

• 1. 若  $K: \mathbb{R}^d \times \mathbb{R}^d \rightarrow \mathbb{R}$  是 kernel 且  $x, z \in \mathbb{R}^d$ .  $K$  为正定核

$$|K(x, z)|^2 \leq K(x, x) K(z, z)$$

$$K = \begin{bmatrix} K(x, x) & K(x, z) \\ K(z, x) & K(z, z) \end{bmatrix} = \begin{bmatrix} K(x, x) & K(x, z) \\ K(x, z) & K(z, z) \end{bmatrix} \geq 0$$

$$\det(K) = K(x, x)K(z, z) - |K(x, z)|^2 \geq 0$$

$$\text{则 } |K(x, z)|^2 \leq K(x, x)K(z, z).$$

$$\text{即 } (\langle \phi(x), \phi(z) \rangle)^2 \leq \langle \phi(x), \phi(x) \rangle \langle \phi(z), \phi(z) \rangle.$$



