计算高维空间的几何性质

高维空间距离

$$\begin{aligned}\|\phi(x)-\phi(x')\|^2 &= (\phi(x)-\phi(x'))^T(\phi(x)-\phi(x')) \\ &= \phi(x)^T\phi(x) - 2\phi(x)^T\phi(x') + \phi^T(x')\phi(x') \\ &= \langle\phi(x),\phi(x)\rangle - 2\langle\phi(x),\phi(x')\rangle + \langle\phi(x'),\phi(x')\rangle \\ &= k(x,x) - 2k(x,x') + k(x',x')\end{aligned}$$

高维空间角度

$$\langle\phi(x),\phi(x')\rangle = \|\phi(x)\|\cdot\|\phi(x')\|\cos\theta$$

$$\Rightarrow \cos\theta = \frac{\langle\phi(x),\phi(x')\rangle}{\|\phi(x)\|\|\phi(x')\|} = \frac{\langle\phi(x),\phi(x')\rangle}{\sqrt{\langle\phi(x),\phi(x)\rangle}\cdot\sqrt{\phi(x')\cdot\phi(x')}}$$

$$= \frac{k(x,x')}{\sqrt{k(x,x)}\cdot\sqrt{k(x',x')}}$$