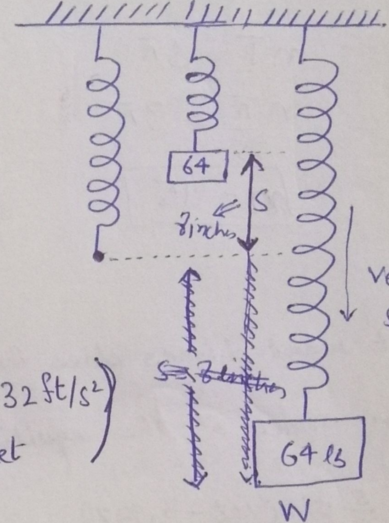
Practice Problem:– A mass weighing 64 pounds stretches a spring 0.32 foot. The mass is initially released from a point 8 inches above the equilibrium position with a downward velocity of 5 ft/s.

(a) Find the Equation of Motion?–



$W = 64\ lb$; $S = 0.32 ft$

$W \propto S$

$W = kS$

$k = W/S = 64/0.32 = 200$.

$W = mg$

$m = W/g = 64/32 = 2$ $\left(\because g = 9.8\ m/s^2 = 32\ ft/s^2 \quad 1m = 3.281 feet \right)$

Equation of motion is

$$m\frac{d^2x}{dt^2} + kx = 0.$$

at $t=0$

$$2\frac{d^2x}{dt^2} + 200x = 0 \ ; \quad x(0) = 8 \text{ inches} = \frac{-8}{12} ft = \frac{2}{3} ft \qquad \left(\because 1 \text{ inch} = \frac{1}{12} \text{ feet}\right)$$

$$x'(0) = 5\ ft/s.$$

(b) What are the amplitude and period of motion?

$\underline{A.E}$: $2m^2 + 200 = 0$.

$m^2 + 100 = 0$

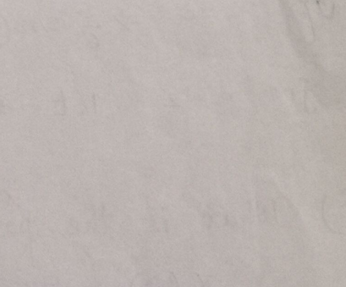
$m^2 = -100$

$m = \pm 10i$

$x(t) = C_1 \cos 10t + C_2 \sin 10t$; $x'(t) = -C_1 10 \sin 10t + C_2 10 \cos 10t$.

$x(0) = -2/3 \Rightarrow -2/3 = C_1$

$x'(0) = 5 \Rightarrow 5 = 10C_2 \Rightarrow C_2 = 1/2$



$$\boxed{x(t) = -\frac{2}{3}\cos 10t + \frac{1}{2}\sin 10t}$$

$$= \frac{5}{6}\sin(10t \mp 0.927)$$

$\tan\phi = \frac{-2/3}{1/2} = \frac{-2}{3} \times \frac{2}{1} = -\frac{4}{3}$

$\phi = -0.927$

A = Amplitude $= \frac{5}{6}$, T = Period $= \frac{2\pi}{\omega} = \frac{2\pi}{10} = \frac{\pi}{5}$

c) How many Complete cycles will the mass have Completed at the end of 3π Seconds.

Let n be the no. of Total Cycles at the end of 3π s.

Let T be the time to Complete one oscillation/cycle $= \pi/5$ ~~at some time~~.

$\Rightarrow$ at 3π Seconds.

$$\left\{\begin{array}{l} T = \pi/5 \; ; 1 \\ T = 2\pi/5 \; ; 2 \\ \vdots \\ T = n\pi/5 \; ; n \end{array}\right.$$

$$nT = 3\pi$$

$$n\frac{\pi}{5} = 3\pi$$

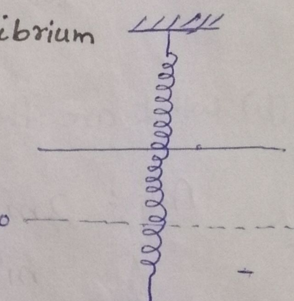
$$\boxed{n = 15}$$

e). At what times does the mass attain its extreme displacements on either side of the equilibrium position. ($x' = 0$)

$x = \frac{5}{6}\sin(10t - 0.927) \Rightarrow x' = \frac{25}{3}\cos(10t - 0.927) = 0 \Rightarrow 10t - 0.927 = \pi/2 + n\pi$

$$\boxed{t = \frac{(2n+1)\pi}{20} + 0.0927} \quad \text{for } n = 0, 1, 2, 3, \ldots$$

d). At what time does the mass pass through the equilibrium position heading downward for the second time.



If $x = 0 \Rightarrow \frac{5}{6}\sin(10t - 0.927) = 0 \Rightarrow \boxed{10t - 0.927 = n\pi}$

for $n = 2$ $\quad 10t - 0.927 = 2\pi \Rightarrow t = \frac{2\pi + 0.927}{10} = 0.721\text{s}.$

f) What is the position of the mass at $t = 3$s.

$x(3) = 5/6\sin(10\times3 - 0.927) = -0.597$ ft.

g) What is the instantaneous velocity at $t = 3$s.

$x'(3) = \frac{25}{3}\cos(10\times3 - 0.927) = -5.814$ ft/s.

h) What is the acceleration at $t = 3$sec.

$x''(3) = \frac{-250}{3}\sin(10\times3 - 0.927) = 59.702$ ft/s^2

i) What is the instantaneous velocity at the times when the mass passes through the equilibrium position.

If $x = 0 \Rightarrow 10t - 0.927 = n\pi \Rightarrow \boxed{t = \frac{n\pi + 0.927}{10}}$, $n = 0, 1, 2, 3, \ldots$

we get $\boxed{x' = \pm 8.33 \text{ ft/s.}}$ by Putting values of t in $x' = \frac{25}{3}\cos(10t - 0.927)$

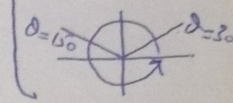
j) At what times is the mass 5 inches below the equilibrium position.

$x = \frac{5}{6}\sin(10t - 0.927)$

$x = \frac{5}{12}$ ft

$\frac{5}{12} = \frac{5}{6}\sin(10t - 0.927)$

$\sin 30 = \frac{1}{2}$
$\sin 150 = \frac{1}{2}$
$\sin 210 = -\frac{1}{2}$
$\sin 330 = -\frac{1}{2}$



$\frac{1}{2} = \sin(10t - 0.927) \Rightarrow \frac{\pi}{6} + 2n\pi = 10t - 0.927$

$t = \frac{1}{10}\left(\pi/6 + 2n\pi + 0.927\right)$; $n = 0, 1, 2, \ldots$

and $t = \frac{1}{10}\left(5\pi/6 + 2n\pi + 0.927\right)$

k). At what times is the mass 5 inches below the equilibrium position heading in the upward direction.

$x = \frac{5}{12}$ ft, $x' < 0$. $\frac{5}{12} = \frac{5}{6}\sin(10t - 0.927)$.

$\frac{1}{2} = \sin(10t - 0.927) \Rightarrow \pi - \pi/6 = 10t - 0.927.$

for $x' < 0$.

$\frac{5\pi}{6} = 10t - 0.927 \Rightarrow \boxed{t = \frac{1}{10}\left(\frac{5\pi}{6} + 2n\pi + 0.927\right)}$ $n = 0, 1, 2, \ldots$