NAME: JIYAD KHAN

ROLL NO: 19I - 1771

SECTION: BS - DS[N]

SUBJECT: PROBABILITY & STATISTICS

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ASSIGNMENT 3

Q1.

a. The person presently violates all 7 rules. So he has to adopt 5 out of the 7 rules. Number of ways in which a person can adopt 5 of these 7 rule is equal to the number of combination of 7 distinct elements taken 5 at a time. This can be given as,

$$\text{Number of ways} = {}^7C_5 = \frac{7!}{(7-5)!\,5!} = \frac{5040}{2\times120}$$

$$\boxed{=21}$$

b. The person never drinks and eat breakfast. So he has already adopted two rules. Now he has to adopt 3 more rules. He has 7-2=5 rules to select form. Number of ways of ways of adopting 3 rules from these 5 is equal to the number of combinations of 5 distinct elements taken 3 at a time. This can be given as,

$$\text{Number of ways} = {}^5C_3 = \frac{5!}{(5-3)!\,3!} = \frac{120}{2\times6}$$

$$\boxed{=10}$$

Q2.

a. With no restrictions:

When there is no restriction, then the number of arrangements of 8 people is,

$$8! = 8\times7\times6\times5\times4\times3\times2\times1$$
$$= 40{,}320 \text{ different ways}$$

b. If all the man sits together to the right of all women:

All the men sit together to the right of the women then the men can arrange in,

$$4! = 4\times3\times2\times1$$
$$= 24 \text{ ways}$$

And the women can have arranged in,

$$4! = 4\times3\times2\times1$$
$$= 24! \text{ way}$$

c. If each couple is to sit together:

Each couple is to sit together, then 4 different couple can be arranged in,

$$4! = 4\times3\times2\times1$$
$$= 24$$

For each such arrangement, each member of couple can change their seats in $2! = 2$ ways. So,

Total number of arrangements $= 24\times(2)^4$
$= 384$ different ways

Therefore, the required answer is,

$$4!\times4! = 24\times24$$
$$= 576 \text{ different ways}$$

Q3. Here X_1, X_2, X_3 and X_4 jointly have multivariate hypergeometric distribution with parameter $N=12, n=4$, and $a_1=2$, $a_2=3$, $a_3=5$ and $a_4=2$

The joint probability mass function of X_1, X_2, X_3 and X_4 is given by,

$$= \frac{\binom{a_1}{x_1}\binom{a_2}{x_2}\binom{a_3}{x_3}\binom{a_4}{x_4}}{\binom{N}{n}}$$

a. P[all nationalities represented in the committee of 4]

$= P[X_1=1, X_2=1, X_3=1, X_4=1]$

$= h[1,1,1,1: 2,3,5,2, N=12, n=4]$

$$= \frac{\binom{2}{1}\binom{3}{1}\binom{5}{1}\binom{2}{1}}{\binom{12}{4}} = \frac{\frac{2!}{1!1!}\times\frac{3!}{1!2!}\times\frac{5!}{1!4!}\times\frac{2!}{1!1!}}{\frac{12!}{4!8!}} = \frac{60}{495}$$

$$= \frac{4}{33}$$

Therefore, the probability that all nationalities are represented is $\boxed{\frac{4}{33}}$

b. Probabilities that all nationalities except italian are represented

So, we need to find,

$= P[X_1=2, X_2=1, X_3=0, X_4=1] + P[X_1=1, X_2=2, X_3=0, X_4=1] + P[X_1=1, X_2=1, X_3=0, X_4=2]$

$$= \frac{\binom{2}{2}\binom{3}{1}\binom{5}{0}\binom{2}{1}}{\binom{12}{4}} + \frac{\binom{2}{1}\binom{3}{2}\binom{5}{0}\binom{2}{1}}{\binom{12}{4}} + \frac{\binom{2}{1}\binom{3}{1}\binom{5}{0}\binom{2}{2}}{\binom{12}{4}}$$

$$= \frac{\frac{2!}{2!0!}\times\frac{3!}{1!2!}\times\frac{5!}{0!5!}\times\frac{2!}{1!1!}}{\frac{12!}{4!8!}} + \frac{\frac{2!}{1!1!}\times\frac{3!}{2!1!}\times\frac{5!}{0!5!}\times\frac{2!}{1!1!}}{\frac{12!}{4!8!}} + \frac{\frac{2!}{1!1!}\times\frac{3!}{1!2!}\times\frac{5!}{0!5!}\times\frac{2!}{2!0!}}{\frac{12!}{4!8!}}$$

$$= \frac{6 + 12+6}{495}$$

$$= \frac{24}{495}$$

$$\boxed{= \frac{8}{165}}$$

Q4. From the given information,

Shelf contains 5 novels, 3 poem and 1 dictionary.

So, the number of books in a shelf is,

$n = 5+3+1 = 9$

The total number of ways that picking 3 books from the 9 books is equal is, $\begin{bmatrix}9\\3\end{bmatrix}$ ways

a. $P[\text{DICTIONARY IS SELECTED}] = \dfrac{\begin{bmatrix}1\\1\end{bmatrix}\times\begin{bmatrix}8\\2\end{bmatrix}}{\begin{bmatrix}9\\3\end{bmatrix}} = \dfrac{1 \times \frac{8!}{2!(8-2)!}}{\frac{9!}{3!(9-3)!}} = \dfrac{28}{84}$

$$\boxed{= 0.3333}$$

b. $P[\text{2 NOVELS AND 1 BOOK OF POEMS ARE SELECTED}] = \dfrac{\begin{bmatrix}5\\2\end{bmatrix}\times\begin{bmatrix}3\\1\end{bmatrix}}{\begin{bmatrix}9\\3\end{bmatrix}}$

$$= \frac{\frac{5!}{2!(5-2)!} \times \frac{3!}{1!(3-1)!}}{\frac{9!}{3!(9-3)!}} = \frac{10 \times 3}{84}$$

$$\boxed{= 0.3571}$$

Q5.

X	0	1	2	3	4	5 or more
P[X]	0.03	0.18	0.24	0.28	0.10	0.17

a. $P[x>2] = 1 - P[x \le 2]$

$= 1 - \{P[x=0] + P[x=1] + P[x=2]\}$

$= 1 - \{0.03 + 0.18 + 0.24\}$

$= 1 - 0.45$

$\boxed{= 0.55}$

b. $P[X \le 4] = \{P[x=0] + P[x=1] + P[x=2] + P[x=3]\} + P[x=4]$

$= \{0.03 + 0.18 + 0.24 + 0.28\} + 0.10$

$\boxed{= 0.83}$

c. $P[x \ge 4] = 1 - P[x < 4]$

$= 1 - \{P[x=0] + P[x=1] + P[x=2] + P[x=3]\}$

$= 1 - \{0.03 + 0.18 + 0.24 + 0.28\}$

$\boxed{= 0.27}$

Q6. Imoles $P(A) = 0.50$

a. Probability that exactly 3 of the next 4 patients are allergic to weeds

$P[3 \text{ of } 4 \text{ are allergic to weeds}] = P(AAAA'] + P[AAA'A) + P(AA'AA] + P(A'AAA]$

$= P(A').[P(A)]^3 + P(A') \{P(A)\}^3 + P(A').[P(A)]^3 + P(A') \{P(A)\}^3$

$= 4.P[A').\{P(A))^3 = 4.(0.5).(0.5)^3 = 4.(0.5)^4$

$\boxed{= 0.25}$

b. Probability that none of next 4 patients is allergic to weed

$P(B) = P(B'B'B'B')$

$= [P(B')]^4$

$= (0.5)^4$

$\boxed{= 0.0625}$

Q7.

a. $P_r = \dfrac{\text{Men Bachelor's + Woman Bachelor's}}{\text{Total number of cases}}$

$= \dfrac{573079 + 775424}{1907172} = \dfrac{1348503}{1907172}$

$\boxed{= 0.707}$

b. $P_r = \dfrac{\text{Doctorate + Degree awarded to a Woman} - [\text{Doctorate} \cap \text{Degree awarded to a Woman's Bachelor's}]}{\text{Total number of cases}}$

$= \dfrac{45024 + 1098371 - 21683}{1907172} = \dfrac{1122712}{1907172}$

$\boxed{= 0.589}$

c. $P_r = \dfrac{\text{Woman Doctorate}}{\text{Total number of cases}}$

$= \dfrac{21683}{1907172}$

$\boxed{= 0.011}$

d. $P_r = \dfrac{\text{Not a Master's Degree}}{\text{Total number of cases}}$

$= \dfrac{573079 + 775424 + 24341 + 21683}{1907172} = \dfrac{1394527}{1907172}$

$\boxed{= 0.731}$

Q8.

Doctor	Frequency
Dermatologist	4
Surgeons	7
General Practices	5
Psychiatrist	3
Orthopedic Specialists	3
Total	22

a. P[Psychiatrist, Surgeon or Dermatologist] = $\frac{\text{TOTAL NUMBER OF PSYCHIATRIST, SURGEON OR DERMATOLOGIST}}{\text{TOTAL STAFF}}$

$$= \frac{3+7+4}{22} = \frac{14}{22}$$

$$\boxed{= 0.63}$$

b. P[General Practitioner or Surgeon] = $\frac{\text{Total number of general practitioner or surgeon}}{\text{Total Staff}}$

$$= \frac{5+7}{22} = \frac{12}{22}$$

$$\boxed{= 0.54}$$

c. P[Orthopedic Specialist, Surgeon, or a dermatological] = $\frac{\text{Total Number of orthopedic Specialist, Surgeon or dermatologist}}{\text{Total Staff}}$

$$= \frac{3+7+4}{22} = \frac{14}{22}$$

$$\boxed{= 0.63}$$

d. P (Surgeon or Dermatologist] = $\frac{\text{(Total number of Surgeon or Dermatologist)}}{\text{Total Staff}}$

$$= \frac{7+4}{22} = \frac{11}{22}$$

$$\boxed{= 0.5}$$

Q9.

a. There are 4 Kings, 4 queens and 4 jacks; hence $P = \frac{4+4+4}{52} = \frac{12}{52}$

$$\boxed{P = \frac{2}{13}}$$

b. There are 13 clubs, 13 hearts and 13 spades; hence $P = \frac{13+13+13}{52} = \frac{39}{52}$

$$\boxed{P = \frac{3}{4}}$$

c. There are 4 Kings, 4 queens and 13 diamonds but the King and the queen of diamonds were counted twice. Hence

$$P = \frac{4+4+13-2}{52}$$

$$\boxed{P = \frac{19}{52}}$$

d. There are 4 aces, 13 diamonds and 13 hearts. There is one ace of diamonds and one ace of hearts. Hence

$$P = \frac{4+13+13-2}{52} = \frac{28}{52}$$

$$\boxed{P = \frac{7}{13}}$$

e. There are 4 nines, 4 tens, 13 spades and 13 clubs There is one nine of spades, one ten of spades, one nine of clubs and one ten of clubs

$$P = \frac{4+4+13+13-4}{52} = \frac{30}{52}$$

$$\boxed{P = \frac{15}{26}}$$

Q10. Let us consider:

T : A customer will invest in tax-free bonds

M : A customer will invest in mutual-funds

$P(T) = 0.6$

$P(M) = 0.3$

$P(T \cap M) = 0.15$

a. Probability that a customer will invest in either tax-free bond or mutual funds is,

$$P[T \cup M] = P[T] + P[M] - P[T \cap M]$$
$$= 0.6 + 0.3 - 0.15$$
$$\boxed{= 0.75}$$

b. Probability that a customer will invest in neither tax-free bond nor mutual funds is,

$$P[T' \cap M'] = 1 - P[T \cup M] \qquad [\because P[T \cup M] = 0.75]$$
$$= 1 - 0.75$$
$$\boxed{= 0.25}$$

Q11.

a. Probability that a person selected at random from the group will be obese.

$$P[\text{Obese}] \boxed{= 0.33}$$

b. P[Young boy less then 30 given that he is from obese]

$$= \frac{P[\text{less than } 30 \text{ \& obese}]}{P[\text{obese}]} = \frac{0.05}{0.33}$$

$$= 0.1515$$

Q12. Given:

$P[D] = 0.83$
$P[A] = 0.82$
$P[A \cap D] = 0.78$

a. P[arrives on time given that it departed on time] $= P[A \mid D]$

$$P[A|D] = \frac{P[A \cap D]}{P(D)} = \frac{0.78}{0.83} \boxed{= 0.94}$$

b. P[departed on time given that it has arrived at time] = P[D|A]

$$P[D|A] = \frac{P[A \cap D]}{P[A]} = \frac{0.78}{0.82} \boxed{= 0.95}$$

13.

	Gold	Silver	Bronze	Total
United States	36	38	36	110
Russia	23	21	28	72
China	51	21	28	100
Great Britain	19	13	15	47
Others	173	209	246	628
Total	302	302	353	957

a. Probability P[G and U] is the number of gold medalists, who are from, divided by number of winners

$$P[G \text{ and } U] = \frac{36}{957}$$

Probability P[U] is the number of medal winners from, divided by total number of winners

$$P[U] = \frac{110}{957} \rightarrow 1$$

Therefore probability of G given U is shown below

$$P[G|U] = \frac{P[G \text{ and } U]}{P[U]} = \frac{36/957}{110/957}$$

$$= \frac{36}{110}$$

$$\boxed{P[G|U] = 0.327}$$