#1.

① the possible cases is calculated by Combination of repetition

$$\underbrace{a+b+c+\cdots+m+n+o}_{15\ \text{students}} = 8 \Rightarrow {}_{15}H_8 = {}_{(15+8-1)}C_8 = {}_{22}C_8$$

② number of cases that no student will have to answer more than 1 question

= 8 students answer one question each

= choosing 8 student out of 15 considering the order

$= {}_{15}P_8 = \dfrac{15!}{(15-8)!}$

So, the probability is $\dfrac{\frac{15!}{7!}}{{}_{22}C_8}$

#2

odd odd even

① ② ③ ④ ⑤

odd: 1, 3, 5, 7, 9

even: 0, 2, 4, 6, 8.

∴ Combination of the integers that meet our criteria:

$$\overline{①\,②}: {}_5P_2 \times \overline{⑤}: 5 \times \overline{③\,④}: {}_7P_2 = 5 \times 4 \times 5 \times 7 \times 6 = 4200$$

out of 4200, randomly choose 5 and place them in any of 8 spots (order matters)

$$= {}_{4200}C_5 \times {}_8P_5$$

out of $(100000 - 4200)$ randomly choose 3 and place them in any of 3 spots (order matters)

$$= {}_{95800}C_3 \times 3!$$

$$\Rightarrow \frac{{}_{4200}C_5 \times {}_8P_5 \times {}_{95800}C_3 \times 3!}{{}_{100000}P_8}$$

#3

$$P(A) = \frac{\text{at least 2 dice have number} \geq 4}{\text{3 different dices combination}}$$

$$= 1 - \frac{\text{no dice have number} \geq 4}{6 \times 6 \times 6} - \frac{\text{only one dice have number} \geq 4}{6 \times 6 \times 6}$$

$$= 1 - \frac{3 \times 3 \times 3}{6 \times 6 \times 6} - \frac{3 \times 3 \times 3}{6 \times 6 \times 6} = \frac{3^3(2^3 - 2)}{6^3}$$

$$= \frac{3^3(6)}{6^3} = \frac{27}{36} = \frac{3}{4}$$

$$P(B) = \frac{6}{6 \times 6 \times 6} = \frac{1}{36}$$

$$P(A \cap B) = \frac{3}{6^3} = \frac{1}{72}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{52}}{\frac{3}{4}} = \frac{4}{3 \times 12} = \frac{1}{4}$$

$$\neq P(B) = \frac{1}{36}$$

So, they are not independent.

#4

$$\underbrace{\frac{1}{C(52,5)}}_{P} \times \underbrace{(4 \times C(13,5))}_{n} = \frac{5108}{2598960}$$

$$\approx \frac{1}{509} \therefore 509$$

#5

superstar plays 5

$$\therefore \frac{0.75 \times 0.7^4 \times 0.3^1 \times {}_5C_1}{(0.75 \times 0.7^4 \times 0.3^1 \times {}_5C_1) + (0.25 \times 0.5^5 \times {}_5C_1)}$$

$$= \frac{0.75 \times 0.7^4 \times 0.3}{(0.75 \times 0.7^4 \times 0.3) + (0.25 \times 0.5^5)}$$