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Section: BS-DS(N)

Subject: Probability & Statistics

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Assignment 4

Q1.

a. P[all flights are on time] = [90%][80%][50%][90%]

= [0.90][0.80][0.50][0.90]

$\boxed{= 0.324}$

b. P[at least 1 flight is not on time] = 1 - P[all flights are on time]

= 1 - 0.324

$\boxed{= 0.676}$

c. P[at least 1 flight is on time] = 1 - P[all 4 flights are not on time]

= 1 - {[1-0.9][1-0.8][1-0.5][1-0.9]}

= 1 - {[0.1][0.2][0.5][0.1]}

= 1 - 0.001

$\boxed{= 0.999}$

d. From the above result, it is clear that the events in part a and b are complementary.

Q2.

Probability of an item being defective = $P[D] = 15\% = 0.15$

Hence, Probability of an item being good = $P[G] = 1 - 0.15 = 0.85$

Also, Probability of mis classfication = $P[M] = 10\% = 0.10$

Hence, Probability of correct classfication = $P[C] = 1 - 0.10 = 0.90$

Proportion of items "classified as good" = $P[\text{classed good}] = P[C] \times P[G] + P[M] \times P[D]$

$P[\text{classed good}] = 0.90 \times 0.85 + 0.10 \times 0.15$

$P[\text{classed good}] \boxed{= 0.78}$

Probability that an item is defective given that it was classfied as given

$$P[D \mid \text{classed good}] = \frac{P[D] \times P[C]}{P[\text{classed good}]} = \frac{0.15 \times 0.10}{0.78}$$

$P[D \mid \text{classed good}] \boxed{= 0.0192}$

Q3.

	Non Smokers	Moderate Smokers	Heavy Smokers	Total
Hypertension	21	36	30	87
Non Hypertension	48	26	19	93
Total	69	62	49	180

a.

$$P[\text{Heavy Smoker}] = \frac{\text{a total number of heavy smoker in the group}}{\text{a total number of individuals in the tested group}} = \frac{49}{180}$$

$$P[\text{Hypertension} \cap \text{Heavy Smoker}] = \frac{\text{Number of heavy smoker that have experienced hypertension}}{\text{total number of individuals in the tested group}} = \frac{30}{180}$$

$$P[\text{Hyper Tension} \mid \text{Heavy Smoker}] = \frac{P[\text{Hyper Tension} \cap \text{Heavy Smoker}]}{P[\text{Heavy Smoker}]}$$

$$= \frac{30/180}{49/180}$$

$$\boxed{= \frac{30}{49}}$$

b. $P[\text{Non Hyper Tension}] = \frac{\text{total number of individuals that have not experienced hyper tension}}{\text{total number of individual in the tested group}} = \frac{93}{180}$

$P[\text{Non Smoker} \cap \text{Non Hyper Tension}] = \frac{\text{number of nonsmoker that have not experienced hyper tension}}{\text{total number of individual in the tested group}} = \frac{48}{180}$

$$P[\text{Non Smoker} \mid \text{Non Hyper Tension}] = \frac{P[\text{Non Smoker} \cap \text{Non Hyper Tension}]}{P[\text{Non Hyper Tension}]}$$

$$= \frac{48/180}{93/180}$$

$$\boxed{= \frac{48}{93}}$$

Q4. We are given events:

A = the first container is defective

B = the second container is defective

Total Containers = 500

No of Defective = 5

No of Non Defective = 495

a. If the first selected container is defective, there are 4 defective container remaining in the batch of 499:

$$P[B \mid A] \boxed{= \frac{4}{499}}$$

b. The Probability that the first and the second container are defective is:

$P[A \cap B] = P[A].P[B|A]$

$\therefore P[A] = \frac{5}{500} = \frac{1}{100}$

$P(A \cap B) = \frac{1}{100} \times \frac{4}{499} \boxed{= \frac{1}{12475}}$

C. The Probability that the first and the second Container are acceptable is:

$P[A' \cap B'] = P[A'] . P[B'/A']$

$\therefore P[A'] = 1 - P[A] = 1 - \frac{1}{100} = \frac{99}{100}$

Given A' - that a non defective container is chosen first, the probability of B' is the probability that one of 494 non defective containers will be selected from the 499 containers remaining after the first draw:

$P[B'|A'] = \frac{494}{499}$

$P[A' \cap B'] = \frac{99}{100} . \frac{494}{499} = \frac{24453}{24950}$

$\boxed{= 0.98}$

d. If the 1^{st} and the 2^{nd} Selected Container are defective, there are 3 defective container remaining in the batch of 498:

$P[C|_{B \cap A}] = \frac{3}{498} \boxed{= \frac{1}{166}}$ $\therefore$ C = the third container is defective

e. If the 1^{st} selected container is defective and the second is not, there are 4 defective container remaining in the batch of 498:

$$P[C \mid B' \cap A] = \frac{4}{498} = \boxed{\frac{2}{249}}$$

f. The Probility that all three are defective:

$$P[A \cap B \cap C] = P[C \mid B \cap A] \times P[A \cap B]$$

$$= \frac{1}{166} \times \frac{1}{12475}$$

$$\boxed{= \frac{1}{2070850}}$$

Q5. Let D be the event which denotes that the Company experenices a Lost over run.

Given that:

$$P[A] = 0.40, \; P[B] = 0.35, \; P[C] = 0.25 \quad \&$$

$$P[D \mid A] = 0.05, \; P[D \mid B] = 0.03, \; P[D \mid C] = 0.15$$

a. Now, we have to find the probability that the consulting firm involved is Company C :

$$P[C \mid D] = \frac{P[D \mid C] \cdot P[C]}{P[D \mid C] \cdot P[C] + P[D \mid B] \cdot P[B] + P[D \mid A] \cdot P[A]}$$

$$= \frac{0.15 \times 0.25}{(0.15 \times 0.25) + (0.03 \times 0.35) + (0.05 \times 0.40)}$$

$$= \frac{0.0375}{0.068}$$

$$\boxed{= 0.5515}$$

b. Now we have to find the probability that the consulting firm involved is company A:

$$P[A|D] = \frac{P[D|A].P[A]}{P[D|A].P[A] + P[D|B].P[B] + P[D|C].P[C]}$$

$$= \frac{0.05 \times 0.40}{[0.15 \times 0.25] + [0.03 \times 0.35] + [0.05 \times 0.40]}$$

$$= \frac{0.02}{0.068}$$

$$\boxed{= 0.29}$$

Q6. Let E_i be the event which denotes that i^{th} [$\forall i = 1,2$] engineer does the work. Given that:

$P[E_1] = 70\% = 0.7$
$P[E_2] = 30\% = 0.3$

Let A be the event which denotes that an error occur in estimating the cost. Given that

$P[A|E_1] = 0.02$
$P[A|E_2] = 0.04$

Now, find the probability that an error occur when work done by engineer 1

$= P[E_1|A]$

$$= \frac{P[A|E_1]P[E_1]}{P[A|E_1].P[E_1] + P[A|E_2].P[E_2]}$$

$$= \frac{0.02 \times 0.7}{[0.02 \times 0.7] + [0.04 \times 0.3]} = \frac{0.014}{0.026}$$

$$\boxed{= 0.54}$$

Now, find the Probability that an error occur when work done by engineer 1

$= P[E_2 | A]$

$$= \frac{P[A|E_2] . P[E_2]}{P[A|E_1] . P[E_1] + P[A|E_2] . P[E_2]}$$

$$= \frac{0.04 \times 0.3}{(0.02 \times 0.7) + (0.04 \times 0.3)}$$

$$= \frac{0.012}{0.026} \boxed{= 0.46}$$

Q7. Let us Consider:

H : The husbond will vote on a bond referendum

W : The wife will vote on a bond referendum

Given that:

$P(H) = 0.21$

$P(W) = 0.28$

$P(H \cap W) = 0.15$

a. Now, we want to find the probability that at least one member of a married couple will vote:

$$P[H \cup W] = P(H) + P[W] - P[H \cap W]$$

$$= 0.21 + 0.28 - 0.15$$

$$\boxed{= 0.34}$$

b. Now, we want to find the probability that a wife will vote, given that her husband will vote:

$$P[W | H] = \frac{P[W \cap H]}{P(H)} = \frac{0.15}{0.21}$$

$\boxed{= 0.714}$

C. Now, we want to find the probability that a husband will vote given that his wife does not vote:

$$P[H \mid W'] = \frac{P[H \cap W']}{P[W']}$$

$$\Rightarrow P[H \cap W'] = P[H] - P[H \cap W]$$
$$= 0.21 - 0.15$$
$$= 0.06$$

And $P[W'] = 1 - P[W]$
$$= 1 - 0.28 = 0.72$$

$$P[H \mid W'] = \frac{0.06}{0.72}$$
$$\boxed{= 0.083}$$

Q8. Event that bolt is defective.. Given that:

$P[A] = 25\% = 0.25$
$P[B] = 35\% = 0.35$
$P[C] = 40\% = 0.40$
$P[D \mid A] = 5\% = 0.05$
$P[D \mid B] = 4\% = 0.04$
$P[D \mid C] = 2\% = 0.02$

a. $P[D] = P[A].P[D \mid A] + P[B].P[B \mid B] + P[C].P[D \mid C]$
$$= 0.25 \times 0.05 + 0.35 \times 0.04 + 0.40 \times 0.02$$
$$\boxed{= 0.0345}$$

$$P(A|D) = \frac{P(D|A) \cdot P(A)}{P(D)}$$

$$= \frac{0.05 \times 0.25}{0.0345} = \frac{0.0125}{0.0345}$$

$$\boxed{= 0.362}$$

Q9.

a. Probability that the system does not have a type 1 defect, i.e.

$P[A1'] = 1 - P[A1]$

$= 1 - 0.12 \qquad \therefore P(A1) = 0.12$

$\boxed{= 0.88}$

b. Probability that the system has both type 1 and type 2 defect, i.e.

$P[A1 \cap A2] = P(A1) + P(A2) - P[A1 \cup A2)$

$= 0.12 + 0.07 - 0.13 \qquad \therefore P(A2) = 0.07$ and $P(A1 \cup A2) = 0.13$

$\boxed{= 0.06}$

c. Probability that the system has both type 1 and type 2, but not a type 3 defect i.e,

$P(A1 \cap A2 \cap A3'] = P[A1 \cap A2) - P(A1 \cap A2 \cap A3]$

$= 0.06 - 0.01 \qquad \therefore P[A1 \cap A2 \cap A3] = 0.01$

$\boxed{= 0.05}$

d. Probability that the system has at most two of these defects, i.e.

$P[\text{at most 2 errors}] = 1 - P[\text{all three errors}]$

$= 1 - P[A1 \cap A2 \cap A3)$

$= 1 - 0.01 \boxed{= 0.99}$

Q10. GIVEN THAT:

$P[A] = 0.3$, $P[B] = 0.5$, $P[C] = 0.2$ &

$P[\text{Increase} \mid A] = 0.8$, $P[\text{Increase} \mid B] = 0.1$, $P[\text{Increase} \mid C] = 0.4$

So,

$$P[\text{Increase}] = P[\text{Increase} \mid A] \cdot P[A] + P[\text{Increase} \mid B] \cdot P[B] + P[\text{Increase} \mid C] \cdot P[C]$$

$$= 0.8 \times 0.3 + 0.5 \times 0.1 + 0.2 \times 0.4$$

$$= 0.24 + 0.05 + 0.08$$

$$\boxed{= 0.37}$$

Q11. GIVEN THAT:

$P[\text{account } 1] = 0.70$, $P[\text{account } 2] = 0.20$, $P[\text{account } 3] = 0.10$ &

$P[\text{spam} \mid \text{account } 1] = 0.01$, $P[\text{Spam} \mid \text{account } 2] = 0.02$, $P[\text{spam} \mid \text{account } 3] = 0.05$

So,

$$P[\text{Spam}] = P[\text{Spam} \mid \text{account } 1] \cdot P[\text{account } 1] + P[\text{account } 2] \cdot P[\text{Spam} \mid \text{account } 2] + P[\text{account } 3] \cdot P[\text{Spam} \mid \text{account } 3]$$

$$= 0.70 \times 0.01 + 0.20 \times 0.02 + 0.10 \times 0.05$$

$$\boxed{= 0.016}$$

Q12. Let A = Strips fail the lengthe test

B = Strips fail the texture test

GIVEN THAT:

$P[A] = 0.1$

$P[B] = 0.05$

& $P[A \cap B] = 0.008$

So

$$P[B \mid A] = \frac{P[B \cap A]}{P[A]} = \frac{0.008}{0.1}$$

$$\boxed{= 0.08}$$