0.0458 |

We see that as $x$ increase $p(x)$ decreases, However the probabilities are all same for discrete uniform distribution

i.e $1/9 = 0.1111$ for $x = 1, 2, 3, 4, 5, 6, 7, 8, 9$

c. Skewness $\alpha_3(x)$ is defined by the formula

$$\alpha_3(x) = \frac{\mu_3(x)}{\sigma(x)^3}$$

where $\mu_3(x) = \sum [x_i - E(x)]^3 . P(x_i)$

Now, Calculate $E(x)$

$$E(x) = \sum_{i=1}^{9} x_i P(x_i)$$

$$\therefore E(x) = 1P(1) + 2P(2) + \cdots + 9P(9)$$

$$= 0.3010 + 2(0.1760) + \cdots + 9(0.0457)$$

$$\boxed{E(x) = 3.3167}$$

$$\therefore \mu_3(x) = \sum (X_i - 3.3167)^3 . p(X)$$

$= (1-3.3167)^3 P(1) + (2-3.316)^3 P(2) + \dots +$
$(9-3.3167)^3$

$\boxed{= 4.102}$

Now calculate

$E(x^2) = \sum x_i^2 P(x_i)$

$= 1^2 P(1) + 2^2 P(2) + \dots + 9^2 P(9)$

$\boxed{= 18.8746}$

$\sigma(x) = E(x^2) - [E(x)]^2$

$= 18.8746 - 3.3167$

$\boxed{= 7.8741}$

$\sigma_3(x) = \dfrac{14.102}{(7.874)^3}$

$\boxed{= 0.028}$

Here

$\sigma_3(x) = 0.028 > 0$

The distribution is skewed to left

d. Mode $= 1$, Since $P(1)$ is maximum

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| $P(x)$ | 0.301 | 0.176 | 0.1249 | 0.0969 | 0.079 | 0.066 | 0.058 | 0.0512 | 0.0458 |
| $F(x)$ | 0.301 | 0.477 | 0.602 | 0.698 | 0.778 | 0.845 | 0.903 | 0.9542 | 1 |

$Q_1$ = First Quartile

$\boxed{= 1}$ (Since $F(5) > 0.25$)

$Q_3$ = Third Quartile

$\boxed{= 5}$ (Since $F(5) > 0.75$)

Inter-Quartile Range $=$

$Q_3 - Q_1$

$= 5 - 1$

$\boxed{= 4}$

Q9.

a. Probability that a parcel would be replaced is computed here as

$= P(X>1)$
$= 1 - P(X=0) - P(X=1)$
$= 1-(1-0.01)^{10} - 10\times 0.01\times (1-0.01)^{9}$ $\boxed{= 0.0043}$

Therefore about 0.43 % of the parcels are expected to be replaced here. The expected number of packages that the companies need to sell to have a replaced one is computed here as

$= 1$ / Probability of replacement
$= 1 / 0.0043$
$\boxed{= 232.56}$

b. The probability that the receiver makes the wrong decision is computed here as

$= P(X=3) + P(X=4) + P(X=5)$
$= \left[\begin{matrix}5\\3\end{matrix}\right](0.1)^3(0.9)^2 + p\left[\begin{matrix}5\\4\end{matrix}\right](0.1)^4(0.9) + (0.1)5$

$\boxed{= 0.00856}$

Therefore 0.00856 is the probability that the receiver make the wrong decision here