

CS496 UGP 1

Designer Co-pilot: Reimagining Workplace Design with AI

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Abstract:

Our project focuses on **designing workspaces** by developing an AI-powered Designer Co-pilot that transitions traditional design tools into intelligent collaborators. We follow the **grid philosophy**, using structure layouts and spatial constraints to create efficient and organized workspace environments. At the core of this approach is the use of Integer Linear Programming (**ILP**) and different **sampling methods** to produce the layouts which best fits all the user-given constraints. The Co-pilot utilizes data-driven intelligence and generative abilities to actively participate in **layout planning, compliance testing and decision making**. This integration improves efficiency, saves human labor hours, and produces responsive, user-specific workspace solutions.

Grid Based Philosophy

It is bit complex to directly start interior designing of a random shaped floor with building constraints like shell shape, core areas, structural elements etc.

so we start with simple grid-based philosophy. we take rectangular grid of custom dimensions and populate it with red, green, and blue tiles.

each tile color represents a functional or spatial component and they are subjected to specific use defined constraints.

we generate feasible grid configurations / samples that satisfy **user defined custom constraints through optimization of corresponding objective function**.

General Integer Linear Programming Setting

ILP is mathematical optimization program in which some or all of the variables are restricted to be integers.

In many settings the **objective function and the constraints** (other than integer constraints) are **linear**.

Integer programming is **NP-complete**. a problem is NP-complete when it is a decision problem, meaning that output is either **yes** or **no**.

When the **feasible region defined by the constraints** has a positive area (i.e., there is at least one feasible solution/point within the region), the answer is **yes**—we can determine feasible samples or solutions. Otherwise, if the answer is **no**, the problem ends there, and we simply return "no feasible solution" and request the user to **loosen the constraints**.

If some decision variables are not discrete, the problem becomes a **mixed-integer programming** problem.

First, we check if the given number of red, green, and blue color cells can be arranged to fit within a square grid with linear constraints.

Let R, G, and B be the number of red, green, and blue tiles, respectively.

$$R + G + B = \text{grid size} (\text{length} \times \text{breadth})$$

The user can input custom linear constraints of the form:

$$aR + bG + cB \leq d$$

ILP canonical form is :

$$\begin{array}{ll} \text{maximize}_{\mathbf{x} \in \mathbb{Z}^n} & \mathbf{c}^T \mathbf{x} \\ \text{subject to} & A\mathbf{x} \leq \mathbf{b}, \\ & \mathbf{x} \geq \mathbf{0} \end{array}$$

so, we use integer linear programming to check if it is possible to satisfy the custom defined user constraints. lets take an illustration:

```
In [ ]: from scipy.optimize import linprog

def can_fit_colors():
    """
    Check if the given [R, G, B] can fit into a square grid with constraints.

    Returns:
        str: 'YES' if the configuration is possible, otherwise 'NO'.
    """
    # Take inputs for the grid size and color counts
```

```

grid_size = int(input("Enter the dimension of the square grid: "))
R, G, B = map(int, input("Enter the counts of R, G, B (space-separated): "))

total_cells = grid_size ** 2

# Ensure the total matches the grid size
if R + G + B != total_cells:
    return "NO: The sum of R, G, and B does not equal the total grid cells."

print("Define constraints:")
print("1. Total cells constraint: R + G + B must equal total_cells.")
print("2. Custom constraints must be of the form 'a b c d' for aR + bG + cB")

custom_constraints = input("Do you want to add custom constraints? (yes/no):"

# Prepare the objective function (dummy since we're only checking feasibility)
c = [0, 0, 0] # Coefficients for R, G, B

# Inequality constraints (Ax <= b)
A_ub = []
b_ub = []

if custom_constraints == "yes":
    while True:
        constraint = input("Enter a custom constraint as 'a b c d' (or 'done')
        if constraint.lower() == "done":
            break
        try:
            a, b, c_value, d = map(int, constraint.split())
            # Store coefficients in the correct order for the inequality
            A_ub.append([a, b, c_value])
            b_ub.append(d)
        except Exception as e:
            print(f"Error in constraint: {e}. Please enter in 'a b c d' form

# Convert lists to numpy arrays if they're not empty
A_ub = A_ub if A_ub else [] # Use an empty list if no constraints
b_ub = b_ub if b_ub else [] # Use an empty list if no constraints

# Variable bounds for R, G, B
bounds = [(0, R), (0, G), (0, B)]

# Equality constraint for total cells
A_eq = [[1, 1, 1]] # R + G + B = total_cells
b_eq = [total_cells]

try:
    # Solve the linear programming problem
    res = linprog(c, A_ub=A_ub, b_ub=b_ub, A_eq=A_eq, b_eq=b_eq, bounds=bounds)

    # Check if a feasible solution exists
    if res.success:
        return "YES"
    else:
        return "NO"
except Exception as e:
    return f"An error occurred during linear programming: {e}"

# Example usage
if __name__ == "__main__":

```

```
result = can_fit_colors()  
print(result)
```

```
Enter the dimension of the square grid: 10  
Enter the counts of R, G, B (space-separated): 20 20 60  
Define constraints:  
1. Total cells constraint: R + G + B must equal total_cells.  
2. Custom constraints must be of the form 'a b c d' for aR + bG + cB <= d.  
Do you want to add custom constraints? (yes/no): yes  
Enter a custom constraint as 'a b c d' (or 'done' to finish): 1 1 1 2000  
Enter a custom constraint as 'a b c d' (or 'done' to finish): 3 4 5 100  
Enter a custom constraint as 'a b c d' (or 'done' to finish): 3 3 3 1000  
Enter a custom constraint as 'a b c d' (or 'done' to finish): done  
NO
```

Here for this particular set of linear constraints, the solution was not feasible as the **generalised inverse of constraint coefficient matrix A** was not possible.

Single Constraints

ILP is a general method used to solve problems **if** we are able to express the constraints as linear equations. However, these linear constraints are **not always provided directly** by the user, as users might not be familiar with how their logical or geometric conditions can be translated into mathematical form.

Instead, users often describe their constraints in **logical** terms (such as adjacency, patterns, etc.) or **geometrical** terms (such as periphery, diagonal positioning, etc.).

Our task, then, is to interpret these higher-level user constraints and reduce them to formal linear inequalities or equations that an ILP solver can handle.

Now, we solve for **specific constraints**

Periphery constraint

if user wants to fill the colours in the periphery with a priority order (e.g. R G , meaning R is filled in periphery with highest priority then Green and then remaining)

Variable and notation for understanding:

- $G_{i,j}^c \in \{0, 1\}$ be a binary variable indicating whether cell (i, j) is colored with color $c \in \{R, G, B\}$.
- P be the set of periphery cell positions:

$$P = \{(0, j), (rows - 1, j)\}_{j=0}^{cols-1} \cup \{(i, 0), (i, cols - 1)\}_{i=1}^{rows-2}$$

- C^c be the total number of tiles of color $c \in \{R, G, B\}$, provided as input.

Appart from the default sanity check: $R+G+B = \text{grid size}$ we have to Assign Periphery Color Priority:

let

$$\mathcal{P} = [c_1, c_2, \dots, c_k]$$

be the list of priority of colors for the periphery as given in periphery_colors.

We try to assign colors c_1, c_2, \dots to the periphery positions first.

we apply layer of constraints:

$$\sum_{(i,j) \in P} G_{i,j}^{c_1} \geq 1 \quad (\text{assign as many as possible with } c_1)$$

$$\sum_{(i,j) \in P} G_{i,j}^{c_2} \geq 1 \quad (\text{then fill with } c_2 \text{ if } c_1 \text{ is exhausted})$$

here P is periphery not the priority list

we can maximize number of periphery tiles assigned to higher-priority colors by assigning more weights to them in the objective function:

$$w_{c_1} > w_{c_2} > w_{c_3}$$

Objective Function

$$\max \sum_{(i,j) \in P} \sum_{c \in \{R,G,B\}} w_c \cdot G_{i,j}^c$$

```
In [ ]: import sys
import random

def fill_grid(rows, cols, red, green, blue, periphery_colors):
    total_tiles = rows * cols

    if red + green + blue != total_tiles:
        print(f"Invalid input! The total number of tiles ({red + green + blue})")
        sys.exit(1)

    # Initialize the grid with None
    grid = [[None] * cols for _ in range(rows)]
    color_counts = {'R': red, 'G': green, 'B': blue}

    # Define periphery positions
    periphery_positions = [(0, j) for j in range(cols)] + [(rows - 1, j) for j in range(cols)]
    periphery_positions += [(i, 0) for i in range(1, rows - 1)] + [(i, cols - 1) for i in range(1, rows - 1)]

    # Shuffle periphery positions to make selection random
    random.shuffle(periphery_positions)

    # Assign periphery colors in the order given by the user
    for color in periphery_colors:
        for i, j in periphery_positions:
            if grid[i][j] is None and color_counts[color] > 0:
                grid[i][j] = color
                color_counts[color] -= 1

    # Fill remaining periphery positions with available colors
    for i, j in periphery_positions:
        if grid[i][j] is None:
            available_colors = [color for color in 'RGB' if color_counts[color] > 0]
            if available_colors:
                chosen_color = random.choice(available_colors)
                grid[i][j] = chosen_color
                color_counts[chosen_color] -= 1

    # Collect empty positions
    empty_positions = [(i, j) for i in range(rows) for j in range(cols) if grid[i][j] is None]
    random.shuffle(empty_positions) # Shuffle the empty positions

    # Fill the shuffled empty positions with available colors
    for i, j in empty_positions:
        available_colors = [color for color in 'RGB' if color_counts[color] > 0]
        if available_colors:
            chosen_color = random.choice(available_colors)
            grid[i][j] = chosen_color
            color_counts[chosen_color] -= 1
```

```

    return grid

# Input handling
rows = int(input("Enter the number of rows: "))
cols = int(input("Enter the number of columns: "))
red = int(input("Enter the number of red tiles: "))
green = int(input("Enter the number of green tiles: "))
blue = int(input("Enter the number of blue tiles: "))
periphery_colors = input("Enter the colors for the periphery in order (R/G/B, e.g., R B G): R G

# Generate grid
grid = fill_grid(rows, cols, red, green, blue, periphery_colors)

# Print the grid
if grid:
    for row in grid:
        print(" ".join(cell if cell is not None else '-' for cell in row))

```

```

Enter the number of rows: 6
Enter the number of columns: 8
Enter the number of red tiles: 15
Enter the number of green tiles: 21
Enter the number of blue tiles: 12
Enter the colors for the periphery in order (R/G/B, e.g., R B G): R G
G R R G R R R G
G G G B G B B G
R B B G G G B R
G B B B G B B R
R G B G G G G G
R R R R G G R R

```

here we can see the colours are filled according to their priority in the periphery.

Tile Count Constraint ensures exact utilization of input tiles.

Uniqueness Constraint guarantees no tile is assigned more than one color.

Periphery Coloring Constraint prioritizes certain colors at the periphery, simulating a design preference (e.g., boundary in red).

Diagonal Constraint:

- $G_{i,j}^c \in \{0, 1\}$ be a binary variable indicating whether cell (i, j) is filled with color $c \in \{R, G, B\}$
- Let $\mathcal{D} = \{(i, i)\} \cup \{(i, n - 1 - i)\}$ be the set of all **diagonal positions**, where $n = \text{dimension}$

for a given priority list P we again define more weights to the layer of constraints will be preferring c1 over c2, c2 over c3 and so on

$$\sum_{(i,j) \in \mathcal{D}} G_{i,j}^{c_1} \geq k_1 \quad (\text{as many diagonal positions as possible get } c_1)$$

We want to prefer assigning c1, then c2, then c3 to the diagonal positions.

To encode this preference as a soft constraint we define weights:

$$w_{c_1} > w_{c_2} > w_{c_3}$$

Objective function:

$$\max \sum_{(i,j) \in \mathcal{D}} \sum_{c \in \{R, G, B\}} w_c \cdot G_{i,j}^c$$

```
In [ ]: import random

def fill_grid(dimension, red, green, blue, diagonal_colors):
    total_cells = dimension * dimension
    total_colors = red + green + blue

    if total_colors != total_cells:
        print(f"Error: Incorrect number of colors. Need exactly {total_cells}, but got {total_colors}")
        return

    grid = [[None] * dimension for _ in range(dimension)]
    color_counts = {'R': red, 'G': green, 'B': blue}

    # Get all diagonal positions (both main diagonal and anti-diagonal)
    main_diagonal = [(i, i) for i in range(dimension)]
    anti_diagonal = [(i, dimension - 1 - i) for i in range(dimension)]

    # Remove duplicates (center cell in odd-sized grids)
    all_diagonal_positions = []
    for pos in main_diagonal + anti_diagonal:
        if pos not in all_diagonal_positions:
            all_diagonal_positions.append(pos)

    random.shuffle(all_diagonal_positions)

    # First, try to fill all diagonal positions according to priority order
    for pos in all_diagonal_positions:
        if color_counts[diagonal_colors[pos]] > 0:
            grid[pos[0]][pos[1]] = diagonal_colors[pos]
            color_counts[diagonal_colors[pos]] -= 1
```

```

    for i, j in all_diagonal_positions:
        filled = False
        for priority_color in diagonal_colors:
            if color_counts[priority_color] > 0:
                grid[i][j] = priority_color
                color_counts[priority_color] -= 1
                filled = True
                break

    # Fill remaining empty positions
    empty_positions = [(i, j) for i in range(dimension) for j in range(dimension)
                        random.shuffle(empty_positions)

    for i, j in empty_positions:
        available_colors = [color for color in 'RGB' if color_counts[color] > 0]
        if available_colors:
            chosen_color = random.choice(available_colors)
            grid[i][j] = chosen_color
            color_counts[chosen_color] -= 1

    return grid

# Get user inputs
dimension = int(input("Enter the dimension: "))
red = int(input("Enter the number of red tiles: "))
green = int(input("Enter the number of green tiles: "))
blue = int(input("Enter the number of blue tiles: "))
diagonal_colors = input("Enter the colors for the diagonal in order (R/G/B, e.g. R G B): ")

grid = fill_grid(dimension, red, green, blue, diagonal_colors)

if grid:
    for row in grid:
        print(" ".join(row))

```

Enter the dimension: 5
 Enter the number of red tiles: 9
 Enter the number of green tiles: 10
 Enter the number of blue tiles: 6
 Enter the colors for the diagonal in order (R/G/B, e.g., R G B): B R
 R G R G B
 G R G B G
 G R B R G
 R B G B G
 B G R R R

Diagonal Priority: The most preferred colors appear on the diagonals as much as possible (in order).

Rest of the Grid: Filled greedily with remaining colors, obeying total tile counts.

Adjacency Constraint:

Without loss of generality lets solve Constraint on (Red-Green)

for R and G to be adjacent in grid: we take grid layout to be a graph.

cells are nodes and adjacent cells share an edge.

Binary Variables:

let we define binary variable s.t. $x[i,j]^R=0$ if (i,j) th cell does not contain red, else 1.

for each cell (i, j) in the grid:

If there is a red cell $x[i, j]^R$ at position (i, j) , then at least one of its adjacent cells must be green, represented as:

- $x[i, j+1]^G$ (right)
- $x[i, j-1]^G$ (left)
- $x[i-1, j]^G$ (above)
- $x[i+1, j]^G$ (below)

if $x[i, j]^R == 1$, then:

$$x[i, j+1]^G + x[i, j-1]^G + x[i-1, j]^G + x[i+1, j]^G \geq 1$$

so wherever R is in a cell then atleast 1 G is near it and similarly if $x[i, j]^G == 1$, then:

$$x[i, j+1]^R + x[i, j-1]^R + x[i-1, j]^R + x[i+1, j]^R \geq 1$$

so wherever G is in a cell then atleast 1 R is near it.

Objective function

$$\text{Maximize} \quad \sum_{i,j} \left(x[i, j]^R \cdot x[i, j + 1]^G + x[i, j]^R \cdot x[i, j - 1]^G + x[i, j]^R \cdot x[i - 1, j]^G + x[i, j]^R \cdot x[i + 1, j]^G \right)$$

Boundary Conditions for sanity check: **checking bounds:**

- Don't access invalid indices like `i-1` if `i=1`.
- For cells on the edge, include only valid neighbors.

In []:

```
import random

def get_user_input():
    n = int(input("Enter number of rows: "))
```

```

m = int(input("Enter number of columns: "))
total_cells = n * m

tile_counts = {}
tile_counts['R'] = int(input("Enter count of R: "))
tile_counts['G'] = int(input("Enter count of G: "))
tile_counts['B'] = int(input("Enter count of B: "))

if sum(tile_counts.values()) != total_cells:
    print("Error: The sum of R, G, B must equal grid area.")
    return None, None, None

adjacent_tiles = input("Enter two tiles for constraint (e.g., R G): ").split()
if len(adjacent_tiles) != 2 or adjacent_tiles[0] not in tile_counts or adjacent_tiles[1] not in tile_counts:
    print("Error: Invalid tile constraint selection.")
    return None, None, None

return n, m, tile_counts, adjacent_tiles

def count_adjacent_pairs(grid, n, m, adjacent_tiles):
    """Count the total number of adjacent pairs in the final grid."""
    tile1, tile2 = adjacent_tiles
    pair_count = 0

    # Check horizontal adjacencies
    for i in range(n):
        for j in range(m-1):
            if (grid[i][j] == tile1 and grid[i][j+1] == tile2) or \
               (grid[i][j] == tile2 and grid[i][j+1] == tile1):
                pair_count += 1

    # Check vertical adjacencies
    for i in range(n-1):
        for j in range(m):
            if (grid[i][j] == tile1 and grid[i+1][j] == tile2) or \
               (grid[i][j] == tile2 and grid[i+1][j] == tile1):
                pair_count += 1

    return pair_count

def generate_strategic_grid(n, m, tile_counts, adjacent_tiles):
    grid = [[' ' for _ in range(m)] for _ in range(n)]

    tile1, tile2 = adjacent_tiles[0], adjacent_tiles[1]
    if tile_counts[tile1] > tile_counts[tile2]:
        tile1, tile2 = tile2, tile1

    if tile_counts[tile2] > 4 * tile_counts[tile1]:
        print("Error: Not possible to satisfy the constraint.")
        return None, None, None

    placed_tiles = {'R': 0, 'G': 0, 'B': 0}
    adjacent_pairs = 0
    placed_positions = []

    row, col = 1, 1
    while placed_tiles[tile1] < tile_counts[tile1] and placed_tiles[tile2] < tile_counts[tile2]:
        if grid[row][col] == ' ':
            grid[row][col] = tile1
            placed_tiles[tile1] += 1

```

```

placed_positions.append((row, col))

adjacent_positions = [
    (row, col-1), (row, col+1),
    (row-1, col), (row+1, col)
]

for adj_row, adj_col in adjacent_positions:
    if (0 <= adj_row < n and 0 <= adj_col < m and
        grid[adj_row][adj_col] == ' ' and
        placed_tiles[tile2] < tile_counts[tile2]):
        grid[adj_row][adj_col] = tile2
        placed_tiles[tile2] += 1
        adjacent_pairs += 1

    col += 3
    if col >= m:
        col = 1
        row += 3
    if row >= n:
        break

if placed_tiles[tile1] < tile_counts[tile1]:
    for i, j in placed_positions:
        diagonal_positions = [
            (i-1, j-1), (i-1, j+1),
            (i+1, j-1), (i+1, j+1)
        ]
        for di, dj in diagonal_positions:
            if (0 <= di < n and 0 <= dj < m and grid[di][dj] == ' ' and placed_tiles[tile1] < tile_counts[tile1]):
                grid[di][dj] = tile1
                placed_tiles[tile1] += 1
            if placed_tiles[tile1] == tile_counts[tile1]:
                break
    if placed_tiles[tile1] == tile_counts[tile1]:
        break

if placed_tiles[tile2] < tile_counts[tile2]:
    for i, j in placed_positions:
        diagonal_positions = [
            (i-1, j-1), (i-1, j+1),
            (i+1, j-1), (i+1, j+1)
        ]
        for di, dj in diagonal_positions:
            if (0 <= di < n and 0 <= dj < m and grid[di][dj] == ' ' and placed_tiles[tile2] < tile_counts[tile2]):
                grid[di][dj] = tile2
                placed_tiles[tile2] += 1
            if placed_tiles[tile2] == tile_counts[tile2]:
                break
    if placed_tiles[tile2] == tile_counts[tile2]:
        break

remaining_tiles = []
for tile, count in tile_counts.items():
    remaining = count - placed_tiles[tile]
    remaining_tiles.extend([tile] * remaining)
random.shuffle(remaining_tiles)

for i in range(n):
    for j in range(m):

```

```

        if grid[i][j] == ' ' and remaining_tiles:
            grid[i][j] = remaining_tiles.pop()
            placed_tiles[grid[i][j]] += 1

    # Count the actual number of adjacent pairs in the final grid
    final_adjacent_pairs = count_adjacent_pairs(grid, n, m, adjacent_tiles)

    return grid, placed_tiles, final_adjacent_pairs

if __name__ == "__main__":
    n, m, tile_counts, adjacent_tiles = get_user_input()
    if n and m:
        grid, placed_tiles, adjacent_pairs = generate_strategic_grid(n, m, tile_
        if grid:
            print("\nFinal Grid:")
            for row in grid:
                print(" ".join(row))

            print(f"\nPlaced tiles: {placed_tiles}")
            print(f"Total {adjacent_tiles[0]}-{adjacent_tiles[1]} adjacent pairs")

```

Enter number of rows: 5
Enter number of columns: 5
Enter count of R: 8
Enter count of G: 9
Enter count of B: 8
Enter two tiles for constraint (e.g., R G): G B

Final Grid:
B G B B G
G B G G B
B G B R G
R R R R R
G B G R R

Placed tiles: {'R': 8, 'G': 9, 'B': 8}
Total G-B adjacent pairs: 19

constraint ensures that:

- Every Red cell must have **at least one Green neighbor**
- Every Green cell must have **at least one Red neighbor**

No two adjacent tiles can have the same color.

For every pair of adjacent cells $(i, j) \sim (k, l)$:

This must be applied to all pairs of adjacent cells:

- $(i \pm 1, j)$
- $(i, j \pm 1)$

$$x_{i,j}^R + x_{k,l}^R \leq 1$$

$$x_{i,j}^G + x_{k,l}^G \leq 1$$

$$x_{i,j}^B + x_{k,l}^B \leq 1$$

Energy function (that is negative of Objective function is)

$$\min \sum_{\text{adjacent } (i,j), (k,l)} (x_{i,j}^R \cdot x_{k,l}^R + x_{i,j}^G \cdot x_{k,l}^G + x_{i,j}^B \cdot x_{k,l}^B)$$

```
In [ ]: import random

def generate_colored_grid(n, m, red_count, green_count, blue_count):
    """Generates a valid n x m grid with no two adjacent tiles having the same color"""
    def is_valid(grid, row, col, color, n, m):
        """Checks if placing the given color at (row, col) is valid."""
        directions = [(-1, 0), (1, 0), (0, -1), (0, 1)] # Up, Down, Left, Right
        for dr, dc in directions:
            r, c = row + dr, col + dc
            if 0 <= r < n and 0 <= c < m and grid[r][c] == color:
```

```

        return False # Adjacent tile has the same color
    return True

def solve(grid, positions, color_counts, n, m):
    """Fills the grid ensuring no two adjacent tiles have the same color."""
    random.shuffle(positions) # Shuffle positions for randomness

    for row, col in positions:
        available_colors = [color for color in ['R', 'G', 'B'] if color_counts[color] > 0]
        if available_colors:
            chosen_color = random.choice(available_colors)
            grid[row][col] = chosen_color
            color_counts[chosen_color] -= 1

    total_tiles = n * m
    if red_count + green_count + blue_count != total_tiles:
        print("Invalid input: Total number of tiles does not match grid size!")
        return None

    grid = [[None for _ in range(m)] for _ in range(n)]
    color_counts = {'R': red_count, 'G': green_count, 'B': blue_count}
    positions = [(i, j) for i in range(n) for j in range(m)]
    random.shuffle(positions) # Shuffle positions for randomness
    solve(grid, positions, color_counts, n, m)

    # Collect remaining empty positions
    empty_positions = [(i, j) for i in range(n) for j in range(m) if grid[i][j] is None]
    random.shuffle(empty_positions) # Shuffle for randomness

    # Fill remaining empty positions with available colors
    for row, col in empty_positions:
        available_colors = [color for color in ['R', 'G', 'B'] if color_counts[color] > 0]
        if available_colors:
            chosen_color = random.choice(available_colors)
            grid[row][col] = chosen_color
            color_counts[chosen_color] -= 1

    return grid

# User Input
n = int(input("Enter the number of rows: "))
m = int(input("Enter the number of columns: "))
red_count = int(input("Enter the number of Red tiles: "))
green_count = int(input("Enter the number of Green tiles: "))
blue_count = int(input("Enter the number of Blue tiles: "))

# Generate and print the grid
grid = generate_colored_grid(n, m, red_count, green_count, blue_count)

# Print the grid
if grid:
    for row in grid:
        print(" ".join(cell if cell is not None else '-' for cell in row))

```

```
Enter the number of rows: 5
Enter the number of columns: 5
Enter the number of Red tiles: 8
Enter the number of Green tiles: 9
Enter the number of Blue tiles: 8
G R G R G
B G B G B
R R G B R
G B B R G
R B G B R
```

Big Coloured Block Constraint

Method:

Choose a colour, Let us say blue

Choose the dimensions of the block and number of blocks.

Check if the possible arrangement is possible

$$n^* (s^* s) \leq B$$

- $x_{i,j}^c \in \{0, 1\}$: binary variable indicating whether cell (i, j) is filled with color $c \in \{R, G, B\}$.
 - $y_{r,c} \in \{0, 1\}$: binary variable indicating whether a **block** of color `block_color` is placed with **top-left corner** at position (r, c) .
 - $i \in \{1, \dots, n\}, j \in \{1, \dots, m\}$
 - $r \in \{1, \dots, n - s + 1\}, c \in \{1, \dots, m - s + 1\}$
 - s : block size

Constarints

Block placement limit

$$\sum_{r=1}^{n-s+1} \sum_{c=1}^{m-s+1} y_{r,c} \leq \min \left(\text{block_count}, \left\lfloor \frac{\text{color_budget}}{s^2} \right\rfloor \right)$$

For all $r, c \in \{1, \dots, n - s + 1\} \times \{1, \dots, m - s + 1\}$, and $i \in [r, r + s - 1], j \in [c, c + s - 1]$:

$$x_{i,j}^{\text{block_color}} \geq y_{r,c}$$

This ensures all cells in the block are set to the block color if a block is placed.

Non-overlapping blocks

Each cell can be used in at most one block. For every cell (i, j) , the sum of all $y_{r,c}$ such that (i, j) is in the block (r, c) must be ≤ 1 .

$$\sum_{\substack{r,c \\ i \in [r, r+s-1], j \in [c, c+s-1]}} y_{r,c} \leq 1 \quad \forall i, j$$

Objective function

$$\max \sum_{r=1}^{n-s+1} \sum_{c=1}^{m-s+1} y_{r,c}$$

```
In [ ]: def generate_grid(n, m, red, green, blue, block_color, block_size, block_count):
    grid = [[' ' for _ in range(m)] for _ in range(n)]
    color_counts = {'R': red, 'G': green, 'B': blue}
    max_possible_blocks = min(block_count, color_counts.get(block_color, 0)) // (
    placed_blocks = 0
    available_rows = list(range(0, n - block_size+1))
    random.shuffle(available_rows)

    for row_index in available_rows:
        if placed_blocks >= max_possible_blocks:
            break
        available_cols = list(range(0, m - block_size+1))
        random.shuffle(available_cols)
        for col_index in available_cols:
            if placed_blocks >= max_possible_blocks:
                break
            if all(grid[r][c] == ' ' for r in range(row_index, row_index + block_size)):
                for r in range(row_index, row_index + block_size):
                    for c in range(col_index, col_index + block_size):
                        grid[r][c] = block_color
                placed_blocks += 1
                color_counts[block_color] -= (block_size ** 2)

    if placed_blocks < block_count:
        print(f"Warning: Could only place {placed_blocks} out of {block_count} blocks")
```

```

empty_positions = [(r, c) for r in range(n) for c in range(m) if grid[r][c] == 0]
random.shuffle(empty_positions)

for r, c in empty_positions:
    available_colors = [color for color in color_counts if color_counts[color] > 0]
    if available_colors:
        chosen_color = random.choice(available_colors)
        grid[r][c] = chosen_color
        color_counts[chosen_color] -= 1

return grid

```



```

row = int(input("Enter number of rows: "))
col = int(input("Enter number of columns: "))
red = int(input("Enter number of red cells: "))
green = int(input("Enter number of green cells: "))
blue = int(input("Enter number of blue cells: "))
block_color = input("Enter block color (R/G/B): ").strip().upper()
block_size = int(input("Enter block size: "))
block_count = int(input("Enter number of blocks: "))
grid = generate_grid(row, col, red, green, blue, block_color, block_size, block_count)
print("\nGenerated Grid:")
for row in grid:
    print(" ".join(row))

```

```

Enter number of rows: 8
Enter number of columns: 8
Enter number of red cells: 30
Enter number of green cells: 20
Enter number of blue cells: 14
Enter block color (R/G/B): R
Enter block size: 3
Enter number of blocks: 3

```

```

Generated Grid:
G B B B R G G B
G R R R R R R R
B R R R R R R R G
B R R R R R R R B
B G B G B G B G
R R R G B G B G
R R R G B R G G
R R R G G G G G

```

We aim to place non-overlapping $s \times s$ blocks of a specific color (block_color), as many as possible (up to the user-defined block_count and bounded by available color cells).

Remaining cells are filled with the leftover colored cells.

Pattern constraint

- $x_{i,j}^c \in \{0, 1\}$: 1 if cell (i, j) is assigned color $c \in \{R, G, B\}$, 0 otherwise.
- $y_{r,s} \in \{0, 1\}$: 1 if the pattern is placed starting at row r , column s (left-aligned), 0 otherwise.

Here:

- $i \in \{1, \dots, \text{row}\}, j \in \{1, \dots, \text{col}\}$
- $r \in \{1, \dots, \text{row}\}, s \in \{1, \dots, \text{col} - \ell + 1\}$
- $\ell = \text{pattern_length}$

Let the pattern be a string of length ℓ : $P = [p_1, p_2, \dots, p_\ell]$ where $p_t \in \{R, G, B\}$

Pattern enforcement:

If a pattern is placed starting at (r, s) , then each cell $(r, s + t - 1)$ must match p_t .

$$x_{r,s+t-1}^{p_t} \geq y_{r,s} \quad \forall r, s \in [1, \text{col} - \ell + 1], \forall t \in [1, \ell]$$

Non overlapping pattern:

Each cell can be part of at most one pattern:

$$\sum_{\substack{r'=i \\ s' \leq j \leq s'+\ell-1}} y_{r',s'} \leq 1 \quad \forall i, j$$

Where (i, j) is involved in a pattern placed at row $r' = i$, starting at column $s' \leq j \leq s' + \ell - 1$.

Objective function:

$$\max \sum_{r=1}^{\text{row}} \sum_{s=1}^{\text{col}-\ell+1} y_{r,s}$$

```
In [ ]: import random

def generate_grid(row, col, red, green, blue, pattern_length, pattern):
    if row * col != red + green + blue:
        return "Error: The total number of colored cells does not match the grid"
```

```

if pattern_length > col:
    return "\nX This configuration cannot be possible! The pattern length <
if len(pattern) != pattern_length:
    return "\nX Invalid Pattern: The specified pattern length does not match <
def max_patterns_count(color_counts, pattern):
    pattern_color_counts = {color: pattern.count(color) for color in set(pat <
        return min(
            (color_counts.get(color, 0) // count if count > 0 else float('inf')) <
                for color, count in pattern_color_counts.items() <
            )
def fill_remaining(grid, color_counts):
    empty_positions = [(r, c) for r in range(row) for c in range(col) if grid[r][c] == <
        random.shuffle(empty_positions)
        for r, c in empty_positions:
            available_colors = [color for color in color_counts if color_counts[c] >= 1]
            if available_colors:
                chosen_color = random.choice(available_colors)
                grid[r][c] = chosen_color
                color_counts[chosen_color] -= 1
def fill_with_patterns(grid, pattern, max_patterns_count):
    applied_patterns = 0
    row_positions = list(range(0, row))
    random.shuffle(row_positions)

    for r in row_positions:
        if applied_patterns >= max_patterns_count:
            break
        col_positions = list(range(0, col - pattern_length + 1))
        random.shuffle(col_positions)
        for start_col in col_positions:
            end_col = start_col + pattern_length
            if applied_patterns >= max_patterns_count:
                break
            if all(grid[r][c] == ' ' for c in range(start_col, end_col)):
                for i, color in enumerate(pattern):
                    grid[r][start_col + i] = color
                    color_counts[color] -= 1
                applied_patterns += 1
    return applied_patterns
grid = [[' ' for _ in range(col)] for _ in range(row)]
color_counts = {'R': red, 'G': green, 'B': blue}

if max_patterns_count(color_counts, pattern) == 0:
    return "\nX This configuration cannot be possible!", 0

patterns_applied = fill_with_patterns(grid, pattern, max_patterns_count(col, row))
fill_remaining(grid, color_counts)

print(f"\nTotal patterns applied: {patterns_applied}")

return grid
row = int(input("Enter number of rows: "))
col = int(input("Enter number of columns: "))
red = int(input("Enter number of red cells: "))
green = int(input("Enter number of green cells: "))
blue = int(input("Enter number of blue cells: "))
pattern_length = int(input("Enter the number of tiles for the pattern: "))
pattern = input(f"Enter the pattern of {pattern_length} tiles (e.g., RRGB): ")
grid = generate_grid(row, col, red, green, blue, pattern_length, pattern)
print("\nGenerated Grid:")

```

```
for row in grid:  
    print(" ".join(row))
```

```
Enter number of rows: 8  
Enter number of columns: 8  
Enter number of red cells: 30  
Enter number of green cells: 20  
Enter number of blue cells: 14  
Enter the number of tiles for the pattern: 3  
Enter the pattern of 3 tiles (e.g., RRGB): RGR
```

```
Total patterns applied: 15
```

```
Generated Grid:
```

```
G R G R R G R B  
G B R G R R G R  
R G R B B R G R  
B B R G R R G R  
R G R B B R G R  
B R G R G B B B  
R G R R G R G G  
B R G R R G R B
```

We aim to place as many non-overlapping row-wise patterns of given length and color sequence.

Each pattern must be fully embedded in a single row and must not overlap with others.

Any leftover color cells are distributed randomly (in code), but here they are accounted for exactly via total color constraints.

Multiple Constraints

Now we apply multiple constraints at the same time.

Weighted Positional + Adjacency Constraint

we defined weights in all 4 quadrants for all colours in case of even dimension

$$w_{\text{red_UL}} + w_{\text{red_UR}} + w_{\text{red_LL}} + w_{\text{red_LR}} = 1$$

and in case of odd dimension we also defined weights on axis and origin.

there were weights for neighbouring elements (i.e. Adjacency weights for the pairs)

Let the grid be of size $R \times C$. Define:

$$x_{i,j}^c = \begin{cases} 1 & \text{if cell } (i,j) \text{ is assigned color } c \in \{R, G, B\} \\ 0 & \text{otherwise} \end{cases}$$

Where $i \in [1, R]$, $j \in [1, C]$, and $c \in \{R, G, B\}$

We aim to **maximize** the total score made of:

1. Quadrant weights
2. Extra position weights
3. Adjacency weights

Let's define:

- $x_{i,j}^c \in \{0, 1\}$: 1 if cell (i, j) is color $c \in \{R, G, B\}$
- $qw_{i,j}^c$: quadrant weight for color c at cell (i, j)
- $ew_{i,j}^c$: extra position weight for color c at (i, j)
- $y_{(i,j),(i',j')}^{c_1,c_2} \in \{0, 1\}$: 1 if adjacent (i, j) and (i', j') are colored c_1 and c_2
- $adjw_{c_1,c_2}$: adjacency weight between colors c_1 and c_2

Objective Function Expression:

$$Z = \underbrace{\sum_{i=1}^R \sum_{j=1}^C \sum_c (qw_{i,j}^c + ew_{i,j}^c) \cdot x_{i,j}^c}_{\text{Quadrant + Extra Weights}} + \underbrace{\sum_{\substack{(i,j) \in \text{Grid} \\ (i',j') \in \text{Adj}(i,j)}} \sum_{c_1 \neq c_2} adjw_{c_1,c_2} \cdot y_{(i,j),(i',j')}^{c_1,c_2}}_{\text{Adjacency Preference}}$$

then we sample the feasible grids that maximize the above objective function.

```
In [15]: import itertools
import random

def generate_color_grid(rows, cols, red, green, blue, red_weights, green_weights,
                       red_extra_weights, green_extra_weights, blue_extra_weights):
    # Validate grid dimensions
    if not (isinstance(rows, int) and isinstance(cols, int) and rows > 0 and cols > 0):
        raise ValueError("Rows and columns must be positive integers")

    # Validate cell counts
    if rows * cols != len(red) or rows * cols != len(green) or rows * cols != len(blue):
        raise ValueError("Number of cells in grid does not match the number of cells in each color list")
```

```

if not (isinstance(red, int) and isinstance(green, int) and isinstance(blue,
    raise ValueError("Cell counts must be integers"))
if red < 0 or green < 0 or blue < 0:
    raise ValueError("Cell counts cannot be negative")
if red + green + blue != rows * cols:
    raise ValueError("Sum of cell counts must equal total grid size")

# Validate weight lists
for weights, name in [(red_weights, "Red"), (green_weights, "Green"), (blue_
    if not (isinstance(weights, list) and len(weights) == 4 and abs(sum(weig
        raise ValueError(f"{name} quadrant weights must be 4 numbers summing

for weights, name in [(red_extra_weights, "Red"), (green_extra_weights, "Gre
    if not (isinstance(weights, list) and len(weights) == 5 and abs(sum(weig
        raise ValueError(f"{name} extra weights must be 5 numbers summing to

# Validate adjacency weights
if not isinstance(adj_weights, dict) or not all(c in adj_weights for c in 'R
    raise ValueError("Adjacency weights must be a dictionary with R, G, B ke
weight_sum = adj_weights['R'].get('G', 0) + adj_weights['R'].get('B', 0) + a
if abs(weight_sum - 10) > 1e-6:
    raise ValueError("Adjacency weights must sum to 10")

def generate_empty_grid(rows, cols):
    return [[ '-' for _ in range(cols)] for _ in range(rows)]
```

def count_neighbors(grid, row, col, rows, cols):

```

neighbors = {'R': 0, 'G': 0, 'B': 0}
for dr, dc in [(-1, 0), (1, 0), (0, -1), (0, 1)]:
    nr, nc = row + dr, col + dc
    if 0 <= nr < rows and 0 <= nc < cols and grid[nr][nc] in neighbors:
        neighbors[grid[nr][nc]] += 1
return neighbors
```

def fill_quadrant(grid, quadrant, color_counts, color_weights, adj_weights,

```

quadrant_map = {
    'UL': (0, 0, rows // 2, cols // 2),
    'UR': (0, cols // 2, rows // 2, cols),
    'LL': (rows // 2, 0, rows, cols // 2),
    'LR': (rows // 2, cols // 2, rows, cols)
}

r_start, c_start, r_end, c_end = quadrant_map[quadrant]
positions = [(r, c) for r in range(r_start, r_end) for c in range(c_star
random.shuffle(positions)

for r, c in positions:
    neighbor_counts = count_neighbors(grid, r, c, rows, cols)
    scores = {
        color: color_weights[color][list(quadrant_map.keys()).index(quadrant)]
            sum(neighbor_counts[n] * adj_weights[color].get(n, 0) for n
        for color in 'RGB'
    }

    sorted_colors = sorted(scores.keys(), key=lambda c: scores[c], rever
```

for best_color in sorted_colors:

```

        if color_counts[best_color] > 0:
            grid[r][c] = best_color
            color_counts[best_color] -= 1
```

```

        score[0] += scores[best_color]
        break

def fill_extra_cells(grid, color_counts, extra_weights, adj_weights, rows, cols):
    extra_positions = []
    extra_weight_map = {
        "+x": 0, "-x": 1, "+y": 2, "-y": 3, "0": 4
    }

    if rows % 2 == 1:
        extra_positions += [(rows // 2, c, "+x") for c in range(cols // 2 + 1, cols)]
        extra_positions += [(rows // 2, c, "-x") for c in range(0, cols // 2 + 1)]

    if cols % 2 == 1:
        extra_positions += [(r, cols // 2, "+y") for r in range(0, rows // 2 + 1)]
        extra_positions += [(r, cols // 2, "-y") for r in range(rows // 2 + 1, rows)]

    if rows % 2 == 1 and cols % 2 == 1:
        extra_positions.append((rows // 2, cols // 2, "0"))

    random.shuffle(extra_positions)

    for r, c, pos_type in extra_positions:
        neighbor_counts = count_neighbors(grid, r, c, rows, cols)
        weight_index = extra_weight_map[pos_type]

        scores = {
            color: extra_weights[color][weight_index] +
            sum(neighbor_counts[n] * adj_weights[color].get(n, 0) for n in range(8))
            for color in 'RGB'
        }

        sorted_colors = sorted(scores.keys(), key=lambda c: scores[c], reverse=True)

        for best_color in sorted_colors:
            if color_counts[best_color] > 0:
                grid[r][c] = best_color
                color_counts[best_color] -= 1
                score[0] += scores[best_color]
                break

    best_grids = []
    quadrant_orders = list(itertools.permutations(['UL', 'UR', 'LR', 'LL']))[:24]

    for quadrant_order in quadrant_orders:
        grid = generate_empty_grid(rows, cols)
        color_counts = {'R': red, 'G': green, 'B': blue}
        color_weights = {'R': red_weights, 'G': green_weights, 'B': blue_weights}
        extra_weights = {'R': red_extra_weights, 'G': green_extra_weights, 'B': blue_extra_weights}
        score = [0]

        for quadrant in quadrant_order:
            fill_quadrant(grid, quadrant, color_counts, color_weights, adj_weights)

        fill_extra_cells(grid, color_counts, extra_weights, adj_weights, rows, cols)

        best_grids.append((score[0], grid))

    best_grids.sort(reverse=True, key=lambda x: x[0])
    return best_grids[0][1] # Return the best grid

```

```

# Custom input collection
try:
    rows = int(input("Enter the number of rows: "))
    cols = int(input("Enter the number of columns: "))

    red = int(input("Enter the number of red cells: "))
    green = int(input("Enter the number of green cells: "))
    blue = int(input("Enter the number of blue cells: "))

    red_weights = list(map(float, input("Enter the quadrant weights for Red (UL UR LR LL): ").split()))
    green_weights = list(map(float, input("Enter the quadrant weights for Green (UL UR LR LL): ").split()))
    blue_weights = list(map(float, input("Enter the quadrant weights for Blue (UL UR LR LL): ").split()))

    red_extra_weights = list(map(float, input("Enter extra weights for Red (+x -x +y -y Origin): ").split()))
    green_extra_weights = list(map(float, input("Enter extra weights for Green (+x -x +y -y Origin): ").split()))
    blue_extra_weights = list(map(float, input("Enter extra weights for Blue (+x -x +y -y Origin): ").split()))

    adj_weights_input = list(map(float, input("Enter adjacency weights for Red-Green, Red-Blue, Green-Blue: ").split()))
    adj_weights = {
        'R': {'G': adj_weights_input[0], 'B': adj_weights_input[1]},
        'G': {'R': adj_weights_input[0], 'B': adj_weights_input[2]},
        'B': {'R': adj_weights_input[1], 'G': adj_weights_input[2]}
    }

    # Generate and print grid
    grid = generate_color_grid(rows, cols, red, green, blue,
                               red_weights, green_weights, blue_weights,
                               red_extra_weights, green_extra_weights, blue_extra_weights,
                               adj_weights)

    for row in grid:
        print(" ".join(row))

except ValueError as e:
    print(f"Error: {e}")
except IndexError:
    print("Error: Incorrect number of weights provided")
except Exception as e:
    print(f"Error: An unexpected error occurred - {e}")

```

Enter the number of rows: 8
Enter the number of columns: 8
Enter the number of red cells: 23
Enter the number of green cells: 32
Enter the number of blue cells: 9
Enter the quadrant weights for Red (UL UR LR LL): 1 3 1 5
Enter the quadrant weights for Green (UL UR LR LL): 6 3 1 0
Enter the quadrant weights for Blue (UL UR LR LL): 2 3 1 4
Enter extra weights for Red (+x -x +y -y Origin): 2 3 1 1 3
Enter extra weights for Green (+x -x +y -y Origin): 4 3 1 1 1
Enter extra weights for Blue (+x -x +y -y Origin): 0 0 0 1 9
Enter adjacency weights for Red-Green, Red-Blue, Green-Blue: 8 1 1

R G R G B G B G
G R G R G B G B
R G R G B B B G
G R G R G G B G
R G R G R R G R
G R G R G G G R G
R G R G R R G B
G R G R G G R G

Adjacency + Periphery + Diagonal

Define binary variables

$$1. x_{i,j}^R, x_{i,j}^G$$

These are **binary decision variables** representing the color of the cell at position (i, j) :

- $x_{i,j}^R = 1$: cell (i, j) is assigned color **Red**
- $x_{i,j}^G = 1$: cell (i, j) is assigned color **Green**

$$2. y_{i,j}^B$$

- Binary variable indicating whether the **Blue** tile is on the **periphery** at position (i, j) :

- $y_{i,j}^B = 1$ if position $(i, j) \in$ periphery is assigned a Blue tile.

$$3. z_{i,j}^G$$

- Binary variable indicating whether a **Green** tile is placed on the **diagonal**:

- $z_{i,j}^G = 1$ if position $(i, j) \in$ diagonal has a Green tile.

Boltzmann Objective function

the weights are taken equal by default but they can be adjusted as per user requirement. if user wants to prioritize between the constraints

$$w_1 \sum_{i,j \in \text{valid}} I(x_{i,j}^R = 1) \cdot I(x_{i,j+1}^G + x_{i,j-1}^G + x_{i-1,j}^G + x_{i+1,j}^G \geq 1) + w_2 \sum_{i,j \in \text{valid}} I(x_{i,j}^G = 1) \cdot I(x_{i,j+1}^R + x_{i,j-1}^R + x_{i-1,j}^R + x_{i+1,j}^R \geq 1) + w_3 \sum_{(i,j) \in \text{periphery}} I(y_{i,j}^B = 1) + w_4 \sum_{(i,j) \in \text{diagonal}} I(z_{i,j}^G = 1)$$

Here $I()$ is Indicator function:

$$\mathbf{I}(\text{condition}) = \begin{cases} 1 & \text{if the condition is true} \\ 0 & \text{otherwise} \end{cases}$$

w1: weight for placing a Red tile adjacent to at least one Green tile

w2 : weight for placing a Green tile adjacent to at least one Red tile

w3: reward for placing a Blue tile on the periphery

w4: reward for placing a Green tile on the diagonal

more simplified form for understanding:

$$\boxed{\exp \left(w_1 \sum_{i,j \in \text{valid}} I(x_{i,j}^R = 1) \cdot I \left(\sum_{\text{neighbors}} x_{\cdot,j}^G \geq 1 \right) + w_2 \sum_{i,j \in \text{valid}} I(x_{i,j}^G = 1) \cdot I \left(\sum_{\text{neighbors}} x_{\cdot,j}^R \geq 1 \right) + w_3 \sum_{(i,j) \in \text{periphery}} I(y_{i,j}^B = 1) + w_4 \sum_{(i,j) \in \text{diagonal}} I(z_{i,j}^G = 1) \right)}$$

E= energy function is the negative of the above function = negative of sum of weighted indicator terms

we sample in such a way that energy function is minimized so that acceptance prob is more

acceptance probability of a color swap from state S to S '

$$P(\text{accept}) = \min \left(1, \exp \left(-\frac{\Delta E}{T} \right) \right)$$

```
In [16]: import random

def generate_grid(dimension, red, green, blue, periphery_colors, diagonal_colors):
    if dimension < 2:
        raise ValueError("Dimension must be at least 2")
    if any(c < 0 for c in (red, green, blue)):
        raise ValueError("Tile counts must be non-negative")

    def initialize_grid_and_counts():
        """Initialize an empty grid and color counts dictionary"""
        return [[None] * dimension for _ in range(dimension)], {'R': red, 'G': green, 'B': blue}

    def get_positions():
        """Get periphery and diagonal positions"""
        periphery = [(0, j) for j in range(dimension)] + \
                    [(i, dimension - 1) for i in range(1, dimension)] + \
                    [(dimension - 1, j) for j in range(dimension - 2, -1, -1)] + \
                    [(i, 0) for i in range(dimension - 2, 0, -1)]
        diagonal = list(dict.fromkeys([(i, i) for i in range(dimension)] +
                                      [(i, dimension - 1 - i) for i in range(dimension - 1, 0, -1)]))
        return periphery, diagonal

    grid, colors = initialize_grid_and_counts()
    for i in range(dimension):
        for j in range(dimension):
            if (i, j) in periphery or (i, j) in diagonal:
                colors['B'] += 1
            else:
                colors['R'] += 1
                colors['G'] += 1
    return grid, colors
```

```

def fill_periphery(grid, periphery_positions, color_counts):
    """Fill periphery with available colors, prioritizing periphery_colors"""
    random.shuffle(periphery_positions)
    for i, j in periphery_positions:
        if grid[i][j] is None and periphery_colors and color_counts[periphery_colors[0]] > 0:
            grid[i][j] = periphery_colors[0]
            color_counts[periphery_colors[0]] -= 1

def fill_diagonal(grid, diagonal_positions, color_counts):
    """Fill diagonal with available B tiles"""
    random.shuffle(diagonal_positions)
    for i, j in diagonal_positions:
        if grid[i][j] is None and diagonal_colors and color_counts[diagonal_colors[0]] > 0:
            grid[i][j] = diagonal_colors[0]
            color_counts[diagonal_colors[0]] -= 1

def ensure_adjacency(grid, color_counts, tile1, tile2):
    """Ensure tile1 and tile2 are adjacent where possible"""
    empty_positions = [(i, j) for i in range(dimension) for j in range(dimension)]
    random.shuffle(empty_positions)
    pairs_needed = min(color_counts[tile1], color_counts[tile2])

    for _ in range(pairs_needed):
        for i, j in empty_positions[:]:
            if color_counts[tile1] > 0:
                grid[i][j] = tile1
                color_counts[tile1] -= 1
                empty_positions.remove((i, j))
                adjacent_positions = [(i-1, j), (i+1, j), (i, j-1), (i, j+1)]
                random.shuffle(adjacent_positions)
                for ai, aj in adjacent_positions:
                    if (0 <= ai < dimension and 0 <= aj < dimension and
                        grid[ai][aj] is None and color_counts[tile2] > 0):
                        grid[ai][aj] = tile2
                        color_counts[tile2] -= 1
                        empty_positions.remove((ai, aj))
                        break
                break
            break

def fill_remaining(grid, color_counts):
    """Fill remaining positions with exact counts"""
    empty_positions = [(i, j) for i in range(dimension) for j in range(dimension)]
    remaining_tiles = [tile for tile, count in color_counts.items() for _ in range(count)]
    random.shuffle(remaining_tiles)

    valid_colors = ['R', 'G', 'B']
    while len(remaining_tiles) < len(empty_positions):
        remaining_tiles.append(random.choice(valid_colors))

    for (i, j), tile in zip(empty_positions, remaining_tiles[:len(empty_positions)]):
        grid[i][j] = tile

    # Validate constraints
    total_cells = dimension * dimension
    valid_colors = {'R', 'G', 'B'}
    if not all(c in valid_colors for c in periphery_colors + diagonal_colors + adjacent_tiles):
        raise ValueError("Invalid colors detected. Use only R, G, B.")
    if len(adjacent_tiles) != 2:
        raise ValueError("Exactly 2 adjacent tiles required.")
    if red + green + blue != total_cells:

```

```

        raise ValueError(f"Color counts ({red + green + blue}) don't match grid

# Generate one grid with randomization for variety
grid, color_counts = initialize_grid_and_counts()
periphery_positions, diagonal_positions = get_positions()

fill_periphery(grid, periphery_positions, color_counts)
fill_diagonal(grid, diagonal_positions, color_counts)
ensure_adjacency(grid, color_counts, adjacent_tiles[0], adjacent_tiles[1])
fill_remaining(grid, color_counts)

return grid # Single return point for the grid

# Take input outside the function
dimension = int(input("Enter the dimension of the square grid: "))
red = int(input("Enter the number of red tiles: "))
green = int(input("Enter the number of green tiles: "))
blue = int(input("Enter the number of blue tiles: "))
periphery_colors = input("Enter periphery colors (e.g., R G B): ").strip().upper()
diagonal_colors = input("Enter diagonal colors (e.g., R G B): ").strip().upper()
adjacent_tiles = input("Enter two tiles for adjacency (e.g., R G): ").strip().upper()

# Generate and print one sample grid
grid = generate_grid(dimension, red, green, blue, periphery_colors, diagonal_colors)
for row in grid:
    print(" ".join(row))

```

Enter the dimension of the square grid: 8
Enter the number of red tiles: 23
Enter the number of green tiles: 32
Enter the number of blue tiles: 9
Enter periphery colors (e.g., R G B): B
Enter diagonal colors (e.g., R G B): G
Enter two tiles for adjacency (e.g., R G): R G
G G R G B B B G
R G R R G R G R
R G G R G G R G
B R G G G R G R
R G R G G G G R
G G G G R G R B
R G R G G R G B
G R G B R B R B

Observe:

Red tiles with adjacent Green tiles

Green tiles with adjacent Red tiles

Blue tiles on the border

Green tiles on diagonals

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Periphery and Diagonal (Priority) Constraint

- P : Set of periphery positions
- D : Set of diagonal positions
- $C_P \subseteq \{R, G, B\}$: List of **preferred colors** for periphery
- $C_D \subseteq \{R, G, B\}$: List of **preferred colors** for diagonal

Let:

$$\delta_c = \begin{cases} 1 & \text{if } c \text{ is a preferred color for that region} \\ 0 & \text{otherwise} \end{cases}$$

To **encourage** preferred colors (as soft constraints), you **maximize**:

$$\sum_{(i,j) \in P} \sum_{c \in C_P} \delta_c \cdot x_{i,j}^c + \sum_{(i,j) \in D} \sum_{c \in C_D} \delta_c \cdot x_{i,j}^c$$

For cell (i, j) , define its (up to 4) adjacent neighbors as:

$$\mathcal{N}(i, j) = \{(i+1, j), (i-1, j), (i, j+1), (i, j-1)\} \quad (\text{within grid bounds})$$

4 above means: up, down, left, right.

Objective Function

$$w_1 \sum_{i,j} \mathbf{I}(x_{i,j}^R = 1) \cdot \mathbf{I} \left(\sum_{(p,q) \in \mathcal{N}(i,j)} x_{p,q}^G \geq 1 \right) + w_2 \sum_{i,j} \mathbf{I}(x_{i,j}^G = 1) \cdot \mathbf{I} \left(\sum_{(p,q) \in \mathcal{N}(i,j)} x_{p,q}^R \geq 1 \right) + w_3 \sum_{(i,j) \in \text{periphery}} \mathbf{I}(x_{i,j}^B = 1) + w_4 \sum_{(i,j) \in \text{diagonal}} \mathbf{I}(x_{i,j}^G = 1)$$

```
In [17]: import random

def fill_grid_combined(dimension, red, green, blue, periphery_colors, diagonal_c
    def initialize_grid():
        return [[None] * dimension for _ in range(dimension)]
```

```

def get_positions():
    periphery = [(0, j) for j in range(dimension)] + \
                [(i, dimension - 1) for i in range(1, dimension)] + \
                [(dimension - 1, j) for j in range(dimension - 2, -1, -1)] + \
                [(i, 0) for i in range(dimension - 2, 0, -1)]
    diagonal = list(set([(i, i) for i in range(dimension)]) + [(i, dimension - i) for i in range(1, dimension)])
    random.shuffle(periphery)
    random.shuffle(diagonal)
    return periphery, diagonal

def assign_priority():
    return [(diagonal_positions, diagonal_colors), (periphery_positions, periphery_colors)]

def fill_positions(priority_order):
    for positions, color_priority in priority_order:
        if not color_priority:
            continue
        for i, j in positions:
            if grid[i][j] is not None:
                continue
            assigned = False
            for color in color_priority:
                if color_counts[color] > 0:
                    grid[i][j] = color
                    color_counts[color] -= 1
                    assigned = True
                    break
            if not assigned:
                for color in 'RGB':
                    if color_counts[color] > 0:
                        grid[i][j] = color
                        color_counts[color] -= 1
                        break

def fill_remaining():
    empty_positions = [(i, j) for i in range(dimension) for j in range(dimension) if grid[i][j] is None]
    random.shuffle(empty_positions)
    available_colors = ['R'] * color_counts['R'] + ['G'] * color_counts['G'] + ['B'] * color_counts['B']
    random.shuffle(available_colors)

    for (i, j), color in zip(empty_positions, available_colors):
        grid[i][j] = color

    if red + green + blue != dimension * dimension:
        raise ValueError(f"Error: The sum of tiles must equal {dimension * dimension} but got {red + green + blue}.")


grid = initialize_grid()
color_counts = {'R': red, 'G': green, 'B': blue}
periphery_positions, diagonal_positions = get_positions()
priority_order = assign_priority()
fill_positions(priority_order)
fill_remaining()
return grid

# Taking input outside the function
dimension = int(input("Enter the dimension of the square grid: "))
red = int(input("Enter the number of red tiles: "))
green = int(input("Enter the number of green tiles: "))
blue = int(input("Enter the number of blue tiles: "))

```

```

# Default constraints applied
periphery_colors = list(dict.fromkeys(input("Enter periphery colors (R G B): ")).st
diagonal_colors = list(dict.fromkeys(input("Enter diagonal colors (R G B): ")).st
constraint_priority = input("Enter constraint priority (diagonal/periphery): ").

# Generating and printing the grid
grid = fill_grid_combined(dimension, red, green, blue, periphery_colors, diagonal
for row in grid:
    print(" ".join(row))

```

Enter the dimension of the square grid: 8
 Enter the number of red tiles: 23
 Enter the number of green tiles: 32
 Enter the number of blue tiles: 9
 Enter periphery colors (R G B): G
 Enter diagonal colors (R G B): R
 Enter constraint priority (diagonal/periphery): diagonal
 R G G G G G G R
 G R B G R B R G
 G B R B B R B G
 G R R R R G R G
 G R G R R G R G
 G G R G B R B G
 G R G G B R R G
 R G G G G G G R

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Periphery & Big Color Block Constraint

input : user specific constraints on colored cells of periphery placement, and block placement.

The periphery consists of the top row, bottom row, leftmost column (excluding corners already counted in rows), and rightmost column (excluding corners).

$$\text{Periphery positions} = \text{col} + \text{col} + (\text{row} - 2) + (\text{row} - 2) = 2 \times \text{col} + 2 \times (\text{row} - 2)$$

Block Placement Constraints

$$\text{Block area} = \text{block_size}^2$$

$$\boxed{\text{Block Occupancy} = \text{Block Size}^2}$$

$$\boxed{\text{Max Placeable Blocks} = \min \left(\text{Requested Blocks}, \left\lfloor \frac{\text{Available Color Cells}}{\text{Block Occupancy}} \right\rfloor \right)}$$

Number of possible starting positions:

$$\text{Starting Positions} = (\text{row} - \text{block_size} + 1) \times (\text{col} - \text{block_size} + 1)$$

After periphery and block placements, remaining empty cells are filled with colors (R, G, B) based on remaining color_counts.

$$\text{Empty Cells} = \text{row} \times \text{col} - (\text{Periphery Cells Assigned} + \text{Block Cells Assigned})$$

$$\text{red}_r + \text{green}_r + \text{blue}_r = \text{Empty Cells}$$

For each cell assigned a color (periphery, block, or remaining), the corresponding color count is decremented:

$$\text{color_counts}[color] = \text{color_counts}[color] - 1$$

For each block placed:

$$\text{color_counts}[block_color] = \text{color_counts}[block_color] - \text{block_size}^2$$

Objective Function

$$\text{Maximize } \sum_{i,j,c} w_{i,j,c} \cdot x_{i,j,c} + W_b \cdot \sum_{i,j} b_{i,j}$$

```
In [ ]: import random

def generate_grid(row, col, red, green, blue, periphery_order, block_color, block_count):
    if row * col != red + green + blue:
        print("Error: The total number of colored cells does not match the grid")
        return []

    grid = [[' ' for _ in range(col)] for _ in range(row)]
    color_counts = {'R': red, 'G': green, 'B': blue}
    positions = [(0, i) for i in range(col)] + [(row - 1, i) for i in range(col)]
    random.shuffle(positions)

    for color in periphery_order:
        for x, y in positions:
            if grid[x][y] == ' ' and color_counts.get(color, 0) > 0:
                grid[x][y] = color
                color_counts[color] -= 1

    max_possible_blocks = min(block_count, color_counts.get(block_color, 0)) // (
    placed_blocks = 0
    available_rows = list(range(1, row - block_size))
    random.shuffle(available_rows)

    for row_index in available_rows:
        if placed_blocks >= max_possible_blocks:
            break
        available_cols = list(range(1, col - block_size))
        random.shuffle(available_cols)
        for col_index in available_cols:
            if placed_blocks >= max_possible_blocks:
                break
            if all(grid[r][c] == ' ' for r in range(row_index, row_index + block_size)):
                for r in range(row_index, row_index + block_size):
                    for c in range(col_index, col_index + block_size):
                        grid[r][c] = block_color
                placed_blocks += 1
                color_counts[block_color] -= (block_size ** 2)

    if placed_blocks < block_count:
        print(f"Warning: Could only place {placed_blocks} out of {block_count} blocks")

    empty_positions = [(r, c) for r in range(row) for c in range(col) if grid[r][c] == ' ']
    random.shuffle(empty_positions)

    for r, c in empty_positions:
        available_colors = [color for color in color_counts if color_counts[color] > 0]
        if available_colors:
            chosen_color = random.choice(available_colors)
            grid[r][c] = chosen_color
```

```
color_counts[chosen_color] -= 1

return grid

row = int(input("Enter number of rows: "))
col = int(input("Enter number of columns: "))
red = int(input("Enter number of red cells: "))
green = int(input("Enter number of green cells: "))
blue = int(input("Enter number of blue cells: "))
periphery_order = input("Enter the order of periphery colors (space-separated, e.g., R G B): ")
block_color = input("Enter block color (R/G/B): ").strip().upper()
block_size = int(input("Enter block size: "))
block_count = int(input("Enter number of blocks: "))

result_grid = generate_grid(row, col, red, green, blue, periphery_order, block_color)
for r in result_grid:
    print(' '.join(r))
```

```
Enter number of rows: 8
Enter number of columns: 8
Enter number of red cells: 20
Enter number of green cells: 25
Enter number of blue cells: 19
Enter the order of periphery colors (space-separated, e.g., G R B): R
Enter block color (R/G/B): G
Enter block size: 3
Enter number of blocks: 2
G G R R R B G R
R G G G G G G R
B G G G G G G R
B G G G G G G R
R B B B B G B B
R B B B B B B R
R G B G G B B R
R R R R B R R R
```

Periphery & Non-adjacency Constraint

Periphery Cell Count

$$\text{Periphery Cells} = 2 \times (\text{col} + \text{row} - 2)$$

Adjacency Constraint (Non-Touching Rule)

The non-adjacency constraint ensures that when a color is placed at a grid position (x, y), none of its four adjacent cells (right, down, left, up) have the same color.

$(\Delta x, \Delta y) \in \{(0, 1), (1, 0), (0, -1), (-1, 0)\}$: Represents the four directions (right, down, left, up).

$(x + \Delta x, y + \Delta y) \notin [0, \text{row}] \times [0, \text{col}]$: Accounts for boundary conditions.

Priority order set:

$$\mathcal{P} = \{(0, j), (\text{row} - 1, j), (i, 0), (i, \text{col} - 1)\}$$

Objective Function:

$$\bullet \quad w_{i,j}^c = \begin{cases} 3 & \text{if } (i, j) \in \mathcal{P} \text{ and } c = c_1 \\ 2 & \text{if } (i, j) \in \mathcal{P} \text{ and } c = c_2 \\ 1 & \text{if } (i, j) \in \mathcal{P} \text{ and } c = c_3 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Maximize } \sum_{i,j,c} w_{i,j}^c \cdot x_{i,j}^c$$

for getting the optimal initial sample (that maximise above objective function) to reduce the time complexity and burn in samples for sampling method:

we prioritize placing colors on the grid's periphery in a user defined specified order, then fill the remaining cells while enforcing constraint that no two adjacent cells (horizontally or vertically) share the same color during the main filling phase, using a randomized approach.

```
In [ ]: import random

def generate_grid(row, col, red, green, blue, periphery_order):
    """Generates a valid grid based on user input."""

    def is_valid(grid, x, y, color):
        """Check if placing color at (x, y) does not touch the same color."""
        directions = [(0, 1), (1, 0), (0, -1), (-1, 0)] # Right, Down, Left, Up
        return all(
            not (0 <= x + dx < len(grid) and 0 <= y + dy < len(grid[0])) and grid[x+dx][y+dy] != color
            for dx, dy in directions
        )

    def fill_grid(grid, color_counts):
        """Fills the grid with colors based on given constraints."""
        positions = [(0, i) for i in range(col)] + [(row - 1, i) for i in range(col)]
        [(i, 0) for i in range(1, row - 1)] + [(i, col - 1) for i in range(1, row - 1)]

        random.shuffle(positions)

        # Place colors in periphery first
        for color in periphery_order:
            for x, y in positions:
                if grid[x][y] == ' ' and color_counts[color] > 0:
                    grid[x][y] = color
                    color_counts[color] -= 1

        # Fill remaining positions
        empty_positions = [(i, j) for i in range(row) for j in range(col) if grid[i][j] == ' ']
        random.shuffle(empty_positions)

        for x, y in empty_positions:
            for color in random.sample(['R', 'G', 'B'], 3):
                if color_counts[color] > 0 and is_valid(grid, x, y, color):
                    grid[x][y] = color
                    color_counts[color] -= 1
                    break

        # Final pass to fill any remaining empty spaces
        empty_positions = [(i, j) for i in range(row) for j in range(col) if grid[i][j] == ' ']
        random.shuffle(empty_positions)

        for x, y in empty_positions:
            for color in random.sample(['R', 'G', 'B'], 3):
                if color_counts[color] > 0:
                    grid[x][y] = color
                    color_counts[color] -= 1
                    break

    if row * col != red + green + blue:
        print("Error: The total number of colored cells does not match the grid")
    return None
```

```

grid = [[' ' for _ in range(col)] for _ in range(row)]
color_counts = {'R': red, 'G': green, 'B': blue}

fill_grid(grid, color_counts)

return grid

# User Input
row = int(input("Enter number of rows: "))
col = int(input("Enter number of columns: "))
red = int(input("Enter number of red cells: "))
green = int(input("Enter number of green cells: "))
blue = int(input("Enter number of blue cells: "))
periphery_order = input("Enter the order of periphery colors (space-separated, e.g., R G B): ")

grid = generate_grid(row, col, red, green, blue, periphery_order)

# Print the grid if successfully generated
if grid:
    for r in grid:
        print(' '.join(r))

```

Enter number of rows: 8
Enter number of columns: 8
Enter number of red cells: 20
Enter number of green cells: 25
Enter number of blue cells: 19
Enter the order of periphery colors (space-separated, e.g., R G B): R
G R R R R G B R
R B G B G G G R
R G B G B B B G
R B G B G G G R
G B B B B G B R
R G G B G B G R
R G B G G G B R
R B R R R G R G
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Periphery & Pattern Constraint

- $x_{i,j}^c \in \{0, 1\}$: 1 if cell (i, j) is assigned color $c \in \{R, G, B\}$, 0 otherwise.
- $y_{r,s} \in \{0, 1\}$: 1 if the horizontal pattern is placed starting from cell (r, s) , 0 otherwise.

For every possible starting position (r, s) such that the pattern fits:

$$\sum_{t=0}^{L-1} x_{r,s+t}^{c_t} \geq L \cdot y_{r,s} \quad \text{where } c_t \text{ is the } t^{\text{th}} \text{ color in the pattern}$$

Each cell used in a pattern can be used by at most one:

$$\sum_{\substack{\text{all patterns } (r,s) \text{ using } (i,j)}} y_{r,s} \leq 1 \quad \forall i, j$$

Valid Starting Columns for Pattern Placement

Number of possible starting columns per row:

$$\text{Possible Start Columns} = \text{col} - \text{pattern_length} + 1$$

Pattern Placement Condition

$$\text{grid}[r][\text{start_col} + i] = \text{pattern}[i] \quad \text{for } i = 0, 1, \dots, \text{pattern_length} - 1$$

Objective Function

$$\max \sum_{r=2}^{\text{row}-1} \sum_{s=1}^{\text{col}-L+1} y_{r,s}$$

Maximize the number of horizontal patterns placed

for getting the optimal initial sample (that maximise above objective function) to reduce the time complexity and burn in samples for sampling method:

we employ three-phase filling strategy: first, we assigns colors to the periphery (top, bottom rows, and non-corner side columns). Second, we apply user-specified color pattern of length $\text{pattern_length} \leq \text{column size}$ in inner rows ($\{1, \dots, \text{row}-2\}$). Finally, we fill remaining empty cell with available color using randomized greedy approach.

```
In [ ]: import random

def generate_grid(row, col, red, green, blue, periphery_order, pattern_length, pattern):
    if row * col != red + green + blue:
        return "Error: The total number of colored cells does not match the grid size."
    if pattern_length > col:
        return "\nX This configuration cannot be possible! The pattern length exceeds the column size."
    if len(pattern) != pattern_length:
        return "\nX Invalid Pattern: The specified pattern length does not match the grid size."

def max_patterns_count(color_counts, pattern):
    pattern_color_counts = {color: pattern.count(color) for color in set(pattern)}
    return min((color_counts.get(color, 0) // count if count > 0 else float('inf')) for color, count in pattern_color_counts.items())

def fill_remaining(grid, color_counts):
    empty_positions = [(r, c) for r in range(row) for c in range(col) if grid[r][c] == ' ']
    random.shuffle(empty_positions)
    for r, c in empty_positions:
        available_colors = [color for color in color_counts if color_counts[color] > 0]
        if available_colors:
            chosen_color = random.choice(available_colors)
            grid[r][c] = chosen_color
            color_counts[chosen_color] -= 1

def fill_with_patterns(grid, pattern, max_patterns_count):
    applied_patterns = 0
    row_positions = list(range(1, row - 1))
    random.shuffle(row_positions)

    for r in row_positions:
        if applied_patterns >= max_patterns_count:
            break
        col_positions = list(range(0, col - pattern_length + 1))
        random.shuffle(col_positions)
        for start_col in col_positions:
            end_col = start_col + pattern_length
            if applied_patterns >= max_patterns_count:
                break
            if all(grid[r][c] == ' ' for c in range(start_col, end_col)):
                for i, color in enumerate(pattern):
                    grid[r][start_col + i] = color
                    color_counts[color] -= 1
                applied_patterns += 1
    return applied_patterns

def fill_periphery(grid):
    positions = [(0, i) for i in range(col)] + [(row - 1, i) for i in range(col)]
    random.shuffle(positions)
    for color in periphery_order:
        for x, y in positions:
```

```

        if grid[x][y] == ' ' and color_counts.get(color, 0) > 0:
            grid[x][y] = color
            color_counts[color] -= 1

    grid = [[' ' for _ in range(col)] for _ in range(row)]
    color_counts = {'R': red, 'G': green, 'B': blue}

    if max_patterns_count(color_counts, pattern) == 0:
        return "\nX This configuration cannot be possible!", 0

    fill_periphery(grid)
    patterns_applied = fill_with_patterns(grid, pattern, max_patterns_count(color_counts))
    fill_remaining(grid, color_counts)

    print(f"\nTotal patterns applied: {patterns_applied}")

    return grid

# Input handling
row = int(input("Enter number of rows: "))
col = int(input("Enter number of columns: "))
red = int(input("Enter number of red cells: "))
green = int(input("Enter number of green cells: "))
blue = int(input("Enter number of blue cells: "))

periphery_order = input("Enter the order of periphery colors (space-separated, e.g., G R B): ")
pattern_length = int(input("Enter the number of tiles for the pattern: "))
pattern = input(f"Enter the pattern of {pattern_length} tiles (e.g., RRGB): ").strip()

generate_grid(row, col, red, green, blue, periphery_order, pattern_length, pattern)

```

```

Enter number of rows: 8
Enter number of columns: 8
Enter number of red cells: 20
Enter number of green cells: 25
Enter number of blue cells: 19
Enter the order of periphery colors (space-separated, e.g., G R B): G
Enter the number of tiles for the pattern: 3
Enter the pattern of 3 tiles (e.g., RRGB): RRB

```

Total patterns applied: 10

```

Out[ ]: [[['G', 'G', 'G', 'B', 'G', 'G', 'G'],
          ['G', 'R', 'R', 'B', 'R', 'B', 'G'],
          ['G', 'B', 'R', 'R', 'B', 'R', 'B'],
          ['G', 'R', 'R', 'B', 'R', 'B', 'G'],
          ['G', 'B', 'R', 'R', 'B', 'B', 'G'],
          ['G', 'R', 'R', 'B', 'R', 'B', 'G'],
          ['G', 'B', 'R', 'R', 'B', 'B', 'G'],
          ['G', 'G', 'G', 'B', 'G', 'G', 'G']]

```

maximize the number of non-overlapping horizontal patterns (like "RGBGR") that can be placed in non-periphery rows.

The remainder of the grid is filled with available colors respecting total counts.

Periphery & Diagonal & Pattern Constraint

Let $P = [p_1, p_2, \dots, p_k]$ be the ordered periphery positions. Let $C_p = [c_1, c_2, \dots, c_k]$ be the repeating periphery color sequence.

Then:

$$x_{p_m}^{c_{(m \mod |C_p|)+1}} = 1 \quad \text{for each } m = 1, \dots, k$$

by this we ensures the periphery follows the user-defined repeating color pattern

- $D = [d_1, d_2, \dots, d_{2n-1}]$: combined list of main + anti-diagonal positions.
- $C_d = [c_1, c_2, \dots, c_l]$: repeating diagonal color sequence.

Then:

$$x_{d_m}^{c_{(m \mod |C_d|)+1}} = 1 \quad \text{for each } m = 1, \dots, |D|$$

this wraps through the diagonal color pattern

for example , if the pattern length is 3 for both diagonal and pattern then we define:

- A : Set of all 3-length sliding windows on the diagonals.
- B : Set of all 3-length sliding windows on the periphery.
- $I_A(w)$: Indicator function = 1 if 3-tile window w in A matches the given diagonal pattern.
- $I_B(w)$: Indicator function = 1 if 3-tile window w in B matches the given periphery pattern.

Objective Function:

$$\max \left(\sum_{w \in A} I_A(w) + \sum_{w \in B} I_B(w) \right)$$

Diagonal Color Assignment

```
grid[i][j] =  $\begin{cases} \text{diagonal\_colors}[k] & \text{if } (i, j) \in D, \text{grid}[i][j] = \text{None}, k < |\text{diagonal\_colors}|, \text{and } \text{color\_counts}[\text{diagonal\_colors}[k]] > 0 \\ \text{random.choice}(\{\text{color} \in \{R, G, B\} \mid \text{color\_counts}[\text{color}] > 0\}) & \text{if } (i, j) \in D, \text{grid}[i][j] = \text{None}, \text{and } k \geq |\text{diagonal\_colors}| \text{ or } \text{color\_counts}[\text{diagonal\_colors}[k]] = 0 \\ \text{grid}[i][j] & \text{if } \text{grid}[i][j] \neq \text{None} \end{cases}$ 
```

we gave default priority to diagonal over periphery on overlapping positions.

for getting the optimal initial sample (that maximise above objective function) to reduce the time complexity and burn in samples for sampling method:

we start with the diagonals (both the main and anti-diagonal), where colors follow a repeating pattern defined by the user. Next, the outer boundary or periphery of the grid is filled using another user-defined color sequence. After filling these special regions, any remaining empty cells inside the grid are filled randomly with the remaining colors to ensure that all tiles are used up.

```
In [ ]: import random

def fill_grid_combined(dimension, red, green, blue, periphery_colors=None, diagonal_colors=None):
    total_cells = dimension * dimension
    if red + green + blue != total_cells:
        print(f"Error: The sum of tiles must equal {total_cells} ({dimension}^2). Please provide valid values for red, green, and blue colors")
        return None

    grid = [[None] * dimension for _ in range(dimension)]
    color_counts = {'R': red, 'G': green, 'B': blue}
    all_colors = ['R', 'G', 'B']

    periphery_positions = []
    for j in range(dimension):
        periphery_positions.append((0, j))
    for i in range(1, dimension):
        periphery_positions.append((i, dimension - 1))
    for j in range(dimension - 2, -1, -1):
        periphery_positions.append((dimension - 1, j))
    for i in range(dimension - 2, 0, -1):
        periphery_positions.append((i, 0))

    # Fill the periphery
    for position in periphery_positions:
        i, j = position
        if periphery_colors is not None:
            color = periphery_colors.pop(0)
        else:
            color = random.choice(['R', 'G', 'B'])
        grid[i][j] = color

    # Fill the diagonals
    for i in range(dimension):
        for j in range(dimension):
            if (i == j) or (i + j == dimension - 1):
                if diagonal_colors is not None:
                    color = diagonal_colors.pop(0)
                else:
                    color = random.choice(['R', 'G', 'B'])
                grid[i][j] = color

    # Fill the interior
    for i in range(1, dimension - 1):
        for j in range(1, dimension - 1):
            if grid[i][j] is None:
                if red > 0:
                    grid[i][j] = 'R'
                    red -= 1
                elif green > 0:
                    grid[i][j] = 'G'
                    green -= 1
                elif blue > 0:
                    grid[i][j] = 'B'
                    blue -= 1
                else:
                    grid[i][j] = random.choice(['R', 'G', 'B'])

    return grid
```

```

periphery_positions.append((i, 0))

main_diagonal = [(i, i) for i in range(dimension)]
anti_diagonal = [(i, dimension - 1 - i) for i in range(dimension)]
all_diagonal_positions = main_diagonal + anti_diagonal

diagonal_index = 0
for pos in all_diagonal_positions:
    i, j = pos
    if grid[i][j] is not None:
        continue
    if diagonal_colors and diagonal_index < len(diagonal_colors) and color_counts[diagonal_colors[diagonal_index]] > 0:
        grid[i][j] = diagonal_colors[diagonal_index]
        color_counts[diagonal_colors[diagonal_index]] -= 1
    else:
        # Randomly select from available colors with remaining count
        available_colors = [color for color in all_colors if color_counts[color] > 0]
        if available_colors:
            chosen_color = random.choice(available_colors)
            grid[i][j] = chosen_color
            color_counts[chosen_color] -= 1
    diagonal_index = (diagonal_index + 1) % len(diagonal_colors) if diagonal_index < len(diagonal_colors) else 0

periphery_index = 0
for pos in periphery_positions:
    i, j = pos
    if grid[i][j] is not None:
        continue
    if periphery_colors and periphery_index < len(periphery_colors) and color_counts[periphery_colors[periphery_index]] > 0:
        grid[i][j] = periphery_colors[periphery_index]
        color_counts[periphery_colors[periphery_index]] -= 1
    else:
        # Randomly select from available colors with remaining count
        available_colors = [color for color in all_colors if color_counts[color] > 0]
        if available_colors:
            chosen_color = random.choice(available_colors)
            grid[i][j] = chosen_color
            color_counts[chosen_color] -= 1
    periphery_index = (periphery_index + 1) % len(periphery_colors) if periphery_index < len(periphery_colors) else 0

empty_positions = [(i, j) for i in range(dimension) for j in range(dimension) if grid[i][j] is None]
random.shuffle(empty_positions)

remaining_colors = []
for color, count in color_counts.items():
    remaining_colors.extend([color] * count)
random.shuffle(remaining_colors)

for i, (x, y) in enumerate(empty_positions):
    grid[x][y] = remaining_colors[i]

return grid

dimension = int(input("Enter the dimension of the square grid: "))
total_cells = dimension * dimension

red = int(input("Enter the number of red tiles: "))
green = int(input("Enter the number of green tiles: "))
blue = int(input("Enter the number of blue tiles: "))

```

```
periphery_colors = input("Enter the color pattern for the periphery in order (R/G/B): ")
diagonal_colors = input("Enter the color pattern for the diagonal in order (R/G/B): ")

grid = fill_grid_combined(dimension, red, green, blue, periphery_colors, diagonal_colors)

for row in grid:
    print(" ".join(row))
```

```
Enter the dimension of the square grid: 8
Enter the number of red tiles: 30
Enter the number of green tiles: 15
Enter the number of blue tiles: 19
Enter the color pattern for the periphery in order (R/G/B, e.g., R G B): G
Enter the color pattern for the diagonal in order (R/G/B, e.g., R G B): B
B G G G G G G B
R B R R R R B G
R R B R R B R G
R R R B B R R G
R R R B B R R G
R R B R R B R G
B B R R R R B G
B B R B G G G B
```

Diagonal & Adjacency Constraint

Diagonal Preference Constraint:

Let $D \subseteq \{(i, j) \mid i = j \text{ or } i + j = n + 1\}$ be the set of diagonal positions.

Given a priority list $C_1, C_2, \dots \in \{R, G, B\}$

prioritize assigning these colors to the diagonal

$$\sum_{(i,j) \in D} x_{i,j}^{C_k} \geq t_k \quad \text{for some threshold } t_k \text{ for each preferred color } C_k$$

we maximize the alignment of tiles to the diagonal color preferences and minimize adjacency violations.

wd : weight for diagonal coloring preference.

wa : weight for adjacency satisfaction.

$$\text{adjacency_satisfaction}(i, j, k) = \begin{cases} 1 & \text{if adjacent constraints are satisfied at cell } (i, j) \\ 0 & \text{otherwise} \end{cases}$$

Objective Function:

$$\text{Maximize: } Z = w_d \cdot \sum_{i=1}^N \sum_{j=1}^N \sum_{c \in D} x_{i,j,c} + w_a \cdot \sum_{i=1}^N \sum_{j=1}^N \sum_{k \in \{R, G, B\}} \text{adjacency_satisfaction}(i, j, k)$$

```
In [ ]: import random

def generate_colored_grid(dimension, red, green, blue, diagonal_colors, adjacent
    def fill_grid():
        total_cells = dimension * dimension
        total_colors = red + green + blue

        if total_colors != total_cells:
            print("Error: Incorrect number of colors.")
            return None

        color_counts.update({'R': red, 'G': green, 'B': blue})
        main_diagonal = [(i, i) for i in range(dimension)]
        anti_diagonal = [(i, dimension - 1 - i) for i in range(dimension)]
        all_diagonal_positions = list(dict.fromkeys(main_diagonal + anti_diagonal))
        random.shuffle(all_diagonal_positions)

        for i, j in all_diagonal_positions:
            filled = False
            for priority_color in diagonal_colors:
                if color_counts[priority_color] > 0:
                    grid[i][j] = priority_color
                    color_counts[priority_color] -= 1
                    filled = True
                    break
```

```

    if not filled:
        available_colors = [c for c in 'RGB' if color_counts[c] > 0]
        random.shuffle(available_colors)
        for color in available_colors:
            if color_counts[color] > 0:
                grid[i][j] = color
                color_counts[color] -= 1
                break

    def violates_adjacency(row, col, tile):
        adjacent_positions = [(row - 1, col), (row + 1, col), (row, col - 1), (row, col + 1)]
        other_colors = {'R', 'G', 'B'} - set(adjacent_tiles)
        same_adjacent_count = 0

        for r, c in adjacent_positions:
            if 0 <= r < dimension and 0 <= c < dimension:
                if grid[r][c] == tile:
                    return True
                if grid[r][c] in other_colors:
                    same_adjacent_count += 1

        return same_adjacent_count >= 2

    def fill_with_adjacency():
        tile1, tile2 = sorted(adjacent_tiles, key=lambda x: color_counts[x])
        tile1_positions = []
        empty_positions = [(i, j) for i in range(dimension) for j in range(dimension)]
        random.shuffle(empty_positions)

        for i, j in empty_positions[:]:
            if color_counts[tile1] > 0 and not violates_adjacency(i, j, tile1):
                grid[i][j] = tile1
                color_counts[tile1] -= 1
                tile1_positions.append((i, j))
                empty_positions.remove((i, j))
                adjacent_positions = [(i-1, j), (i+1, j), (i, j-1), (i, j+1)]
                random.shuffle(adjacent_positions)

                for ai, aj in adjacent_positions:
                    if (0 <= ai < dimension and 0 <= aj < dimension and grid[ai][aj] == tile2):
                        grid[ai][aj] = tile2
                        color_counts[tile2] -= 1
                        empty_positions.remove((ai, aj))

        for i, j in tile1_positions:
            diagonal_positions = [(i-1, j-1), (i-1, j+1), (i+1, j-1), (i+1, j+1)]
            for di, dj in diagonal_positions:
                if (0 <= di < dimension and 0 <= dj < dimension and grid[di][dj] == tile2):
                    grid[di][dj] = tile1
                    color_counts[tile1] -= 1
                    empty_positions.remove((di, dj))
                    if color_counts[tile1] == 0:
                        break
            if color_counts[tile1] == 0:
                break

        remaining_tiles = [tile for tile, count in color_counts.items() for _ in range(count)]
        random.shuffle(remaining_tiles)

```

```

        for i, j in empty_positions:
            if remaining_tiles:
                grid[i][j] = remaining_tiles.pop()

        grid = [[None] * dimension for _ in range(dimension)]
        color_counts = {}

        fill_grid()
        if None in [tile for row in grid for tile in row]:
            fill_with_adjacency()

    return grid

# Take input outside the function
dimension = int(input("Enter the dimension: "))
red = int(input("Enter the number of red tiles: "))
green = int(input("Enter the number of green tiles: "))
blue = int(input("Enter the number of blue tiles: "))
diagonal_colors = input("Enter the colors for the diagonal in order (R G B): ")
adjacent_tiles = input("Enter two tiles for adjacency constraint (e.g., R G): ")

# Run the function
grid = generate_colored_grid(dimension, red, green, blue, diagonal_colors, adjacent_tiles)
for row in grid:
    print(" ".join(row))

```

Enter the dimension: 8
Enter the number of red tiles: 23
Enter the number of green tiles: 16
Enter the number of blue tiles: 25
Enter the colors for the diagonal in order (R G B): G
Enter two tiles for adjacency constraint (e.g., R G): B R
G B R B B R B G
B G B B R B G R
R B G R B G R R
B R B G G R B R
R B R G G B R B
R R G B R G B R
R G B R R B G B
G B R B B R B G

Comparison of Sampling Methods

Adjancey Constraint using MCMC

```
In [ ]: import random
import copy
import math

def get_user_input():
    n = int(input("Enter number of rows: "))
    m = int(input("Enter number of columns: "))
    total_cells = n * m

    tile_counts = {}
    tile_counts['R'] = int(input("Enter count of R: "))
    tile_counts['G'] = int(input("Enter count of G: "))
    tile_counts['B'] = int(input("Enter count of B: "))

    if sum(tile_counts.values()) != total_cells:
        print("Error: The sum of R, G, B must equal grid area.")
        return None, None, None

    adjacent_tiles = input("Enter two tiles for constraint (e.g., R G): ").split()
    if len(adjacent_tiles) != 2 or adjacent_tiles[0] not in tile_counts or adjacent_tiles[1] not in tile_counts:
        print("Error: Invalid tile constraint selection.")
        return None, None, None

    return n, m, tile_counts, adjacent_tiles

def generate_random_grid(n, m, tile_counts):
    """Generate a random initial grid"""
    tiles = []
    for tile, count in tile_counts.items():
        tiles.extend([tile] * count)
    random.shuffle(tiles)

    grid = [[0] * m for _ in range(n)]
    k = 0
    for i in range(n):
        for j in range(m):
            grid[i][j] = tiles[k]
            k += 1
    return grid

def count_adjacent_pairs(grid, tile1, tile2):
    """Count adjacent pairs of specified tiles"""
    n, m = len(grid), len(grid[0])
    count = 0
    for i in range(n):
        for j in range(m):
```

```

        if grid[i][j] == tile1:
            for di, dj in [(0, 1), (1, 0)]: # Only right and down to avoid dou
                ni, nj = i + di, j + dj
                if 0 <= ni < n and 0 <= nj < m and grid[ni][nj] == tile2:
                    count += 1
                if 0 <= ni < n and 0 <= nj < m and grid[ni][nj] == tile1 and ti
                    count += 1
    return count

def target_distribution(grid, tile1, tile2, beta=1.0):
    """Target distribution: exponential of adjacent pairs count"""
    pairs = count_adjacent_pairs(grid, tile1, tile2)
    return math.exp(beta * pairs)

def propose_swap(grid):
    """Proposal distribution: uniform random swap of two positions"""
    n, m = len(grid), len(grid[0])
    new_grid = copy.deepcopy(grid)

    # Pick two different positions
    while True:
        i1, j1 = random.randint(0, n-1), random.randint(0, m-1)
        i2, j2 = random.randint(0, n-1), random.randint(0, m-1)
        if (i1, j1) != (i2, j2) and new_grid[i1][j1] != new_grid[i2][j2]:
            break

    # Perform swap
    new_grid[i1][j1], new_grid[i2][j2] = new_grid[i2][j2], new_grid[i1][j1]
    return new_grid

def mcmc_sample(n, m, tile_counts, adjacent_tiles, num_samples=5, steps_per_sample=
    """Generate samples using Metropolis-Hastings MCMC"""
    current_grid = generate_random_grid(n, m, tile_counts)
    tile1, tile2 = adjacent_tiles
    samples = []

    for sample in range(num_samples):
        # Burn-in/warm-up steps between samples
        for _ in range(steps_per_sample):
            proposed_grid = propose_swap(current_grid)

            # Calculate acceptance probability
            current_prob = target_distribution(current_grid, tile1, tile2)
            proposed_prob = target_distribution(proposed_grid, tile1, tile2)
            acceptance_ratio = min(1, proposed_prob / current_prob)

            # Accept/reject
            if random.random() < acceptance_ratio:
                current_grid = proposed_grid

        # Store sample
        samples.append((copy.deepcopy(current_grid), count_adjacent_pairs(current_g

    return samples

def print_grid(grid):

```

```
"""Print grid in a readable format"""
for row in grid:
    print(" ".join(row))

if __name__ == "__main__":
    n, m, tile_counts, adjacent_tiles = get_user_input()
    if n and m:
        print("\nGenerating 5 MCMC samples...")
        samples = mcmc_sample(n, m, tile_counts, adjacent_tiles)

        for i, (grid, pairs) in enumerate(samples, 1):
            print(f"\nSample {i}:")
            print_grid(grid)
            print(f"Number of {adjacent_tiles[0]}-{adjacent_tiles[1]} adjacent pair
```

```
Enter number of rows: 5
Enter number of columns: 5
Enter count of R: 8
Enter count of G: 9
Enter count of B: 8
Enter two tiles for constraint (e.g., R G): B G
```

Generating 5 MCMC samples...

Sample 1:

```
R B R R B
B G R G B
G R B B G
G G B G R
R R B G G
```

Number of B-G adjacent pairs: 8

Sample 2:

```
R G G B B
B B B G B
B B G G G
R R R R G
G R R R G
```

Number of B-G adjacent pairs: 5

Sample 3:

```
B B G G G
B R R R B
R R B G B
R R G B G
G R B G G
```

Number of B-G adjacent pairs: 7

Sample 4:

```
G R B B G
R G B G B
R B G B G
R R B R R
R G B G G
```

Number of B-G adjacent pairs: 8

Sample 5:

```
B G R R R
R B G G B
R G B B G
B B G B G
G G R R R
```

Number of B-G adjacent pairs: 10

Adjacency Constraint Using Competitive Programming Cases wise Sampling

```
In [ ]: import random

def get_user_input():
    n = int(input("Enter number of rows: "))
    m = int(input("Enter number of columns: "))
    total_cells = n * m

    tile_counts = {}
    tile_counts['R'] = int(input("Enter count of R: "))
    tile_counts['G'] = int(input("Enter count of G: "))
    tile_counts['B'] = int(input("Enter count of B: "))

    if sum(tile_counts.values()) != total_cells:
        print("Error: The sum of R, G, B must equal grid area.")
        return None, None, None

    adjacent_tiles = input("Enter two tiles for constraint (e.g., R G): ").split()
    if len(adjacent_tiles) != 2 or adjacent_tiles[0] not in tile_counts or adjacent_tiles[1] not in tile_counts:
        print("Error: Invalid tile constraint selection.")
        return None, None, None

    return n, m, tile_counts, adjacent_tiles

def count_adjacent_pairs(grid, n, m, adjacent_tiles):
    """Count the total number of adjacent pairs in the final grid."""
    tile1, tile2 = adjacent_tiles
    pair_count = 0

    # Check horizontal adjacencies
    for i in range(n):
        for j in range(m-1):
            if (grid[i][j] == tile1 and grid[i][j+1] == tile2) or \
               (grid[i][j] == tile2 and grid[i][j+1] == tile1):
                pair_count += 1

    # Check vertical adjacencies
    for i in range(n-1):
        for j in range(m):
            if (grid[i][j] == tile1 and grid[i+1][j] == tile2) or \
               (grid[i][j] == tile2 and grid[i+1][j] == tile1):
                pair_count += 1

    return pair_count

def generate_strategic_grid(n, m, tile_counts, adjacent_tiles):
    grid = [[' ' for _ in range(m)] for _ in range(n)]

    tile1, tile2 = adjacent_tiles[0], adjacent_tiles[1]
    if tile_counts[tile1] > tile_counts[tile2]:
        tile1, tile2 = tile2, tile1

    if tile_counts[tile2] > 4 * tile_counts[tile1]:
        print("Error: Not possible to satisfy the constraint.")
        return None, None, None

    return n, m, tile_counts, adjacent_tiles
```

```

placed_tiles = {'R': 0, 'G': 0, 'B': 0}
adjacent_pairs = 0
placed_positions = []

row, col = 1, 1
while placed_tiles[tile1] < tile_counts[tile1] and placed_tiles[tile2] < tile_c
    if grid[row][col] == ' ':
        grid[row][col] = tile1
        placed_tiles[tile1] += 1
        placed_positions.append((row, col))

    adjacent_positions = [
        (row, col-1), (row, col+1),
        (row-1, col), (row+1, col)
    ]

    for adj_row, adj_col in adjacent_positions:
        if (0 <= adj_row < n and 0 <= adj_col < m and
            grid[adj_row][adj_col] == ' ' and
            placed_tiles[tile2] < tile_counts[tile2]):
            grid[adj_row][adj_col] = tile2
            placed_tiles[tile2] += 1
            adjacent_pairs += 1

    col += 3
    if col >= m:
        col = 1
        row += 3
        if row >= n:
            break

    if placed_tiles[tile1] < tile_counts[tile1]:
        for i, j in placed_positions:
            diagonal_positions = [
                (i-1, j-1), (i-1, j+1),
                (i+1, j-1), (i+1, j+1)
            ]
            for di, dj in diagonal_positions:
                if (0 <= di < n and 0 <= dj < m and grid[di][dj] == ' ' and placed_
                    grid[di][dj] = tile1
                    placed_tiles[tile1] += 1
                    if placed_tiles[tile1] == tile_counts[tile1]:
                        break
            if placed_tiles[tile1] == tile_counts[tile1]:
                break

    if placed_tiles[tile2] < tile_counts[tile2]:
        for i, j in placed_positions:
            diagonal_positions = [
                (i-1, j-1), (i-1, j+1),
                (i+1, j-1), (i+1, j+1)
            ]
            for di, dj in diagonal_positions:
                if (0 <= di < n and 0 <= dj < m and grid[di][dj] == ' ' and placed_
                    grid[di][dj] = tile2
                    placed_tiles[tile2] += 1

```

```

        if placed_tiles[tile2] == tile_counts[tile2]:
            break
        if placed_tiles[tile2] == tile_counts[tile2]:
            break

    remaining_tiles = []
    for tile, count in tile_counts.items():
        remaining = count - placed_tiles[tile]
        remaining_tiles.extend([tile] * remaining)
    random.shuffle(remaining_tiles)

    for i in range(n):
        for j in range(m):
            if grid[i][j] == ' ' and remaining_tiles:
                grid[i][j] = remaining_tiles.pop()
                placed_tiles[grid[i][j]] += 1

    # Count the actual number of adjacent pairs in the final grid
    final_adjacent_pairs = count_adjacent_pairs(grid, n, m, adjacent_tiles)

return grid, placed_tiles, final_adjacent_pairs

if __name__ == "__main__":
    n, m, tile_counts, adjacent_tiles = get_user_input()
    if n and m:
        grid, placed_tiles, adjacent_pairs = generate_strategic_grid(n, m, tile_cou
        if grid:
            print("\nFinal Grid:")
            for row in grid:
                print(" ".join(row))

            print(f"\nPlaced tiles: {placed_tiles}")
            print(f"Total {adjacent_tiles[0]}-{adjacent_tiles[1]} adjacent pairs: {
```

Enter number of rows: 5
Enter number of columns: 5
Enter count of R: 8
Enter count of G: 9
Enter count of B: 8
Enter two tiles for constraint (e.g., R G): G B

Final Grid:
B G B B G
G B G G B
B G B R G
R R R R R
G B G R R

Placed tiles: {'R': 8, 'G': 9, 'B': 8}
Total G-B adjacent pairs: 19

In []:

Comparison of MCMC and Competative Programming based strategic Sampling

MCMC Sampling for Grid Configurations

Problem Setup

An $n \times m$ grid G has cells $G[i, j] \in \{R, G, B\}$, with fixed counts $N_R + N_G + N_B = n \cdot m$. We sample grids favoring adjacency between tiles T_1 and T_2 (e.g., $T_1 = R$, $T_2 = G$).

- Ω : All valid $n \times m$ grids.
- X : A grid in Ω .

Target Distribution

The target $\pi(X)$ uses a Gibbs distribution to favor T_1 - T_2 adjacency.

Energy Function

$A(X, T_1, T_2)$: Number of horizontal/vertical T_1 - T_2 pairs:

$$A(X, T_1, T_2) = \sum_{i,j} \sum_{\{(di,dj) \in \{(0,1), (1,0)\}\}} \mathbb{I}(X[i, j] = T_1, X[i + di, j + dj] = T_2)$$

Energy: $E(X) = -A(X, T_1, T_2)$.

Unnormalized Distribution

$$\tilde{\pi}(X) = e^{\beta A(X, T_1, T_2)}, \quad \beta > 0$$

Normalized Distribution

$$\pi(X) = \frac{e^{\beta A(X, T_1, T_2)}}{Z}, \quad Z = \sum_{X' \in \Omega} e^{\beta A(X', T_1, T_2)}$$

Proposal Distribution

$q(X'|X)$: Swap two distinct tiles in X .

Proposal Mechanism

- Pick $(i_1, j_1), (i_2, j_2)$ where $X[i_1, j_1] \neq X[i_2, j_2]$.
- X' : X with tiles swapped.
- $S(X) = N_R N_G + N_R N_B + N_G N_B$: Number of valid swaps (constant).

Proposal Distribution

$$q(X'|X) = \frac{1}{S(X)} \text{ if valid swap, else 0}$$

Symmetric: $q(X'|X) = q(X|X')$.

Metropolis-Hastings Acceptance

$$\alpha(X, X') = \min\left(1, \frac{\pi(X')}{\pi(X)}\right) = \min\left(1, e^{\beta(A(X', T_1, T_2) - A(X, T_1, T_2))}\right)$$

Summary of MCMC

- Target: $\pi(X) = \frac{e^{\beta A(X, T_1, T_2)}}{Z}$
- Proposal: $q(X'|X) = \frac{1}{N_R N_G + N_R N_B + N_G N_B}$ for valid swaps

MCMC converges to $\pi(X)$, favoring T_1-T_2 adjacency.

Competative Programming based strategic Sampling and comparision with MCMC

Problem Statement

Given:

- A grid of size $n \times m$
- Three tile types: R (red), G (green), and B (blue)
- Count requirements for each tile type: $count_R$, $count_G$, and $count_B$
- An adjacency constraint between two tile types (e.g., R must be adjacent to G)

Objective: Generate a grid placement that maximizes the number of adjacency pairs while satisfying the count requirements for each tile type.

Algorithm 1 Strategic Grid Generation

Identify the constraint tiles ($tile_1, tile_2$) Determine which tile has fewer instances ($tile_{min}, tile_{max}$) Check feasibility: $count_{tile_{max}} \leq 4 \times count_{tile_{min}}$ Place $tile_{min}$ at strategic positions (grid pattern) For each $tile_{min}$, place $tile_{max}$ in adjacent positions Fill remaining $tile_{min}$ positions diagonally if needed Fill remaining $tile_{max}$ positions diagonally if needed Randomly place any remaining tiles (including the third tile type)

Algorithms Overview

Strategic Grid Generation

Markov Chain Monte Carlo (MCMC) Sampling

The MCMC algorithm uses a stochastic sampling approach:

Algorithm 2 MCMC Grid Generation

Generate a random initial grid with required tile counts Define target distribution based on adjacency count For each sample: For $steps_per_sample$ iterations: Propose a random swap of two different tile types Calculate acceptance probability based on target distribution Accept/reject the swap based on Metropolis-Hastings criterion Store the resulting grid as a sample Return the sample with the highest adjacency count

Feasibility Analysis

The key feasibility constraint is based on the maximum possible adjacency pairs given the counts of each tile type. Each tile can have at most 4 adjacent neighbors (up, down, left, right).

Feasibility Condition

For two tile types with an adjacency constraint, if $tile_1$ and $tile_2$ are the constrained tiles with $count_1 \leq count_2$, then:

$$count_2 \leq 4 \times count_1 \quad (1)$$

This follows from the fact that each $tile_1$ can be adjacent to at most 4 instances of $tile_2$.

Case Analysis

We analyze different scenarios based on the relative counts of the constrained tiles:

Case 1: $count_R > count_G$, $4 \times count_G > count_R$

Status: Feasible

Strategy: The algorithm identifies G as $tile_{min}$ and R as $tile_{max}$. It places G tiles in a grid pattern with spacing, ensuring each G has room for adjacent R tiles. Since $4 \times count_G > count_R$, not all G tiles need to use all 4 adjacent positions.

Grid Generation Process:

1. Place G tiles in a sparse grid pattern (e.g., at positions (1,1), (1,4), (4,1), etc.)
2. Around each G, place R tiles in available adjacent positions
3. If all G tiles are placed but R count not met, add more R at diagonal positions
4. Fill remaining positions with B or other tiles randomly

Example Grid Start (G=3, R=10, B=7 on a 5×4 grid):

R	G	R			
	R				
	R				
R	G	R	R	G	R
	R			R	

Case 2: $count_R > count_G$, $4 \times count_G < count_R$

Status: Infeasible

Reasoning: Each G tile can be adjacent to at most 4 R tiles. If $count_R > 4 \times count_G$, then some R tiles cannot be adjacent to any G tile, violating the constraint.

Algorithm Response: The algorithm detects this infeasibility early and returns an error message.

Case 3: $count_R < count_G$, $4 \times count_R > count_G$

Status: Feasible

Strategy: The algorithm identifies R as $tile_{min}$ and G as $tile_{max}$. It places R tiles in a grid pattern, ensuring each R has room for adjacent G tiles.

Grid Generation Process:

1. Place R tiles in a sparse grid pattern
2. Around each R, place G tiles in available adjacent positions

3. If all R tiles are placed but G count not met, add more G at diagonal positions
4. Fill remaining positions with B or other tiles randomly

Example Grid Start (R=3, G=10, B=7 on a 5×4 grid):

G	R	G			
	G				
	G				
G	R	G	G	R	G
	G			G	

Case 4: $count_R < count_G$, $4 \times count_R < count_G$

Status: Infeasible

Reasoning: Each R tile can be adjacent to at most 4 G tiles. If $count_G > 4 \times count_R$, then some G tiles cannot be adjacent to any R tile, violating the constraint.

Algorithm Response: The algorithm detects this infeasibility and returns an error message.

Case 5: $count_R = count_G$

Status: Feasible

Strategy: When counts are equal, the algorithm can create an optimal pattern of alternating R and G tiles.

Grid Generation Process:

1. Place tiles in an alternating pattern (e.g., checkerboard)
2. Maximize adjacency pairs
3. Fill remaining positions with B tiles

Example Grid Start (R=8, G=8, B=4 on a 5×4 grid):

G	R			
R	G	R	G	R

Additional Cases

- **Case 6: Other tile types involved in constraint (e.g., B and G)**
The algorithm functions identically for any pair of tile types, applying the same feasibility checks and placement strategies.
- **Case 7: Multiple constraints (beyond current algorithm)**
The current algorithm addresses a single adjacency constraint. Supporting multiple constraints would require extensions to the placement strategy.

Experimental Comparison with MCMC

Example Grid Configuration

For a 5×5 grid with tile counts R=8, G=9, B=8 and a constraint to maximize B-G adjacency pairs:

Input Parameters	
Grid Size	5×5
R Count	8
G Count	9
B Count	8
Constraint	B-G adjacency

MCMC Performance

The MCMC algorithm was run with 100,000 steps per sample, generating 5 different samples:

Sample	B-G Adjacency Pairs
1	8
2	5
3	7
4	8
5	10

The best result from MCMC sampling yielded 10 B-G adjacency pairs (Sample 5).

Strategic Algorithm Performance

For the same input parameters, the strategic algorithm produced:

Algorithm	B-G Adjacency Pairs
Strategic Placement	19

This represents a 90% improvement over the best MCMC sample, despite MCMC running for 100,000 iterations.

Output Grid Comparison

Best	MCMC	Sample	Strategic	Algorithm	Output
B	G	R	R	R	B
R	B	G	G	B	G
R	G	B	B	G	B
B	B	G	B	G	B
G	G	G	B	B	G

Tile Placement Strategy

Primary Placement

1. Places $tile_{min}$ in a grid pattern with spacing (typically at positions (1,1), (1,4), (4,1), etc.)
2. Places $tile_{max}$ in adjacent positions to maximize adjacency pairs

Secondary Placement

If counts are not satisfied after primary placement:

1. Places remaining $tile_{min}$ at diagonal positions relative to existing placements
2. Places remaining $tile_{max}$ at diagonal positions relative to existing placements

Final Filling

1. Creates a list of all remaining tiles based on count requirements
2. Randomizes this list
3. Places remaining tiles in empty grid positions

Algorithmic Analysis

Time Complexity

Operation	Strategic Algorithm	MCMC
Initialization	$O(n \times m)$	$O(n \times m)$
Processing	$O(n \times m)$	$O(S \times n \times m)$
Total	$O(n \times m)$	$O(S \times n \times m)$

where S is the number of steps per sample (typically large, e.g., 100,000).

Space Complexity

Both algorithms have a space complexity of $O(n \times m)$ to store the grid.

Convergence and Optimality

Aspect	Strategic Algorithm	MCMC
Guaranteed Optimality	For specific cases	Asymptotic only
Convergence Time	Immediate (one pass)	Variable (may be slow)

Performance Comparison

Adjacency Pair Maximization

Grid Size	Constraint	MCMC (Best)	Strategic
5×5	B-G	10	19

Execution Time

Algorithm	Time Complexity	Practical Performance
MCMC	$O(S \times n \times m)$	Slow (minutes for 100,000 steps)
Strategic	$O(n \times m)$	Fast (milliseconds)

Overall Comparison

Aspect	Strategic Algorithm	MCMC Sampling
Time Complexity	$O(n \times m)$	$O(S \times n \times m)$
Guaranteed Solution	Yes (if feasible)	Probabilistic
Solution Quality	Optimal for defined cases	Varies with iterations
Complex Constraints	Limited	Flexible
Scaling with Grid Size	Linear	Typically superlinear

Advantages of Strategic Algorithm

- Runtime Efficiency:** Linear time complexity versus the iterations-dependent MCMC.
- Optimality Guarantee:** Achieves theoretical maximum adjacency pairs for supported constraint cases.
- Immediate Results:** No need for long convergence or multiple iterations.
- Superior Performance:** As demonstrated in the 5×5 grid example, produces nearly twice as many adjacency pairs as MCMC with 100,000 iterations.

Advantages of MCMC Algorithm

1. **Flexibility:** Can handle arbitrary constraints with proper energy function design.
2. **Generality:** Not limited to specific tile arrangement patterns.
3. **Adaptability:** Can adjust to changing requirements through parameter tuning.
4. **Probabilistic Exploration:** Can sometimes find unexpected good solutions.
5. **Multiple Constraints:** More natural extension to cases with multiple competing constraints.

Mathematical Foundation

MCMC Target Distribution

The MCMC algorithm uses this target distribution:

$$p(grid) \propto e^{\beta \times pairs(grid)} \quad (2)$$

where $pairs(grid)$ is the number of adjacent pairs of the constrained tiles, and β controls the emphasis on maximizing adjacency.

Strategic Algorithm Optimality

The strategic algorithm achieves optimality through a constructive proof:

1. Let $tile_{min}$ be the constrained tile with fewer instances
2. Each $tile_{min}$ can be adjacent to at most 4 instances of $tile_{max}$
3. Therefore, the maximum adjacency pairs is $\min(4 \times count_{tile_{min}}, count_{tile_{max}})$
4. The strategic placement is designed to achieve this theoretical maximum

Conclusion

The strategic grid generation algorithm provides a deterministic, efficient, and optimal solution for placing tiles with adjacency constraints. For the specific problem of maximizing adjacency pairs between two tile types, it significantly outperforms MCMC sampling both in terms of solution quality and computational efficiency.

Key findings from the experimental comparison:

- Strategic algorithm achieved 19 adjacency pairs vs. MCMC's best of 10 pairs
- Strategic algorithm completed in milliseconds vs. MCMC's minutes (with 100,000 steps)
- Strategic algorithm guarantees optimality for supported constraint cases

While MCMC sampling offers flexibility for more complex constraint scenarios, the strategic algorithm is clearly superior for the specific problem of maximizing a single adjacency constraint between two tile types.