Q2.

a. For the first weighing median = 20.83

Here we prepare the table taking continuous classes as follows:

Class	Freq	N
0 - 4.5	a	a
4.5 - 9.5	b	a+b
9.5 - 14.5	11	a+b+11
14.5 - 19.5	52	a+b+63
19.5 - 24.5	75	a+b+138
24.5 - 29.5	22	a+b+160
Total	a+b+160	

Here median = 20.83 belongs to class 19.5 - 24.5

$$\text{Median} = L + \left[\frac{N/2 - cf}{f} \times h\right]$$

where L = Lower bound of median class

N = total freq = a+b+160

Cf = Commulative freq of previous class = a+b+63

f = freq of median class = 75

h = class width = 5

$$\therefore 20.83 = 19.5 + \left[\frac{\frac{a+b+160}{2} - [a+b+63]}{75}\right] \times 5$$

$$\therefore 20.83 - 19.5 = \frac{\frac{a+b+160}{2} - (a+b+63)}{15}$$

$$\therefore (20.83 - 19.5)15 = \frac{a}{2} + \frac{b}{2} + 80 - a - b - 63$$

$$19.95 = -\frac{a}{2} - \frac{b}{2} + 17 \Rightarrow 19.95 - 17 = -\frac{a}{2} - \frac{b}{2}$$

$$2.95 = -\frac{a}{2} - \frac{b}{2} \Rightarrow 5.9 = -a - b \Rightarrow \boxed{a+b = -5.9}$$

which is inconsistent $\therefore$ we take a = b = 0

For Second weighing median = 17.35

We prepare the table as follow:

Class	Freq	N
0 - 4.5	x	x
4.5 - 9.5	y	x+y
9.5 - 14.5	40	x+y+40
14.5 - 19.5	50	x+y+90
19.5 - 24.5	30	x+y+120
24.5 - 29.5	28	x+y+148
Total	x+y+148	

Here median belongs to class 14.5 - 19.5

$L = 14.5$, $N = x+y+148$,

$cf = x+y+40$, $f = 50$

$h = 5$

Consider

$$\text{Median} = L + \left[\frac{\frac{N}{2} - cf}{f} \times h\right]$$

$$\therefore 17.35 = 14.5 + \left[\frac{\frac{x+y+148}{2} - (x+y+40)}{50} \times 5\right]$$

$$\therefore 2.85 = \frac{\frac{x+y+148}{2} - (x+y+40)}{10} \Rightarrow 28.5 = \frac{x}{2} + \frac{y}{2} + 74 - x - y - 40$$

$$\therefore 28.5 = -\frac{x}{2} - \frac{y}{2} + 34 \Rightarrow -5.5 = -\frac{x}{2} - \frac{y}{2} \Rightarrow \boxed{x+y = 11}$$

$\therefore N = 11 + 148 = 159$

Total number of articals = 159

6. Let X_1, X_2, X_3 be three r.v.s with

$E(X_1) = 3$, $E(X_2) = 4$, $E(X_3) = 1$ (last Digital of Jayad's Roll No)

$V(X_1) = 10$, $V(X_2) = 20$, $V(X_3) = 30$

$Cov(X_1, X_2) = Cov(X_1, X_3) = Cov(X_2, X_3) = 0$

Let $Y = 2X_1 + 3X_2 - 4X_3$

$E(Y) = \text{mean} = 2E(X_1) + 3E(X_2) - 4E(X_3)$

$= 2(3) + 3(4) - 4(1) = 6 + 12 - 4 = 14$

$V(Y) = 4V(X_1) + 9V(X_2) + 16V(X_3)$

$= 4(10) + 9(20) + 16(30)$

$= 700$

mean$(Y) = 14$

Variance $(Y) = 700$

Q4.

1.

23rd Percentile:

$P_{23} = $ Size of $\left[23\left[\frac{n+1}{100}\right]^{th}\right]$ item

$= $ Size of $\left[23\left[\frac{29+1}{100}\right]^{th}\right]$ item

$= $ Size of $(23(0.3))$ item

$= $ Size of 6.9 item $= 6^{th}$ item $+ 0.9$ item

$= 73 + 0.9\,(7^{th} - 6^{th}$ item$) = 73 + 0.9\,(75-73)$

$\boxed{= 74.9}$

48th Percentile:

$P_{48} = $ Size of $\left[48\left[\frac{n+1}{100}\right]^{th}\right]$ item

$= $ Size of $\left[48\left[\frac{30}{100}\right]\right]$ item

$= $ Size of 14.4 item

$= 14^{th}$ item $+ 0.4$ item $= 80 + 0.4\,(15^{th} - 14^{th})$

$= 80 + 0.4\,(80-80)$

$\boxed{= 80}$

64th Percentile:

$P_{64} = $ Size of $\left[64\left[\frac{n+1}{100}\right]^{th}\right]$ item

$= $ Size of $64\left[\frac{30}{100}\right]$

$= $ Size of 19.2 item $= $ Size of 19^{th} item $+ 0.2\,(20^{th} - 19^{th}$ item$)$

$= 85 + 0.2\,(86-85)$

$= 85.2$

82th Percentile:

$P_{64} = $ Size of $\left[82\left[\frac{n+1}{100}\right]^{th}\right]$ item

$= \text{Size of } 82\left[\frac{30}{100}\right]$

$= \text{Size of } 24.6^{th} \text{ item}$

$= \text{Size of } 24^{th} \text{ item} + 0.6\,[25^{th} - 24^{th}] \text{ item}$

$= 88 + 0.6\,(90 - 88)$

$= 89.2$

3rd DECILE:

$= \text{Size of } \left[3\left[\frac{n+1}{10}\right]^{th}\right] \text{ item}$

$= \text{Size of } \left[3\left[\frac{30}{10}\right]\right] \text{ item}$

$= \text{Size of } 9^{th} \text{ item}$

$\boxed{= 27}$

6th DECILE:

$= \text{Size of } \left[6\left[\frac{n+1}{10}\right]^{th}\right] \text{ item}$

$= \text{Size of } \left[6\left[\frac{30}{10}\right]\right] \text{ item} = \text{Size of } 18^{th} \text{ item}$

$\boxed{= 85}$

9th DECILE:

$= \text{Size of } \left[9\left[\frac{n+1}{10}\right]^{th}\right] \text{ item} = \text{Size of } \left[9\left[\frac{30}{10}\right]\right] \text{ item}$

$= \text{Size of } 27^{th} \text{ item}$

$\boxed{= 93}$

11. Last Three Digits of Jiyad's Roll No is 7, 7, 1
Sum of Last Three Digits is $7 + 7 + 1 = 15$

a. Now, Data will be:

71, 80, 80, 82, 87, 88, 90, 92, 92, 93, 93, 93, 95, 95, 95, 97, 98

100, 100, 101, 102, 102, 102, 103, 105, 107, 108, 110, 113

MEAN:

$$\text{MEAN} = \frac{\text{SUM OF THE TERMS}}{\text{NUMBER OF TERMS}}$$

SUM OF THE TERMS = 2774

NUMBER OF TERMS = 29

$$\text{MEAN} = \frac{2774}{29} \boxed{= 95.655172}$$

STANDARD DERIVATION:

$$SD = \sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}[x_i - \mu]^2} = \sqrt{\frac{(71 - 95.655172)^2 + \dots + (113 - 95.655174)^2}{29}}$$

$$\sigma = \sqrt{\frac{2622.551724}{29}} = \sqrt{90.432818}$$

$$\sigma = \boxed{9.509617}$$

Variance:

$$\sigma^2 = (9.509612)^2 \boxed{= 90.432818}$$

b. Now, the data will be:

840, 975, 975, 1005, 1080, 1095, 1125, 1155, 1155, 1170, 1170, 1170, 1200, 1200, 1200, 1230, 1245, 1275, 1275, 1290, 1305, 1305, 1305, 1320, 1350, 1380, 1395, 1425, 1470

MEAN:

No of term = N = 29

Sum of term = Σx = 35085

$$\text{MEAN} = \frac{\Sigma x}{N} = \frac{35085}{29} \boxed{= 1209.827586}$$

STANDRAD DERIVATION:

$$SD = \sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2} = \sqrt{\frac{(840 - 1207.827586)^2 + \ldots + (1470 - 1207.827586)^2}{29}}$$

$$= \sqrt{\frac{590074.137931}{29}} \boxed{= 142.644257}$$

VARIANCE:

$$\sigma^2 = (SD)^2 = (142.644257)^2$$

$$\boxed{= 20347.384066}$$

Q6.

a. $P_1 = 0.95 \qquad q_1 = 0.05 = (1 - P_1)$

$P_2 = 0.75 \qquad q_2 = 0.25 = (1 - P_2)$

$P_3 = 0.55 \qquad q_3 = (1 - P_3) = 0.45$

$\therefore \; P_1 \cdot P_2 \cdot P_3 = 0.95 \times 0.75 \times 0.55 = 0.391875$

$\therefore \; P_1 q_2 P_2 = 0.95 \times 0.25 \times 0.75 = 0.178125$

$\therefore \; q_1 P_1 P_2 = 0.05 \times 0.95 \times 0.75 = 0.035625$

$\therefore \; q_1 q_1 P_1 = 0.05 \times 0.05 \times 0.95 = 0.002375$

P [Third is Correct] $= 0.391875 + 0.178125 + 0.035625 + 0.002375$

$\boxed{= 0.608}$

$$P\left[\frac{\text{Second is Correct}}{\text{Third is Correct}}\right] = \frac{0.391875 + 0.035625}{0.608}$$

$\boxed{= 0.703125}$

b. $P(t < y) = y/3, \quad 0 \le y \le 2$

The conditional probability that the full time is used will be

$$P(t > 2 \mid t > 1.75) = \frac{P(t > 2 \cap t > 1.75)}{P(t > 1.75)} = \frac{P(t > 2)}{P(t > 1.75)}$$

$$= \frac{1 - P(t \le 2)}{1 - P(t \le 1.75)} = \frac{1 - 2/3}{1 - 1.75/3}$$

$\boxed{= 0.8}$

C. There are 4 choices per question and 7 questions.
So there are:

$$4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^7 = 16384$$

PROBABILITY that two papers have the same answer to the first question:

$$P = \frac{1}{4}$$

Q7. $f(x) = \begin{cases} ax - bx^2 & \text{for } 0 \le x \le 2 \\ 0 & \text{elsewhere} \end{cases}$

i. To be a valid probability density function,

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_0^2 (ax - bx^2)\,dx = 1$$

$$\Rightarrow \left[\frac{ax^2}{2} - \frac{bx^3}{3}\right]_0^2 = 1$$

$$\Rightarrow \left[\frac{a \cdot 2^2}{2} - \frac{b \cdot 2^3}{3}\right] - \left[\frac{a \cdot 0^2}{2} - \frac{b \cdot 0^3}{3}\right] = 1$$

$$\Rightarrow 2a - \frac{8b}{3} - 0 + 0 = 1 \Rightarrow 6a - 8b = 3 \rightarrow (i)$$

From the given condition,, Jiyad's last two digit roll number is

7, 1

$$E(x) = 7 + 1 = 8$$

$$\Rightarrow \int_{-\infty}^{\infty} x\, f(x)\,dx = 8 \Rightarrow \int_0^2 x(ax - bx^2)\,dx = 8$$

$$\Rightarrow \int_0^2 (ax^2 - bx^3)\,dx = 8 \Rightarrow \left[\frac{ax^3}{3} - \frac{bx^4}{4}\right]_0^2 = 8$$

$$\Rightarrow \frac{8a}{3} - 4b = 8 \Rightarrow 8a - 12b = 24 \rightarrow ii$$

Using i and ii, we have

$$4(6a - 8b) - 3(8a - 12b) = 4(3) - 3(24)$$

$$24a - 32b - 24a + 36b = 12 - 72$$

$$4b = -60 \Rightarrow \boxed{b = -15}$$

Using i, we have

$$6a = 8b + 3$$

$$6a = -8(15) + 3$$

$$6a = -120 + 3 \Rightarrow 6a = -117$$

$$a = \frac{-117}{6} \Rightarrow \boxed{a = -19.5}$$

ii. P [at least one of them is ≤ 0.5]

$= 1 - P$[none of them is ≤ 0.5]

$$= 1 - P[X > 0.5]^2$$

$$= 1 - \left[\int_{0.5}^{\infty} f(x)\,dx\right]^2 = 1 - \left[\int_{0.5}^{2} (-19.5x + 15x^2]\,dx\right]$$

$$= 1 - \left[-19.5\int_{0.5}^{2} x\,dx + 15\int_{0.5}^{2} x^2\,dx\right]$$

$$= 1 - \left[-19.5 \times \frac{1}{2}[x^2]_{0.5}^{2} + 15 \times \frac{1}{3}[x^3]_{0.5}^{2}\right]$$

$$= 1 - \left[-9.75\left[4 - \frac{1}{4}\right] + 5\left[8 - \frac{1}{8}\right]\right]$$

$$= 1 + 39 - 2.4375 - 40 + \frac{5}{8} \boxed{= -1.8125}$$