引入

Outline

- Given a kernel function, how to construct a feature space, i.e., a feature mapping, such that the kernel function value between two samples in the original space is the inner product between the corresponding samples in the feature space.
- Given a feature mapping, how to construct the corresponding kernel function.

定义

- R^{R^d} 是所有函数从 $R^d \to R$. $R^{R^d} = \{f : R^d \to R\}$

Example △ 给定 $K(x,z) = \exp\{-\frac{(x-z)^2}{2}\}$

$\phi_0(x) = K(x,0) = \exp\{-\frac{x^2}{2}\} \in R^R$

$\phi_1(x) = K(x,1) = \exp\{-\frac{(x-1)^2}{2}\} \in R^R$.

△ 给定 $K(x,z) = (x_1 z_1 + x_2 z_2)^2$

$\phi_{[0,0]^T}(x) = K(x, [0,0]^T) = 0 \in R^{R^2}$

$\phi_{[1,1]^T}(x) = K(x, [1,1]^T) = (x_1+x_2)^2 \in R^{R^2}$

再生核 map

- 假设 $K: R^d \times R^d \to R$ 的核.

  $\phi: R^d \to R^{R^d}$ $\quad \Phi(z) = \phi_z = k(\cdot, z)$ 为再生核 map

  其中 $\phi(z)(x) = \phi_z(x) = k(x, z)$

Example

△ $k(x,z) = \exp\{-\frac{(x-z)^2}{2}\}$

$\phi_0(x) = k(x, 0) = \exp\{-\frac{x^2}{2}\} \in R^R$

$\Phi(0) = \phi_0$

$\Phi(0)(x) = \phi_0(x) = \exp\{-\frac{x^2}{2}\}$

$\Phi(0)(0) = \phi_0(0) = \exp\{-\frac{0}{2}\} = 1 = k(0,0)$

△ $\phi_1(x) = k(x, 1) = \exp\{-\frac{(x-1)^2}{2}\} \in R^R$

$\Phi(1) = \phi_1$

$\Phi(1)(x) = \phi_1(x) = \exp\{-\frac{(x-1)^2}{2}\}$

$\Phi(1)(0) = \phi_1(0) = \exp\{-\frac{1}{2}\}$

$\Phi(1)(1) = \phi_1(1) = \exp\{0\} = 1$

△ Given $k(x,z) = (x_1 z_1 + x_2 z_2)^2$

$\phi_{[0,0]^T}(x) = k(x, 0) = 0 \in R^{R^2}$

$\Phi(0) = \phi_{[0,0]^T}$

$\Phi(0)(x) = \phi_{[0,0]^T}(x) = k(x, 0) = 0$

$\Phi(0)([5,-5]^T) = \phi_{[0,0]^T}([5,-5]^T) = k([5,-5]^T, [0,0]^T) = 0$

$\phi_{[1,1]^T}(x) = k(x, [1,1]^T) = (x_1 + x_2)^2 \in R^{R^2}$

$\Phi(1) = \phi_{[1,1]^T}$.