

e. 对子降维.

$$S_t = S_b + S_w = \sum_{i=1}^m (x_i - \mu)(x_i - \mu)^T$$

其中  $\mu$  是所给子集均值向量,  $N$  个类

$$S_w = \sum_{i=1}^N S_{w_i} \quad S_{w_i} = \sum_{x \in X_i} (x - \mu_i)(x - \mu_i)^T$$

$$S_b = S_t - S_w = \sum_{i=1}^N \mu_i (\mu_i - \mu)(\mu_i - \mu)^T$$

优化目标  $\max_W \frac{\text{tr}(W^T S_b W)}{\text{tr}(W^T S_w W)} \quad W \in \mathbb{R}^{d \times (N-1)}$

通过  $S_b W = \lambda S_w W \rightarrow d'$  个最大非零广义特征值.  $d' \in N-1$

从而  $W$  为投影矩阵, 多类 LDA 投影到  $d'$  维,  $< d$ .

降维.









