Department of Economics
Keio University

Masa Watanabe

Exercise 3: Asset Pricing Tests (Cross-section & time-series)

Due 1pm July 9 (at the beginning of class)

Upload a PDF copy of your answer with well-documented code in separate, runnable, human-readable files on Canvas; your result should be verifiable by a third party, as many top finance and economics journals require or recommend. Keep a copy for class discussion. Use a programming language of your choice.

This exercise asks you to perform important asset-pricing tests, both in the cross section and in time series. Consider a time-series regression of asset *i*'s excess return, $\tilde{R}_i^e \equiv \tilde{R}_i - R_f$, on a vector of factors, $\tilde{F}$:

$$\tilde{R}_i^e = \alpha_i + \beta_i' \tilde{F} + \tilde{\varepsilon}, \tag{1}$$

where α_i is the intercept, β_i is a vector of the asset's loadings on the factors, and $\tilde{\varepsilon}$ is the residual. If all the factors are excess returns and the model is "correct" (to be clarified in "Gibbons-Ross-Shanken test" below), the intercept should be zero. The cross-sectional relation is then

$$E[\tilde{R}_i^e] = \lambda' \beta_i, \tag{2}$$

where λ is the vector of prices of risk. Recall that if the *j*'th factor is an excess return, its price of risk λ_j must simply equal the mean factor, i.e., the mean excess return (see Slide "Multifactor Models" in Topic 6 "Factor Models").

As factors, use the value-weighted Fama-French three factors (SAS dataset `factors_monthly.sas7bdat` or its corresponding CSV file from Exercise 1). It also includes the risk-free rate. For asset returns, use the 25 value-weighted Fama-French size-BM portfolios (SAS dataset `portfolios25.sas7bdat`, ditto). Subtract the risk-free rate to compute excess returns on the portfolios. Perform the following tests (the last two are optional for extra credits) for two specifications, the CAPM ($\tilde{F} = MKTRF$) and the Fama-French three-factor model ($\tilde{F} = [MKTRF \;\; SMB \;\; HML]'$), for each of the sample periods, July 1963-December 1991 and January 1927-December 2024 (the same period as Exercise 1 "Fama-French Factors and Portfolios" for comparison below).

1) **Gibbons-Ross-Shanken test**. Following Gibbons, Ross, and Shanken (1989, GRS), conduct a multivariate joint test of all the alphas in (1) being zero. This is equivalent to testing the mean-variance efficiency of a portfolio of the portfolios in the excess-return factors, $\tilde{F}$ (the model being "correct"). This multivariate efficiency test has a nice geometric interpretation in terms of the mean-variance frontier when there is a single factor. For a textbook exposition of the GRS test, see Cochrane (2005, Section 12.1).

Report the value of the test-statistic, its distribution with the degrees of freedom, p-value, as well as the numbers of periods, assets, and factors.

2) **Fama-MacBeth (FM) procedure**. In the first pass, estimate the vector of betas for each asset by the time-series regression in (1). In the second pass, run the cross-sectional regression of $\tilde{R}_i^e$ on the estimated betas from the first pass with an intercept in (2) each month (you can include the intercept and see if it is zero). This will give time series of factor premia, λ_t. Perform the t-test of the mean of λ_t. This is the estimate of λ. Report the mean, standard error, t-statistic, p-value, and the number of observations (periods) for each factor. Casually compare the mean premia to the mean factors from Exercise 1 (no statistical test required). Do they match according to (2), especially for significant factors?

3) (Optional, bonus points) **FM with Newey-West correction for heteroscedasticity and autocorrelation**. Following Newey and West (1987), correct the standard errors of the mean premia in the FM procedure to address the heteroscedasticity and autocorrelation of premia. For the number of lags, use floor(4*(T/100)$^{2/9}$)) for T observations as in Table IIC(6) of Newey and West (1994). Report the number of lags and the same set of statistics as 2) with the Newey-West correction.

4) (Optional, bonus points) **FM with Shanken correction for errors-in-variables problem**. Following Shanken (1992), correct the standard errors of the mean premia in the FM procedure to address the errors-in-variable problem in the two-step procedure. Report the same set of statistics as 2) with the Shanken correction.

References

Cochrane, John, 2005, *Asset Pricing* (revised edition), Princeton University Press.

Gibbons, Michael R., Stephen A. Ross, and Jay Shanken, 1989, "A Test of the Efficiency of a Given Portfolio," *Econometrica* 57 (5), 1121-1152.

Newey, Whitney K., and Kenneth D. West, 1987, "A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica* 55 (3), 703-708.

Newey, Whitney K., and Kenneth D. West, 1994, "Automatic Lag Selection in Covariance Matrix Estimation," *Review of Economic Studies* 61 (4), 631-653.

Shanken, Jay, 1992, "On the Estimation of Beta-Pricing Models," *Review of Financial Studies* 5 (1), 1-33.