Final Case Study Report

Bubble Wrap Experiment

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Introduction

With the prevalence of online shopping and the need for transportation of commodities from place to place for wholesale and retail sales, the demand for protective packaging products also rises to a new height because protective packaging is crucial to product manufacturing, storing, and transporting. Therefore, there exists stiff competition among the companies which produce protective packaging. It is beneficial and significant for companies to test and adapt to the most efficient production method to maximize their production and make the most profits from everyday sales. Among all the protective packaging products, bubble wrap introduces a new light to the market since it is a light-weighted protective material but meanwhile provides excellent protection for commodities, especially for fragile products.

In this case study, we will use the provided dataset to suggest a strategy for the bubble wrap production for a protective packaging manufacturer, BubbleBobble, to maximize their production capability based on two factors, line speed and percent loading of additives, after the engineers' pre-selection of production line factors.

Methodology

Our group started the model selection process by cleaning up the bubblewrap.csv dataset first, then we did the model selection based on the significance of factors. Then, the model assumptions were tested for the first model. After testing the model assumptions, we performed model transformation and tested model assumptions for our new model. Ultimately, we ended our analysis with the model selection process for our new model.

1 Data Processing

As indicated by the experiments, for Replication 2 with run orders 2, 3, and 5, the qualities of the bubble wraps produced were deficient; therefore, we decided to delete the rows that match the second replication with run order 2, 3, and 5. In addition, the line speed and the percent loading of additives in the dataset are the only two key factors we need to consider, while the production rate is the response variable for our model.

Firstly, we visualized our data with box-plots for each factor separately. From the box-plot for the line speed factor (Appendix 1.1), we observed that the production rates produced by different levels of line speed are in the same level (350-400), so we could not tell whether there are statistical differences. On the other hand, according to the box-plot for the loading of additives (Appendix 1.2), we found that the factor at different levels produces different production rates, as the 4% loading of additives produces a much larger production rate than other levels. However, we still could not decide whether there is a statistical difference since there are boxes overlapping.

To investigate whether interactions are present, we construct the interaction plots. Intersecting lines are presented in both in the interaction plot for line speed (Appendix 1.3) and that for loading (Appendix 1.4) , indicating the presence of interactions; however, we need to proceed to test whether the interactions are statistically significant.

2 Model Selection

Starting with the model with interaction terms, we decided to take an unbalanced ANOVA approach because the treatment sample sizes are unequal, recalling that we decided to drop the rows with replication 2 with run times 2, 3, and 5. More specifically, we used the Anova() command with Type = "III" specification from the car library to do a Type III test. Under the null hypothesis for this test, we select the additive model, while under the alternative hypothesis, we select the interaction model. The p-value here is 0.6426, which is larger than the significance level of 0.05, so we fail to reject the null and conclude that the additive model is better.

We fitted an additive model for the next step and did the Type III test again. We then observed that loading is statistically significant with a p-value of $0.00575 < 0.05$, while the other factor, line speed, is not statistically significant with a p-value of $0.10016 > 0.05$. Therefore, we removed line speed and declared the model with the intercept and loading to be our final model.

3 Diagnostic

3.1 Checking Model Assumptions

After selecting the appropriate model, we moved on to check model assumptions for it, specifically checking for constant variance and normality. If these model assumptions fail, then we perform a transformation on our model.

3.1.1 Checking for Constant Variance

By observing the residuals vs. fitted values plot (Appendix 3.1), we observed that the variance is larger for lower fitted values and smaller for higher fitted values, indicating that the variance is not constant. Moreover, we decided to further verify our statement through the Breusch-Pagan test. Since we had a p-value of 0.01466, which is smaller than the significance level of 0.05. We concluded that the variance is not constant.

3.1.2 Checking for Normality

Next, we check for the assumption of normality through the Normal Q-Q Plot (Appendix 3.2) and the histogram of residuals (Appendix 3.3). It is seemingly that the points in the QQ-plot fall on a straight line, so we proceeded to check the histogram of residuals. The histogram of residuals suggests that the model has no departure from the normality assumption. In addition, we performed the Shapiro-Wilk normality test and got a p-value of 0.06379. Since the p-value is greater than the 0.05 significance level, we concluded that the normality assumption is satisfied.

3.1.3 Checking for Serial Dependence

We proceeded to check for the serial dependence assumption since the order in which the data were collected is provided to us.

Firstly, we performed a sequence plot (Appendix 3.4), using which we observed no clear pattern between residuals and time. We further performed the Durbin-Watson test, getting a DW score of 2.1712 and a p-value of 0.6626. Since the DW score is greater than 2, and the p-value is greater than 0.05, there is not enough evidence for the presence of positive serial dependence. To conclude in other words, the error terms are not auto-correlated.

3.2 Box-Cox Transformation

Since the constant variance assumption fails, we had to perform transformations on our model. We decided to conduct a Box-Cox transformation. In order to find the appropriate value of λ so that Y^λ follows a normal distribution, we plotted the log-likelihood function vs. λ graph (Appendix 3.5). Based on the output graph, we picked a lambda value of 5 and successfully transformed our model. Later, we will check the model assumptions again on our new model.

4 Result

4.1 Diagnostic for Transformed Model

Since we performed transformation on our model, we need to perform tests on model assumptions again and come up with a final model. To be more specific, we performed tests on the assumptions of normality and constant variance for the transformed model.

By observing the side-by-side normal Q-Q plot and residuals plot (Appendix 4.1), we could see that there is seemingly no pattern for the residuals, but the transformed model seems to fail the normality assumption. We moved on to check model assumptions using more specific approaches.

4.1.1 Checking Assumptions

Based on the output of the Breusch-Pagan test, the p-value is 0.5032, larger than the 0.05 significance level. Therefore, we conclude that the variance is constant. On the other hand, the p-value derived from the Shapiro-Wilk normality test is 0.04961, which is slightly smaller than our 0.05 significance level, but we still had to reject the null and conclude that the normality assumption is not satisfied. After consideration, we decided to pick the current model instead of the model that fails the constant variance assumption, as if the constant variance assumption fails, the residuals will be heteroscedastic, making the estimations of our model not as reliable as those when the constant variance assumption is met.

4.2 Final Model Selection

After transforming our model, we performed the same model selection processes. Constructing the Type III test on the model which contains the interaction term, we got a p-value of 0.935083 for the interaction term, larger than the significance level. Therefore, we removed the interaction term and continued on with the additive model. Doing the Type III test again, the p-value for line speed is 0.0435287 and the p-value for loading is 0.0006235. Both values are smaller than the significance level, so we decided to keep both factors in our new model since they are all statistically significant.

4.3 Pairwise Comparisons & Analyzing Optimal Combination

To view the effects different levels of each factor have on production rate, we decided to perform tests of pairwise differences for both loading and line speed. Namely, we used the Tukey's method and Scheffe's method.

For the factor "loading", observing the output of the test, we stated that the pairs 4-0 and 4-2 are statistically significant since the p-values of both pairs are smaller than 0.05 (Appendix 4.2). The same pattern can be observed from the Tukey's family confidence intervals plot (Appendix 4.3) as the only interval including 0 is that for the level pairs 2-0. On the other hand, the output of Tukey's method for the factor "line speed" suggested that no levels are statistically significant different from each other since none of the p-values is smaller than 0.05 (Appendix 4.4); the intervals plot for line speed agrees with this observation (Appendix 4.5) as all three intervals include 0. After analyzing pairwise comparisons obtained from Tukey's method, to further confirm our decision about which combination will produce the highest production rate, we calculated the confidence interval of the mean difference for each factor using Scheffe's Method.

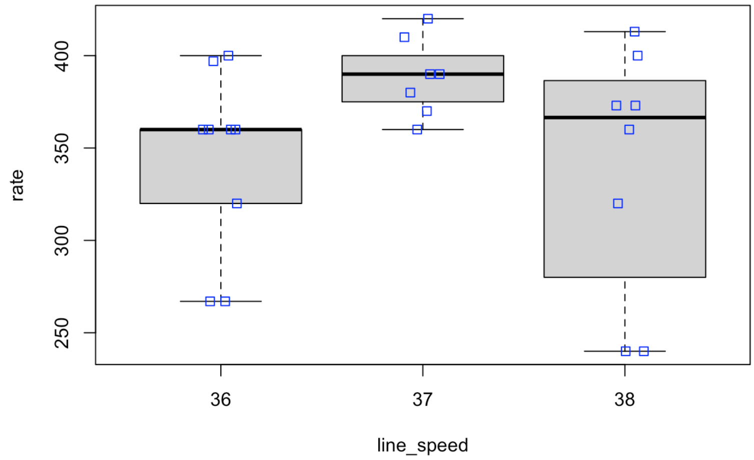
For the factor, percent loading of additives, the level that corresponds to the highest production rate is level 4 (4 %) as the difference between level 4 and level 0 is 4.207018e+12 and the difference between level 4 and level 2 is 5.488291e+12. By using a similar comparing approach, we could see that the level of line speed which corresponds to the highest production rate is level 37 (37 m/mm). However, from the Scheffe's method table (Appendix 4.6), we could see that although there are statistically significant differences between level 4 and level 2, as well as level 4 and level 0 for the factor percent loading for additives, there is no statistical difference between 0% loading and 2% loading. When proceeding to the line speed part in Scheffe's test table, we observed there are no statistically significant differences between all three levels of the line speed factor.

5 Conclusion

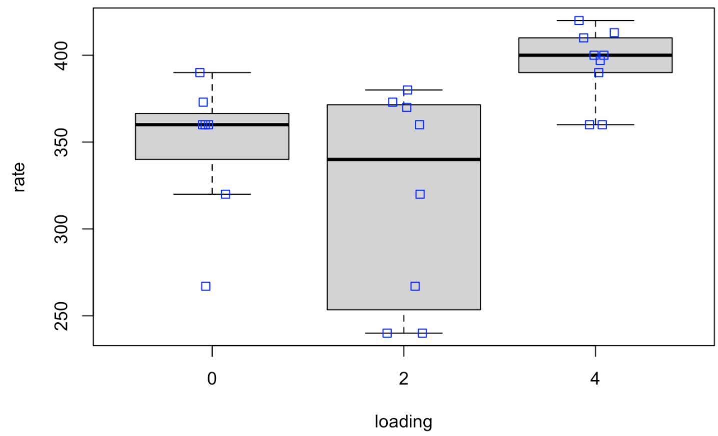
According to the estimations for production rate we obtained from different line speed and percent loading of additives, we highly recommend that BubbleBobble uses a combination of 4 % loading of additives and line speed of 37 meters per minute to achieve the most optimal production rate. Although all three line speeds have no difference from one another, we think the line speed of 37 m/mm may be the best fit for the manufacturer because it may protect the production line machine with a medium-rate speed (if the manufacturer wants to achieve their weekly, monthly, or yearly production goal, they may adjust to different line speeds other than the one being suggested).

Although there are no differences in the three different line speeds in statistical terms, it does not mean that the speed of the production line does not have an effect on production rate on a daily basis. Another reason why we would not suggest using the lowest or the highest line speed is that we consider the situation in which the production of the plastic products fails to meet or exceeds the demand of the consumers or the need of the manufacturer. A deficit or a surplus in production may lead to unexpectable results, and we do not want that to happen since we already have some predictions made based on the data provided to us. We suggest BubbleBobble starts with a modest line speed of 37 meters per minute with 4 % loading of additives, and adjust them further in practice.

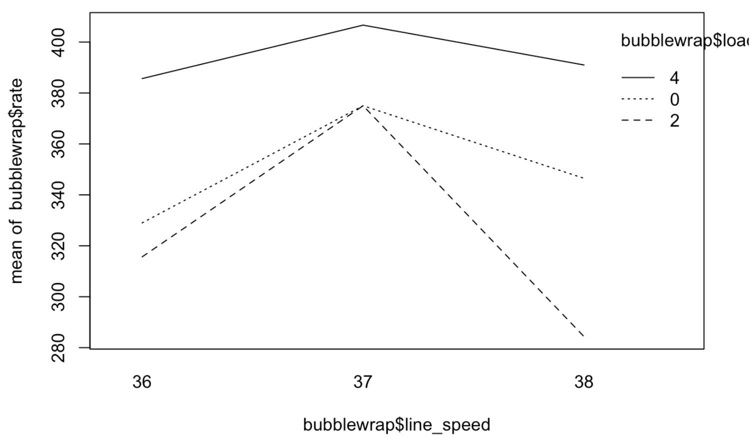
Appendix



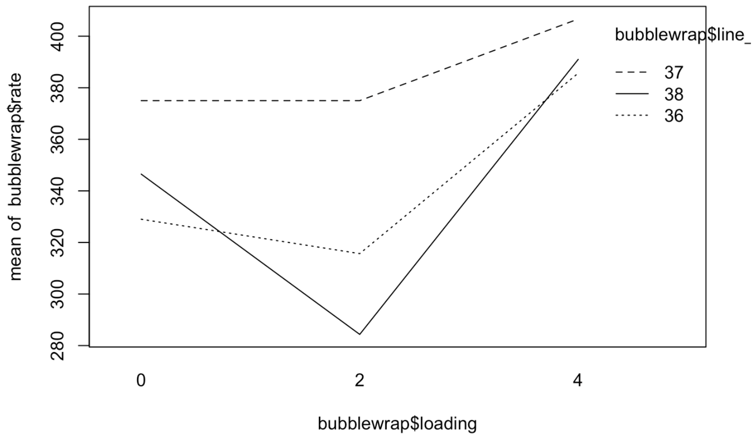
1.1 box-plot for line_speed



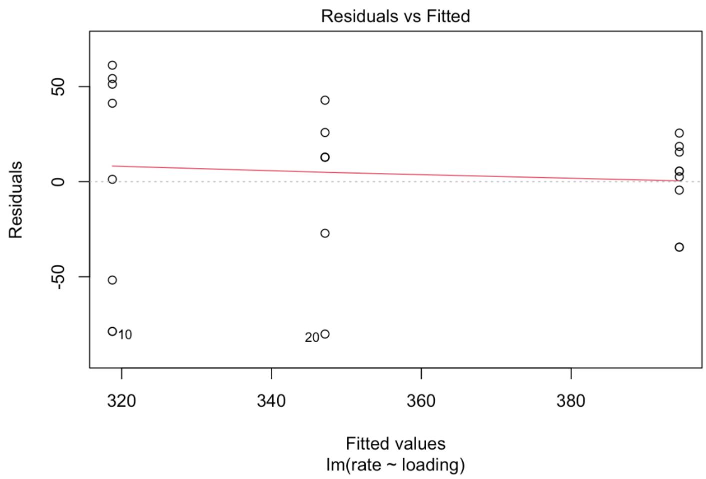
1.2 box-plot for loading



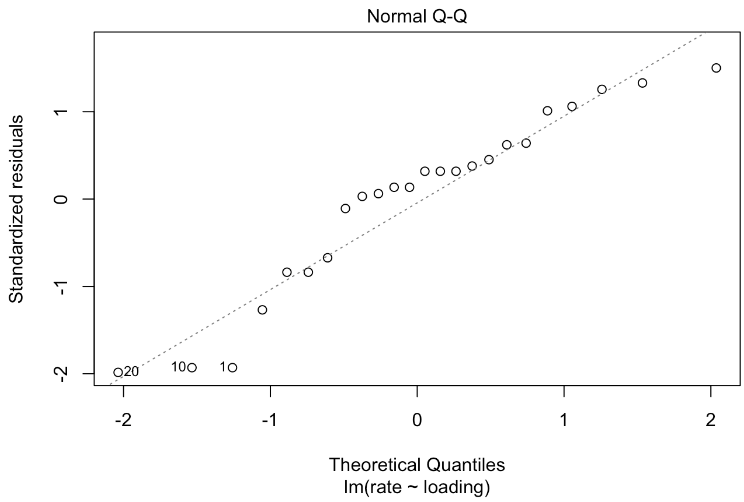
1.3 interaction plot for line_speed



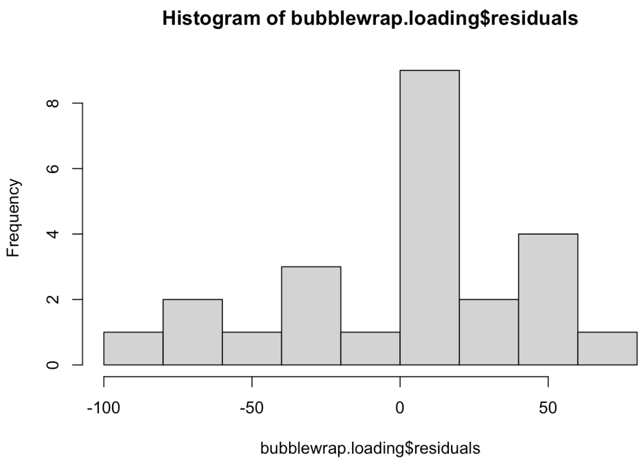
1.4 interaction plot for loading



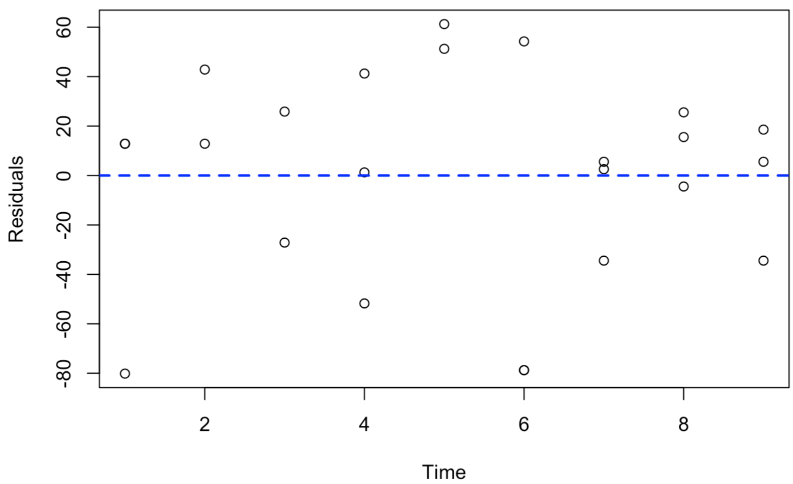
3.1 residuals plot for the first final model, residuals against fitted values



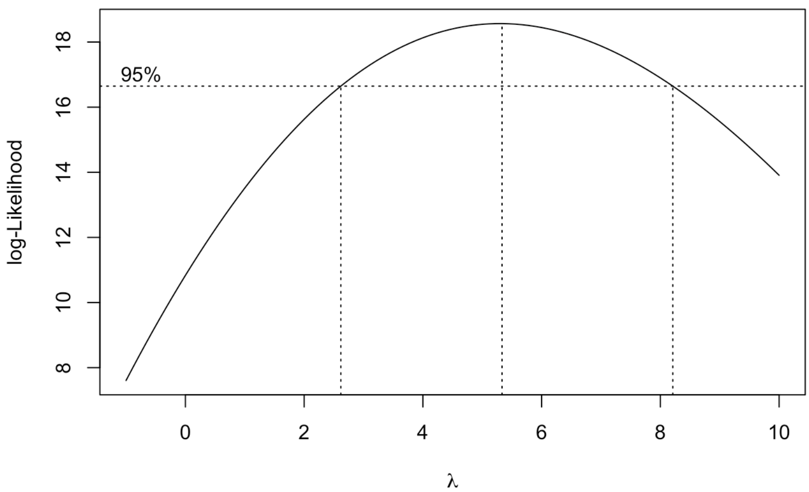
3.2 QQ-plot for the first final model



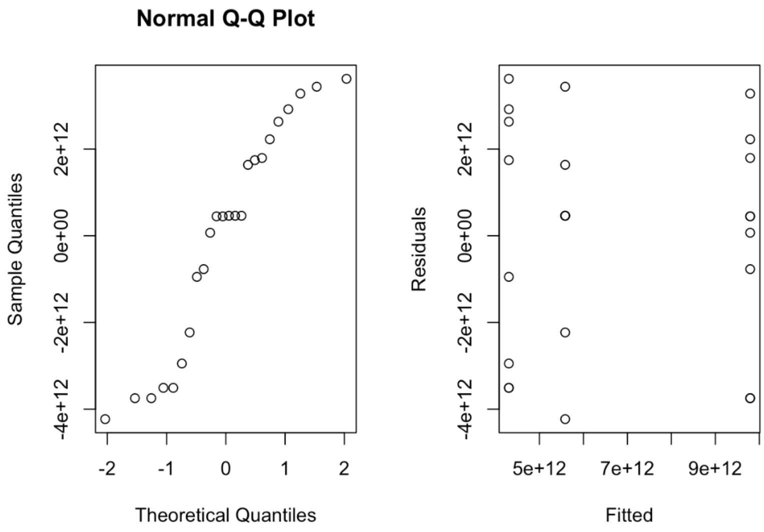
3.3 histogram of residuals for the first final model



3.4 sequence plot for the first final model



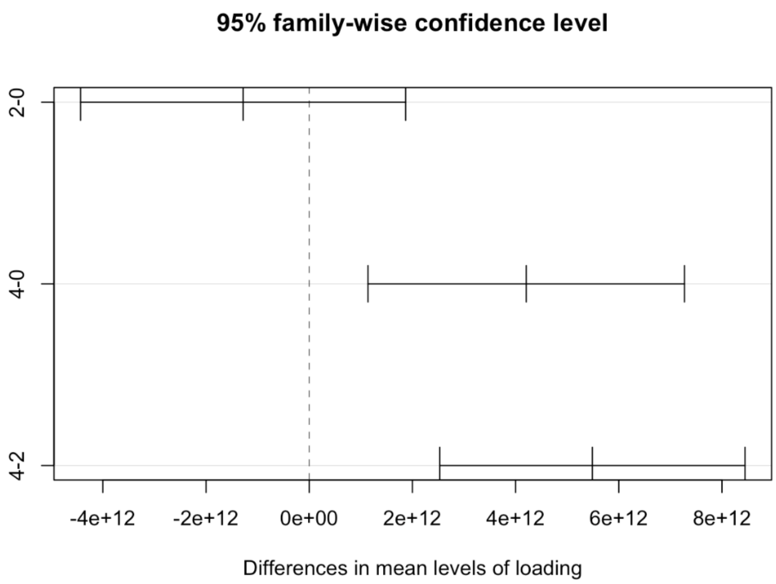
3.5 log-likelihood function vs. λ graph



4.1 side-by-side QQ plot, residuals plot

```
##   Tukey multiple comparisons of means
##     95% family-wise confidence level
##
## Fit: aov(formula = rate^5 ~ loading + line_speed, data = bubblewra
p)
##
## $loading
##              diff           lwr          upr     p adj
## 2-0 -1.281273e+12 -4.431303e+12 1.868758e+12 0.5655894
## 4-0  4.207018e+12  1.139742e+12 7.274294e+12 0.0066843
## 4-2  5.488291e+12  2.530814e+12 8.445767e+12 0.0004241
```

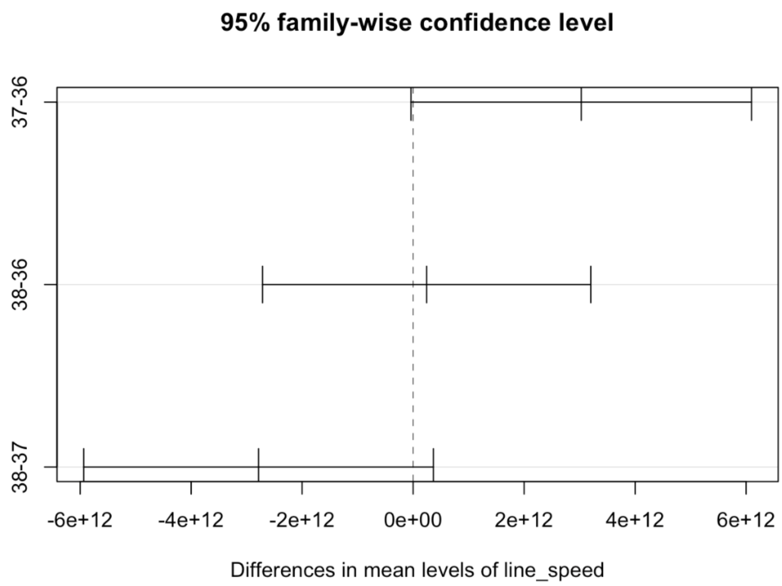
4.2 Tukey’s method for loading



4.3 Tukey’s family confidence intervals plot for loading

```
##   Tukey multiple comparisons of means
##     95% family-wise confidence level
##
## Fit: aov(formula = rate^5 ~ loading + line_speed, data = bubblewra
p)
##
## $line_speed
##                diff           lwr          upr     p adj
## 37-36  3.029345e+12 -3.793140e+10 6.096621e+12 0.0532477
## 38-36  2.441367e+11 -2.713340e+12 3.201614e+12 0.9760773
## 38-37 -2.785208e+12 -5.935238e+12 3.648226e+11 0.0888049
```

4.4 Tukey's method for line_speed



4.5 Tukey's family confidence intervals plot for line_speed

```
##
##   Posthoc multiple comparisons of means: Scheffe Test
##     95% family-wise confidence level
##
## $loading
##              diff        lwr.ci       upr.ci   pval
## 2-0 -1.281273e+12 -5.500826e+12 2.938280e+12 0.8956
## 4-0  4.207018e+12  9.831674e+10 8.315719e+12 0.0430 *
## 4-2  5.488291e+12  1.526669e+12 9.449913e+12 0.0039 **
##
## $line_speed
##                diff        lwr.ci       upr.ci   pval
## 37-36  3.029345e+12 -1.079356e+12 7.138046e+12 0.2219
## 38-36  2.441367e+11 -3.717485e+12 4.205759e+12 0.9997
## 38-37 -2.785208e+12 -7.004761e+12 1.434345e+12 0.3195
##
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

4.6 Scheffe's test