Question # 1:

[2+2+2]=6 marks

If $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^3$, under what conditions $span\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ will represent

a) a line

If u, v, w are ~~product~~ multiple of each other then span{u, v, w} = line.

b) a plane

If two vectors are linearly independent but third is linear combination of other two then span{u, v, w} = plane.

c) whole $\mathbb{R}^3$

If all three vectors are linearly independent then Span{u, v, w} = $\mathbb{R}^3$

Question # 2:

[2+2+2+2+2+2+2+2]=16 marks

For the matrix

$$A = \begin{bmatrix} 1 & 0 & -2 & 5 \\ 0 & 1 & 4 & -2 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & 5 \\ 0 & 1 & 4 & -2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & -7 \end{bmatrix}$$

$R_3 \leftrightarrow R_4$
$R_4 - 3R_3$

Prove or disprove the following statements (give detailed justification of your answer)

a) Columns of A are linearly dependent.

Columns of A are linearly independent since $Ax = 0$ has only trivial solution

b) Matrix is invertible.

Matrix is invertible since it is 4×4 matrix with 4 pivot positions and columns are linearly idependtlet so it is row equivalent to I_4

c) Columns of A will span $\mathbb{R}^3$.

No columns of A can not span $\mathbb{R}^3$ as each column is element of $\mathbb{R}^4$.

d) Columns of A will span $\mathbb{R}^4$.

Yes, It will span $\mathbb{R}^4$ since $Ax = b$ is consistent for $x, b \in \mathbb{R}^4$

e) The system $A\mathbf{x} = \mathbf{b}$ is inconsistent for any $\mathbf{x}, \mathbf{b} \in \mathbb{R}^4$.

$Ax = b$ is consistent for any $x, b \in \mathbb{R}^4$

f) The system $A\mathbf{x} = \mathbf{0}$ has only trivial solution.

Yes $Ax = 0$ has only trivial solution since there is no free variable involved.

g) $Nul(A) = \{0\}$.

Since $Ax = 0$ only if $\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow Nul(A) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \{\vec{0}\}$

h) $Col(A) = \mathbb{R}^4$.

Yes $Col(A) = \mathbb{R}^4$ since for each $b \in \mathbb{R}^4$, $Ax = b$ is consistent.

Question # 3: [2+2+2+2]=8 marks

If we have following information about the matrices A, B, C and D,

matrix A is of order 2×3 with 2 pivot position,
matrix B is of order 3×4 with 2 pivot position,
matrix C is of order 3×2 with 2 pivot position,
matrix D is of order 3×2 with 1 pivot position,

then in each case determine the value of k and m such that $\mathbf{x} \in \mathbb{R}^k$ and $\mathbf{b} \in \mathbb{R}^m$ and also comment (with reasoning) on the solution set of

a) $A\mathbf{x} = \mathbf{b}$ $A_{2\times3}\, x_{3\times1} = b_{2\times1}$

$x \in \mathbb{R}^3$ and $b \in \mathbb{R}^2$.

and $Ax = b$ is consistent for all $b \in \mathbb{R}^2$

b) $B\mathbf{x} = 0$ $\Rightarrow B_{3\times4}\, x_{4\times1} = 0_{3\times1}$

$\Rightarrow x \in \mathbb{R}^4$

$Bx = 0$ will have infinite solution as it involve 2 free variables.

c) $C\mathbf{x} = 0$

$C_{3\times2}\, x_{2\times1} = 0_{3\times1}$

$\Rightarrow x \in \mathbb{R}^2$.

$Cx = 0$ will have only trivial solution

d) $D\mathbf{x} = \mathbf{b}$ $D_{3\times2}\, x_{2\times1} = b_{3\times1}$

$\Rightarrow x \in \mathbb{R}^2$ $b \in \mathbb{R}^3$

$Dx = b$ will not be consistent for all $b \in \mathbb{R}^3$

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Question # 4:   [4+2]=6 marks

Find the inverse of following matrices:

a) $\begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

a)

$$\left[\begin{array}{cc|cc} 0 & 2 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|cc} 1 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{array}\right] \quad R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{cc|cc} 1 & 1 & 0 & 1 \\ 0 & 1 & \frac{1}{2} & 0 \end{array}\right] \quad \frac{1}{2}R_2$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -\frac{1}{2} & 1 \\ 0 & 1 & \frac{1}{2} & 0 \end{array}\right] \quad R_1 - R_2 .$$

b) Inverse of identity is identity itself.

so

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question # 5: [8+2+4]=14 marks

a) Find the rank, basis of column space, null space and basis of null space for the following matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad R_2 - 3R_1$$

Rank = 1

Basis of col(A) = $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

Nul(A)

Let $x_2 = t$

$x_3 = s$.

then $x_1 = -2t - s$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2t-s \\ t \\ s \end{bmatrix} = t\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + s\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Hence Nul(A) = $\left\{ t\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + s\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} ; t, s \in \mathbb{R} \right\}$

Basis of Nul(A) = $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

b) For what value of b_2 does

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & 3 \end{bmatrix} \mathrm{x} = \begin{bmatrix} 3 \\ b_2 \end{bmatrix}$$

has a solution?

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 3 & 6 & 3 & b_3 \end{array}\right]$$

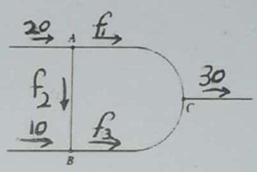
$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 0 & 0 & b_2-9 \end{array}\right] \quad R_2-3R_1$$

given system is consistent only if

$b_2 - 9 = 0$

$\Rightarrow b_2 = 9$.

c) Network of water pipes with flows measured in litres per minute is shown by the figure



Set up the linear system (**DON'T SOLVE**) to find possible flows.

Inflow = Outflow

at A $\quad 20 = f_1 + f_2$

at B $\quad 10 + f_2 = f_3$

at C $\quad f_1 + f_3 = 30$

Total Inflow = Total outflow

$30 = 30$.

So to find flow we need to solve the linear system

$$\begin{aligned} f_1 + f_2 &= 20 \\ -f_2 + f_3 &= 10 \\ f_1 + f_3 &= 30 \end{aligned}$$