

C. L1 正则化.

$$\bar{J}(w; X, y) = \alpha \|w\|_1 + J(w; X, y).$$

$$\nabla_w \bar{J}(w; X, y) = \alpha \operatorname{sign}(w) + \nabla_w J(w; X, y).$$

$$\bar{J}(w; X, y) = J(w^*; X, y) + \sum_i \left[\frac{1}{2} H_{i,i} (w_i - w_i^*)^2 + \alpha |w_i| \right]$$

$$w_i = \operatorname{sign}(w_i^*) \max \left\{ |w_i^*| - \frac{\alpha}{H_{i,i}}, 0 \right\}$$

对于 $w_i^* > 0$ 有

$$(1) \ w_i^* \leq \frac{\alpha}{H_{i,i}}, \quad w_i = 0$$

$$(2) \ w_i^* > \frac{\alpha}{H_{i,i}}, \quad \text{仅仅在那个方向上移动 } \frac{\alpha}{H_{i,i}} \text{ 距离}$$

对于 $w_i^* \leq 0$ 有 $w_i = 0$, 其中 w_i^* 为不加正则化时的最优解

