1. $\frac{7!}{3!} = 7 \times 6 \times 5 \times 4 = 42 \times 20 = 840$

2. ☆☆♡♡◇◇ → $\frac{5!}{2!2!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 2} = 30$

3. Case one: fighting couple don't want any serenade:

total 16 songs will be distributed to 6 couples

couple A will get a number of songs,

" B " " b " " " , ... etc

so $a+b+c+d+e+f = 16$.

if we assume $a, b, c, d, e, f \geq 0$ then

$${}_{21}C_6 = \frac{16!}{6!} = 8008$$

Case two: the fighting couple would allow 1 song:

total 15 songs will be distributed to 6 couples.

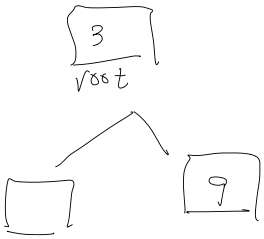
so $a+b+c+d+e+f = 15$

likewise, $a, b, c, d, e, f \geq 0$ then

$${}_{15}C_6 = \frac{15!}{6!} = 5005$$

therefore total = $8008 + 5005 = 13013$

4.



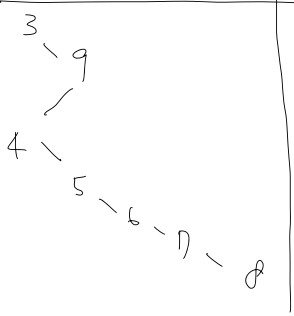
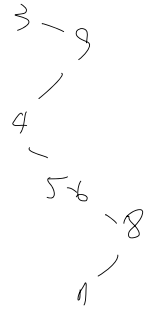
① order of number 1. and 2 is called matters

if 1 is called first, then

if 2 is called first then

likewise,

② order of (4.5.6.7.8) being called matters

order:	(4→5→6→7→8),	(4→5→6→8→7),	...etc
BST			

⇒ each cases are unique

③ likewise, (10.11.12) being called matters and each cases are unique.

however, order between th numbers in different group doesn't matter

ex) 1→8 or 8→1 doesn't matter.

so the total possibility is :

group ① : 2!

group ② : 5!

group ③ : 3!

$2! \times 5! \times 3!$

(#5 is below. here!
↓)

5.

① case 1 : 1 nurse is taking a rest.

then, there are 3 nurses serving 10 friends

let's assume we named nurses as A, B, and C

number of Patients A, B, and C serves are a, b, and C respectively.

then, $a+b+c \leq 10$ where $a, b, C \geq 1$

to solve this, we will say $a+b+c+x = 10$ and $\begin{pmatrix} a, b, C \geq 1 \\ x \geq 0 \end{pmatrix}$

because it is compatible : $\begin{cases} \text{if } a+b+c = 10, \text{ then } x = 0 \\ \text{if } a+b+c = 9, \text{ then } x = 1 \\ \vdots \end{cases}$

so, $(a+b+c+x = 10$ where $a, b,$ and $C \geq 1, x \geq 0)$

since a, b, C are bigger than 1, let's think as if we took one of a, b, and C each aside out of 10 choise.

then the formula will be compatible as

$(a+b+c+x = 10 \;\&\; a, b, C \geq 1 \;\&\; x \geq 0)$

$= (a+b+c+x = 7 \;\&\; a, b, C, x \geq 0$

this is the case of combination with repetition.

so, we will use $n H r = (n+r-1) C r$

$4 H 7 = (4+7-1) C 7 = 10 C 7$

finally consider that all nurses are actually identical: we need to ignore their order.

which is $3!$

therefore, the answer for case1 is $\frac{10 C 7}{3!}$

② case 2: no nurse is having a break time

name the nurses: A, B, C, D

# of patients each nurse will take: a, b, c, d respectively.

then $(a+b+c+d \leq 10 \quad \& \quad a, b, c, d \geq 1)$

$= (a+b+c+d+x = 10 \ \& \ a, b, c, d \geq 1, x \geq 0)$

$= (a+b+c+d+x = 6 \ \& \ a, b, c, d, x \geq 0)$

$= 5 H 6 = 10 C 6$

then, ignore the order of 4 nurses: $\frac{10 C 6}{4!}$

Case ① and case ② do not overlaps

case ② have a combination of numbers which are all ≥ 1 while case ① will always have one zero in the combinations.

therefore total combination is

$$\frac{{}^{10}C_7}{3!} + \frac{{}^{10}C_6}{4!}$$