Question 10:

BINOMIAL DISTRIBUTION:

Example 5.1: The probability that a certain kind of component will survive a shock test is 3/4. Find the probability that exactly 2 of the next 4 components tested survive.

***Solution*:** Assuming that the tests are independent and $p = 3/4$ for each of the 4 tests, we obtain

$$b\left(2; 4, \frac{3}{4}\right) = \binom{4}{2}\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^2 = \left(\frac{4!}{2!\,2!}\right)\left(\frac{3^2}{4^4}\right) = \frac{27}{128}.$$

R code:

a)

```
> n <- 4;
> p<- 3⁄4;
> x <- 0:n
> y=dbinom(x, n, p)
> z=cbind(x, y)
> z
     x          y
[1,] 0 0.00390625
[2,] 1 0.04687500
[3,] 2 0.21093750
[4,] 3 0.42187500
[5,] 4 0.31640625
```

b)

```
> n <- 4;
> p<- 0.75;
> x <- 0:n
> y=pbinom(x,n,p)
> z = cbind(x,y)
> z
```

```
     x         y
[1,] 0 0.00390625
[2,] 1 0.05078125
[3,] 2 0.26171875
[4,] 3 0.68359375
[5,] 4 1.00000000
```

c)

i) Let A be the event that exactly 2 of the next 4 components tested survived

```
> x <- 2;
> n <- 4;
> p<- 0.75;
> y=pbinom(x,n,p)
> y
[1] 0.2617188
```

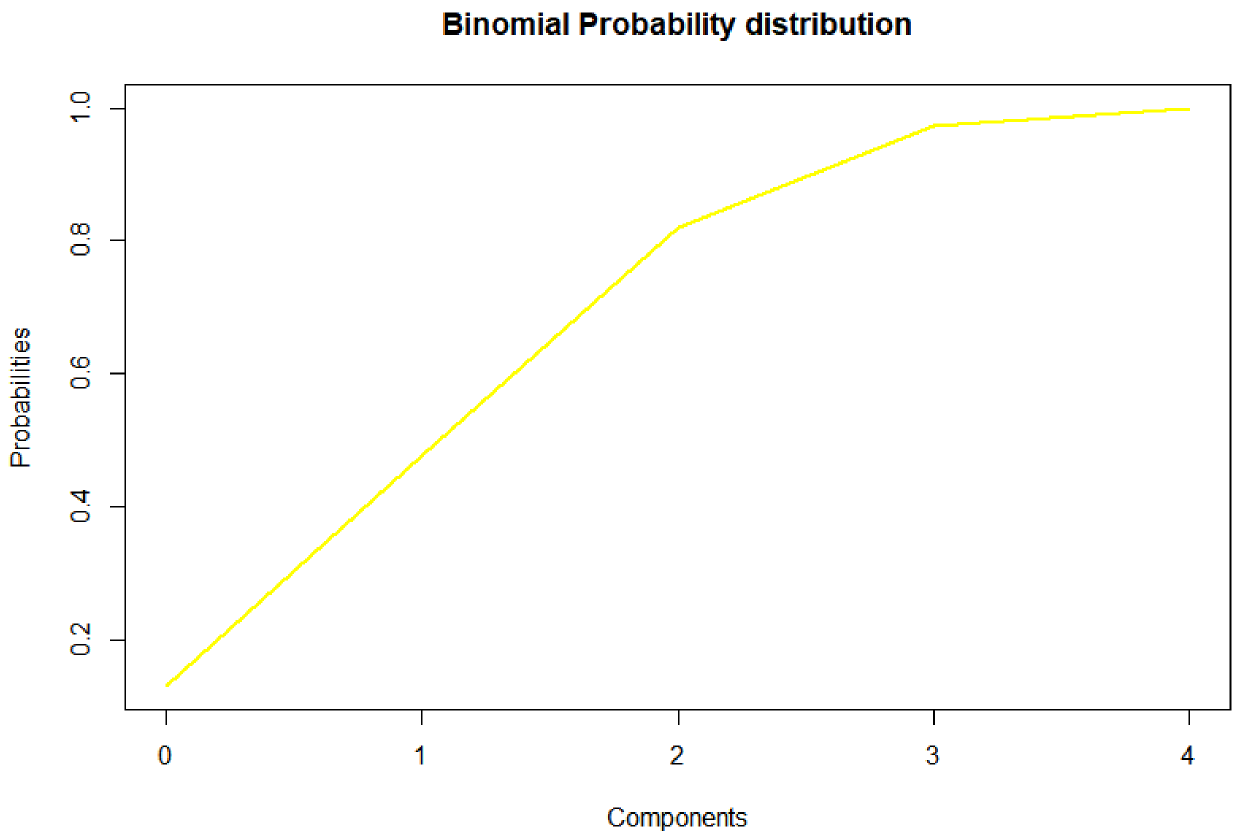
ii) Let A be the event that exactly 3 of the next 4 components tested survived

```
> x<-3;
> n <- 4;
> p<- 0.75;
> y=pbinom(x,n,p)
> y
[1] 0.6835937
```

d)

```
> n <- 4;
> p<- 0.4;
> x <- 0:n
> y=pbinom(x,n,p)
```

> plot(x,y,type="l",xlab="Components", ylab="Probabilities",main = "Binomial Probability distribution",lwd = 2,col = "yellow")



e)

> n <- 4;

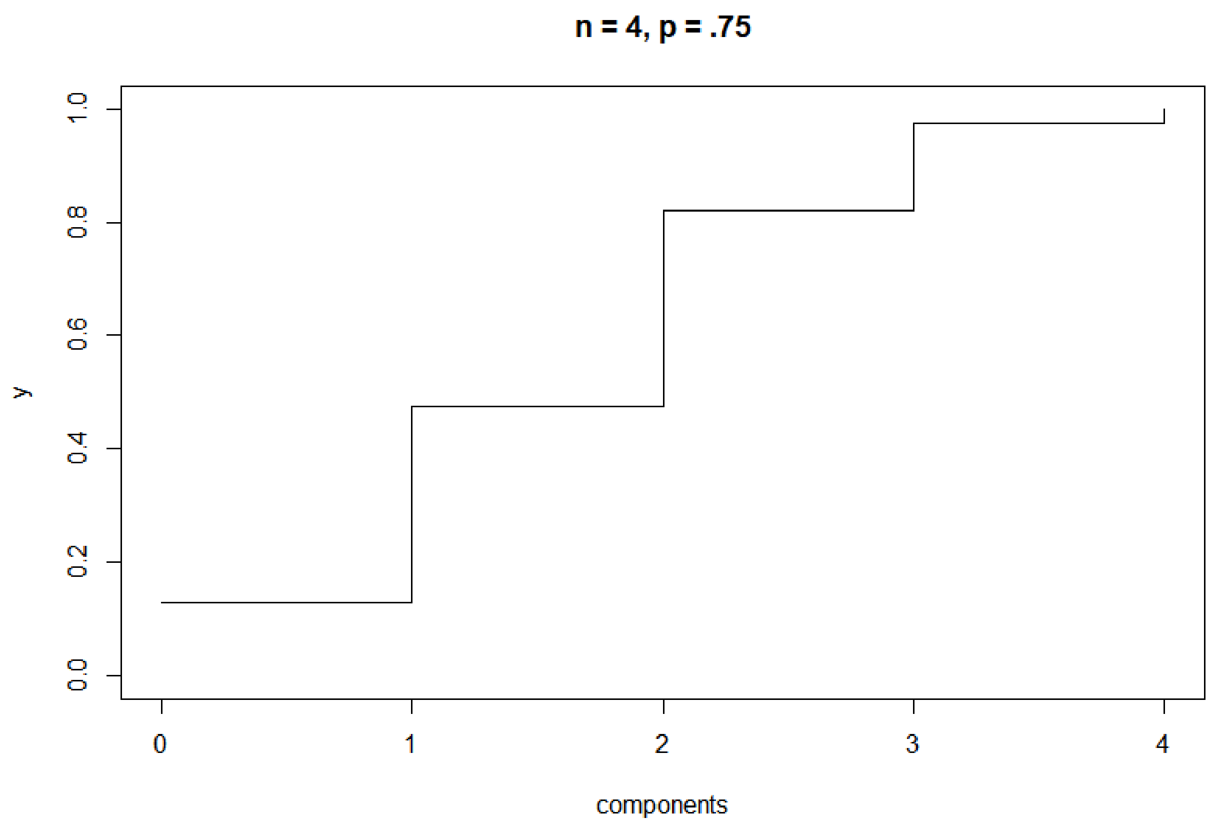
> p<- 0.4;

> x <- 0:n

> y=pbinom(x,n,p)

> p<- 0.4;

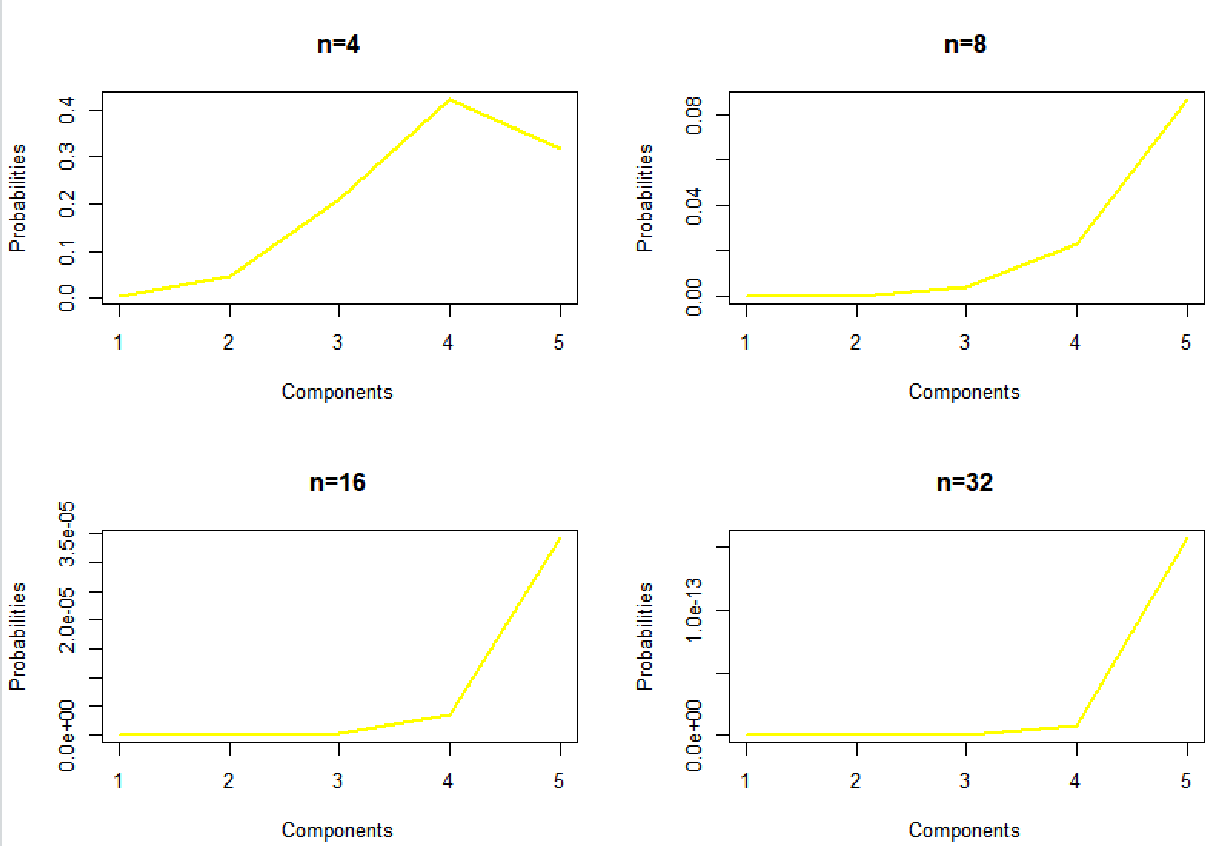
> plot(x, y, xlab = "components", ylab = "y", ylim = c(0, 1), type = "s", main = "n = 4, p = .75")



f)

```
> n_1 <- 4;

> n_2<- 8;

> n_3<-16;

> n_4 <-32;

> p<- 0.75;

> x <- 0:n;

> a=dbinom(x,n_1,p)

> b=dbinom(x,n_2,p)

> c=dbinom(x,n_3,p)

> d=dbinom(x,n_4,p)

> par(mfrow=c(2,2))

> plot(a,type="l",xlab="Components", ylab = "Probabilities", main="n=4",lwd=2,col="yellow")
```

```
> plot(b,type="l",xlab="Components", ylab = "Probabilities", main="n=8",lwd=2,col="yellow")
> plot(c,type="l",xlab="Components", ylab = "Probabilities", main="n=16",lwd=2,col="yellow")
> plot(d,type="l",xlab="Components", ylab="Probabilities", main="n=32",lwd=2,col="yellow")
```



g)

```
> n <- 4;
> p<- 0.75;
> rbinom(50,n,p)
[1] 4 4 3 4 3 4 4 1 2 3 2 3 4 3 4 3 4 1 4 3 3 4 3 4 3 3 3 3 3 4 3 3 4 4 2 4 4 2 3 4 4 4 4 4 2 3 4 3 3 3
```

h)

```
> n <- 4;
> p<- 0.75;
> zx=c(.25,.5,.75)
```

```
> qbinom(x.n,p)
[1] 2 3 4
```

i)

```
> n <- 4; p<- 0.75;
> vec=c(.5,.7,.95)
> qbinom(vec,n,p)
[1] 3 4 4
```

j)

```
> n <- 4;
> p<- 0.75;
> vec=c(.2,.6)
> qbinom(vec,n,p)
[1] 2 3
```

HYPERGEOMETRIC DISTRIBUTION:

Example 5.8: A particular part that is used as an injection device is sold in lots of 10. The producer deems a lot acceptable if no more than one defective is in the lot. A sampling plan involves random sampling and testing 3 of the parts out of 10. If none of the 3 is defective, the lot is accepted. Comment on the utility of this plan.

***Solution*:** Let us assume that the lot is truly **unacceptable** (i.e., that 2 out of 10 parts are defective). The probability that the sampling plan finds the lot acceptable is

$$P(X=0) = \frac{\binom{2}{0}\binom{8}{3}}{\binom{10}{3}} = 0.467.$$

R code:

a)

```
> n <- 3;
> x<-0:n;
 > k<-2 ;
```

```
> N<-10 ;
> N1=N-k
> y=dhyper(x,k,N1,n)
> z=cbind(x,y)
> z
     x          y
[1,] 0 0.46666667
[2,] 1 0.46666667
[3,] 2 0.06666667
[4,] 3 0.00000000
```

b)

```
> n<-3;
> x<-0:n;
> k<-2;
> N<-10;
> N1= N-k
> y=phyper(x,k,N1,n)
> z=cbind(x,y)
> z
     x         y
[1,] 0 0.4666667
[2,] 1 0.9333333
[3,] 2 1.0000000
[4,] 3 1.0000000
```

c)

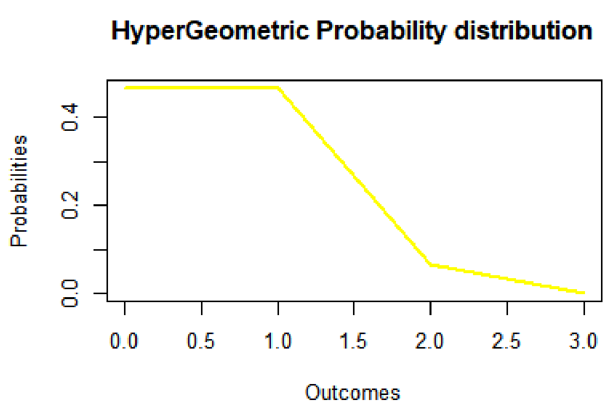
"If none of the 3 is defective, the lot is accepted. Comment on the utility of this plan."

```
> n <- 3;
```

```
> x<-0;
> k<-2;
> N<-10;
> N1=N-k
> y=dhyper(x,k,N1,n)
> z=cbind(x,y)
> z
     x         y
[1,] 0 0.4666667
```

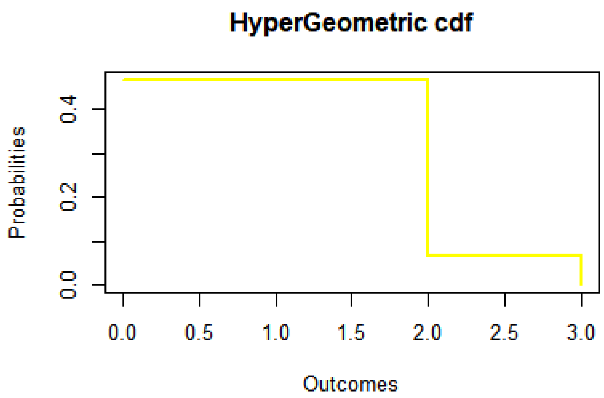
d)

```
> n<-3;
> x<-0:n;
> k<-2;
> N<-10;
> N1= N-k
> y=dhyper(x,k,N1,n)
> par(mfrow=c(2,2))
> plot(x, y ,type="l", xlab="Outcomes", ylab="Probabilities", main="HyperGeometric Probability distribution",lwd=2,col="yellow")
```



e)

```
> n<-3; x<-0:n; k<-2; N<-10; N1= N-k

> y=dhyper(x,k,N1,n)

> par(mfrow=c(2,2))

> plot(x, y, type="s", xlab="Outcomes", ylab="Probabilities", main = "HyperGeometric cdf",lwd=2,col="yellow")
```

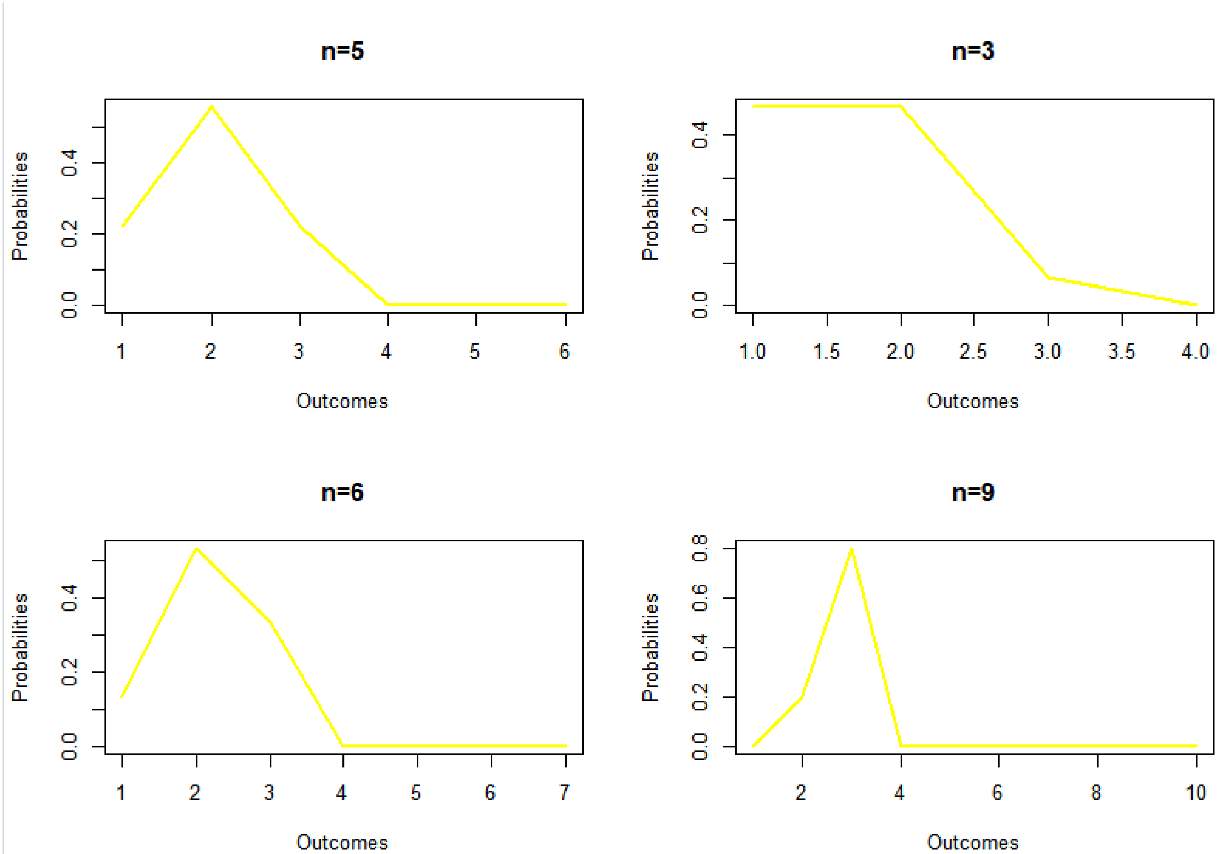


f)

```
> n_1<- 5;

> n_2<-3;

> n_3<-6;

> n_4<-9;

> x1<-0:n_1;

> x2<-0:n_2; x3<-0:n_3; x4<-0:n_4;

> k<-2; N<-10; N_1= N-k

> a=dhyper(x1,k,N_1,n_1)

> b=dhyper(x2,k,N_1,n_2)

> c=dhyper(x3,k,N_1,n_3)

> d=dhyper(x4,k,N_1,n_4)

> par(mfrow=c(2,2))

> plot(a, type="l",xlab="Outcomes", ylab="Probabilities", main="n=5",lwd=2,col="yellow")

> plot(b, type="l",xlab="Outcomes", ylab="Probabilities", main="n=3",lwd=2,col="yellow")
```

> plot(c, type="l", xlab="Outcomes", ylab="Probabilities", main="n=6",lwd=2,col="yellow")

> plot(d, type="l", xlab="Outcomes", ylab="Probabilities", main="n=9",lwd=2,col="yellow")



g)

> n<-3;

> x<-50;

> k<-2;

> N<-10;

> N1= N-k

> rhyper(x,k,N1,n)

[1] 0 1 0 0 0 1 0 0 2 0 1 0 0 0 0 1 0 0 0 0 0 0 0 0 1 1 0 1 1 0 1 0 0 0 1 0 1 0 1 0 1 0 1 0 1 0 1 1 0 0

h)

> n<-3;

```
> x<-50;

> k<-2;

> N<-10;

> N1= N-k

> z=c(.25,.5,.75)

> qhyper(z,k,N1,n)

[1] 0 1 1
```

i)

```
> n<-3;

> x<-50;

> k<-2;

> N<-10;

> N1= N-k

> z=c(.5,.7,.95)

> qhyper(z,k,N1,n)

[1] 1 1 2
```

j)

```
> n<-3;

> x<-50;

> k<-2;

> N<-10;

> N1= N-k

> z=c(.2,.6)

> qhyper(z,k,N1,n)

[1] 0 1
```

POISSON DISTRIBUTION:

Example 5.19: In a certain industrial facility, accidents occur infrequently. It is known that the probability of an accident on any given day is 0.005 and accidents are independent of each other.

(a) What is the probability that in any given period of 400 days there will be an accident on one day?

(b) What is the probability that there are at most three days with an accident?

***Solution*:** Let X be a binomial random variable with $n = 400$ and $p = 0.005$. Thus, $np = 2$. Using the Poisson approximation,

(a) $P(X = 1) = e^{-2}2^1 = 0.271$ and

(b) $P(X \leq 3) = \sum_{x=0}^{3} e^{-2}2^x/x! = 0.857$.

R code:

a)

```
> x=0:20;
> n<-400;
> p<-0.005;
> lambdha=n*p;
> y=dpois(x,lambdha)
> z=cbind(x,y)
> z
      x            y
 [1,]  0 1.353353e-01
 [2,]  1 2.706706e-01
 [3,]  2 2.706706e-01
 [4,]  3 1.804470e-01
 [5,]  4 9.022352e-02
 [6,]  5 3.608941e-02
 [7,]  6 1.202980e-02
 [8,]  7 3.437087e-03
 [9,]  8 8.592716e-04
[10,]  9 1.909493e-04
```

```
[11,] 10 3.818985e-05
[12,] 11 6.943609e-06
[13,] 12 1.157268e-06
[14,] 13 1.780413e-07
[15,] 14 2.543447e-08
[16,] 15 3.391262e-09
[17,] 16 4.239078e-10
[18,] 17 4.987150e-11
[19,] 18 5.541278e-12
[20,] 19 5.832924e-13
[21,] 20 5.832924e-14
```

b)

```
> x=0:20;
> n<-400;
> p<-0.005;
> lambdha=n*p;
> y=ppois(x,lambdha)
> z=cbind(x,y)
> z
      x         y
 [1,]  0 0.1353353
 [2,]  1 0.4060058
 [3,]  2 0.6766764
 [4,]  3 0.8571235
 [5,]  4 0.9473470
 [6,]  5 0.9834364
 [7,]  6 0.9954662
```

```
 [8,]  7 0.9989033
 [9,]  8 0.9997626
[10,]  9 0.9999535
[11,] 10 0.9999917
[12,] 11 0.9999986
[13,] 12 0.9999998
[14,] 13 1.0000000
[15,] 14 1.0000000
[16,] 15 1.0000000
[17,] 16 1.0000000
[18,] 17 1.0000000
[19,] 18 1.0000000
[20,] 19 1.0000000
[21,] 20 1.0000000
```

c)

i) P(X = 1)

```
> x=1;
> n<-400;
> p<-0.005;
> y=n*p;
> dpois(x,y)
[1] 0.2706706
```

ii) P(X ≤ 3)

```
> x=3;
> n<-400;
> p<-0.005;
> y=n*p;
```

> ppois(x,y)

[1] 0.8571235

d)

> x=0:20;

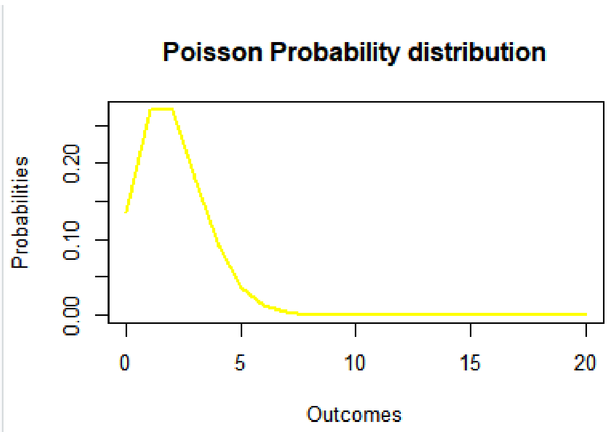
> n<-400;

> p<-0.005;

> z=n*p;

> y=dpois(x,z)

> plot(x, y, type="l", xlab="Outcomes", ylab="Probabilities", main="Poisson Probability distribution",lwd = 2, col = "yellow")



e)

> x=0:20;

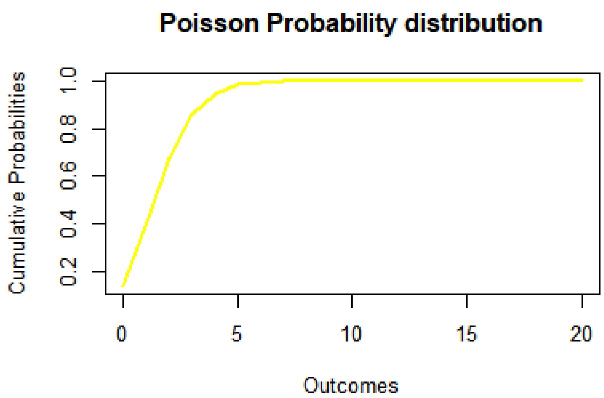
> n<-400;

> p<-0.005;

> z=n*p;

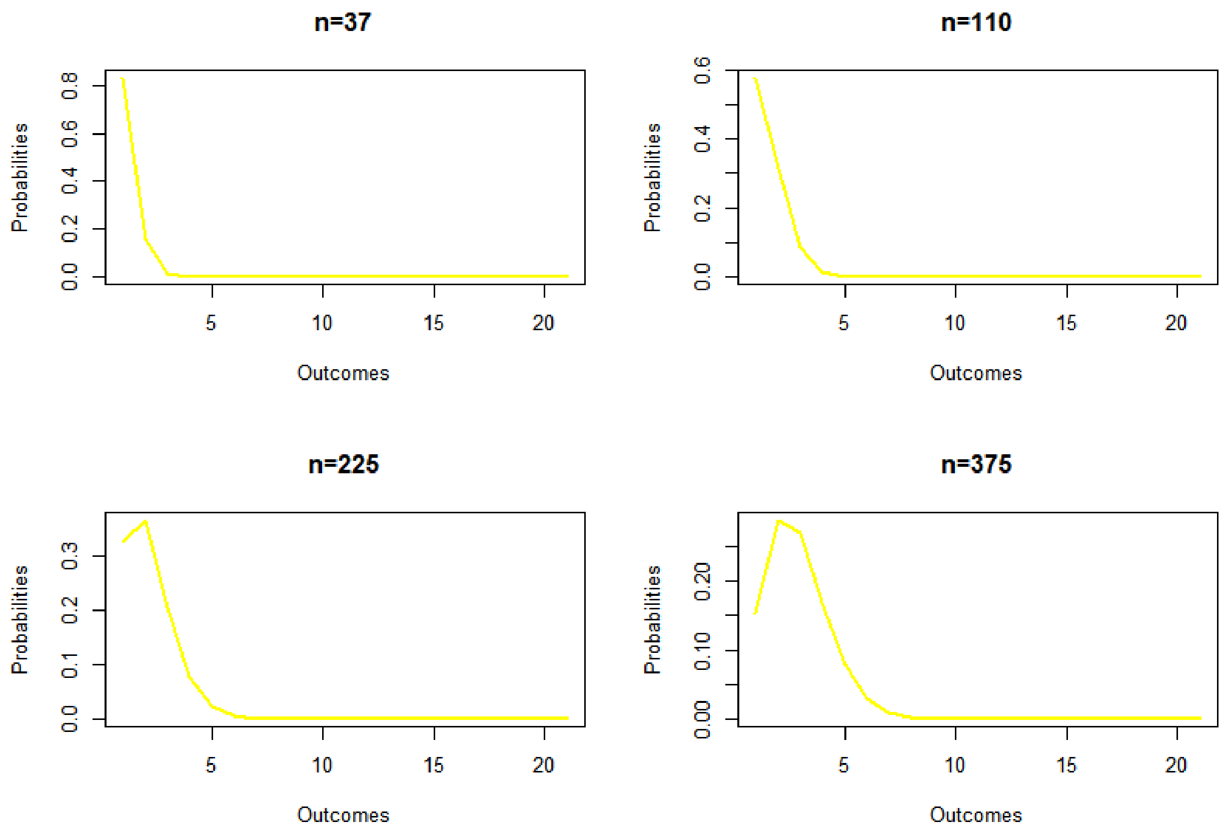
> y=ppois(x,z)

> plot(x, y, type="l", xlab="Outcomes", ylab="Cumulative Probabilities", main="Poisson Probability distribution",lwd = 2, col = "yellow")



f)

```
> n_1<-37;

> n_2<-110;

> n_3<-225;

> n_4<-375;

> N_1=n_1*p;

> N_2=n_2*p;

> N_3=n_3*p;

> N_4=n_4*p;

> a=dpois(x,N_1)

> b=dpois(x,N_2)

> c=dpois(x,N_3)

> d=dpois(x,N_4)

> par(mfrow=c(2,2))

> plot(a, type="l", xlab="Outcomes", ylab="Probabilities", main="n=37",lwd=2,col="yellow")

> plot(b, type="l", xlab="Outcomes", ylab="Probabilities", main="n=110",lwd=2,col="yellow")

> plot(c, type="l", xlab="Outcomes", ylab="Probabilities", main="n=225",lwd=2,col="yellow")

> plot(d, type="l", xlab="Outcomes", ylab="Probabilities", main="n=375",lwd=2,col="yellow")
```



g)

```
> x=50;
> p<-0.005;
> n<-400;
> z=n*p;
> rpois(x,z)
 [1] 2 1 2 3 2 3 2 6 4 1 3 3 4 0 3 1 0 4 5 4 3 3 2 2 2 1 0 1 0 3 1 1 3 2 2 2 4 2 1 1 2 2 6 0 3 1 1 1 0 2
```

h)

```
> x=c(.25,.5,.75);
> p<-0.005;
> n<-400;
> z=n*p;
> qpois(x,z)
```

```
[1] 1 2 3
```

i)

```
> x=c(.5,.7,.95);
> p<-0.005;
> n<-400;
> z=n*p;
> qpois(x,z)
[1] 2 3 5
```

j)

```
> x=c(.2,.6);
> p<-0.005;
> n<-400;
> la=n*p;
> qpois(x,la)
[1] 1 2
```